COHORT REVENUE & RETENTION ANALYSIS: A BAYESIAN APPROACH

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Abstract. We present a bayesian approach to model cohort-level retention rates and revenue over time. We use bayesian additive regression trees (BART) to model the retention component which we couple with a linear model to model the revenue component. This method is flexible enough to allow adding additional covariates to both model components. This bayesian model allows us to quantify the uncertainty in the estimation, understand the effect of the covariates on the retention through partial dependence plots (PDP), individual conditional expectation (ICE) plots and last but not least forecast the future revenue and retention rates.

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1. Introduction

Retention and customer lifetime value estimation are one of the most important aspects to understand customer behavior. There are many ways to model retention and revenue by modeling individual-level purchase behavior for both the contractual and the non-contractual setting, see for example the work by Fader and Herdie [2] and [1] respectively[1]. In real cases, one is interested in modeling retention and revenue at cohort-level. There are (at least) three options to use the techniques mentioned above to model cohort-level retention:

(1) Pool the cohorts together and model the retention and revenue as a whole.
(2) Un-pool the cohorts and model each cohort separately.
(3) Try to model the cohorts jointly.

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[1] Our definition of retention is what they call survival curve. See precise definitions below.

See [4] for more details on these approaches. One of the limitations of those approaches is that they are not flexible enough to model seasonality and add external regressors[2]. One can argue that seasonality is not that important when trying to estimate the customer-lifetime-value. Nevertheless, in practice, there are business models on which the customer base is very seasonal.

In this work, we present a bayesian approach to model cohort-level retention rates from a top-down perspective. We do not model the individual-level purchase behavior[3] but rather the retention and revenue at cohort matrices. This approach allows for modeling non-linear relationships between cohorts, adding seasonality and external regressors. Concretely, we use bayesian additive regression trees (BART, see [8]) to model the retention component and we couple it with a linear model to model the revenue. The following are the main ingredients behind the model:

Features.

- **Cohort age**: Age of the cohort in months.
- **Age**: Age of the cohort with respect to the observation time. This feature is a numerical encoder for the cohort.
- **Month**: Month of the observation time.

In Figure 1 we show an example of a retention matrix. Note that we are removing the diagonal as it is not informative since it has just ones. As an example, let us assume our observation month is $2022-11$ and consider the cohort $2022-09$. In this case, the age of this cohort is 2 months as it is always relative to the observation period. This cohort was two observation periods $2022-10$ and $2022-11$ with cohort age 1 and 2 respectively.

Model Specification.

- We model the number of active users in the cohort as a binomial random variable $\text{Binomial}(N_{\text{total}}, p)$, where the parameter p represents the retention. We model the latent variable p using a BART model with features cohort age, age and month.
- We model the revenue as a gamma random variable $\text{Gamma}(N_{\text{active}}, \lambda)$. We model the latent variable λ through a linear model with features cohort age, age and a multiplicative interaction. Note that we do not add a seasonality component as we often see most of the seasonality coming from the retention itself. This of course can be added as a feature (plus any other covariates!) to the model.
- Here is a summary of how the retention and revenue components are coupled together:

[2]Actually, one can add regressors in some cases as described in [3] in the non-contractual case.

[3]This is a clear limitation if one is interested at customer-level parameters and predictions.

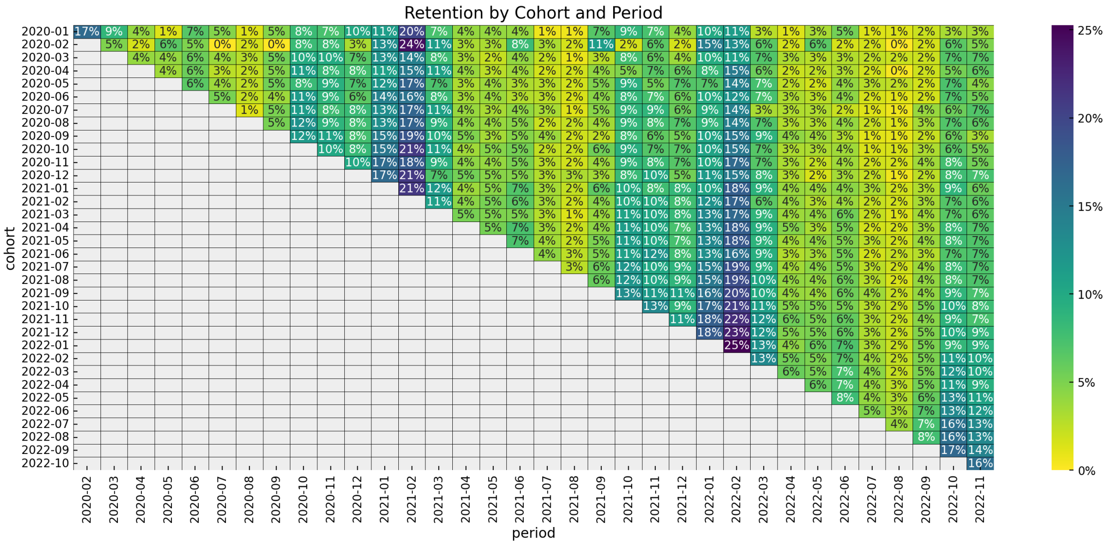


FIGURE 1. Retention matrix example.

$$
\begin{aligned}
\text{Revenue} &\sim \text{Gamma}(N_{\text{active}}, \lambda) \\
\log(\lambda) &= (\text{intercept} \\
&\qquad + \beta_{\text{cohort age}} \text{cohort age} \\
&\qquad + \beta_{\text{age}} \text{age} \\
&\qquad + \beta_{\text{cohort age} \times \text{age}} \text{cohort age} \times \text{age}) \\
N_{\text{active}} &\sim \text{Binomial}(N_{\text{total}}, p) \\
\text{logit}(p) &= \text{BART}(\text{cohort age}, \text{age}, \text{month})
\end{aligned}
$$

Figure 2 summarizes the model structure.

