W271-2 - Spring 2016 - HW 6

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Exercises

Exercise 1

a. Discuss the mean and variance functions and how the similarities and differences from those we studied in classical linear model.

. .

b. Define strict and weak statonarity

. . .

Exercise 2

a. Generate a zero-drift random walk model using 500 simulation.

```
set.seed(123)
N <- 500 # number of simulations / time periods
wn <- rnorm(n = N, mean = 0, sd = 1) # white noise (can use any mean and sd)
rw <- cumsum(wn)</pre>
```

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b. Provide the descriptive statistics of the simulated realizations. The descriptive statistics should include the mean, standard deviation, 25th, 50th, and 75th quantiles, minimum, and maximum.

Table 1: Descriptive statistics of the simulated random walk

	Random walk
Mean	4.58
St. Dev	4.87
1st Quartile	0.97
Median	3.42
3rd Quartile	8.27
Min	-5.16
Max	19.35

c. Plot the time-series plot of the simulated realizations.

See the last part of this Exercise in the following page.

d. Plot the autocorrelation graph.

See the last part of this Exercise in the following page.

e. Plot the partial autocorrelation graph.

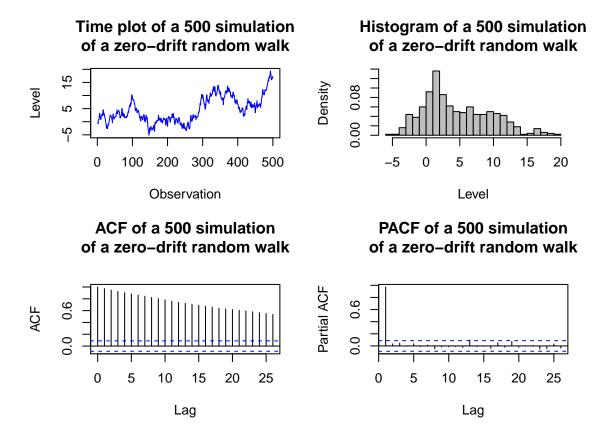


Figure 1: Time-series plot, histogram, correlogram, and partial autocorrelogram of the 500 simulation of a zero-drift random walk

Exercise 3

a. Generate a random walk with drift model using 500 simulation, with the drift = 0.5.

```
drift <- 0.5 # drift
# Use the same GWN that genereate the prev. zero-drift RW
# set.seed(123); wn <- rnorm(n = N, mean = 0, sd = 1)
rw_drift <- cumsum(wn + drift)</pre>
```

b. Provide the descriptive statistics of the simulated realizations. The descriptive statistics should include the mean, standard deviation, 25th, 50th, and 75th quantiles, minimum, and maximum.

Table 2: Descriptive statistics of the two simulated random walks

	Random walk	Random walk with 0.5 drift
Mean	4.58	129.83
St. Dev	4.87	75.43
1st Quartile	0.97	64.58
Median	3.42	122.91
3rd Quartile	8.27	199.68
Min	-5.16	-0.06
Max	19.35	267.30

c. Plot the time-series plot of the simulated realizations.

See the last part of this Exercise in the following page.

d. Plot the autocorrelation graph.

See the last part of this Exercise in the following page.

e. Plot the partial autocorrelation graph.

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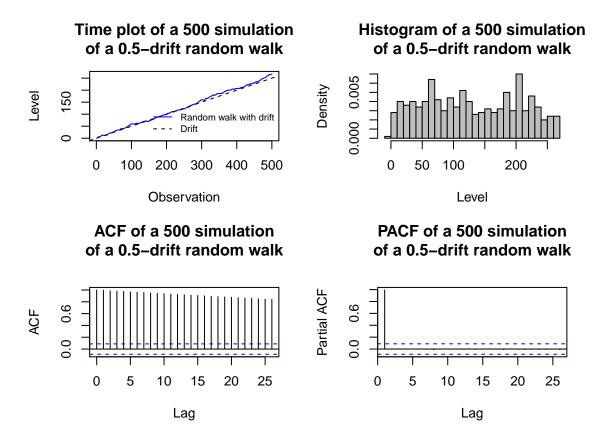


Figure 2: Time-series plot, histogram, correlogram, and partial autocorrelogram of the 500 simulation of a 0.5-drift random walk

Exercise 4

Use the series from INJCJC.csv.

