The Prediction Performance of the FRB-MIT-PENN Model of the U.S. Economy

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This paper presents an evaluation of the prediction performance of the FRB-MIT-PENN (*FMP*) econometric model of the *U.S.* economy using predictions provided by simple time-series models to establish standards of accuracy. The motivation for such an evaluation is two-fold. First, the quality of predictions provides a measure of the success of the model in simulating the behavior of the system under study. Second, we may be interested in the utility of the model for operational forecasting and policy design. The continuing development of the *FMP* model has resulted in a succession of revisions, the subject of this study being Version 4.1 which was released during 1969.[1] It should be emphasized that this evaluation is not intended as an audit of current developmental efforts, rather the objective is to consider a version of the model which has been thoroughly checked for stability and computational anomalies and which has been the subject of considerable interest and research in academic, corporate, and governmental policy contexts.

The study focuses on the one-quarter-ahead predictions of fourteen endogenous variables of general interest; namely nominal *GNP*, its endogenous components, the unemployment rate, two price indices, and three interest rates. Predictions are obtained from the model by simultaneous solution of the equation system, requiring as inputs the historical values of endogenous and exogenous variables and projected *future* values of exogenous variables. In order to avoid ambiguity with regard to a method for projecting exogenous variables, these were set at their *actual* future values. These actual values provide *ex post* information to the model which is exploited by the behavioral relationships and provides some of the accounting components of *GNP*. Such predictions may be thought of as forecasts which would have been made by a user of the model who was endowed with perfect foresight with regard to future values of exogenous variables.

The computation of predictions from the *FMP* model amounts to evaluation of the conditional expectations of future endogenous variables implied by the equation system, since future values of the stochastic disturbances in the system are set at their expected values of zero.[2] If the

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[1] Version 4.1 has been described by Arnold Zellner (1969). The *FMP* model has also been described by Franco Modigliani, Robert Rasche, and J. Phillip Cooper; Frank DeLeeuw and Edward Gramlich; and Rasche and Harold Shapiro.

[2] Solution values so obtained may differ from conditional expectations somewhat due to the nonlinearity of the system. However, explicit reduced form solution of the system is infeasible and we must assume that departures due to disturbances entering into non-linear relationships are relatively minor. For evidence of the approximate linearity of the model, see Zellner and Stephen Peck.

observed data being predicted were generated by the *FMP* system, then the conditional expectation predictions being computed would constitute minimum mean square error predictions amongst those conditioned on the same set of information.[3] In this case, that information set includes historical data on all of the endogenous and exogenous variables of the system, as well as future values of exogenous variables.

The time-series models used in this study to establish standards of accuracy for the *FMP* model are empirical representations of individual endogenous variables as stochastic processes of integrated autoregressive moving average (*ARIMA*) form, following the methodology developed by George E. P. Box and Gwilym M. Jenkins. Given a model for a particular series, predictions are obtained by computing the expected values of future observations which are implied by the model conditional only on the past history of the series. The fact that the information set utilized by the *FMP* model (that is the histories of *all* variables in the system as well as actual future values of exogenous variables) subsumes the set available to the *ARIMA* models (just the past history of the variable being forecast) motivates the choice of the time-series models as standards of accuracy. To the extent that the economy behaves "as if" it were being generated by the *FMP* model, then the larger information set used by the model should be useful in reducing mean square prediction error relative to the *ARIMA* models. Further, if *FMP* and *ARIMA* predictions are combined in a composite prediction to minimize mean square error, we would expect little contribution from the *ARIMA* predictions. On the other hand, if *FMP* predictions prove to be relatively inaccurate and *ARIMA* predictions contribute substantially to a composite prediction, then we would be led to conclude that the *FMP* model had underutilized the information available to it, presumably because of statistical and economic errors of specification and sampling errors in parameter estimates.

Previous authors in the area of prediction evaluation have frequently used "naive" prediction models to obtain standards of accuracy (for example, see Geoffrey Moore, H. O. Steckler, Victor Zarnowitz). The *ARIMA* models used in this study may be viewed as "not-quite-so-naive" models. While they indeed are economically naive (for example, predictions are not constrained to satisfy accounting identities), they are based on statistically sophisticated analysis. The reason for preferring the *ARIMA* models over more naive schemes is that their implementation is founded on statistical theory and places particular emphasis on selection of models appropriate to the stochastic structure of individual time-series. If the fitted models are appropriate representations of the stochastic structures of the variables being predicted, then the implied conditional expectations of future observations will in general provide more accurate predictions than will naive models of arbitrary form.

The results of comparison of *FMP* and *ARIMA* model prediction accuracy reported in this study indicate that the former were more accurate for most of the variables during the sample period over which both models were fitted. When the two sets of predictions are combined into linear composites, most of the ARIMA

[3] To see this, denote by F a prediction of actual value A which is a function of information set Φ and differs from the conditional expectation $E(A\,|\,\Phi)$ by amount d. The expectation of the squared error $(A-F)$ is then

$$\begin{aligned} E[(A-F)^2\,|\,\Phi] &= E\{[A-E(A\,|\,\Phi)]^2\,|\,\Phi\} \\ &\quad + 2dE\{[A-E(A\,|\,\Phi)]\,|\,\Phi\} + d^2 \\ &= E\{[A-E(A\,|\,\Phi)]^2\,|\,\Phi\} + d^2 \end{aligned}$$

so that mean square error is at a minimum when $d=0$; that is, when the prediction F is set at $E(A\,|\,\Phi)$.

predictions make a statistically significant contribution to accuracy over the sample period. Further, when composites are constructed to minimize joint loss across variables, the *ARIMA* models make significant contributions for almost all variables. Examination of post-sample errors suggests, however, that the *ARIMA* models are relatively more robust with respect to prediction outside the sample period. Among the alternatives of *FMP*, *ARIMA*, and composite predictions, the *ARIMA* predictions achieve the smallest mean square error for seven of fourteen variables, the composites for five, and the *FMP* predictions for only two.