Remark 1. *This work is the result of a sequence of blog posts where all the details on the code and implementation are presented, see* [5], [6] *and* [7].

2. Data and Exploratory

3. Model Specification and Diagnostics

4. Predictions

4.1. **In-Sample Predictions.**

4.2. **Out-of-Sample Predictions.**

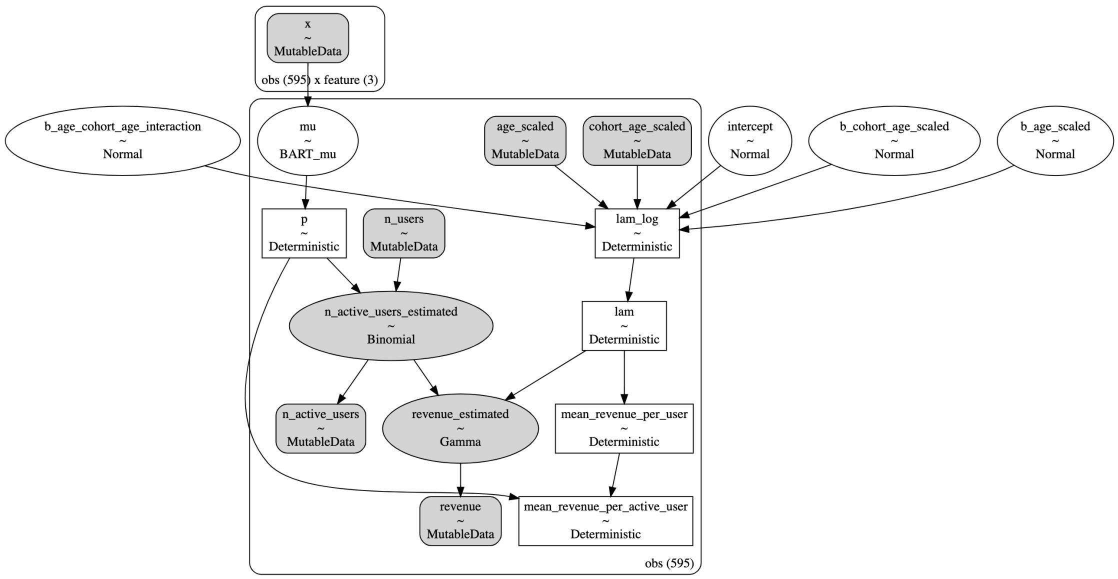
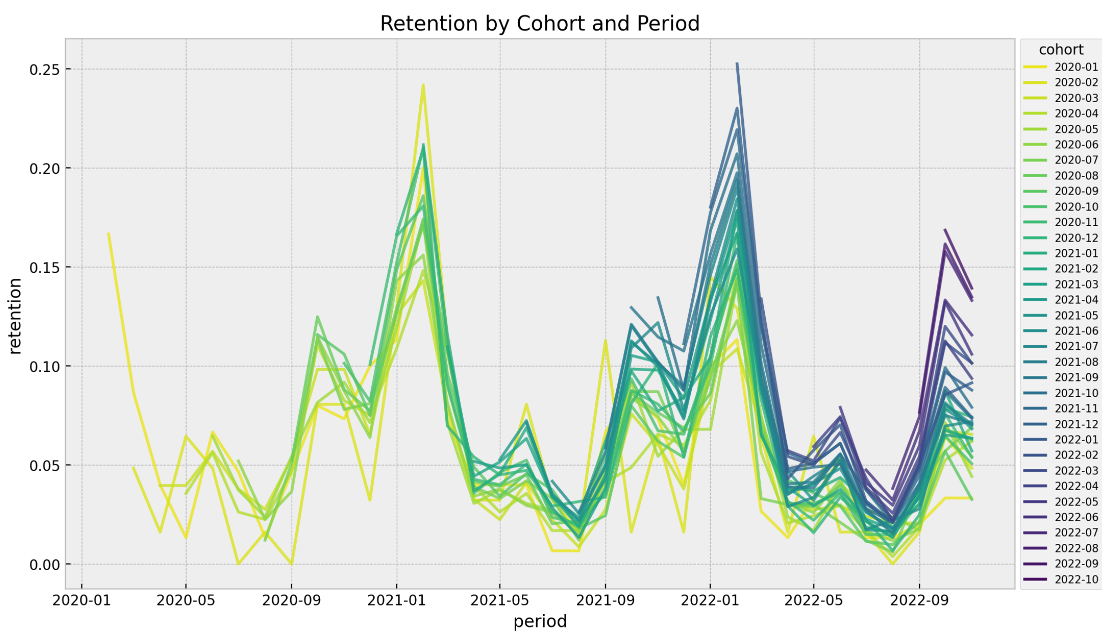