a. Load the data and examine the basic structure of the data using str(), dim(), head(), and tail() functions.

```
INJCJC_df <- read.csv('INJCJC.csv', header = TRUE)</pre>
str(INJCJC_df)
## 'data.frame':
                    1300 obs. of 3 variables:
\#\# $ Date : Factor \#\# 1300 levels "10-Apr-09","10-Apr-92",...: 1102 101 400 741 907 1271 270 611 922
## $ INJCJC : int 355 369 375 345 368 367 348 350 351 349 ...
## $ INJCJC4: num 362 366 364 361 364 ...
dim(INJCJC_df); obs <- dim(INJCJC_df)[1]</pre>
## [1] 1300
head(INJCJC_df)
##
          Date INJCJC INJCJC4
## 1 5-Jan-90
                  355 362.25
## 2 12-Jan-90
                  369 365.75
## 3 19-Jan-90
                  375 364.25
## 4 26-Jan-90
                  345 361.00
## 5 2-Feb-90
                  368 364.25
## 6 9-Feb-90
                  367 363.75
tail(INJCJC_df)
             Date INJCJC INJCJC4
## 1295 24-Oct-14
                     288 281.25
## 1296 31-Oct-14
                     278 279.00
## 1297 7-Nov-14
                     293 285.75
## 1298 14-Nov-14
                     292 294.25
## 1299 21-Nov-14
                     314 294.25
## 1300 28-Nov-14
                     297 299.00
```

Table 3: Descriptive statistics of the INJCJC variables

	INJCJC	INJCJC4
Mean	371.14	371.24
St. Dev	67.38	66.30
1st Quartile	324.00	324.69
Median	353.50	352.12
3rd Quartile	406.00	405.75
Min	259.00	266.25
Max	665.00	659.25

The 1300 observations (of two variables, INJCJC and INJCJC4) correspond to 1,300/52 = 25 periods of 52 weeks, i.e., almost 25 years from January 5, 1990, until November 28, 2014.

All observations correspond to Fridays.

```
levels(as.factor(weekdays(as.Date(as.character(INJCJC_df$Date), '%d-%b-%y'))))
```

```
## [1] "Friday"
```

Since years are slightly longer (by 1 or 2 days) than 52 weeks, some years have 53 Fridays. Those years with 53 Fridays sum up, and as a result (and since the number of observations we have is a multiple of 52) the last year in the sample (2014) does not include all of the Fridays there were that year (there were 1,304 Fridays in that 25-year period from 1990 to 2014).

```
INJCJC_df %>%
  mutate(Date = as.Date(as.character(Date), '%d-%b-%y')) %>%
  mutate(Year = year(Date)) %>%
  group_by(Year) %>%
  summarise(obs = n(), start_date = min(Date), end_date = max(Date)) %>%
  print(n=Inf)
```

```
## Source: local data frame [25 x 4]
##
##
       Year
              obs start_date
                                end_date
##
      (dbl)
            (int)
                       (date)
                                  (date)
## 1
       1990
               52 1990-01-05 1990-12-28
## 2
       1991
               52 1991-01-04 1991-12-27
       1992
## 3
               52 1992-01-03 1992-12-25
## 4
       1993
               53 1993-01-01 1993-12-31
## 5
       1994
               52 1994-01-07 1994-12-30
## 6
       1995
               52 1995-01-06 1995-12-29
## 7
       1996
               52 1996-01-05 1996-12-27
## 8
       1997
               52 1997-01-03 1997-12-26
## 9
               52 1998-01-02 1998-12-25
       1998
## 10
       1999
               53 1999-01-01 1999-12-31
## 11
       2000
               52 2000-01-07 2000-12-29
## 12
       2001
               52 2001-01-05 2001-12-28
       2002
               52 2002-01-04 2002-12-27
## 13
##
  14
       2003
               52 2003-01-03 2003-12-26
## 15
       2004
               53 2004-01-02 2004-12-31
## 16
       2005
               52 2005-01-07 2005-12-30
## 17
       2006
               52 2006-01-06 2006-12-29
## 18
       2007
               52 2007-01-05 2007-12-28
## 19
       2008
               52 2008-01-04 2008-12-26
       2009
               52 2009-01-02 2009-12-25
## 20
## 21
       2010
               53 2010-01-01 2010-12-31
## 22
       2011
               52 2011-01-07 2011-12-30
## 23
       2012
               52 2012-01-06 2012-12-28
## 24
       2013
               52 2013-01-04 2013-12-27
## 25
       2014
               48 2014-01-03 2014-11-28
```

```
# Count weeks
sum(sapply(1990:2014, function(y)
  ifelse(((y %% 4 == 0) & (y %% 100 != 0)) | (y %% 400 == 0), 366, 365))) / 7

## [1] 1304.429

# Count Fridays in that period
ceiling(as.numeric(as.Date('2014-12-31') + 1 - 5 + 4) / 7) -
ceiling(as.numeric(as.Date('1990-01-01') - 5 + 4) / 7)
```