I. *ARIMA* Models for Endogenous Variables of the *FMP* Model

The class of time-series models under consideration for comparison with the *FMP* model is that for which some difference of the observed series may be represented as a stationary stochastic process of autoregressive moving average forms. Letting z_t denote the observed value of series z at time t and B denote the backshift operator (i.e., $B^k z_t = z_{t-k}$) then the sequence $\{z_t\}$ is said to have a representation as an *ARIMA* process if z_t may be expressed as

$$(1) \qquad \phi_p(B)(1 - B)^d z_t = \theta_0 + \theta_q(B) a_t$$

where $\phi(B)$ and $\theta(B)$ are polynomials in the backshift operator of degrees p and q, respectively, having zeros outside the unit circle, θ_0 is a constant, and $\{a_t\}$ is a sequence of random disturbances. Thus, the dth difference of the observed series is a pth order autoregression with a qth order moving average disturbance and is both stationary and invertible. The appeal of this class of processes as empirical models derives from their compatibility with a very wide range of autocorrelation structures and hence a very wide range of stationary and nonstationary behavior.

Empirical application begins with what Box and Jenkins refer to as "identification," that is, specification of dimensions p, d, and q on the basis of sample autocorrelations and partial autocorrelations of successive differences of the raw data. Parameters of an identified model are fitted by iterative minimization of the sum of squared residuals, $\Sigma \hat{a}_t^2$, which provides maximum likelihood estimates under the assumption that disturbances are normal. Various diagnostic procedures, particularly residual analysis, are applied to check the adequacy of the model. Given the fitted model, predictions of any desired horizon are computed by direct evaluation of the expected values of the future observations being predicted conditional only on the past history of the individual series.[4]

As in linear regression, the sum of squared residuals and therefore of one-step-ahead prediction errors may be reduced over the period of fit simply by addition of more "independent variables," that is, more autoregressive or moving average terms. The criterion of model selection followed in this study has been the representation of each series in the most parsimonious form which is consistent with its stochastic structure. The parameters included in the models are those for which estimates are significant or which are required to eliminate serial correlation in residuals. Thus the procedure has not been to minimize the variance of prediction errors over the general class of *ARIMA* models, rather it has been to obtain the simplest adequate representations. It has been my objective to apply the methodology in the most straightforward fashion so that these models would presumably be duplicated, except

[4] For a summary of the Box and Jenkins methodology including an illustrative application, the reader is referred to chs. 2 and 5 of my study, *The Term Structure of Interest Rates*.

for minor differences, by another investigator. This objective required that prior information of various sorts be disregarded. For example, *logs* rather than levels of output variables presumably exhibit spatial stochastic homogeneity. *Logs* were not used, however, since the postwar data to which the study was confined do not by themselves provide strong evidence against homogeneity in the raw levels. Also, inclusion of a constant term in models for interest rates would clearly have improved both sample and post-sample period performance. These terms were omitted, however, because they were not, interestingly enough, statistically significant.

Most equations of Version 4.1 were estimated through 1966–04. In order to maintain comparability with respect to data base, the *ARIMA* models were also estimated through 1966–04. Models for the fourteen endogenous variables included in the study are displayed in the Appendix. It suffices to note that the models generally involve rather few parameters and few lagged values. The range of models represented is quite broad, including both pure autoregressive and pure moving average models as well as mixed models. An interesting by-product of fitting the models was evidence from the autocorrelations of residuals of a *negatively* seasonal component in some of the standard seasonally adjusted series. In the cases of consumer expenditures on non-durable goods, housing expenditures, and the *GNP* deflator and to a lesser extent *GNP* itself, the residual $\hat{a}_t$ for a given quarter tended to be negatively related to the residual appearing four quarters later. The implication of this finding is that the seasonal adjustment procedures in general use may "overadjust" series with particular stochastic structures and thereby introduce a negative seasonal relationship. This would tend to reinforce the idea

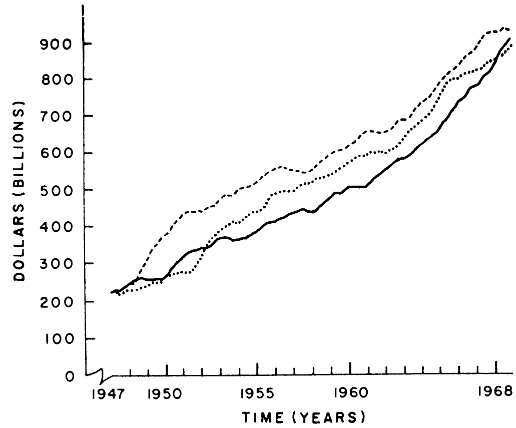

FIGURE 1. NOMINAL GROSS NATIONAL PRODUCT 1947–01 THROUGH 1969–01: HISTORICAL VALUES (SOLID LINE) AND TWO SIMULATIONS OF *ARIMA* MODEL

that unadjusted data should be utilized wherever possible and that seasonality should be accommodated by the econometric or statistical model under construction.[5]

The *ARIMA* model for nominal *GNP* is probably of special interest and is of remarkably simple form, namely

$$(2)\quad GNP_t - GNP_{t-1} = .615(GNP_{t-1} - GNP_{t-2}) + 2.76 + a_t$$

$$\hat{\sigma}_a^2 = 22.9$$

Thus, the change in the current quarter is simply related to the change in the previous quarter. Artificial realizations of postwar *GNP* were generated with the model by drawing random a_ts having the same variance as that of the residuals from estimation and using the initial quarters 1947–01 and 1947–02 as starting values. These simulations appear in Figure 1 along with the historical record through 1969–01. The familiar features of eco-

[5] The problems of seasonal adjustment and interpretation of adjustment procedures have been greatly illuminated by the recent work of David M. Grether and Marc Nerlove.

nomic history, recessions and booms, are easily recognized in the simulations and indistinguishable in character from actual episodes.[6]

II. Analysis of Sample Period Prediction Errors

The system of equations of the *FMP* model requires a substantial number of lagged observations for solution, so that 1956–01 is the first quarter for which one-quarter-ahead predictions may be computed. Housing expenditures are exogenous through 1956 and only become endogenous in 1957 providing a more restricted prediction sample that begins in 1957–01. Thus, while we shall refer to 1956–01 through 1966–04 as the "sample period," both the *FMP* equations and the *ARIMA* models were generally fitted over periods that began before 1956–01. The analysis is confined to one-quarter-ahead predictions although predictions of longer horizon are available from both the *FMP* and *ARIMA* models and are of considerable practical interest. The intention is to concentrate on a more thorough analysis of the one-step-ahead case than would be tractable if a wider range of horizons were included. Hopefully, our qualitative conclusions remain valid for multiperiod prediction.

