

# W271-2 – Spring 2016 – HW 6

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## Exercises

## Warning: package 'knitr' was built under R version 3.1.3

### Exercise 1

- a. **Discuss the mean and variance functions and how the similarities and differences from those we studied in classical linear model.**

The mean function for a time series is defined by the function:

$$\mu_x(t) = E(x_t) = \int_{-\infty}^{+\infty} x_t f_t(x_t) dx_t$$

This function has a time component so the mean could be different in different time periods. This is different from a mean in classical linear models where the mean is constant.

The variance functions for a time series analysis is defined by the function:

$$\sigma_x^2(t) = E(x_t - \mu_x(t))^2 = \int_{-\infty}^{+\infty} (x_t - \mu_x(t))^2 f_t(x_t) dx_t$$

Again this function is time dependant which means it varies with time unlike the variance in a classical linear model.

...

- b. **Define strict and weak stationarity**

Stationarity indicates the parameter is consistent across time.

Strict stationarity is when the joint distributions  $F(x_{t_1}, \dots, x_{t_n})$  and  $F(x_{t_1+m}, \dots, x_{t_n+m})$  are the same implying that the distribution is unchanged for any time shift.

Weak stationarity (also called second-order stationary) is when its mean and variance are stationary and its autocovariance  $Cov(x_t, x_{t+k})$  depends on the time placement  $k$  and can be written as  $\gamma^{(k)}$ . Once a distribution assumption is imposed the series can be completely characterized by the mean and covariance.

...

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## Exercise 2

- a. Generate a zero-drift random walk model using 500 simulation.

```
# QUESTION 2 -----
x<-w<-rnorm(500)
for (t in 2:500) x[t] <- x[t-1] + w[t]

...
```

- b. Provide the descriptive statistics of the simulated realizations. The descriptive statistics should include the mean, standard deviation, 25th, 50th, and 75th quantiles, minimum, and maximum.

```
mean(x)

## [1] -13.07839

sd(x)

## [1] 9.252352

min(x)

## [1] -30.93562

max(x)

## [1] 6.59579

quantile(x, probs=c(.25, .5, .75))

##          25%          50%          75%
## -21.280157 -12.362306  -6.182762
```

- c. Plot the time-series plot of the simulated realizations.

```
plot(x, type= "l")

...
```

- d. Plot the autocorrelation graph.

```
acf(x, main="ACF Zero-drift random walk")

...
```

- e. Plot the partial autocorrelation graph.

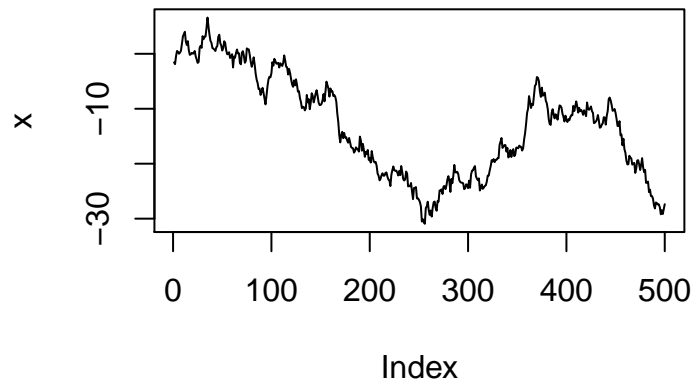


Figure 1:

