Fierz-Pauli Field Theory

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This poster presents a theory that explains how massless particles with spin 2 behave. We start by explaining why we choose a field, and why they are related with a spin, and then we move on to discussing the Fierz-Pauli Field Theory, which is a the only ghost-free model for a spin-2 field. We also show how this theory is related to Einstein's theory of gravity and how it can help us better understand the fundamental forces of the universe.

1 Spin Field Theory

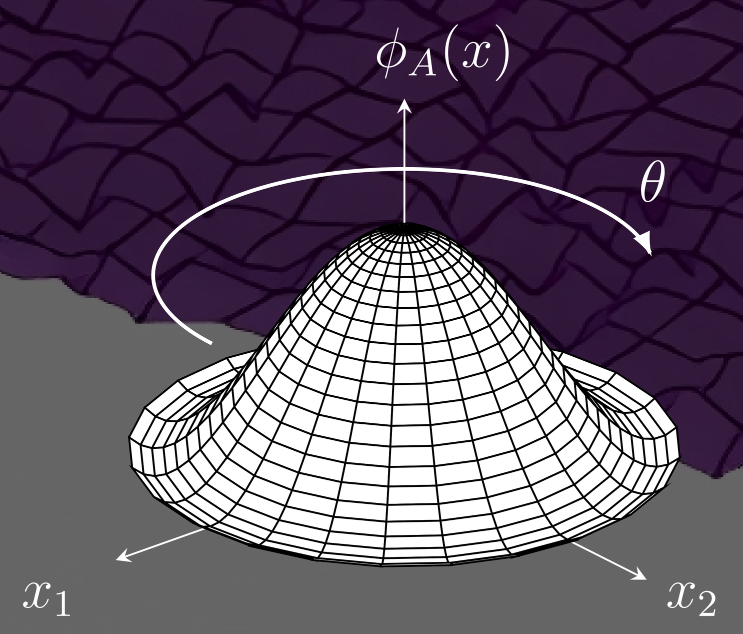
Field theory provides a coherent and consistent explanation for physical interactions. Spin is a fundamental property of fields in physics, characterized by relativistic systems that determine field transformations under rotations. Understanding spin is crucial for comprehending field behavior in different reference frames.

$$\phi_A = \phi_A(x_0, x_1, x_2, x_3) \equiv \phi_A(x). \tag{1}$$

Helicity ξ can be described mathematically:

$$\phi'_A = e^{i\xi\theta}\phi_A, \tag{2}$$

where ϕ_A and ϕ'_A are the original and rotated field, respectively; h is the helicity of each component of the vector and θ is the rotation angle.



Spin field theory plays a crucial role in understanding particles with integer spin and has been vital for the development of various areas of physics.

2 Fierz-Pauli Action

The Fierz-Pauli action describes the dynamics of a spin-2, massless, symmetric, second-order tensor field $h_{\mu\nu}$. This action must satisfy the principles of locality and relativity. To capture the behavior of this field, we use the unique general ghost free Lagrangian [2] that satisfies these characteristics can be expressed as:

$$S_{FP} = \int_{M_4} dx^4 \left(\frac{1}{4}\partial_\mu h_{\nu\rho}\,\partial^\mu h^{\nu\rho} - \frac{1}{2}\,\partial_\mu h_{\nu\rho}\,\partial^\nu h^{\rho\mu} + \frac{1}{2}\,\partial_\mu h\,\partial_\rho\, h^{\mu\rho} - \frac{1}{4}\,\partial_\mu h\,\partial^\mu h.\right), \tag{3}$$

where $h = \eta^{\mu\nu}h_{\mu\nu}$ and M_4 is the four-dimensional Minkowski space-time.

This Lagrangian is invariant under a gauge transformation

$$S_{FP}\left(h_{\mu\nu}\right) = S_{FP}\left(h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu\right). \tag{4}$$

This characteristic is important because it allows us to choose a particular gauge that simplifies calculations. One common choice of gauge is the harmonic gauge, which is defined by the condition

$$\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h = 0. \tag{5}$$

Furthermore, the choice of a full gauge allows the reduction of the degrees of freedom of the field, to such an extent that it matches the values expected by the Wigner classification.

To obtain the equation of motion, we vary the action with respect to $h_{\mu\nu}$

$$\frac{\delta S_{FP}}{\delta h^{\mu\nu}} = 0 \wedge (5) \implies \partial_\alpha\partial^\alpha h_{\mu\nu} = 0. \tag{6}$$

This traceless solution for $h_{\mu\nu}$ is in accordance with a relativistic wave equation.

3 Linear General Relativity

General relativity

The Einstein-Hilbert action (EHA) encapsulates the principles of general relativity, representing spacetime dynamics via geometry (see more in [1]). It is given by:

$$S_{EH}\left(g_{\mu\nu}\right) = \frac{1}{16\pi G}\int_{M_4} \mathrm{d}^4x \sqrt{|det(g_{\mu\nu})|}\,R\left(g_{\mu\nu}\right), \tag{7}$$

where $g_{\mu\nu}$, R, and G denote the metric tensor, scalar curvature, and gravitational constant, respectively. Einstein's field equation is derived from Einstein-Hilbert and material action using the Hamiltonian principle:

$$\frac{\delta\left(S_{EH} + S_{\text{material}}\right)}{\delta g^{\mu\nu}} = 0 \implies R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}. \tag{8}$$

Here, S_{material} represents the action of the rest of the physical system and $T_{\mu\nu}$ its stress-energy tensor, and $R_{\mu\nu}$ is the Ricci tensor.