The mean squares, means, and standard deviations of sample period prediction errors appear in Table 1 and indicate that the *FMP* model provided generally more accurate predictions during that 44-quarter interval, although the differences are surprisingly small. In the cases of State and Local Government Expenditure and the Unemployment Rate, the *ARIMA* model predictions had smaller mean square errors. Mean errors were all small, suggesting that the prediction bias may be characterized as being minor. To investigate further the question of prediction bias, the actual values of endogenous variables were regressed on predictions. The predictions are properly regarded as the independent variables in these regressions since if they were correlated with their respective error terms then their prediction accuracy could be improved merely by exploitation of that correlation.[7] The constant term was significant (at the 5 percent level) only in the case of *FMP* predictions of State and Local Government Expenditures for which it was $1.54 billion. Estimated slopes were significantly different from unity only for *FMP* predictions of Expenditures on Producers' Structures (1.04) and State and Local Government Expenditures (.97) and for *ARIMA* predictions of Expenditures on Producers' Durables (1.05). While these deviations from theoretical values are statistically significant, they are of rather small magnitude, thus reinforcing our previous conclusion that prediction bias is fairly small.

The correlation between *FMP* and *ARIMA* errors provides a measure of similarity between the two sets and is substantial for many of the variables including *GNP*. The highest error correlations are for Consumers' Expenditures on Nondurable Goods, Nonfarm Inventory Investment, and Expenditure on Producers' Structures. The lowest correlations are for State and Local Government Expenditures, where the *FMP* predictions did poorly, and for Yields on U.S. Treasury Bills.

Among desirable properties for predictions are that their errors be successively uncorrelated and that the predictions themselves be uncorrelated with future

[6] For an early demonstration that random disturbances can account for cyclical phenomena, see Eugen Slutzky. Also see Ragnar Frisch. Stochastic simulations of the *FMP* model have been described by Cooper and Stanley Fischer.

[7] As John Muth pointed out this must be a property of rational expectations. Further, this must be a property of all conditional expectation predictions.

errors. Failure to meet either of these conditions implies underutilization of information, that is, predictions may be adjusted to reduce mean square error. If one-step-ahead errors are serially correlated, then predictions could have been improved upon by simply taking into account the relation between past and future errors. If predictions are correlated with corresponding future errors, then this relationship may be used to adjust predictions accordingly. Sample correlations between predictions and errors reported in Table 1 are small except for *FRB* predictions of Expenditures on Producers' Durables and State and Local Government Expenditures for which the mean square error was high. The sample correlation is also substantial for *ARIMA* predictions of Expenditures on Producers' Durables. Sample autocorrelations of *FMP* errors are large at lag one quarter (relative to a standard error of .16 for sample autocorrelations of uncorrelated noise) for variables, Expenditures on Producers' Durables, Expenditures on Producers' Structures, State and Local Government Expenditures, Housing Expenditures, and the *GNP* Deflator. Results for the Consumer Goods Price Index shows substantial correlation of *FMP* errors at longer lags (3 and 4 quarters). *ARIMA* model errors are relatively less autocorrelated but display strong correlation at lags 2 and 4 for the *GNP* Deflator. By way of summarizing these results for the two sets of prediction errors, we note that seven of the fifty-six autocorrelations for *FMP* errors lie outside the bounds $\pm .32$ compared to one value of .32 for *ARIMA* model errors.

From the viewpoint of the operational forecaster, relationships between errors for different variables are important in prediction evaluation since his loss function will depend in general on such relationships. For example, in the case that his loss function is a quadratic form in the prediction errors, as we assume in Section IV, then expected loss is a weighted sum of covariances between errors as well as individual error variances. Correlations across variables for both *FMP* and *ARIMA* errors appear in Table 2. A large correlation between *ARIMA* errors for a pair of variables would suggest that factors accounting for their respective contemporaneous disturbances are common to both. Thus, it is not surprising to find substantial positive correlation between *GNP* errors (disturbances) and those for Consumers' Expenditures on Durable Goods, Nonfarm Inventory Investment, and Expenditures on Producers' Durables, and substantial negative correlation with those for the Unemployment Rate. Errors for Expenditures in Producers' Durables are strongly correlated with those for Consumers' Expenditures on Durable Goods and Nonfarm Inventory Investment. Perhaps surprisingly, these three investment categories show quite strong positive correlation with the disturbances in the three interest rate series.

Correlations between contemporary errors of *FMP* predictions are generally indicative of the structure of the model, and, of course, of accounting relationships. Consequently, in Table 2 errors for *GNP* are positively related to those for its components, and negatively to the Unemployment Rate errors. Errors for Housing Expenditures and Expenditures on Producers' Structures are positively related. Where relationships are not so obvious on prior grounds, the correlations provide indications of structural interaction within the model, and may help to locate problem areas. For example, it is surprising that *GNP* Deflator and Consumer Price Index errors are negatively related to those for *GNP* and Expenditures on Producers' Durables. It is also interesting that errors for the three interest rates are positively