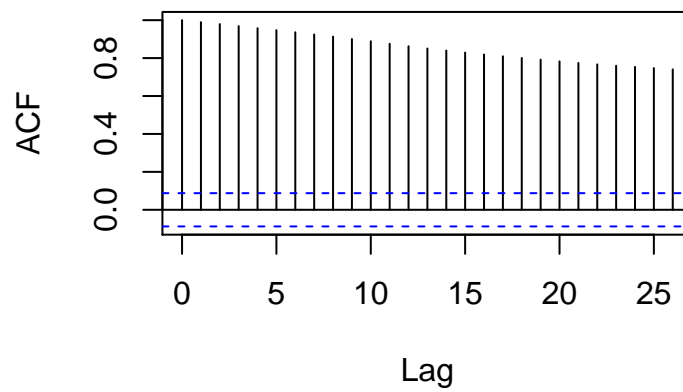
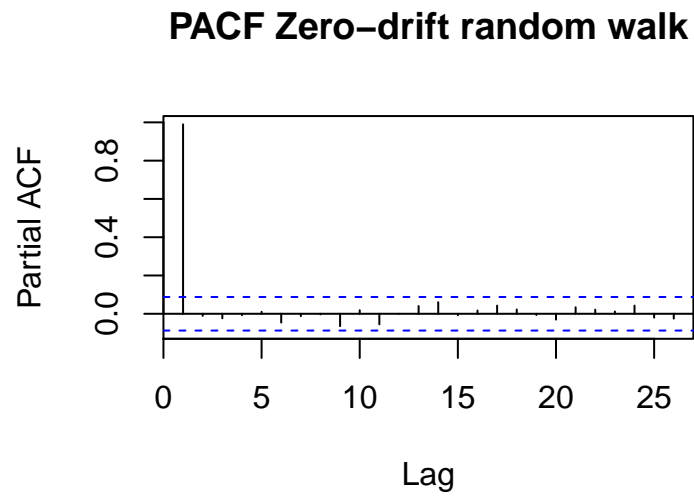
**ACF Zero-drift random walk**

Figure 2:

```
pacf(x, main = "PACF Zero-drift random walk")
```



...

### Exercise 3

- a. Generate arandom walk with drift model using 500 simulation, with the drift = 0.5.

```
x1<-w1<-rnorm(500)
d<-0.5
for (t in 2:500) x1[t] <- x1[t-1] + w1[t] + d
```

...

- b. Provide the descriptive statistics of the simulated realizations. The descriptive statistics should include the mean, standard deviation, 25th, 50th, and 75th quantiles, minimum, and maximum.

```
mean(x1)
```

```
## [1] 161.6304
```

```
sd(x1)
```

```
## [1] 87.48769
```

```
min(x1)
```

```
## [1] 0.1342579
```

```
max(x1)
```

```
## [1] 299.1543
```

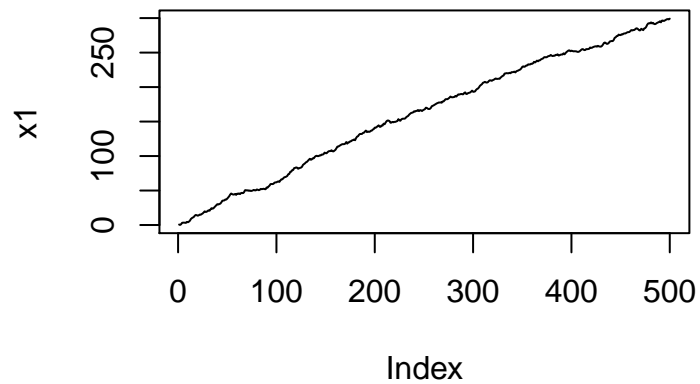
```
quantile(x1, probs=c(.25, .5, .75))
```

```
##      25%      50%      75%
## 84.18863 167.63741 243.74097
```

...

- c. Plot the time-series plot of the simulated realizations.

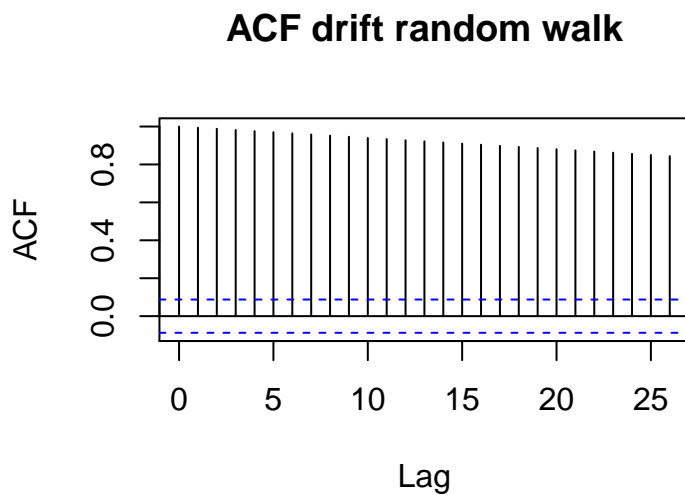
```
plot(x1, type= "l")
```



...

d. Plot the autocorrelation graph.