[1] 1304

Another way to see it: because some years had more than 52 Fridays, each set of 52 observations gradually ends earlier (see the even rows below, that correspond to week == 52: December 28, December 27, December 25, December 24...). Hence, the next set of 52 observations start earlier, even in the previous year (see row 8), and the aggregate effect is that the 25th set of 52 observations—which should correspond to 2014—start as early as December 6, 2013, and ends in November 28, 2014.

```
##
            Date INJCJC INJCJC4 week_num week
## 1
       5-Jan-90
                     355
                          362.25
                                               1
                                         1
## 2
      28-Dec-90
                     454
                          456.00
                                        52
                                              52
                                        53
## 3
       4-Jan-91
                     415
                          447.50
                                               1
## 4
      27-Dec-91
                     441
                          456.75
                                       104
                                              52
                                       105
## 5
       3-Jan-92
                     432
                          446.00
                                               1
      25-Dec-92
                                              52
## 6
                     313
                          339.00
                                       156
## 7
       1-Jan-93
                     341
                          336.75
                                       157
                                               1
## 8
      24-Dec-93
                     290
                          323.00
                                       208
                                              52
      31-Dec-93
                          324.00
## 9
                     341
                                       209
                                               1
## 10 23-Dec-94
                     314
                          324.25
                                       260
                                              52
## 11 30-Dec-94
                                       261
                     319
                          323.00
                                               1
## 12 22-Dec-95
                     374
                          366.50
                                       312
                                              52
## 13 29-Dec-95
                     359
                          363.00
                                       313
                                               1
## 14 20-Dec-96
                     350
                          347.25
                                       364
                                              52
## 15 27-Dec-96
                     357
                          353.50
                                       365
                                               1
## 16 19-Dec-97
                          317.00
                                       416
                                              52
                     310
## 17 26-Dec-97
                     303
                          313.25
                                       417
                                               1
## 18 18-Dec-98
                     297
                          309.50
                                       468
                                              52
## 19 25-Dec-98
                     336
                          316.00
                                       469
                                               1
                                       520
                                              52
## 20 17-Dec-99
                     287
                          283.50
## 21 24-Dec-99
                     268
                          278.50
                                       521
                                               1
## 22 15-Dec-00
                          342.25
                                       572
                                              52
                     354
## 23 22-Dec-00
                          344.25
                     364
                                       573
                                               1
## 24 14-Dec-01
                     389
                          434.50
                                       624
                                              52
## 25 21-Dec-01
                          415.75
                                       625
                     416
                                               1
## 26 13-Dec-02
                     429
                          405.25
                                       676
                                              52
## 27 20-Dec-02
                     394
                          406.25
                                       677
                                               1
## 28 12-Dec-03
                     363
                                       728
                                              52
                          360.25
```

```
## 29 19-Dec-03
                     354
                          360.25
                                       729
                                               1
## 30 10-Dec-04
                                              52
                     316
                          326.75
                                       780
## 31 17-Dec-04
                     322
                          329.00
                                       781
                                               1
## 32
       9-Dec-05
                          320.75
                                       832
                                              52
                     327
## 33 16-Dec-05
                     312
                          317.75
                                       833
                                               1
## 34
       8-Dec-06
                     311
                          328.25
                                       884
                                              52
## 35 15-Dec-06
                     318
                          326.25
                                       885
                                               1
## 36
       7-Dec-07
                     332
                          340.00
                                       936
                                              52
## 37 14-Dec-07
                     350
                          344.50
                                       937
                                               1
## 38
       5-Dec-08
                     570
                          541.75
                                       988
                                              52
## 39 12-Dec-08
                     566
                          549.25
                                       989
                                               1
                                              52
## 40
       4-Dec-09
                     497
                          490.25
                                      1040
## 41 11-Dec-09
                     498
                          488.00
                                      1041
                                               1
                                      1092
## 42
       3-Dec-10
                     431
                          427.50
                                              52
                          426.00
                                      1093
## 43 10-Dec-10
                     428
                                               1
## 44
       2-Dec-11
                     390
                          390.25
                                      1144
                                              52
## 45
       9-Dec-11
                     369
                          386.25
                                      1145
                                               1
## 46 30-Nov-12
                     379
                          406.25
                                      1196
                                              52
                                      1197
## 47
       7-Dec-12
                          380.50
                                               1
                     343
                    317
## 48 29-Nov-13
                          328.75
                                      1248
                                              52
## 49
       6-Dec-13
                     358
                          332.75
                                      1249
                                               1
## 50 28-Nov-14
                          299.00
                                      1300
                                              52
                     297
```

b. Convert the variables INJCJC into a time series object frequency=52, start=c(1990,1,1), end=c(2014,11,28). Examine the converted data series.