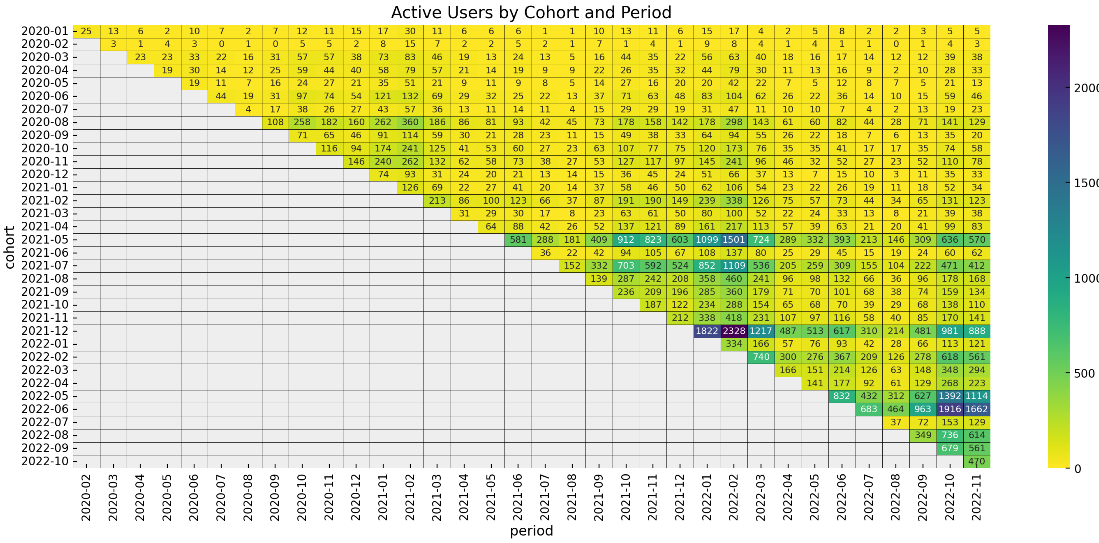
Figure 2. Cohort-revenue-retention model structure.

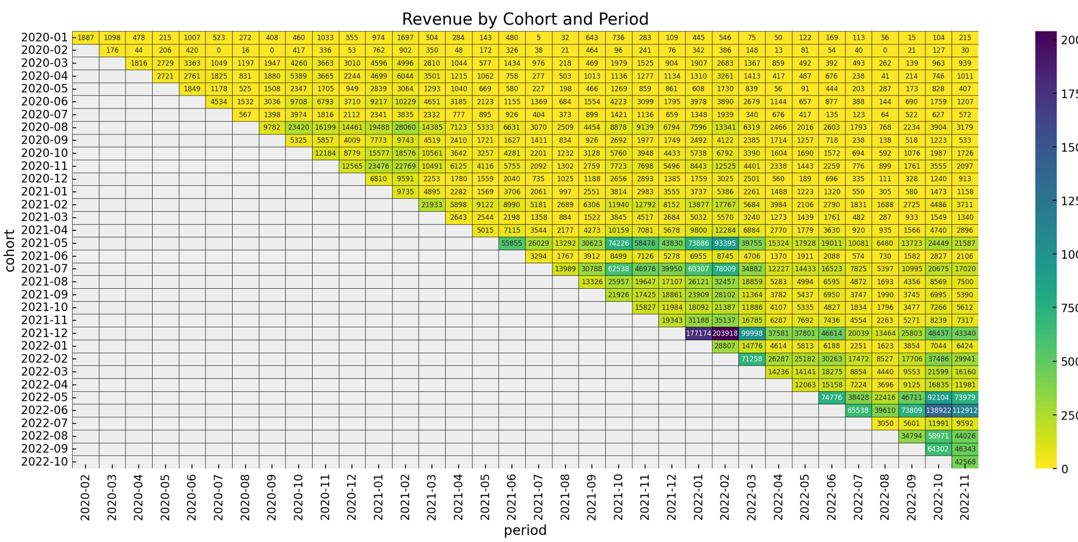


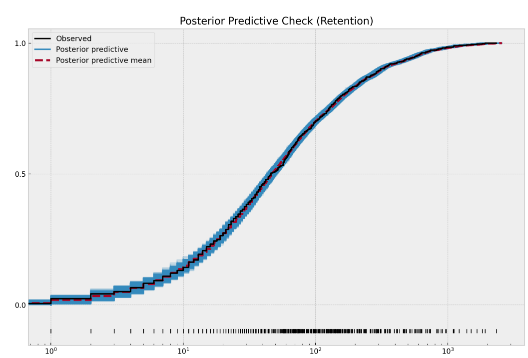
Appendix: Python Code

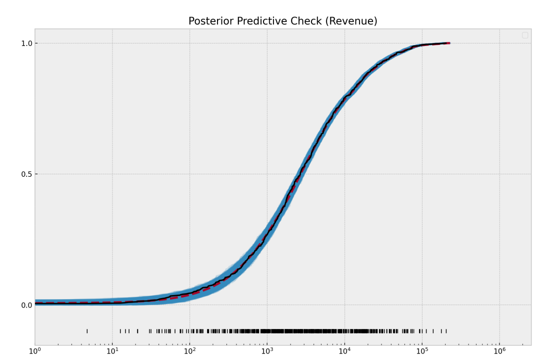
Listing 1. Python example

```python
import pymc_bart as pmb
import pymc as pm

```


Active Users by Cohort and Period
cohort
period
0
500
1000
1500
2000


Revenue by Cohort and Period
cohort
period
0
25000
50000
75000
100000
125000
150000
175000
200000


Posterior Predictive Check (Retention)
Observed
Posterior predictive
Posterior predictive mean
likelihood (n_active_users) - log scale