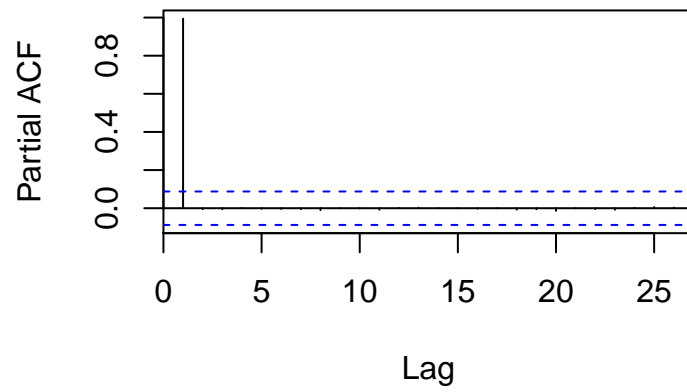
```
acf(x1, main="ACF drift random walk")
```



...

e. Plot the partial autocorrelation graph.

```
pacf(x1, main="PACF drift random walk")
```

**PACF drift random walk**

...



## Exercise 4

Use the series from INJCJC.csv.

- Load the data and examine the basic structure of the data using `str()`, `dim()`, `head()`, and `tail()` functions.

```
data<-read.csv("INJCJC.csv")
str(data)
```

```
## 'data.frame': 1300 obs. of 3 variables:
## $ Date : Factor w/ 1300 levels "1-Apr-05","1-Apr-11",...: 1102 143 442 784 483 1271 312 654 498 12
## $ INJCJC : int 355 369 375 345 368 367 348 350 351 349 ...
## $ INJCJC4: num 362 366 364 361 364 ...
```

```
dim(data)
```

```
## [1] 1300 3
```

```
head(data)
```

```
##      Date INJCJC INJCJC4
## 1 5-Jan-90   355   362.25
## 2 12-Jan-90   369   365.75
## 3 19-Jan-90   375   364.25
## 4 26-Jan-90   345   361.00
## 5 2-Feb-90    368   364.25
## 6 9-Feb-90    367   363.75
```

```
tail(data)
```

```
##      Date INJCJC INJCJC4
## 1295 24-Oct-14   288   281.25
## 1296 31-Oct-14   278   279.00
## 1297 7-Nov-14    293   285.75
## 1298 14-Nov-14   292   294.25
## 1299 21-Nov-14   314   294.25
## 1300 28-Nov-14   297   299.00
```

```
...
```

- Convert the variables INJCJC into a time series object `frequency=52`, `start=c(1990,1,1)`, `end=c(2014,11,28)`. Examine the converted data series.

```
data_ts<-ts(data$INJCJC, start=c(1990, 1, 1), end=c(2014, 11, 28), frequency=52)
length(data_ts)
```

```
## [1] 1259
```

```
head(data_ts)
```

```
## [1] 355 369 375 345 368 367
```

```
tail(data_ts)
```

```
## [1] 329 334 345 328 343 330
```

```
...
```

- c. Define a variable using the command `INJCJC.time<-time(INJCJC)`.

```
data_ts.time<-time(data_ts)
```

```
...
```