General relativity - Spin-2

Consider a small fluctuation $h_{\mu\nu}$ around the Minkowski metric $\eta_{\mu\nu}$ [4] as:

$$g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon h_{\mu\nu} \quad \downarrow \quad \varepsilon^3 \simeq 0, \tag{9}$$

Using this expression, we can re-express the elements of the Einstein-Hilbert action in terms of $h_{\mu\nu}$:

$$g^{\mu\nu} = \eta^{\mu\nu} - \varepsilon h^{\mu\nu} + \varepsilon^2 h^{\mu\lambda}h^\nu_\lambda + \mathcal{O}(\varepsilon^3), \tag{10}$$

$$\sqrt{|det(g_{\mu\nu})|} = 1 + \frac{\varepsilon}{2}h + \frac{\varepsilon}{8}h^2 - \frac{\varepsilon^2}{4}h_{\mu\nu}h^{\mu\nu} + \mathcal{O}(\varepsilon^3), \tag{11}$$

$$R\left(g_{\mu\nu}\right) = \frac{1}{2}\left(\varepsilon R^{(1)} + \varepsilon^2 R^{(2)}\right) + \mathcal{O}(\varepsilon^3), \tag{12}$$

where h is the trace of the metric perturbation and the $R^{(1)}$ and $R^{(2)}$ expression are:

$$R^{(1)}(\varepsilon) = \left(\eta^{\mu\nu} - \varepsilon h^{\mu\nu}\right)\left(\partial^\rho\partial_\mu h_{\nu\rho} + \partial^\rho\partial_\nu h_{\mu\rho} - \partial_\alpha\partial^\alpha h_{\mu\nu} - \partial_\mu\partial_\nu h\right),$$
$$R^{(2)}(\varepsilon^2) = \frac{1}{2}\partial_\rho h\partial^\rho h - \partial_\lambda h\partial_\rho h^{\rho\lambda} + \partial_\rho h_{\sigma\lambda}\partial^\sigma h^{\rho\lambda} - \frac{1}{2}\partial_\rho h_{\sigma\lambda}\partial^\rho h^{\sigma\lambda}.$$

Using the expressions above, we can write the perturbative Lagrangian as:

Newton - Spin-2

By choosing the harmonic gauge (5), the equation of motion of $h_{\mu\nu}$ this perturbative action takes the form:

$$\partial_\alpha\partial^\alpha h_{\mu\nu} = -\frac{1}{\kappa^2}\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right). \tag{13}$$

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In the non-relativistic limit, the energy-momentum tensor for a static particle of mass M can be described like:

$$T_{\mu\nu} = \varepsilon^2 M\,\delta^0_\mu\,\delta^0_\nu\,\delta(\vec{x}), \tag{14}$$

where the first two deltas are Kronecker deltas, and the third one is a Dirac delta for the spatial coordinates. Solving the equation of motion [3] we obtain:

$$h_{00} = \frac{1}{4\kappa^2}\frac{\varepsilon^2 M}{r} = \frac{2GM}{r}, \tag{15}$$

the gravitational potential of Newton's Law.

$$\mathcal{L}_{EH}\left(\eta_{\mu\nu} + \varepsilon h_{\mu\nu}\right) \simeq \varepsilon^2\left(\frac{1}{4}\partial_\mu h_{\nu\rho}\,\partial^\mu h^{\nu\rho} - \frac{1}{2}\,\partial_\mu h_{\nu\rho}\,\partial^\nu h^{\rho\mu} + \frac{1}{2}\,\partial_\mu h\,\partial_\rho\, h^{\mu\rho} - \frac{1}{4}\,\partial_\mu h\,\partial^\mu h\right),$$
$$\therefore S_{EH}\left(\eta_{\mu\nu} + \varepsilon h_{\mu\nu}\right) \simeq \kappa^2 S_{FP}, \quad \kappa^2 = \frac{\varepsilon^2}{16\pi G}$$

Peering into the abyss of quantum gravity, we stand at the precipice of unfathomable revelations that may shatter our comprehension of the cosmos. Delving into this enigmatic domain could unlock the deepest mysteries of existence, rewriting our understanding of reality itself.

References

[1] Sean M Carroll. *Spacetime and Geometry: An Introduction to General Relativity.* Cambridge University Press, 2004.

[2] Juan José Fernández. "Teorías de Campos Bosónicos Una introducción a los campos de Spin-1, Spin-2 y Kalb-Ramond". In: (2023).

[3] Kurt Hinterbichler. "Theoretical Aspects of Massive Gravity". In: (2012). arXiv: `1105.3735 [hep-th]`.

[4] Edouard B. Manoukian. *Quantum field theory: Vol. 2: Introductions to Quantum Gravity, Supersymmetry and String Theory.* Springer, 2016.

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