The above parameters may be wrong: according to the ts documentation, start and end have to be "Either a single number or a vector of two integers, which specify a natural time unit and a (1-based) number of samples into the time unit," i.e., and not a date in y-m-d format. If we use the parameters in the instructions, R discards 28 and takes 11 observations from 2014, not the 52 that there really are.

[1] 1259

```
tail(INJCJC_wrong, 10)
```

[1] 368 339 344 333 329 334 345 328 343 330

```
tail(INJCJC_df$INJCJC[1:length(INJCJC_wrong)], 10)
```

[1] 368 339 344 333 329 334 345 328 343 330

We can plot both the original vector and the time series object to confirm this.

U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

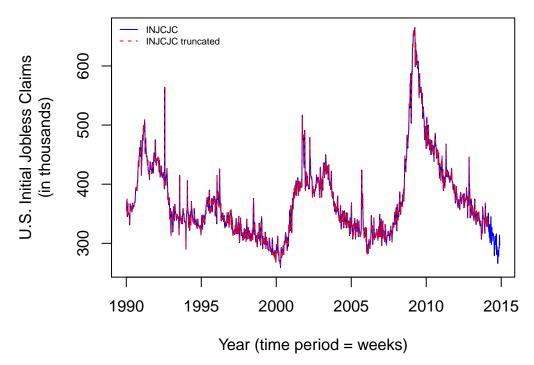


Figure 3: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014

Hence, we slightly change the R command to get the proper time series object:

[1] 1300

The way ts works, our time scale does not correspond to the original dates (every Friday between the start and end date, including both) because we've set a fixed frequency of 52 (instead of 365/7 = 52.143 if the year is not leap, and 366/7 = 52.286 if it is) and the 1st time period is set to the very beginning of the year. Compare the original dates in our dataset with the dates ts sets, for the 1st and 52nd observation (while the day and month of the 1st and 52nd observations of each of the 25 subsets will vary, as we have seen, the day and month—and even time—will be fixed in the ts object, regardless of the year); the date of the last observation in the "wrong/truncated version" of INJCJC is also shown.:

This does not happen with montly or quarterly data (and happens to a lesser extent with daily data).

One way to preserve the sampling dates is by using the xts library:

```
## [,1]

## 1990-01-05 355

## 1990-01-12 369

## 1990-01-19 375

## 1990-01-26 345

## 1990-02-02 368

## 1990-02-09 367
```

U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

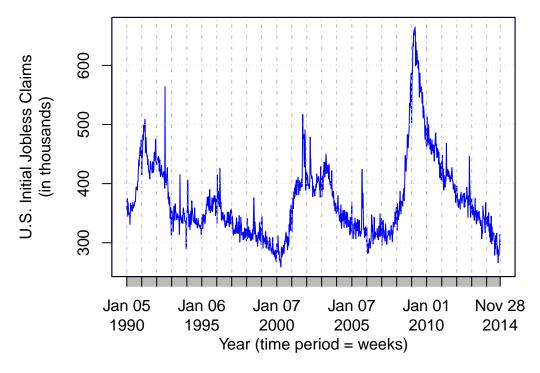


Figure 4: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014, using the xts package