- d. Using the following command to examine the first 10 rows of the data. Change the parameter to examine different number of rows of data.

```
head(cbind(INJCJC.time, INJCJC),10)
```

```
head(cbind(data_ts.time, data_ts), 10)
```

```
##      data_ts.time data_ts
## [1,]      1990.000      355
## [2,]      1990.019      369
## [3,]      1990.038      375
## [4,]      1990.058      345
## [5,]      1990.077      368
## [6,]      1990.096      367
## [7,]      1990.115      348
## [8,]      1990.135      350
## [9,]      1990.154      351
## [10,]     1990.173      349
```

```
head(cbind(data_ts.time, data_ts), 13)
```

```
##      data_ts.time data_ts
## [1,]      1990.000      355
## [2,]      1990.019      369
## [3,]      1990.038      375
## [4,]      1990.058      345
## [5,]      1990.077      368
## [6,]      1990.096      367
## [7,]      1990.115      348
## [8,]      1990.135      350
## [9,]      1990.154      351
## [10,]     1990.173      349
## [11,]     1990.192      349
## [12,]     1990.212      331
## [13,]     1990.231      346
```

```
head(cbind(data_ts.time, data_ts), 6)
```

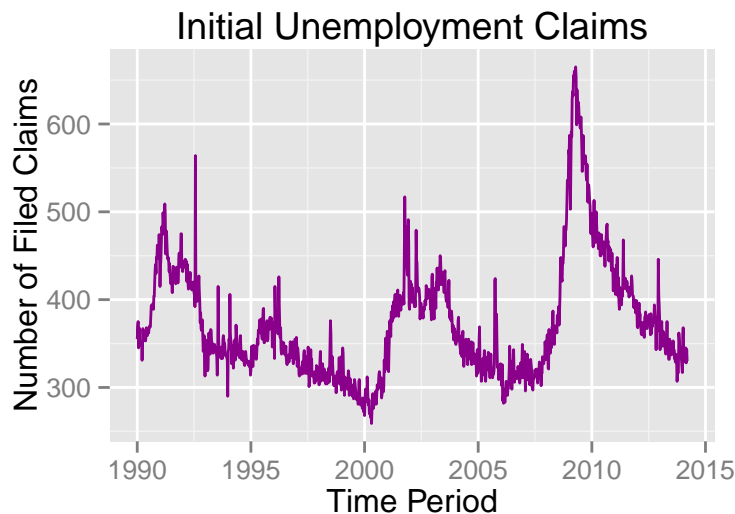
```
##      data_ts.time data_ts
## [1,]      1990.000      355
## [2,]      1990.019      369
## [3,]      1990.038      375
## [4,]      1990.058      345
## [5,]      1990.077      368
## [6,]      1990.096      367
```

...

e.

1. Plot the time series plot of INJCJC. Remember that the graph must be well labelled.

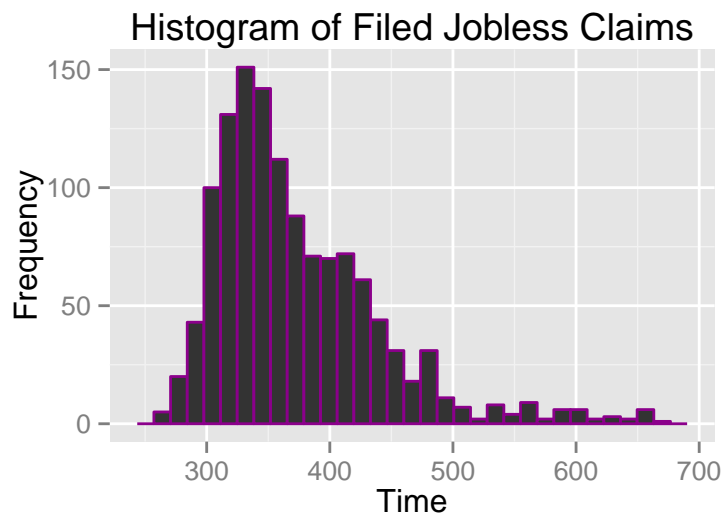
```
autoplot(data_ts , xlab = "Time Period", ylab = "Number of Filed Claims",
          main="Initial Unemployment Claims", ts.colour = 'magenta4' )
```



...

