SINGAPORE MANAGEMENT UNIVERSITY

QF101 Quantitative Finance G1

SPX vs SPY Option Pricing Project

Team 3
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1. Introduction

In this project, we utilised the binomial tree option pricing method to model the prices of European-styled SPX options and American-styled SPY options, both of which are based on the S&P 500 Index. We chose these options as they are heavily traded, and their comparison allows us to evaluate the performance of our model for different types of options. To assess our model's performance, we compared our model's prices to actual market prices and executed trades accordingly, measuring our profit and losses (P/L), Sharpe ratio, Calmar ratio, and maximum drawdown.

2. Methodology

We used options data with 30 days to expiry from January 2020 to December 2022.

Rolling volatility of one year up to before each trading date was used as implied volatility, with Covariance Shrinkage applied to minimise noise.

The average annual interest rate for the period, 2.5%, was used as the risk-free rate, and the average annual dividend yield for the period, 1.5%, was used for underlying SPY.

We used Cox-Ross-Rubinstein (CRR) and Jarrow-Rudd (JR) models to compare their performance in the binomial tree model.

For trading, we performed both long and short positions for call and put options, entering 1 long position when our model's price was greater than the market price and entering 1 short position when our model's price was less than the market price. We bought exactly one option for each transaction to weigh the options according to market demand. The premium spent/received on purchase and the payoff on sale were taken into account when calculating P/L.

3. Initial Results

We made several observations from our results:

1. The outcomes of the CRR and JR models were very similar. The prices predicted and P/L of both models were within 1% deviation. The error terms calculated are the deviations of our model's price from market prices.

```
Root Mean Squared Error of CRR Call model: 20.894495881960978
Root Mean Squared Error of JR Call model: 20.892574928621944
Root Mean Squared Error of CRR Put model: 21.61877256944796
Root Mean Squared Error of JR Put model: 21.6169893240666
Mean Absolute Error of CRR Call model: 12.179502242149622
Mean Absolute Error of JR Call model: 12.178586841592729
Mean Absolute Error of CRR Put model: 13.381022873010346
Mean Absolute Error of JR Put model: 13.3795543630229
Percentage Mean Absolute Error of CRR Call model: 89.90169561712081 %
Percentage Mean Absolute Error of JR Call model: 89.7479891380906 %
Percentage Mean Absolute Error of CRR Put model: 52.08037360182608 %
Percentage Mean Absolute Error of JR Put model: 52.073426146778615 %
```

```
Mean Profit/Loss of CRR model: 8.959887290942877
Mean Profit/Loss of JR model: 8.900853060784788
Volatility of CRR model: 207.2139025439408
Volatility of JR model: 207.2486516839786
Total Profit/Loss of CRR model: 332689.57499999995
Total Profit/Loss of JR model: 330497.57499999995
```

Figure 1. Error and P/L values for SPX options model

2. The errors for the SPX and SPY model are very similar. They differ by a factor of 10x because of the difference in underlying price by a factor of 10x.

```
Root Mean Squared Error of CRR Call model: 2.2014083365619896
Root Mean Squared Error of JR Call model: 2.201059135659596
Root Mean Squared Error of CRR Put model: 2.221104470891091
Root Mean Squared Error of JR Put model: 2.2208519746157314
Mean Absolute Error of CRR Call model: 1.33057266941219
Mean Absolute Error of JR Call model: 1.3303482365065098
Mean Absolute Error of CRR Put model: 1.398773103360051
Mean Absolute Error of JR Put model: 1.3986049776617586
Percentage Mean Absolute Error of CRR Call model: inf %
Percentage Mean Absolute Error of JR Call model: inf %
Percentage Mean Absolute Error of CRR Put model: 46.934619008242365 %
Percentage Mean Absolute Error of JR Put model: 46.93023731003161 %
```

```
Mean Profit/Loss of CRR model: -0.5089830091975012
Mean Profit/Loss of JR model: -0.5032064873973185
Volatility of CRR model: 19.96222973690234
Volatility of JR model: 19.96649186704153
Total Profit/Loss of CRR model: -14498.89000000002
Total Profit/Loss of JR model: -14334.340000000015
```

Figure 2. Error and P/L values for SPY options model

However, we see that the SPX model managed to profit by 330 thousand compared to the 14 thousand loss from the SPY model. Scaling that by the 10x factor, it is a 330 thousand profit compared to a 140 thousand loss.