Posterior Predictive Check (Revenue)
likelihood (revenue) - log scale

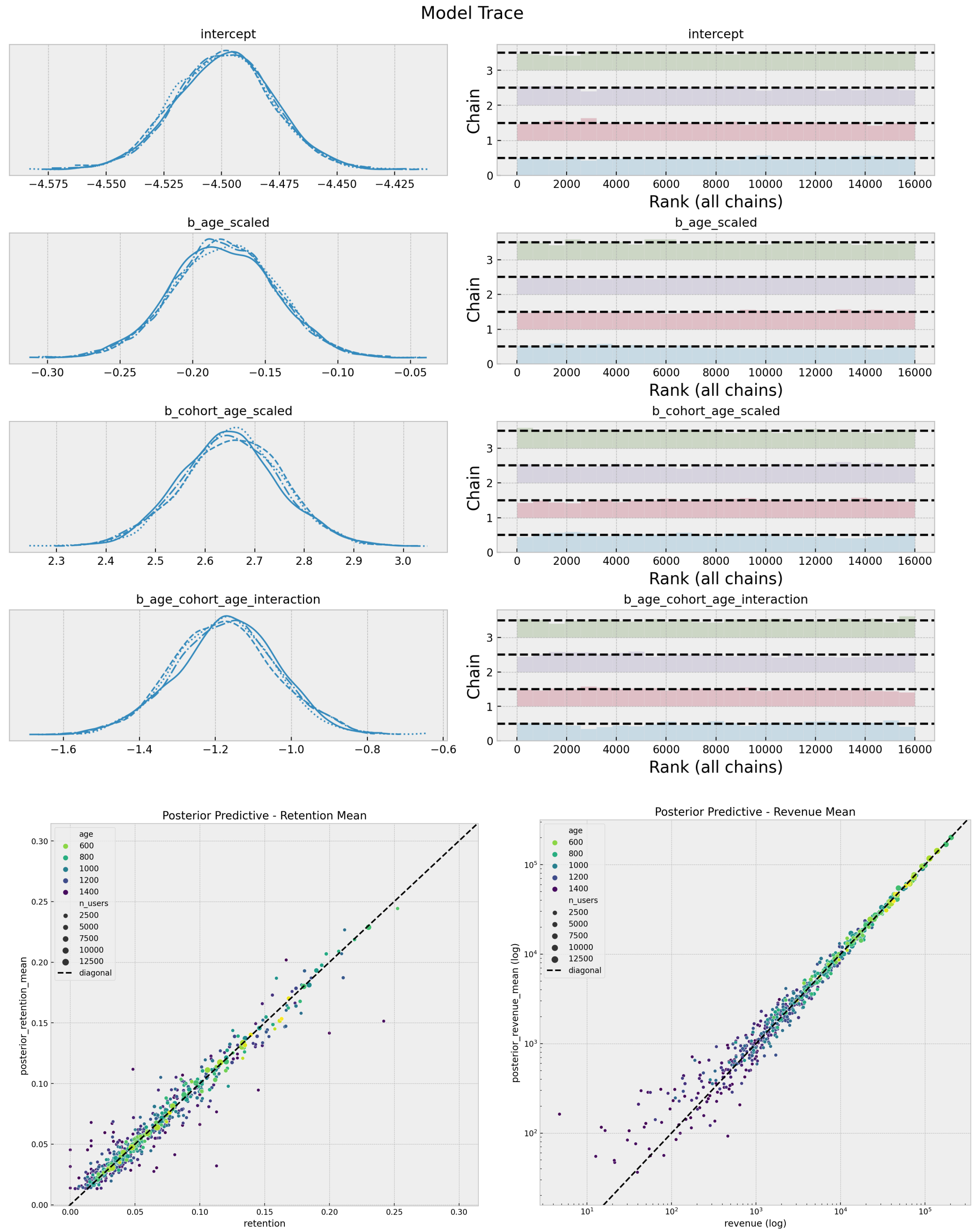


FIGURE 3. My caption