2. Plot the histogram of INJCJC. What is shown and not shown in a histogram?  
How do you decide the number of bins used?

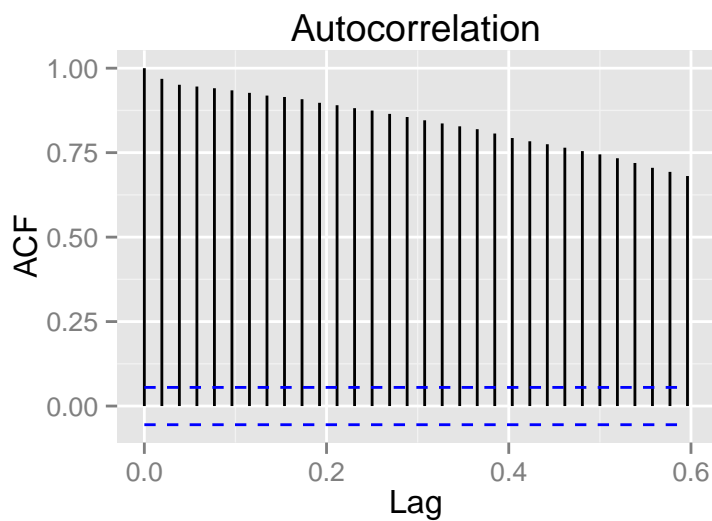
```
qplot(data_ts , geom="histogram", main='Histogram of Filed Jobless Claims',
       ylab='Frequency', xlab='Time', colour = I('magenta4'))
```



...

3. Plot the autocorrelation graph of INJCJC series.

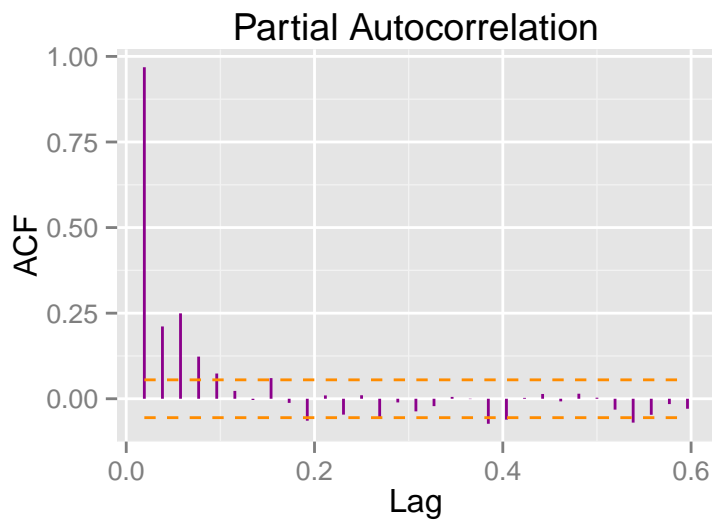
```
autoplot(acf(data_ts, plot = FALSE), xlab = "Lag", main="Autocorrelation",
         ts.colour = 'magenta4')
```



...

4. Plot the partial autocorrelation graph of INJCJC series.

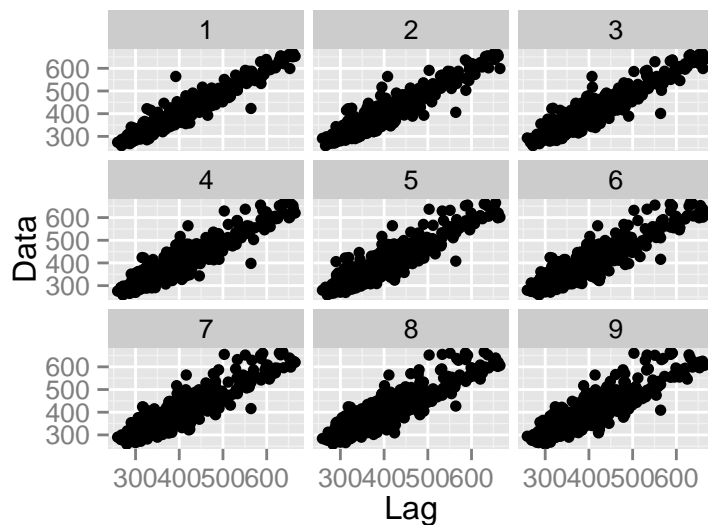
```
autoplot(pacf(data_ts, plot = FALSE), xlab = "Lag", main="Partial Autocorrelation",
         colour = "magenta4", conf.int.colour = "darkorange")
```



...