TABLE 1—SUMMARY STATISTICS FOR *FMP* MODEL AND *ARIMA* MODEL SAMPLE PERIOD PREDICTION ERRORS

Endogenous Variable	*FMP* Model Errors[a] *MSE*	Mean	Standard Deviation	*ARIMA* Model Errors[a] *MSE*	Mean	Standard Deviation	Correlation Between Model Errors and *ARIMA* Model Errors
1. *Gross National Product*	11.665	.695	3.344	25.330	.520	5.006	.468
2. Consumers' Expenditures on Nondurable Goods	1.926	.008	1.388	2.671	.230	1.618	.673
3. Consumers' Expenditures on Durable Goods	1.278	−.034	1.130	3.258	.057	1.804	.581
4. Nonfarm Inventory Investment	6.710	.892	2.432	10.992	.376	3.294	.640
5. Expenditures on Producers' Durables	.682	−.224	.795	1.050	.116	1.018	.238
6. Expenditures on Producers' Structures	.249	.064	.495	.310	.013	.557	.627
7. State and Local Government Expenditures	.570	.018	.755	.294	.136	.525	.121
8. Housing Expenditures[b]	.206	.092	.445	.486	.006	.697	.297
9. Unemployment Rate	.134	−.087	.356	.089	.040	.296	.432
10. *GNP* Deflator-Price Index	.038	.022	.194	.053	.004	.230	.402
11. Consumer Goods Price Index	.052	.018	.227	.062	.028	.247	.321
12. Yields on U.S. Treasury Bills	.061	.012	.247	.124	.015	.352	.173
13. Yields on Commercial Paper	.053	.044	.227	.101	.048	.314	.324
14. Yield on Corporate Bonds	.010	.020	.097	.016	.034	.120	.568

Endogenous Variable	Correlation Between Predictions and Errors *FMP*	*ARIMA*	Serial Correlation Coefficients of Prediction Errors[c] *FMP* r_1	r_2	r_3	r_4	*ARIMA* r_1	r_2	r_3	r_4
1. *Gross National Product*	.200	.263	.09	−.04	.05	.12	−.06	−.03	.05	−.14
2. Consumers' Expenditure on Nondurable Goods	.055	.174	−.19	.04	−.07	−.07	−.13	.00	.13	−.09
3. Consumers' Expenditure on Durable Goods	.005	.161	−.21	.17	−.08	.00	−.09	.12	−.07	.06
4. Nonfarm Inventory Investment	.191	−.096	.11	.01	−.08	−.12	.08	−.21	−.01	.04
5. Expenditure on Producers' Durables	−.152	.406	.38	.15	−.01	.14	.18	.25	−.07	.08
6. Expenditure on Producers' Structures	.303	.032	.21	.26	.28	−.15	−.13	.02	.08	.05
7. State and Local Government Expenditures	−.555	.224	.44	.43	.44	.35	−.09	−.10	.05	.03
8. Housing Expenditures[b]	.216	−.224	.34	.19	.24	.19	.09	−.01	.06	−.04
9. Unemployment Rate	.120	−.039	.01	.12	−.08	−.14	−.03	−.03	−.07	−.16
10. *GNP* Deflator-Price Index	−.160	−.108	.31	.16	.24	.31	−.09	.28	−.05	.32
11. Consumer Goods Price Index	−.199	−.132	.10	.11	.25	.33	.00	.24	.09	.23
12. Yields on U.S. Treasury Bills	−.058	−.179	−.23	−.20	.05	.09	−.01	−.11	−.17	.23
13. Yields on Commercial Paper	.080	−.165	.07	−.18	.01	.15	.04	−.21	−.09	.27
14. Yield on Corporate Bonds	−.067	−.204	.15	−.08	.10	.04	−.01	−.17	.01	.17

[a] Errors are in billions of current dollars for variables 1–8, and percentage points for the remaining variables.
[b] Sample period for Housing Expenditures is 1957–01 through 1966–04.
[c] The estimated standard error of r_1 under the hypothesis that the errors are uncorrelated is .16.

related to errors for Consumers' Expenditures on Durable Goods, Nonfarm Inventory Investment, and Expenditures on Producers' Durables. These correlations may be indicative of the origin of shocks in the real sector and their impact on the financial sector. They may also, of course, be indicative of problems in the structure of the model. The chains of interaction in a model of this size are enormously complex and examination of error correlations may facilitate otherwise unwieldy diagnostic analysis.

The reader may have noted by this point the absence of analysis of "turning point" errors, a topic which has practically become standard in analysis of prediction accuracy (see previous references as well as Henri Theil). The argument for the importance of turning point errors, that

TABLE 2—CORRELATION OF SAMPLE PERIOD PREDICTION ERRORS ACROSS VARIABLES: *FMP* ERRORS ABOVE THE DIAGONAL, *ARIMA* BELOW THE DIAGONAL

Endogenous Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1. *Gross National Product*	1.00	.20	.38	.74	.55	.28	.14	.34	−.35	−.16	−.27	.23	.33	.51
2. Consumers' Expenditures on Nondurable Goods	.13	1.00	−.16	−.19	−.06	.08	−.08	.14	.03	.27	.28	−.09	−.07	.04
3. Consumers' Expenditures on Durable Goods	.48	.09	1.00	.04	.22	.23	.03	.22	−.30	−.20	−.32	.22	.11	.30
4. Nonfarm Inventory Investment	.66	−.03	.18	1.00	.26	.12	−.14	.28	−.32	−.05	−.16	.22	.32	.38
5. Expenditures on Producers Durables	.50	.18	.63	.35	1.00	−.06	.37	.11	.09	−.41	−.37	.02	.20	.26
6. Expenditures on Producers' Structures	.20	.04	.25	.19	.12	1.00	−.19	.35	−.22	−.00	−.02	−.05	−.00	−.02
7. State and Local Government	.06	.11	.13	.14	−.02	.12	1.00	.16	−.01	−.37	−.28	−.08	−.13	−.02
8. Housing Expenditures[a]	.33	−.12	.30	.12	.03	.24	−.18	1.00	−.42	−.10	−.15	.03	.10	.05
9. Unemployment Rate	−.62	−.17	−.32	−.60	−.37	−.41	−.12	−.18	1.00	−.14	−.07	−.31	−.20	−.18
10. *GNP* Deflator-Price Index	−.05	−.06	−.03	.08	.06	.13	.21	−.13	−.23	1.00	.95	.16	.06	−.07
11. Consumer Goods Price Index	−.33	−.20	−.29	−.00	−.08	.05	.16	−.42	.05	.65	1.00	−.00	−.09	−.29
12. Yield on U.S. Treasury Bills	.34	.29	.30	.26	.40	.11	−.22	.19	−.30	−.22	−.31	1.00	.86	.53
13. Yield on Commercial Paper	.34	.20	.24	.44	.45	.12	−.17	.12	−.39	−.06	−.10	.89	1.00	.65
14. Yield on Corporate Bonds	.40	.29	.16	.33	.40	.11	−.21	.13	−.40	−.05	−.10	.77	.74	1.00

[a] Sample period for Housing Expenditures entries is 1957–01 through 1966–04.

is, errors in predicting the direction of change, has been well stated by Zarnowitz, p. 51, and, briefly, goes as follows. Economic time-series show strong systematic movements—trends and cycles. It should, then, be relatively easy to predict the continuation of a rise or fall. Consequently, to predict the end of the current movement and the beginning of the next appears to be a more crucial goal.