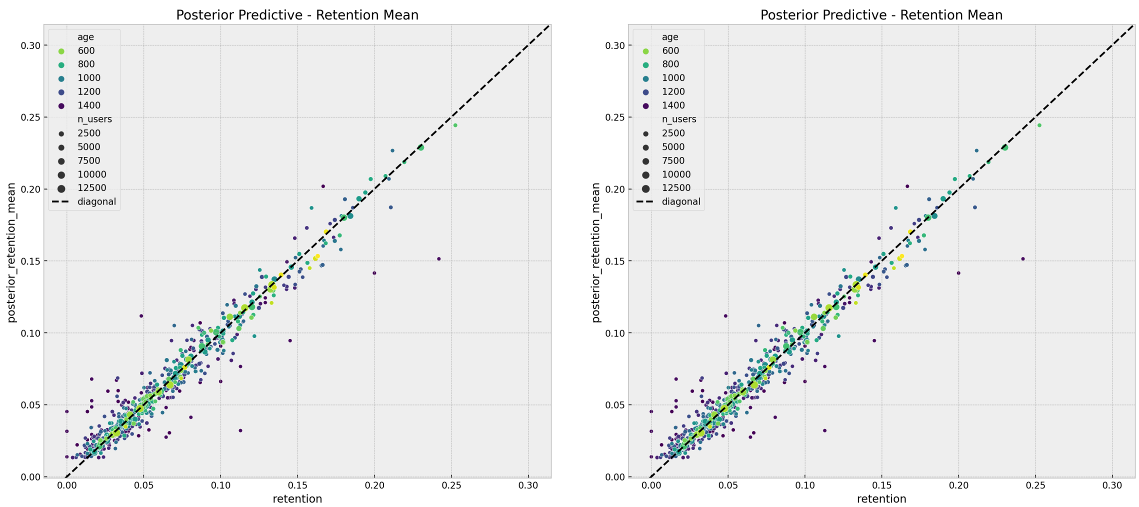
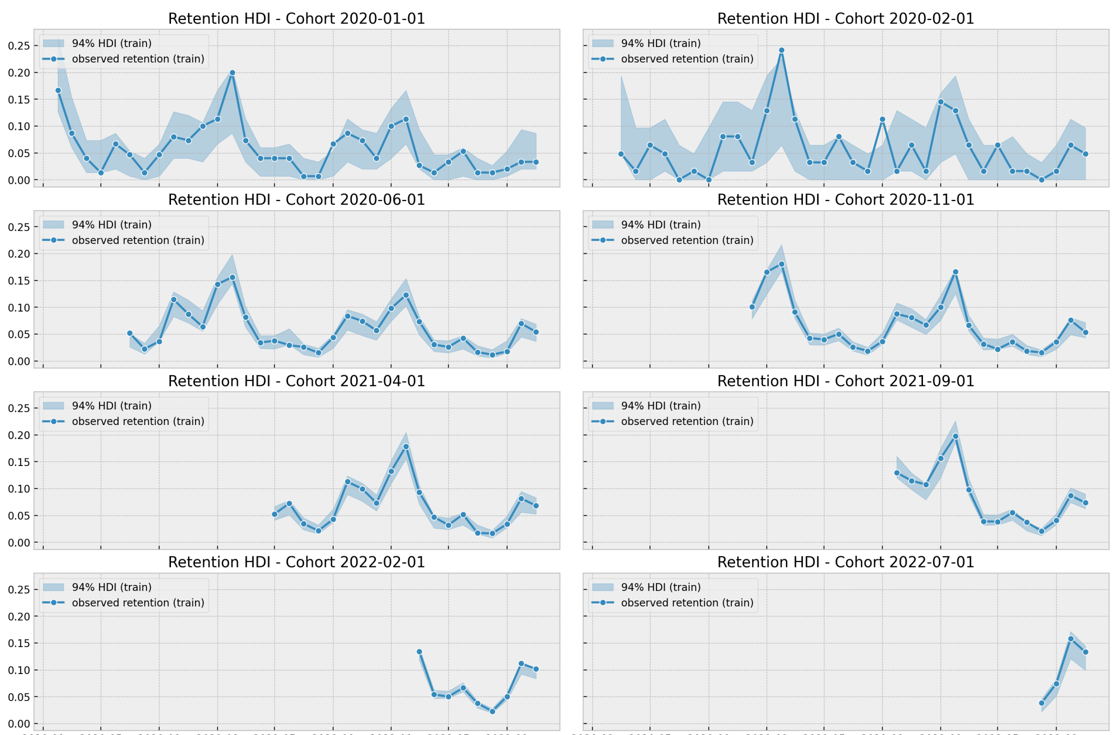
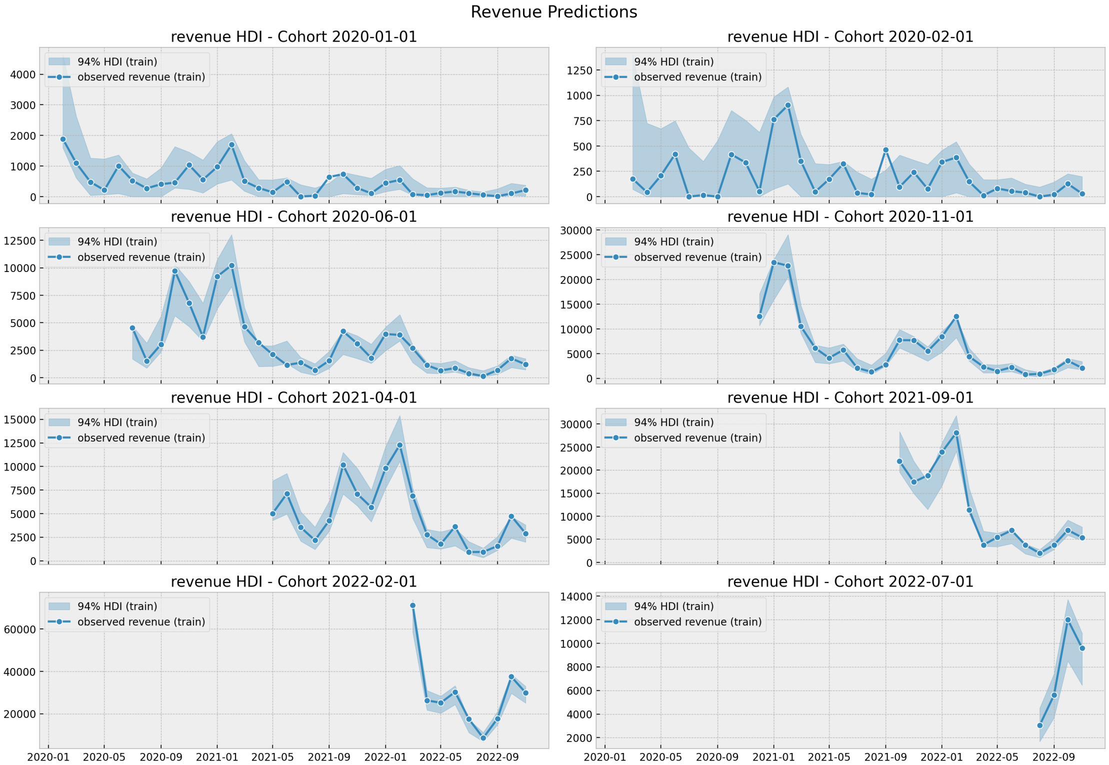