5. Plot a 3x3 Scatterplot Matrix of correlation against lag values.

```
gglagplot(data_ts, lags = 9, nrow = 3, ncol = 3)
```



...

f.

1. Generate two symmetric Moving Average Smoothers. Choose the number of moving average terms such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

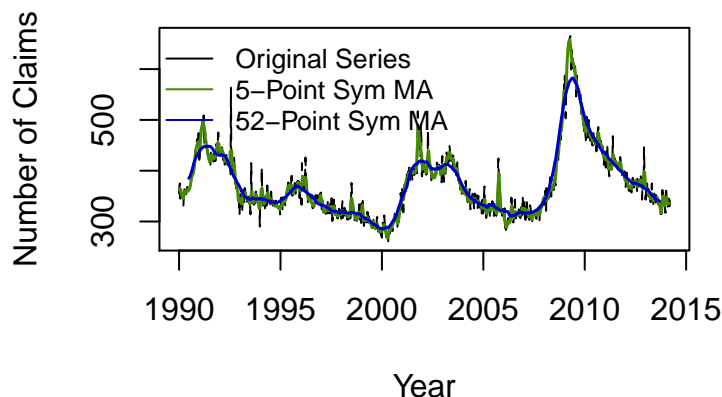
```
data_ts_rep5 = filter(data_ts, sides=2, rep(1,5)/5)
data_ts_rep52 = filter(data_ts, sides=2, rep(1,52)/52)
plot(data_ts, main="Moving Average Smoothing",
```

```

pch=4, lty=5, lwd=1, xlab="Year",
ylab="Number of Claims")
lines(data_ts_rep5, lty=1, lwd=1.5, col="chartreuse4")
lines(data_ts_rep52, lty=1, lwd=1.5, col="mediumblue")
# Add Legend
leg.txt <- c("Original Series", "5-Point Sym MA", "52-Point Sym MA")
legend("topleft", legend=leg.txt, lty=c(1,1,1), col=c("black","chartreuse4","mediumblue"),
      bty='n', cex=0.8, merge = TRUE, bg=336)

```

## Moving Average Smoothing



...

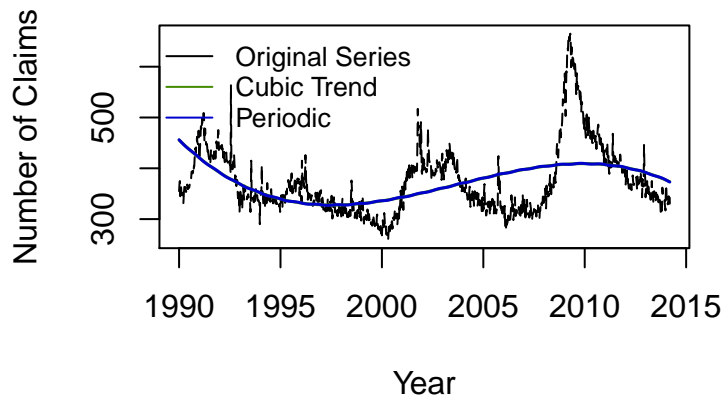
2. Generate two regression smoothers, one being a cubic trend regression and the other being a periodic regression. Plot the smoothers and the original series in one graph.

```

wk = time(data_ts) - mean(time(data_ts))
wk2 = wk^2
wk3 = wk^3
cs = cos(2*pi*wk)
sn = sin(2*pi*wk)
reg1 = lm(data_ts~wk + wk2 + wk3, na.action=NULL)
reg2 = lm(data_ts~wk + wk2 + wk3 + cs + sn, na.action=NULL)
plot(data_ts, main="Regression & Periodic Smoothing",
     pch=4, lty=5, lwd=1, xlab="Year",
     ylab="Number of Claims")
lines(fitted(reg1), lty=1, lwd=1.5, col="chartreuse4")
lines(fitted(reg2), lty=1, lwd=1.5, col="mediumblue")
# Add Legend
leg.txt <- c("Original Series", "Cubic Trend", "Periodic")
legend("topleft", legend=leg.txt, lty=c(1,1,1), col=c("black","chartreuse4","mediumblue"),
      bty='n', cex=0.8, merge = TRUE, bg=336)