The crucial element in the argument for the importance of turning points is the view that cycles and trends in economic time-series are *systematic*. However, as we have seen from simulations of the *ARIMA* representations of *GNP*, "cycles" are not necessarily systematic in nature, but rather may be merely artifacts of random shocks working their way through the economy as Slutzky and Frisch suggested some time ago. Thus, it appears *ex post* that if turning points had been foreseen, prediction errors for subsequent observations could have been reduced. Turning points are usually associated with the occurrence of unusually large shocks to the system, and presumably success in anticipating *any* large disturbance would contribute to the accuracy of predictions of subsequent observations. If that is the case, then we should not restrict our attention only to the large disturbances which produce turning points, but rather should be interested in anticipation of all large disturbances. In other words, to say that turning points are important because they are difficult to predict is only to say that large disturbances are associated with large prediction errors. Statistical decision theory offers further clarification on this point. Namely, once the loss associated with errors have been specified, then conditions for optimal predictions may be stated, as for example in minimization of mean square error. Thereafter, turning point errors are of no special interest in and of themselves.

III. Composite Predictions of Endogenous Variables

Predictions computed from the *FMP* model are essentially the conditional expectations of future realizations implied by the structure of the model and the in-

$$\hat{\beta} = \frac{\sum [(FMP)_t - (ARIMA)_t][A_t - (ARIMA)_t]}{\sum [(FMP)_t - (ARIMA)_t]^2} \tag{5}$$

formation set available to it. If the *FMP* model make efficient use of that information, that is, if it does in fact provide conditional expectation predictions, then the *ARIMA* models which draw on only a subset of the same information should not be able to contribute to the accuracy of composite predictions which combine both.

A linear composite prediction is of the form

$$A_t = \beta_1(FMP)_t + \beta_2(ARIMA)_t + \epsilon_t \tag{3}$$

where A_t denotes the actual value for period t, β_1 and β_2 are fixed coefficients, and ϵ_t is the composite prediction error. Least squares fitting of (3) requires minimization of $\sum \hat{\epsilon}_t^2$ over values of β_1 and β_2 and therefore provides the minimum mean square error linear composite prediction for the sample period. In the case that both *FMP* and *ARIMA* predictions are individually unbiased, then (3) may be rewritten simply as

$$A_t = \beta(FMP)_t + (1-\beta)(ARIMA)_t + \epsilon_t \tag{4}$$

The least squares estimate of β in (4) is then given by equation (5) which is seen to be the coefficient of the regression of *ARIMA* prediction errors, $[A_t - (ARIMA)_t]$, on the difference between the two predictions. As would seem quite reasonable, the greater the ability of the difference between the two predictions to account for errors committed by $(ARIMA)_t$, the larger will be the weight given to $(FMP)_t$.

Consider now the hypothetical case that the *FMP* predictions subsume the *ARIMA* predictions and contain additional information $(FMP')_t$ so that

$$(FMP)_t = (ARIMA)_t + (FMP')_t \tag{6}$$

Then

$$\hat{\beta} = \frac{\sum (FMP')_t[A_t - (FMP)_t + (FMP')]}{\sum (FMP')_t} \tag{7}$$

and it is readily seen that

$$Plim\, \hat{\beta} = 1 \tag{8}$$

since *FMP* predictions are presumably uncorrelated with their associated errors. Thus, if *FMP* predictions do incorporate all of the information provided by *ARIMA* predictions, then estimates $\hat{\beta}_1$ and $\hat{\beta}_2$ in (3) should be approximately unity and zero, respectively.

Further insight into the structure of composite predictions is provided by an analogy to asset portfolios, namely composite predictions may be viewed as "portfolios" of predictions. If we denote individual *FMP* and *ARIMA* errors by u_1 and u_2, respectively, then from (4) the composite prediction error is seen to be

$$\epsilon_t = \beta(u_{1t}) + (1-\beta)(u_{2t}) \tag{9}$$

Thus, just as the return on a portfolio of assets is the weighted average of individual asset returns, the composite error is the weighted average of individual errors. In both cases, the objective is to minimize the variance of the weighted average given its expected value.[8] Construction of efficient asset portfolios requires selection of weights that minimize variance for various values of expected return, while in the case of prediction portfolios the

[8] In the case of two assets this is trivial since specification of a given rate of return determines both weights (except when the expected returns on both assets are identical) and leaves no additional degrees of freedom. The conceptual analogy, however, generalizes to many assets and many predictions.

weighted average always has expectation zero if individual predictions are unbiased or may be given expectation zero by addition of an appropriate constant.