3. The periods where the SPX model makes a profit, the SPY model makes a loss, and vice versa. We theorise that this is due to SPY options being American style while SPX options are European style. This is attributed to the fact that the SPY options holders are able to exercise their options before expiry causing the difference in volatility during the Covid-19 and economic downturn.

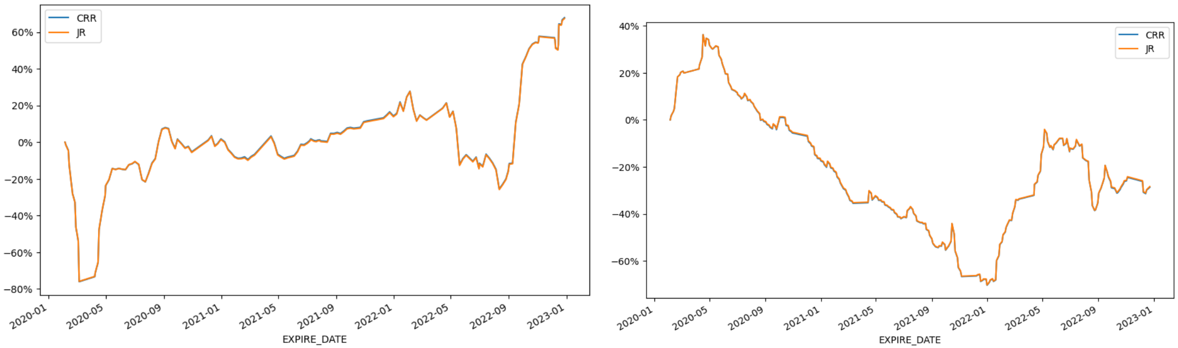

Figure 3. Total P/L for SPX (left) and SPY (right) models

4. Call options generally perform better than put options for both models. We theorise that this is due to the difference in liquidity between calls and puts, puts being much more liquid than calls, thus less spread to profit from.

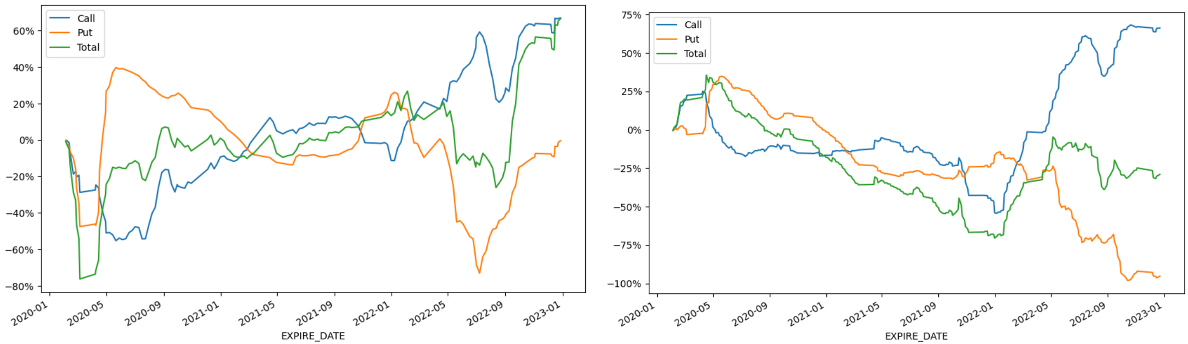

Figure 4. Performance of calls and puts for SPX (left) and SPY (right) models

5. Short positions are much more volatile than long positions. This is due to the potential losses of short positions being much greater than that of long positions.

