Monte Carlo Simulation of Photon Attenuation in Homogeneous Media

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Abstract

This report presents a Monte Carlo simulation framework for modeling the attenuation of monoenergetic photon radiation in homogeneous materials. The model is validated against analytical solutions and its statistical convergence properties are analyzed. All results are obtained from a modular and reproducible numerical implementation developed in Python.

1 Introduction

Monte Carlo methods are widely used in radiation transport simulations due to their flexibility and robustness in modeling stochastic processes. This work presents a minimal but physically consistent Monte Carlo implementation focused on photon attenuation problems with industrial relevance.

Special emphasis is placed on validation, convergence analysis, and software modularity, providing a solid foundation for future extensions towards more complex radiation transport models.

2 Physical Model

We consider monoenergetic photons propagating through a homogeneous material of thickness x. Photon interactions are modeled using an exponential free-path distribution characterized by the linear attenuation coefficient μ.

The analytical transmission probability is given by:

$$T(x) = e^{-\mu x} \tag{1}$$

This expression is used as a reference solution for validation purposes throughout this work.

3 Monte Carlo Method

The Monte Carlo transport algorithm is implemented in a dedicated simulation module developed in Python (`src/montecarlo.py`). This module contains the core routine responsible for generating particle histories and estimating statistical observables.

For each particle history, a random free path s is sampled as:

$$s = -\frac{1}{\mu} \ln r, \tag{2}$$

where r is a uniformly distributed random number in $(0, 1)$.

If $s < x$, an interaction is recorded and the particle energy is deposited within the material. Otherwise, the photon is transmitted through the material without interaction.

The Monte Carlo routine returns key quantities such as the transmitted fraction, absorbed fraction, and statistical uncertainty estimates. All results presented in this report are obtained by repeated sampling over N independent particle histories using this core implementation.

4 Validation

The Monte Carlo results are validated by direct comparison with the analytical solution for photon attenuation.

This validation procedure is implemented in a separate analysis module (`analysis/validation.py`), which executes the Monte Carlo simulation for a given number of histories and compares the estimated transmission probability with the analytical expression:

$$T(x) = e^{-\mu x}. \tag{3}$$

The relative error between numerical and analytical results is computed to assess the accuracy and statistical consistency of the Monte Carlo implementation. The separation between simulation and validation modules ensures clarity, reproducibility, and ease of extension.

5 Convergence Study

A convergence study is performed by increasing the number of Monte Carlo histories N over several orders of magnitude.

This analysis is carried out using a dedicated convergence module (`analysis/convergence.py`), which repeatedly executes the Monte Carlo simulation and evaluates the deviation from the analytical solution as a function of N.

The execution and visualization of the convergence analysis are performed using the script `analysis/run_convergence.py`, which generates the figure included in this report.

As shown in Figure 1, the numerical error decreases proportionally to $1/\sqrt{N}$, in agreement with the theoretical convergence properties of Monte Carlo methods.

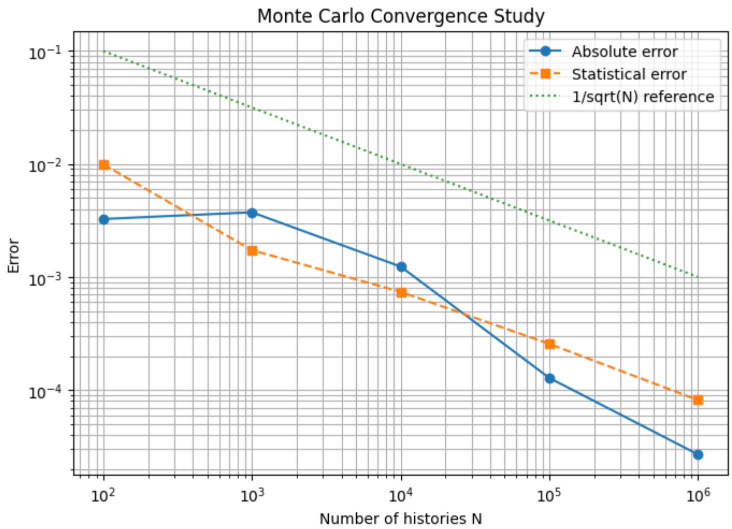

Figure 1: Monte Carlo convergence study showing the expected $1/\sqrt{N}$ error scaling.

6 Conclusions

A physically consistent and statistically validated Monte Carlo framework for photon attenuation has been developed. The implementation reproduces the analytical attenuation law and exhibits the expected Monte Carlo convergence behavior.

The modular structure of the numerical framework enables straightforward extension towards more advanced radiation transport models, including energy-dependent interactions and scattering processes, making it suitable as a foundation for industrial and research-oriented applications.

References

- J. Spanier and E. M. Gelbard, *Monte Carlo Principles and Neutron Transport Problems*
- I. Lux and L. Koblinger, *Monte Carlo Particle Transport Methods*

A Additional Implementations Available in the Repository

In addition to the baseline Monte Carlo implementation described in the main body of this report, the associated code repository includes further developments that extend the physical modeling capabilities of the simulation framework.

These additional implementations are not described in detail in this document in order to preserve clarity and focus, but are fully functional and available for inspection, execution, and extension.

A.1 Photon Transport with Scattering

The repository includes an extended Monte Carlo transport model that accounts for photon scattering processes in addition to absorption. This implementation allows photons to undergo multiple interactions within the material, resulting in more realistic transport behavior compared to the purely absorptive model described in the main text.

The scattering-capable transport algorithm is implemented in the module:

- `src/montecarlo_scattering.py`

This module extends the baseline transport logic by introducing particle directionality, multiple collision histories, and stochastic interaction selection between absorption and scattering events.

A.2 Compton Scattering with Klein–Nishina Sampling

To accurately model inelastic photon scattering, the repository further includes an implementation of Compton scattering based on the exact Klein–Nishina differential cross section.

The angular and energy distributions of scattered photons are sampled using a rejection-based Monte Carlo algorithm, following standard techniques used in radiation transport codes.

This functionality is implemented in the module:

- `src/klein_nishina.py`

The Klein–Nishina sampling routine is integrated into the scattering-enabled transport model, enabling energy-dependent photon transport and forward-peaked scattering behavior consistent with physical expectations.

A.3 Scope of the Present Document

The results and analyses presented in this report are intentionally limited to the baseline photon attenuation model without scattering, in order to:

- clearly demonstrate analytical validation,
- quantify Monte Carlo convergence properties,
- and establish a reliable reference implementation.

The scattering extensions described above represent a natural continuation of this work and are provided in the repository as an advanced feature set for future studies or application-specific developments.