Minimizing composite error variance over a finite sample of observations leads to the estimate of β given by

$$(10)\quad \hat{\beta} = \frac{\sum u_{2t}^2 - \sum u_{1t}u_{2t}}{\sum u_{1t}^2 + \sum u_{2t}^2 - 2\sum u_{1t}\cdot u_{2t}}$$

For large samples, or in the case that the variances $V(u_{1t})$ and $V(u_{2t})$ and the covariance $C(u_{1t}, u_{2t})$ are known, we have

$$(11)\quad \beta = \frac{V(u_{2t}) - C(u_{1t}, u_{2t})}{V(u_{1t}) + V(u_{2t}) - 2C(u_{1t}, u_{2t})}$$

Thus, the minimum variance weight β is seen to depend on the covariance between individual errors as well as on their respective variances, just as the analogous weight for a minimum variance two-asset portfolio depends on the covariance of returns as well as on return variances. Holding covariance constant, the larger the variance of the *ARIMA* error relative to that of the *FMP* error, the larger the weight given to the *FMP* prediction. However, with the exception of the special case $C(u_{1t}, u_{2t}) = V(u_{1t})$, β will always differ from unity and therefore the weight given to the *ARIMA* prediction differ from zero no matter how extreme might be the ratio of their error variances. *In short, relative accuracy is not an appropriate basis for choosing one prediction to the exclusion of the other; rather, even a very inaccurate prediction would generally be included in a minimum variance composite.*

Considering again the limiting case where the *FMP* prediction subsumes all the information in the *ARIMA* prediction, we see from expression (6) that the *ARIMA* error u_{2t} would be given by

$$(12)\quad u_{2t} = u_{1t} + (FMP')_t$$

Since errors are presumably uncorrelated with corresponding predictions, we have

$$(13)\quad V(u_{2t}) = V(u_{1t}) + V(FMP')_t$$

and

$$(14)\quad C(u_{1t}, u_{2t}) = V(u_{1t})$$

Expression (14) implies, as noted above, that $\beta = 1$. The portfolio analysis of composite predictions then also leads to the conclusion that if the *FMP* model succeeds in utilizing the larger information set available to it, subsuming the information contained in *ARIMA* predictions, estimates $\hat{\beta}_1$ and $\hat{\beta}_2$ in (3) should be approximately unity and zero.

Least squares estimates of β_1, and β_2 in (3) for each of the endogenous variables appear in Table 3. Values of $\hat{\beta}_1$ are significant at the 5 percent level for all of the variables. Values of $\hat{\beta}_2$ are significant at the 5 percent level for nine of fourteen variables and at the 10 percent level for a tenth. Durbin-Watson statistics (*D–W*) are generally close to two, and in no case may the hypothesis that the errors of the composite prediction are uncorrelated be rejected at the 5 percent level.[9] These results suggest that the *ARIMA* predictions do embody information which is omitted by *FMP* predictions, in particular, information available from the history of the individual variables themselves.

Since individual predictions are essentially unbiased, we would expect that their coefficients in a composite would add to approximately unity. Tests of the hypothesis that $\beta_1 + \beta_2 = 1$ in each regression led to rejection only in the case of nonfarm inventory investment. The weights given to the *ARIMA* predictions when the weights are reestimated under the

[9] When constant terms were added to regressions (3) none were significantly different from zero.

TABLE 3—COMPOSITE SAMPLE PERIOD PREDICTIONS OF ENDOGENOUS VARIABLES

Endogenous Variables	Minimum Squared Error Composite Predictions: Weights *FMP*	Weights *ARIMA*	Standard Deviation of Error	*D-W*	Weight Given to *ARIMA* Under Constraint $\hat{\beta}_1+\hat{\beta}_2=1$
1. *Gross National Product*	.834	.167	3.27	1.87	.168
2. Consumers' Expenditures on Nondurable Goods	.722	.278	1.36	2.39	.264
3. Consumers' Expenditures on Durable Goods	.958	.042	1.14	2.41	.044
4. Nonfarm Inventory Investment	.975	.194	2.38	1.81	.185
5. Expenditures on Producers' Durables	.658	.340*	.70	1.62	.369*
6. Expenditures on Producers' Structures	.670	.334*	.47	1.99	.355*
7. State and Local Government Expenditures	.290	.712*	.46	2.35	.681*
8. Housing Expenditures[a]	.794	.209*	.42	1.44	.225*
9. Unemployment Rate	.338	.662*	.27	1.91	.659*
10. *GNP* Deflator-Price Index	.641	.359*	.18	1.70	.363*
11. Consumer Goods Price Index	.561	.439*	.19	2.00	.436*
12. Yield on U.S. Treasury Bills	.706	.301*	.22	2.19	.292*
13. Yield on Commercial Paper	.736	.276*	.21	1.86	.273*
14. Yield on Corporate Bonds	.750	.255**	.09	1.85	.225

[a] Sample period for Housing Expenditures is 1957–01 through 1966–04.
* Denotes weight for *ARIMA* prediction significant at the 5 percent level.
** Denotes weight for *ARIMA* prediction significant at the 10 percent level.

constraint $\hat{\beta}_1+\hat{\beta}_2=1$ are also given in Table 3 and differ little from the unconstrained estimates.

IV. Jointly Optimal Composite Predictions

While the composite forecasts given by (3) are of interest in assessing the utilization of information by the *FMP* model, they may not be the optimal composites for a decision maker whose objective is to select weights which minimize expected loss. In particular, the relationships between errors for different variables may be of crucial importance as noted in Section II. A class of loss functions which allows for such interaction is that of the quadratic forms

$$L = \underline{\epsilon}' W \underline{\epsilon} \tag{15}$$

where L is the loss associated with the vector of errors across variables, $\underline{\epsilon}$, and W is a symmetric matrix. Enumeration of plausible choices of W is, of course, impossible. As an illustrative example, however, consider the particular loss function

$$L = \underline{\epsilon}' \Sigma^{-1} \underline{\epsilon} \tag{16}$$

where Σ is Var ($\underline{\epsilon}$), the matrix of variances and covariances of composite errors. Minimization of average loss over a given sample period corresponds in this special case to Aitken's generalized least squares estimation of parameters β_1 and β_2 for each of the equations (3) over the set of endogenous variables of interest. The matrix Σ is, of course, in practice unknown, and must be estimated. Zellner has suggested that Σ be estimated as the matrix of sample moments of residuals from ordinary least square estimation of the individual equations, in this case the individual composite predictions. Estimates of jointly optimal weights obtained by Zellner's procedure appear in Table 4. The weight assigned to the *FMP* prediction is significant at the 5 percent level for each variable. The weight assigned to the *ARIMA* prediction is significant at the 5 percent level for ten of the fourteen variables and at the 10 percent level for another three. Thus, joint estimation of optimal weights for *ARIMA* predictions reinforces our conclusion that these predictions utilize information which is omitted by the *FMP* predictions.