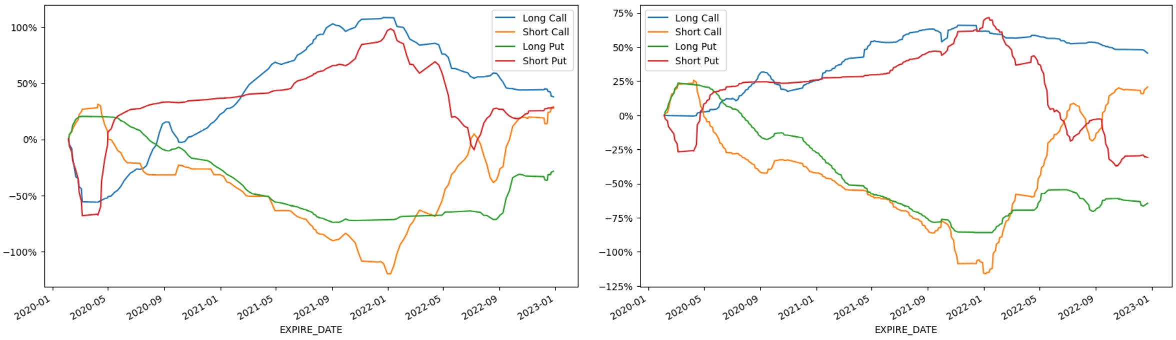

Figure 5. Performance of short and long positions for SPX (left) and SPY (right) models

We used initial capital of $500,000 for SPX model and $50,000 for SPY model to account for the 10x factor in price.

```
Sharpe Ratio of CRR and JR model
Sharpe Ratio of CRR model: 1.3478728504137625
Sharpe Ratio of JR model: 1.3440163187738348
Calmars Ratio of CRR and JR model
Calmars Ratio of CRR model: 2.388381389534184
Calmars Ratio of JR model: 2.3820730929606366
Max Drawdown of CRR and JR model
Max Drawdown of CRR model (in percentage): -75.71047077979915 %
Max Drawdown of JR model (in percentage): -75.71047077979915 %
```

```
Sharpe Ratio of CRR and JR model
Sharpe Ratio of CRR model: -0.3324533907057003
Sharpe Ratio of JR model: -0.3275258227870484
Calmars Ratio of CRR and JR model
Calmars Ratio of CRR model: -0.2573403827934774
Calmars Ratio of JR model: -0.25306157548859476
Max Drawdown of CRR and JR model
Max Drawdown of CRR model (in percentage): -78.2148890562779 %
Max Drawdown of JR model (in percentage): -78.2148890562779 %
```

Figure 6. Performance statistic of SPX (left) and SPY (right) models

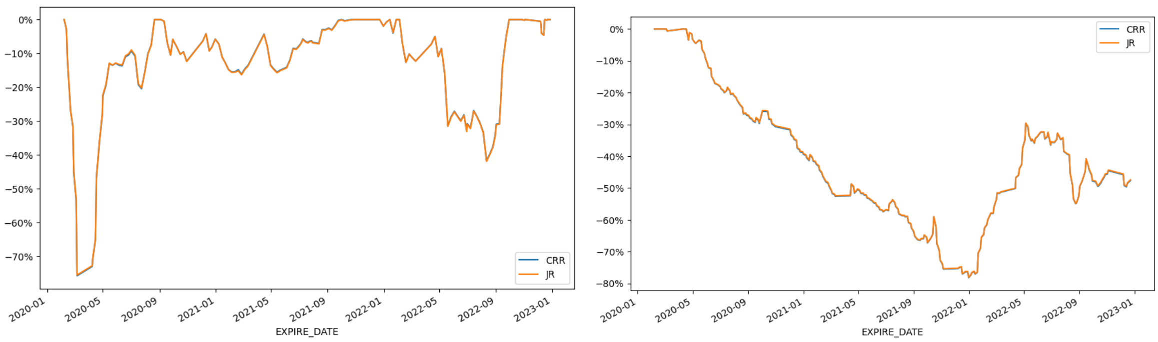

Figure 7. Downturn of SPX (left) and SPY (right) models

The performance statistic of our model was okay for SPX but unsatisfactory for SPY. Especially when it comes the the max drawdown. SPY is also unsatisfactory as sharpe, and calmar are both negative. Thus we choose to work on improvements.