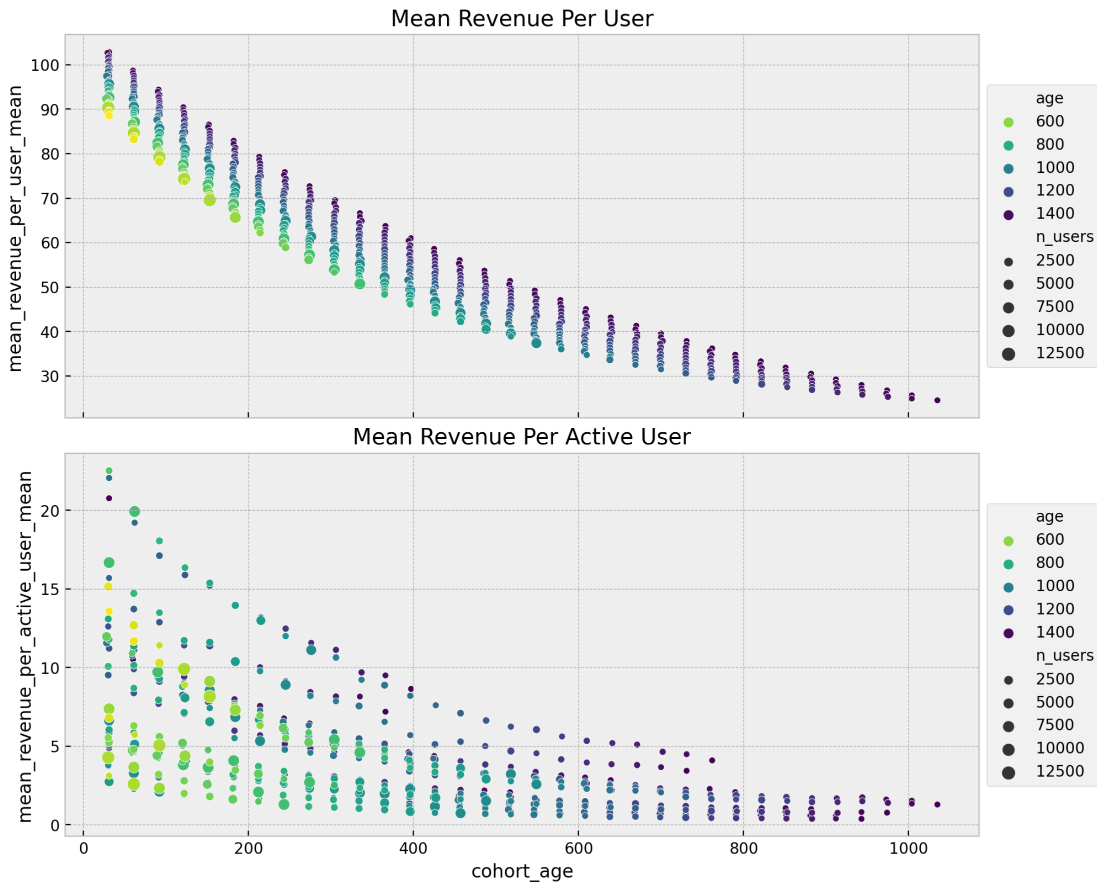
FIGURE 4. My caption



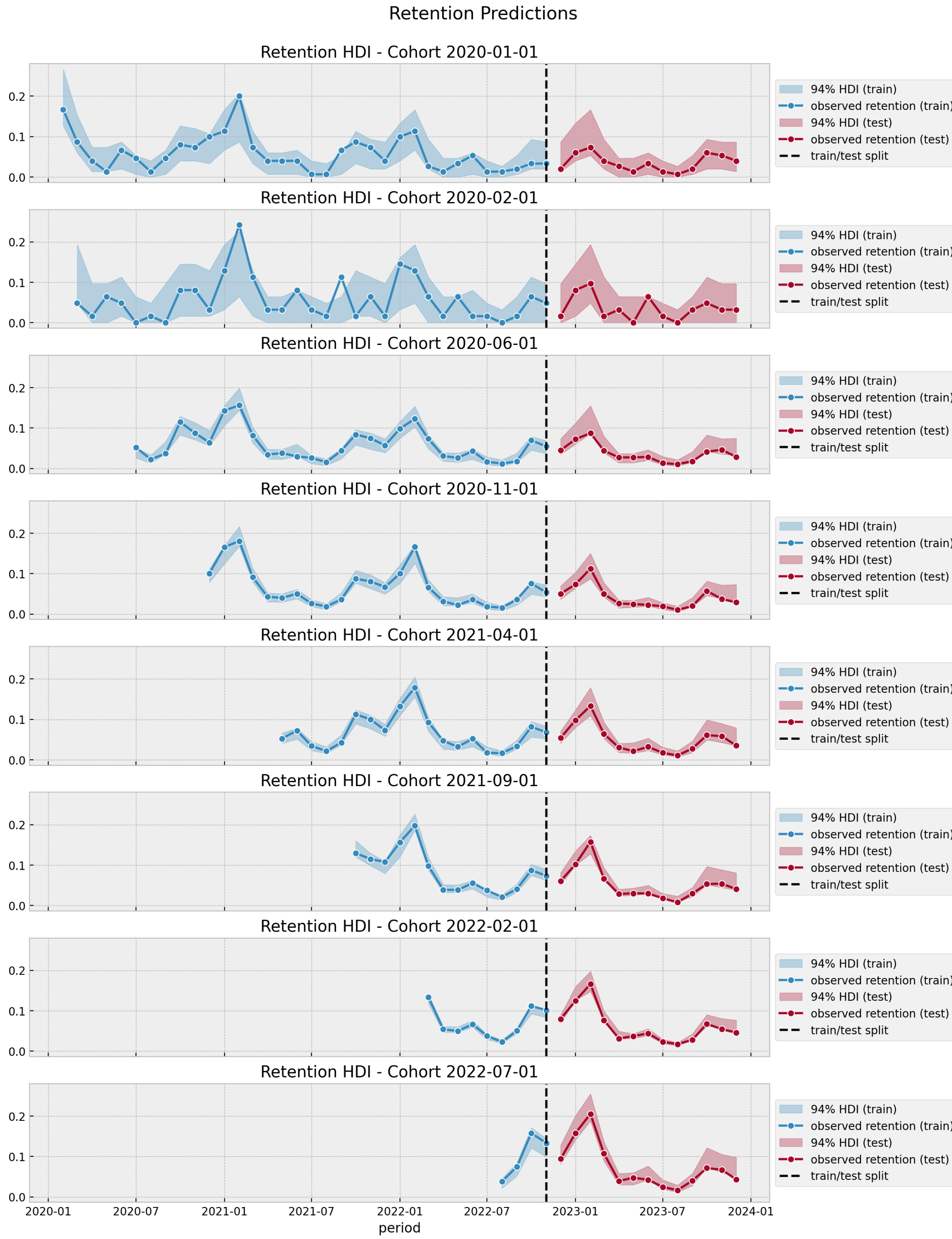

```python

with pm.Model(coords={"feature": features}) as model:

    # --- Data ---
    model.add_coord(name="obs", values=train_obs_idx, mutable=True)
```