The weights given in Table 4 sum to approximately unity for each variable except Nonfarm Inventory Investment. Individual t-statistics for the linear hypotheses $\beta_1+\beta_2=1$ are not significant except in the case of the latter variable. However, the F-statistic for the joint test of $\beta_1+\beta_2=1$ for *all* variables is 4.03 with 14 and 532 degrees of freedom so that we may reject the joint hypothesis at the .01 level. Thus, while departures from unbiasedness over the sample period may not be large in absolute magnitude, they are sufficient to provide rejection of the joint hypothesis of unbiasedness.

The general implication of stating the problem of composite weight selection in a loss function context is that from the viewpoint of the decision maker the question of whether one set of predictions or the other is more accurate is irrelevant. Since his objective is to minimize expected loss, he will purchase any piece of information which reduces expected loss by more than its cost. Thus, the value of the *ARIMA* predictions, for example, is not measured by their individual errors, but rather by the contribution which they are able to make to the reduction in expected loss associated with a composite prediction or a set of composite predictions. This is also true, of course, for the *FMP* predictions. Since the latter are rela-

TABLE 4—JOINTLY OPTIMAL COMPOSITE PREDICTIONS

Endogenous Variable	Weights for Jointly Optimal Composite Predictions[a] *FMP*	*ARIMA*	t-statistic for Hypothesis $\beta_1+\beta_2=1$
1. *Gross National Product*	.881	.119*	1.301
2. Consumers' Expenditures on Nondurable Goods	.807	.194**	.542
3. Consumers' Expenditures on Durable Goods	.936	.065	.056
4. Nonfarm Inventory Investment	1.042	.091**	4.424
5. Expenditures on Producers' Durables	.692	.306*	−.903
6. Expenditures on Producers' Structures	.659	.343*	.720
7. State and Local Government Expenditures	.345	.656	.931
8. Housing Expenditures	.880	.123**	1.263
9. Unemployment Rate	.310	.698*	1.052
10. *GNP* Deflator-Price Index	.791	.209*	.302
11. Consumer Goods Price Index	.711	.289*	.548
12. Yield on U.S. Treasury Bills	.668	.354*	.338
13. Yield on Commercial Paper	.700	.310*	1.121
14. Yield on Corporate Bonds	.816	.187*	.948

[a] Sample period for estimate of jointly optimal weights is 1957–01 through 1966–04.
* Denotes weight for *ARIMA* prediction significant at the 5 percent level.
** Denotes weight for *ARIMA* prediction significant at the 10 percent level.

tively expensive relative to *ARIMA* predictions (including computational expense, updating, etc.), we might expect to find that many decision makers would purchase the less accurate and less expensive set of predictions. Likewise, if the bum on the street corner offers free tips to the decision maker on his way to the office, these will be incorporated in composite predictions if they result in any reduction in expected loss, regardless of presumably gross inaccuracy.

V. Analysis of Post-Sample Prediction Errors

It is scarcely surprising that both sets of predictors as well as their composites achieve reasonable accuracy during the period they were designed to explain. In the operational use of models, however, neither the forecaster nor the policy maker enjoys the luxury of working within the period of fit. Rather, from their point of view it is post-sample performance which is most relevant. Data for quarters 1967–01 through 1969–01 included in the *FMP* data deck provide only a short post-sample record, but nevertheless yield rather interesting and important results.

The mean squares, means, and standard deviations of post-sample one-quarter-ahead errors for both *FMP* and *ARIMA* models appear in Table 5 (*FMP* predictions continue to be conditioned on *true* future values of exogenous variables). It is immediately apparent that the accuracy of both sets of predictions deteriorated substantially during the post-sample period. However, mean square errors are smaller for *ARIMA* than for *FMP* predictions in the case of *GNP*, both categories of Consumer Expenditures, Expenditures on Producers' Durables, State and Local Government Expenditures, the Unemployment Rate, and all three interest rates. Differences are small for the *GNP* Deflator and Consumer Goods Price Index. It would appear, then, that the accuracy of *FMP* predictions deteriorated relative to that of *ARIMA* predictions during the post-sample period. In particular, the *FMP* model appears to have overestimated the effect of the federal tax surcharge enacted in 1968 and applied to personal income taxes in the third quarter of 1968 and to corporate income taxes retroactively to the first quarter of 1968. The *FMP* prediction of *GNP* was low by

TABLE 5—SUMMARY STATISTICS FOR *FMP* MODEL AND *ARIMA* MODEL POST-SAMPLE PREDICTION ERRORS

	FMP Model Errors			*ARIMA* Model Errors			Errors of Jointly Estimated Composite Predictions		
Endogenous Variables	*MSE*	Mean	Standard Deviation	*MSE*	Mean	Standard Deviation	*MSE*	Mean	Standard Deviation
1. *Gross National Product*	77.259	3.782	7.934	36.652	2.632	5.452	55.468	2.979	6.826
2. Consumers' Expenditures on Nondurable Goods	25.540	−3.914	3.197	11.605	1.464	3.076	18.944	−3.050	3.105
3. Consumers' Expenditures on Durable Goods	14.440	2.152	3.132	5.369	.990	2.095	13.270	2.065	3.001
4. Nonfarm Inventory Investment	11.161	1.474	2.998	49.589	−.166	7.040	12.285	.586	3.456
5. Expenditures on Producers' Durables	22.288	2.752	3.836	6.211	.668	2.401	15.939	2.261	3.291
6. Expenditures on Producers' Structures	1.038	.220	.995	4.427	.337	2.077	1.596	.349	1.214
7. State and Local Government Expenditures	8.065	.693	2.754	.766	.001	.875	1.414	.118	1.183
8. Housing Expenditures	1.935	1.002	.965	2.646	.764	1.436	1.572	.873	.899
9. Unemployment Rate	.412	−.522	.374	.081	−.141	.247	.114	−.287	.178
10. *GNP* Deflator-Price Index	.068	−.016	.260	.120	.237	.253	.051	.025	.225
11. Consumer Goods Price Index	.098	−.191	.249	.200	.295	.336	.068	−.074	.249
12. Yield on U.S. Treasury Bills	.425	.176	.628	.305	.091	.545	.280	.132	.512
13. Yield on Commercial Paper	.240	.282	.400	.190	.085	.427	.168	.172	.372
14. Yield on Corporate Bonds	.066	.156	.204	.055	.116	.205	.058	.133	.200

\$4.9 billion and \$4.4 billion in the third and fourth quarters of 1968 compared with \$.5 billion and \$2.4 billion, respectively, for the *ARIMA* model. In the first quarter of 1969 the *FMP* model got very seriously off-track with a prediction that was too low by \$23.2 billion when the *ARIMA* prediction was high by \$1.9 billion.