```

## Regression & Periodic Smoothing

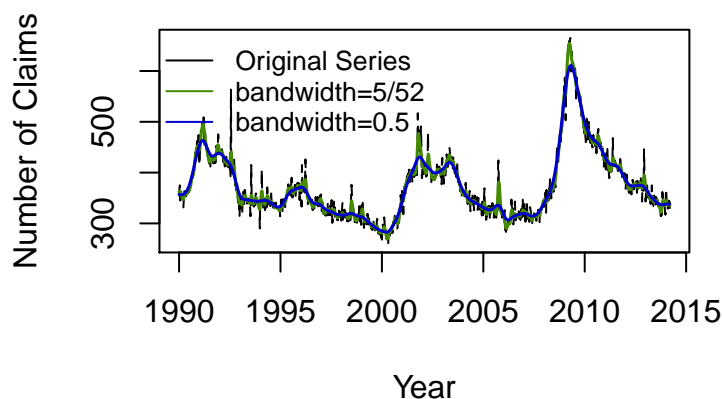


...

3. Generate kernel smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

```
plot(data_ts, main="Kernel Smoothing",
     pch=4, lty=5, lwd=1, xlab="Year",
     ylab="Number of Claims")
lines(ksmooth(time(data_ts), data_ts, "normal", bandwidth=5/52), lty=1, lwd=1.5, col="chartreuse4")
lines(ksmooth(time(data_ts), data_ts, "normal", bandwidth=0.5), lty=1, lwd=1.5, col="mediumblue")
# Add Legend
leg.txt <- c("Original Series", "bandwidth=5/52", "bandwidth=0.5")
legend("topleft", legend=leg.txt, lty=c(1,1,1), col=c("black","chartreuse4","mediumblue"),
      bty='n', cex=0.8, merge = TRUE, bg=336)
```

## Kernel Smoothing



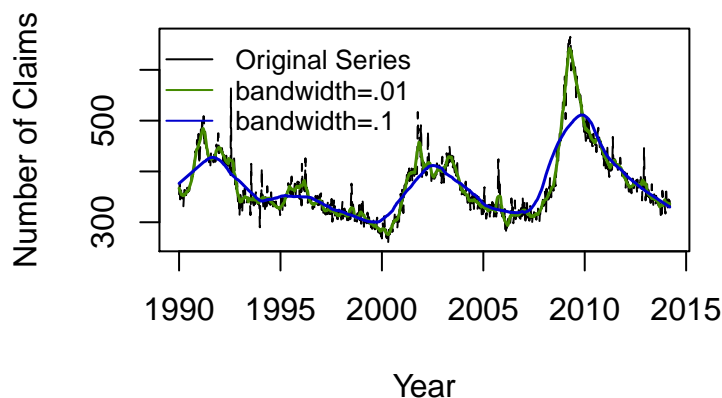
...