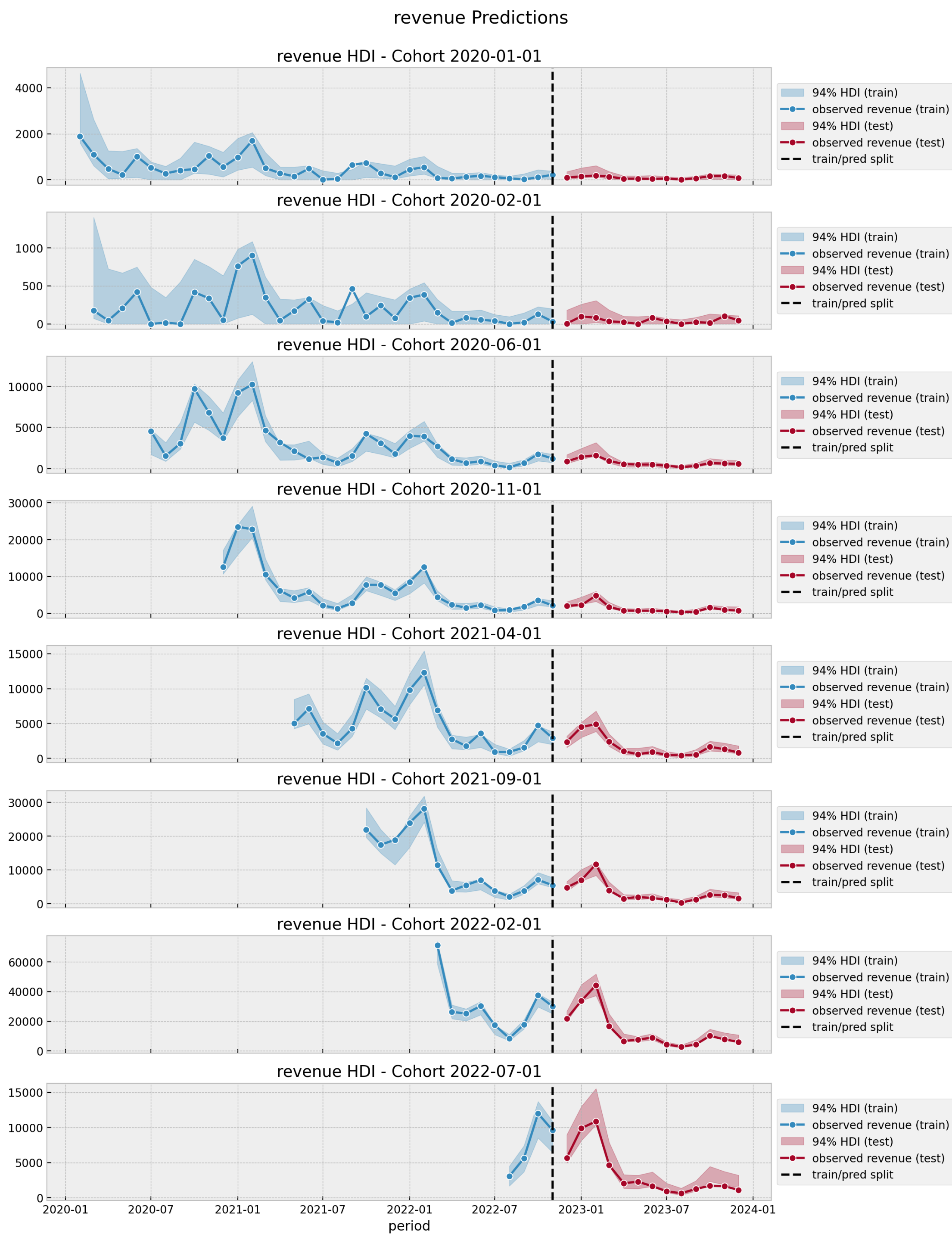



```python
    age_scaled = pm.MutableData(
        name="age_scaled", value=train_age_scaled, dims="obs"
    )
    cohort_age_scaled = pm.MutableData(
        name="cohort_age_scaled", value=train_cohort_age_scaled, dims="obs"
    )
    x = pm.MutableData(name="x", value=x_train, dims=("obs", "feature"))
    n_users = pm.MutableData(name="n_users", value=train_n_users, dims="obs")
    n_active_users = pm.MutableData(
        name="n_active_users", value=train_n_active_users, dims="obs"
    )
    revenue = pm.MutableData(name="revenue", value=train_revenue, dims="obs")

    # --- Priors ---
    intercept = pm.Normal(name="intercept", mu=0, sigma=1)
    b_age_scaled = pm.Normal(name="b_age_scaled", mu=0, sigma=1)
    b_cohort_age_scaled = pm.Normal(name="b_cohort_age_scaled", mu=0, sigma=1)
    b_age_cohort_age_interaction = pm.Normal(
        name="b_age_cohort_age_interaction", mu=0, sigma=1
    )

    # --- Parametrization ---
    # The BART component models the image of the retention rate under the
```




```python
    # logit transform so that the range is not constrained to [0, 1].
    mu = pmb.BART(name="mu", X=x, Y=train_retention_logit, m=50, dims="obs")
    # We use the inverse logit transform to get the retention rate
    # back into [0, 1].
    p = pm.Deterministic(name="p", var=pm.math.invlogit(mu), dims="obs")
    # We add a small epsilon to avoid numerical issues.
    p = pt.switch(pt.eq(p, 0), eps, p)
    p = pt.switch(pt.eq(p, 1), 1 - eps, p)

    # For the revenue component we use a Gamma distribution where we
    # combine the number of estimated active users with the average
    # revenue per user.
    lam_log = pm.Deterministic(
        name="lam_log",
        var=intercept
        + b_age_scaled * age_scaled
        + b_cohort_age_scaled * cohort_age_scaled
        + b_age_cohort_age_interaction * age_scaled * cohort_age_scaled,
        dims="obs",
```