The results described above suggest that the simple *ARIMA* models are relatively more robust with respect to post-sample prediction than is the complex *FMP* model. It is interesting that this comparison generalizes to a considerable extent to relative performance among *ARIMA* models for different variables. In particular, the ratios of post-sample to sample Mean Square Error (*MSE*) are large for the fairly complex models for Expenditures on Producers' Structures and Housing Expenditures. Among the best performers are the very simple models for *GNP* and Consumers' Expenditures on Durable Goods, although the four-parameter model for the Unemployment Rate is the best of all and is the only model with a post-sample *MSE* smaller than its sample period *MSE*.

Finally, it is interesting to compare the post-sample performance of *FMP* and *ARIMA* predictions with that of the composite predictions constructed using the jointly estimated weights of Table 4. The relative magnitudes of mean square errors given in Table 5 indicate that composite predictions were more accurate than *FMP* predictions for twelve of the fourteen variables, the exceptions being Nonfarm Inventory Investment and Housing Expenditures for which the *ARIMA* component had suffered considerable post-sample deterioration. Composite predictions were more accurate than *ARIMA* predictions, however, in only seven cases, reflecting the generally severe deterioration in *FMP* performance. Composite predictions were more accurate than *either* individual prediction in five cases. If we score each of the three predictions by number of first places, the *ARIMA* models earn seven points, the composites five, and the *FMP* model only two. Thus, if mean square error were an appropriate measure of loss, an unweighted assessment clearly indicates that a decision maker would have been best off relying simply on *ARIMA* predictions in the post-sample period. To have ignored the information available from the simple time series models altogether would have been costly indeed.

APPENDIX

The following are the *ARIMA* models fitted on data from the *FMP* data deck for 1947-01 through 1966-04. Variables (1) through (8) are in billions of current dollars, the remaining variables in percentage points. The z_t and a_t are understood to be general notation referring to the observed value and disturbance associated with each respective variable.

1. *Gross National Product*

$$z_t = z_{t-1} + .615(z_{t-1} - z_{t-2}) + 2.76 + a_t$$

$$\hat{\sigma}_a = 4.77$$

2. Consumers' Expenditures on Nondurable Goods

$$z_t = z_{t-1} + .190(z_{t-1} - z_{t-2}) + .504(z_{t-2} - z_{t-3}) + 1.06 + a_t$$

$$\hat{\sigma}_a = 1.72$$

3. Consumers' Expenditures on Durable Goods

$$z_t = z_{t-1} + .666 + a_t$$

$$\hat{\sigma}_a = 1.92$$

4. Nonfarm Inventory Investment

$$z_t = .581 z_{t-1} + a_t + .0013 a_{t-1} + .742 a_{t-2} + 1.69$$

$$\hat{\sigma}_a = 3.14$$

5. Expenditures on Producers' Durables

$$z_t = z_{t-1} + a_t + .347a_{t-1} + .517$$
$$\hat{\sigma}_a = 1.06$$

6. Expenditures on Producers' Structures

$$\begin{aligned} z_t = z_{t-1} &+ .303(z_{t-1} - z_{t-2}) \\ &+ .216(z_{t-2} - z_{t-3}) \\ &+ .297(z_{t-3} - z_{t-4}) \\ &- .442(z_{t-4} - z_{t-5}) \\ &+ .159 + a_t \end{aligned}$$
$$\hat{\sigma}_a = .47$$

7. State and Local Government Expenditures

$$z_t = 2z_{t-1} - z_{t-2} + a_t - .695a_{t-1}$$
$$\hat{\sigma}_a = .52$$

8. Housing Expenditures

$$\begin{aligned} z_t = z_{t-1} &+ .639(z_{t-1} - z_{t-2}) \\ &+ .076(z_{t-2} - z_{t-3}) \\ &- .286(z_{t-3} - z_{t-4}) + a_t \end{aligned}$$
$$\hat{\sigma}_a = .74$$

9. Unemployment Rate

$$\begin{aligned} z_t = 1.46z_{t-1} &- .612z_{t-2} + a_t \\ &+ .284a_{t-1} + .734 \end{aligned}$$
$$\hat{\sigma}_a = .33$$

10. *GNP* Deflator-Price Index

$$z_t = z_{t-1} + .523(z_{t-1} - z_{t-2}) + a_t + .256$$
$$\hat{\sigma}_a = .46$$

11. Consumer Goods Price Index

$$z_t = z_{t-1} + .414(z_{t-1} - z_{t-2}) + a_t + .244$$
$$\hat{\sigma}_a = .48$$

12. Yield on U.S. Treasury Bills

$$\begin{aligned} z_t = z_{t-1} &+ .608(z_{t-1} - z_{t-2}) \\ &- .425(z_{t-2} - z_{t-3}) + a_t \end{aligned}$$
$$\hat{\sigma}_a = .29$$

13. Yield on Commercial Paper

$$\begin{aligned} z_t = z_{t-1} &+ .727(z_{t-1} - z_{t-2}) \\ &- .427(z_{t-2} - z_{t-3}) + a_t \end{aligned}$$
$$\hat{\sigma}_a = .27$$

14. Yield on Corporate Bonds

$$\begin{aligned} z_t = z_{t-1} &+ .490(z_{t-1} - z_{t-2}) \\ &- .169(z_{t-2} - z_{t-3}) + a_t \end{aligned}$$
$$\hat{\sigma}_a = .11$$

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