4. Generate two nearest neighborhood smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one

can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

```
plot(data_ts, main="Nearest Neighborhood Smoothing",
     pch=4, lty=5, lwd=1, xlab="Year",
     ylab="Number of Claims")
lines(supsmu(time(data_ts), data_ts, span=.01), lty=1, lwd=1.5, col="chartreuse4")
lines(supsmu(time(data_ts), data_ts, span=.1), lty=1, lwd=1.5, col="mediumblue")
# Add Legend
leg.txt <- c("Original Series", "bandwidth=.01", "bandwidth=.1")
legend("topleft", legend=leg.txt, lty=c(1,1,1), col=c("black", "chartreuse4", "mediumblue"),
      bty='n', cex=0.8, merge = TRUE, bg=336)
```

## Nearest Neighborhood Smoothing



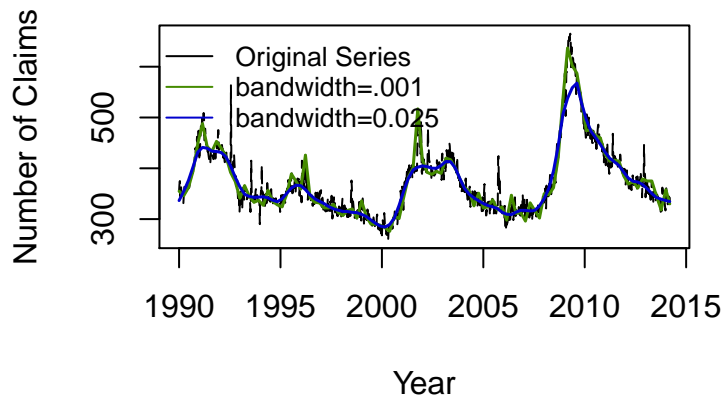
...

5. Generate two LOWESS smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

```
plot(data_ts, main="LOWESS Smoothing",
     pch=4, lty=5, lwd=1, xlab="Year",
     ylab="Number of Claims")
lines(lowess(data_ts, f=.001), lty=1, lwd=1.5, col="chartreuse4")
lines(lowess(data_ts, f=0.05), lty=1, lwd=1.5, col="mediumblue")
# Add Legend
leg.txt <- c("Original Series", "bandwidth=.001", "bandwidth=0.025")
legend("topleft", legend=leg.txt, lty=c(1,1,1), col=c("black", "chartreuse4", "mediumblue"),
      bty='n', cex=0.8, merge = TRUE, bg=336)
```



## LOWESS Smoothing

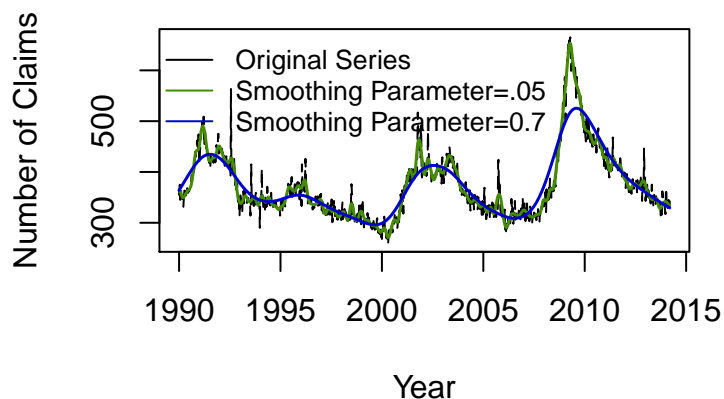


...

6. Generate two spline smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

```
plot(data_ts, main="Spline Smoothing",
     pch=4, lty=5, lwd=1, xlab="Year",
     ylab="Number of Claims")
lines(smooth.spline(time(data_ts), data_ts, spar=0.05), lty=1, lwd=1.5, col="chartreuse4")
lines(smooth.spline(time(data_ts), data_ts, spar=0.75), lty=1, lwd=1.5, col="mediumblue")
# Add Legend
leg.txt <- c("Original Series", "Smoothing Parameter=.05", "Smoothing Parameter=0.7")
legend("topleft", legend=leg.txt, lty=c(1,1,1), col=c("black","chartreuse4","mediumblue"),
      bty='n', cex=0.8, merge = TRUE, bg=336)
```

## Spline Smoothing



...