4. Final Results after Implementing Improvement

One improvement that we made is to go long when our predicted value exceeds market value by 0% to 10% and to go short when our predicted value is below market value by 0% to 10%. The following are the results.

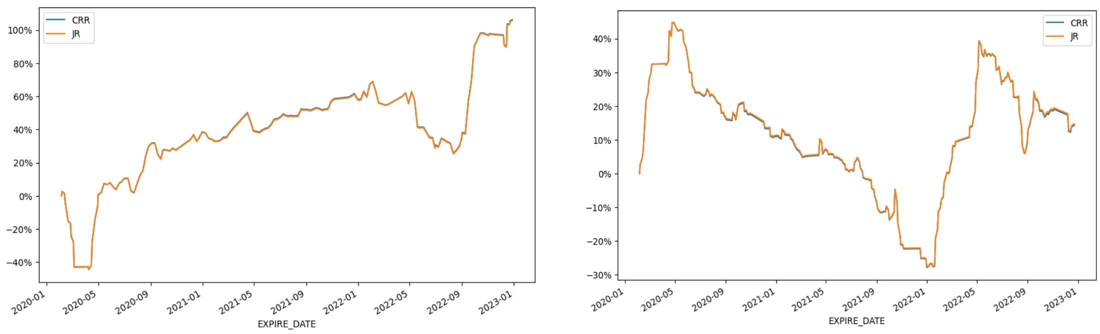

Figure 8. Total P/L for SPX (left) and SPY (right) models

```
Mean Profit/Loss of CRR model: 14.05162047884517
Mean Profit/Loss of JR model: 13.985484500821414
Volatility of CRR model: 198.01085248543916
Volatility of JR model: 198.03539355460376
Total Profit/Loss of CRR model: 521750.72
Total Profit/Loss of JR model: 519295.0249999999
Plotting daily Profit/Loss of CRR and JR model
```

```
Mean Profit/Loss of CRR model: 0.24374078494699913
Mean Profit/Loss of JR model: 0.25121621147230183
Volatility of CRR model: 19.076043535318924
Volatility of JR model: 19.080055467270356
Total Profit/Loss of CRR model: 6943.199999999994
Total Profit/Loss of JR model: 7156.14499999999
Plotting daily Profit/Loss of CRR and JR model
```

Figure 9. P/L values for improved SPX (left) and SPY (right) models

```
Sharpe Ratio of CRR and JR model
Sharpe Ratio of CRR model: 2.0239698705503057
Sharpe Ratio of JR model: 2.0212593456708934
Calmars Ratio of CRR and JR model
Calmars Ratio of CRR model: 3.255370246778414
Calmars Ratio of JR model: 3.2636355788514346
Max Drawdown of CRR and JR model
Max Drawdown of CRR model (in percentage): -45.835832163806 %
Max Drawdown of JR model (in percentage): -45.835832163806 %
```

```
Sharpe Ratio of CRR and JR model
Sharpe Ratio of CRR model: 0.4974259671220377
Sharpe Ratio of JR model: 0.5089276309705297
Calmars Ratio of CRR and JR model
Calmars Ratio of CRR model: 0.3525006399690064
Calmars Ratio of JR model: 0.3609271568987873
Max Drawdown of CRR and JR model
Max Drawdown of CRR model (in percentage): -50.13413973715339 %
Max Drawdown of JR model (in percentage): -50.13413973715339 %
```