Retention Predictions
Retention HDI - Cohort 2020-01-01
Retention HDI - Cohort 2020-02-01
Retention HDI - Cohort 2020-06-01
Retention HDI - Cohort 2020-11-01
Retention HDI - Cohort 2021-04-01
Retention HDI - Cohort 2021-09-01
Retention HDI - Cohort 2022-02-01
Retention HDI - Cohort 2022-07-01
94% HDI (train)
observed retention (train)
94% HDI (test)
observed retention (test)
train/test split
0.0
0.1
0.2
2020-01
2020-07
2021-01
2021-07
2022-01
2022-07
2023-01
2023-07
2024-01
period


revenue Predictions
revenue HDI - Cohort 2020-01-01
revenue HDI - Cohort 2020-02-01
revenue HDI - Cohort 2020-06-01
revenue HDI - Cohort 2020-11-01
revenue HDI - Cohort 2021-04-01
revenue HDI - Cohort 2021-09-01
revenue HDI - Cohort 2022-02-01
revenue HDI - Cohort 2022-07-01
94% HDI (train)
observed revenue (train)
94% HDI (test)
observed revenue (test)
train/pred split
2020-01
2020-07
2021-01
2021-07
2022-01
2022-07
2023-01
2023-07
2024-01
period

```
    )

    lam = pm.Deterministic(name="lam", var=pm.math.exp(lam_log), dims="obs")

    # --- Likelihood ---
    n_active_users_estimated = pm.Binomial(
        name="n_active_users_estimated",
        n=n_users,
        p=p,
        observed=n_active_users,
        dims="obs",
    )

    x = pm.Gamma(
        name="revenue_estimated",
        alpha=n_active_users_estimated + eps,
        beta=lam,
        observed=revenue,
        dims="obs",
    )

    # --- Derived Quantities ---
    mean_revenue_per_user = pm.Deterministic(
        name="mean_revenue_per_user", var=(1 / lam), dims="obs"
    )
    pm.Deterministic(
        name="mean_revenue_per_active_user",
        var=p * mean_revenue_per_user,
        dims="obs"
    )
```

References

[1] Fader, P., Hardie, B., and Lee, K. "Counting Your Customers" the Easy Way: An Alternative to the Pareto/NBD Model. *Marketing Science 24* (05 2005), 275–284.

[2] Fader, P. S., and Hardie, B. G. How to project customer retention. *Journal of Interactive Marketing 21*, 1 (2007), 76–90.

[3] Fader, P. S., and Hardie, B. G. Incorporating Time-Invariant Covariates into the Pareto/NBD and BG/NBD Models. `http://brucehardie.com/notes/019/`, 2007.

[4] Fader, P. S., and Hardie, B. G. Fitting the sBG Model to Multi-Cohort Data. `http://brucehardie.com/notes/017/`, 2017.

[5] Orduz, J. A Simple Cohort Retention Analysis in PyMC. `https://juanitorduz.github.io/retention/`, 12 2022.

[6] Orduz, J. Cohort Retention Analysis with BART. `https://juanitorduz.github.io/retention_bart/`, 01 2023.

[7] Orduz, J. Cohort Revenue & Retention Analysis: A Bayesian Approach. `https://juanitorduz.github.io/revenue_retention/`, 01 2023.

[8] Quiroga, M., Garay, P. G., Alonso, J. M., Loyola, J. M., and Martin, O. A. Bayesian additive regression trees for probabilistic programming, 2022.

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