Generalised Coordination of Multi-robot System

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Preliminary notions

We want to achieve a method to coordinate the evolution of a group of Agents in a common state space $\mathbb{R}^d$

The Generalised Coordination Problem

$$\lim_{t \to \infty} \operatorname{dist}\left(x_{\mathcal{N}}(t), \mathcal{D}\right) = 0,$$

where $\mathcal{D} \subset (\mathbb{R}^d)^n$ is a desired configuration set

Local and Global Coordinate Frames

We define two coordinate frames:

- A Global (fixed to Earth) coordinate frame, Σ, Common to every agent
- A Local (fixed to body) coordinate frame Σ_i for each agent i

A global coordinate $p(t) in \mathbb{R}^d$ and a local coordinate $p^{[i]}(t) \in \mathbb{R}^d$ are transformed into each other as,

$$p((t) = M_i(t)p^{[i]}(t) + b_i(t)$$

Where $(M_i(t), b_i(t)) \in \mathcal{M} \ltimes \mathcal{B}$.
$\mathcal{M} \ltimes \mathcal{B}$ is a frame transformation set, typically a subgroup of, scaled$(O(d)) \ltimes \mathbb{R}^d$

The problem

The Kinematic model

Given a transformation set $\mathcal{M} \ltimes \mathcal{B}$ and the coordinate transformation $p((t) = M_i(t)p^{[i]}(t) + b_i(t) : (M_i(t), b_i(t)) \in \mathcal{M} \ltimes \mathcal{B}$. We define the kinematic model

$$\dot{x}_i(t) = M_i(t)u_i(t)$$

The problem

Local Coordinates

The relative position $x_i^{[j]}$ of neighbor $j \in \mathcal{N}_i$ wrt agent i is given as,

$$x_i^{[j]}(t) = M_i^{-1}(t)(x_j(t) - b_i(t)) = (M_i(t), b_i(t))^{-1} \bullet x_j(t)$$

The problem

The admissible controller

for a graph $G = (\mathcal{N}, \mathcal{E})$ that sets the neighborhood of de agents

$$u_i(t) = c_i\left(x_{\mathcal{N}_i}^{[i]}(t)\right)$$

Where function $c_i : \left(\mathbb{R}^d\right)^{|\mathcal{N}_i|} \to \mathbb{R}^d$ and $\mathcal{N}_i \subset \mathcal{N}$ is the neighbor set of agent i.

c_i is called a *distributed controller with relative measurements*

The problem

To achieve the generalised coordination with respect to a set $\mathcal{D} \subset \left(\mathbb{R}^d\right)^n$ we need to design a controller that asymptotically stabilises $\mathcal{D}$:

Gradient-flow approach

$$x_i(t) = -k_i \frac{\partial v}{\partial x_i}\left(x_{\mathcal{N}}(t)\right)$$

- With a non-negative function $v : \left(\mathbb{R}^d\right)^n \to \mathbb{R}_+$ and a positive constant $k_i > 0$
- The objective function $v\left(x_{\mathcal{N}}\right)$ should be an indicator of $\mathcal{D}$ that is, $v\left(x_{\mathcal{N}}\right) \in \mathcal{V}_{ind}(\mathcal{D})$

The problem

The controller

$$c_i\left(x_{\mathcal{N}_i}^{[i]}(t)\right) = -k_i M_i^{-1}\frac{\partial v}{\partial x_i}\left(x_{\mathcal{N}}(t)\right)$$

Recall:
$x_j^{[i]} = (M_i, b_i)^{-1} \bullet x_j$ represents the relative position of neighbor $j \in \mathcal{N}_i$ for $(M_i, b_i) \in \mathcal{M} \ltimes \mathcal{B}$

The problem

The objective function for the controller should belong to $\mathcal{V}_{rel}(\mathcal{M} \ltimes \mathcal{B})$

$$\mathcal{V}_{rel}(\mathcal{M} \ltimes \mathcal{B}) = \{v(x_{\mathcal{N}}) \in \mathcal{V}_{c1} : \forall i \in \mathcal{N}, \exists \bar{c}_i : (\mathbb{R}^d)^n \rightarrow \mathbb{R}^d \\ s.t.((M_i, b_i)^{-1} \frac{\partial v}{\partial x_i}(x_{\mathcal{N}}) = \bar{c}_i \bullet x_{\mathcal{N}})\}$$

Recall:
$x_j^{[i]} = (M_i, b_i)^{-1} \bullet x_j$ represents the relative position of neighbor $j \in \mathcal{N}_i$ for $(M_i, b_i) \in \mathcal{M} \ltimes \mathcal{B}$