Figure 10. Performance statistic of improved SPX (left) and SPY (right) models

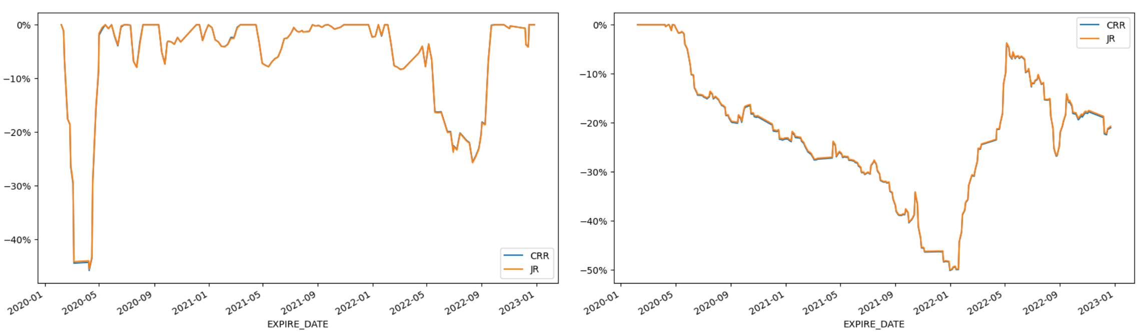

Figure 11. Downturn of improved SPX (left) and SPY (right) models

For both SPX and SPY, there were significant improvements to both our maximum drawdown, calmar, sharpe, and P/L. Maximum drawdown improved from 70% to 40% in the current model. We began to make profits for the SPY (from a loss of $14,334 to a profit of $7,156) and total profits also increased for SPX (from $330,497 to $519,295).

A lesson learnt in this section is that we can easily truncate losses and increase P/L by simply changing our strategy in long/short call/put options given the range of acceptable values that our model churns out. This is based on the theory that the market is pretty efficient, and large or excessive deviation of our model prediction likely means that the model is predicting unreasonable value.

5. Evaluation of Rolling Volatility and its Impact on Model Performance

We aimed to assess how the duration of rolling affects volatility calculations and the accuracy of our model, which would in turn affect our profits and losses. To achieve this, we calculate rolling volatility for four different periods: 2 years, 1 year, 6 months, and 1 month.

```
Root Mean Squared Error of CRR Call model: 23.7041223043019
Root Mean Squared Error of JR Call model: 23.701786447329976
Root Mean Squared Error of CRR Put model: 23.012301365254974
Root Mean Squared Error of JR Put model: 23.00987583923175
```

2-years rolling volatility model error

```
Root Mean Squared Error of CRR Call model: 20.894481878903033
Root Mean Squared Error of JR Call model: 20.89256091639572
Root Mean Squared Error of CRR Put model: 21.61875990512392
Root Mean Squared Error of JR Put model: 21.61697665249206
```

1-year rolling volatility model error

```
Root Mean Squared Error of CRR Call model: 21.757388869649386
Root Mean Squared Error of JR Call model: 21.755105798406873
Root Mean Squared Error of CRR Put model: 22.130746399790954
Root Mean Squared Error of JR Put model: 22.128963222342662
```

6-months rolling volatility model error

```
Root Mean Squared Error of CRR Call model: 21.99948231916816
Root Mean Squared Error of JR Call model: 21.97910547119046
Root Mean Squared Error of CRR Put model: 21.624078225037202
Root Mean Squared Error of JR Put model: 21.60953702613894
```

1-month rolling volatility model error

To evaluate the initial performance of our model, we compared it against actual options prices using the RMS error metric. We wanted to heavily penalise predictions that deviate further from the market price, hence we chose RMS error for comparison. The result indicated that the 1 year rolling volatility had the lowest RMS error among all the options, which led us to use it as the implied volatility for the rest of the project.

However, upon further analysis after the project presentation, we realised that the differences in RMS error between the four different rolling periods were not significant. As a result, we ran the improved model using the other three periods, i.e., 2 years, 6 months, and 1 month, and obtained the following results for SPX.

Evaluating 2-year Rolling Volatility:

```
Mean Profit/Loss of CRR model: 12.534965931431943
Mean Profit/Loss of JR model: 12.534106945678802
Volatility of CRR model: 196.19200729589153
Volatility of JR model: 196.17197577541464
Total Profit/Loss of CRR model: 465435.8199999995
Total Profit/Loss of JR model: 465403.92499999964
Plotting daily Profit/Loss of CRR and JR model
```

```
Sharpe Ratio of CRR and JR model
Sharpe Ratio of CRR model: 1.9975037305341947
Sharpe Ratio of JR model: 1.9933184869402458
Calmars Ratio of CRR and JR model
Calmars Ratio of CRR model: 4.361662509453376
Calmars Ratio of JR model: 4.32699983340629
Max Drawdown of CRR and JR model
Max Drawdown of CRR model (in percentage): -30.7195073056976 %
Max Drawdown of JR model (in percentage): -30.7195073056976 %
```

2-year rolling volatility P/L and Performance Statistic

We found that using a 2-year rolling volatility yielded a P/L of $465,435, which was the closest to our model that utilised a 1-year rolling volatility which achieved a P/L of $519,295. Additionally, the Sharpe ratio of this model (1.99) was similar to our 1-year model (2.02). However, it had a better maximum drawdown of -30% compared to our 1-year model with a maximum drawdown of -45%. Furthermore, this model had a better Calmar ratio of 4.3 compared to our 1-year model with a Calmar ratio of 3.2.

Evaluating 6-months Rolling Volatility:

```
Mean Profit/Loss of CRR model: 6.665854138051757
Mean Profit/Loss of JR model: 6.547153052705283
Volatility of CRR model: 200.91578199835567
Volatility of JR model: 200.99563156099595
Total Profit/Loss of CRR model: 247509.82999999978
Total Profit/Loss of JR model: 243102.33999999985
Plotting daily Profit/Loss of CRR and JR model
```

```
Sharpe Ratio of CRR and JR model
Sharpe Ratio of CRR model: 1.271194268561928
Sharpe Ratio of JR model: 1.2556951735706923
Calmars Ratio of CRR and JR model
Calmars Ratio of CRR model: 2.546729136018665
Calmars Ratio of JR model: 2.528310983472525
Max Drawdown of CRR and JR model
Max Drawdown of CRR model (in percentage): -37.63713555099606 %
Max Drawdown of JR model (in percentage): -37.63713555099606 %
```

6-months rolling volatility P/L and Performance Statistic

As for the 6-month rolling volatility model, we found it did not perform well in general when compared to our 1-year model, except for the maximum drawdown, where it had a slightly better performance of -37% compared to our 1-year model with a maximum drawdown of -45%. However, when we accounted for the annualised return using the Calmar ratio, we observed that the Calmar ratio of the 6-month model was worse than our 1-year model, with a Calmar ratio of 2.5 compared to our 1-year model's Calmar ratio of 3.2.

Evaluating 1-month Rolling Volatility:

```
Mean Profit/Loss of CRR model: -5.430362904311768
Mean Profit/Loss of JR model: -5.568726131803624
Volatility of CRR model: 199.45887247175395
Volatility of JR model: 199.51450402582418
Total Profit/Loss of CRR model: -201634.80500000028
Total Profit/Loss of JR model: -206772.37000000034
Plotting daily Profit/Loss of CRR and JR model
```

```
Sharpe Ratio of CRR and JR model
Sharpe Ratio of CRR model: -0.537885523642852
Sharpe Ratio of JR model: 0.32145860222577993
Calmars Ratio of CRR and JR model
Calmars Ratio of CRR model: -3.423741842472
Calmars Ratio of JR model: 2.027293946893295
Max Drawdown of CRR and JR model
Max Drawdown of CRR model (in percentage): -102.72936189005648 %
Max Drawdown of JR model (in percentage): -102.72936189005648 %
```

1-month rolling volatility P/L and Performance Statistic

Finally, we observed that the 1-month rolling volatility produced poor results, with the model indicating a wiped-out portfolio, and a maximum drawdown exceeding -100% at -102%.

Therefore, based on our analysis, we can conclude that our initial choice of 1-year rolling volatility was a good decision to maximise profits. Nevertheless, if minimising maximum drawdown is a goal of yours, utilising a 2-year rolling volatility period would be a better option.

6. Conclusion

During the project, we aimed to improve our models by applying the general CRR/JR additive binomial models. We observed that both CRR and JR models provided similar results. Additionally, we found that SPY options offered more flexibility for executing options, allowing them to perform better during economic downturns like the one caused by COVID-19.

We also altered our trading strategy by constraining our model's predicted values to market values and experimenting with different rolling periods for volatility. Through these changes, we were able to achieve significant improvements in P/L, Sharpe ratio, Calmar ratio, and maximum drawdown. These outcomes were significant accomplishments for our team.