c. Define a variable using the command INJCJC.time<-time(INJCJC).

```
INJCJC.time <- time(INJCJC)</pre>
head(INJCJC.time)
## [1] 1990.000 1990.019 1990.038 1990.058 1990.077 1990.096
tail(INJCJC.time)
## [1] 2014.885 2014.904 2014.923 2014.942 2014.962 2014.981
tail(time(INJCJC_wrong))
## [1] 2014.096 2014.115 2014.135 2014.154 2014.173 2014.192
  d. Using the following command to examine the first 10 rows of the data. Change the
     parameter to examine different number of rows of data.
     head(cbind(INJCJC.time, INJCJC),10)
head(cbind(INJCJC.time, INJCJC), 10)
##
         INJCJC.time INJCJC
    [1,]
                         355
##
            1990.000
##
   [2,]
            1990.019
                         369
                         375
##
  [3,]
            1990.038
## [4,]
            1990.058
                         345
##
   [5,]
            1990.077
                         368
## [6,]
            1990.096
                         367
## [7,]
            1990.115
                         348
## [8,]
            1990.135
                        350
## [9,]
            1990.154
                         351
## [10,]
                        349
            1990.173
head(cbind(INJCJC.time, INJCJC)) # default: 6
##
        INJCJC.time INJCJC
           1990.000
                       355
## [1,]
## [2,]
           1990.019
                       369
## [3,]
           1990.038
                       375
## [4,]
           1990.058
                       345
## [5,]
           1990.077
                       368
## [6,]
           1990.096
                       367
head(cbind(INJCJC.time, INJCJC), -(length(INJCJC)-6)) # -1294: equivalent
##
        INJCJC.time INJCJC
## [1,]
           1990.000
                       355
## [2,]
           1990.019
                       369
## [3,]
           1990.038
                       375
## [4,]
           1990.058
                       345
## [5,]
           1990.077
                       368
                       367
## [6,]
           1990.096
```

head(cbind(INJCJC.time, INJCJC), 13) # approximately 3 months (1 quarter)

```
##
         INJCJC.time INJCJC
##
   [1,]
            1990.000
                         355
   [2,]
                         369
##
            1990.019
##
   [3,]
            1990.038
                         375
##
   [4,]
            1990.058
                         345
##
   [5,]
            1990.077
                         368
##
   [6,]
            1990.096
                         367
##
   [7,]
            1990.115
                         348
##
   [8,]
            1990.135
                         350
## [9,]
            1990.154
                         351
## [10,]
            1990.173
                         349
## [11,]
            1990.192
                         349
## [12,]
            1990.212
                         331
## [13,]
            1990.231
                         346
```

e.

1. Plot the time series plot of INJCJC. Remember that the graph must be well labelled.

U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

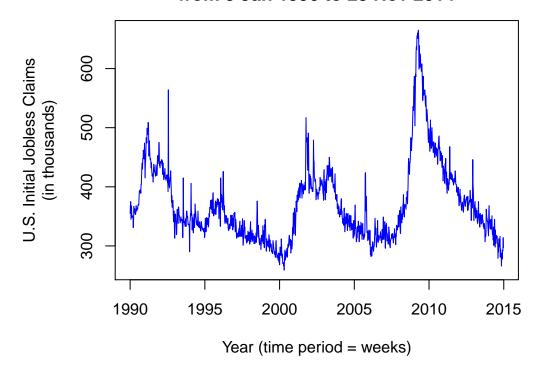
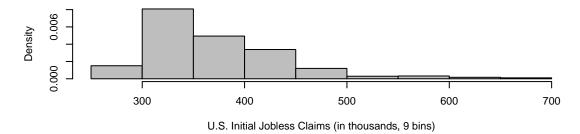


Figure 5: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014

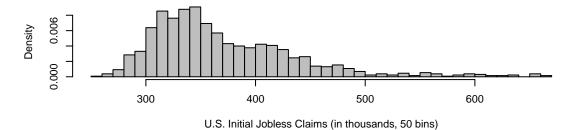
Compare the previous Figure, plotted using xts, with the one above: the former shows the proper time scale (starting in January 5, 1990, and ending in November 28, 2014), while the latter covers the full 25-year period.

2. Plot the histogram of INJCJC. What is shown and not shown in a histogram? How do you decide the number of bins used?

Histogram of the U.S. Initial Jobless Claimsfrom 5 Jan 1990 to 28 Nov 2014



Histogram of the U.S. Initial Jobless Claimsfrom 5 Jan 1990 to 28 Nov 2014



Histogram of the U.S. Initial Jobless Claimsfrom 5 Jan 1990 to 28 Nov 2014

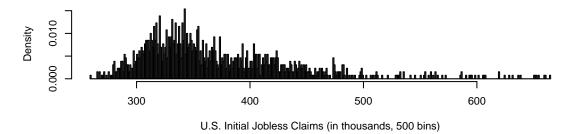


Figure 6: Histogram of the U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014, using different bins

The histograms tells us nothing about the dynamics of the series; e.g., it lets us know that there were a few time periods where the number of Initial Jobless Claims was greater than 600 (thousand), but not when that happened (in the 1st half of 2009).

A good number of bins is one that neither *oversimplifies* the (sample) distribution (i.e., it is not so low that some modes may be hidden) nor shows too much detail that is due to sampling and makes the underlying (real) distribution much more complex than it really is (i.e., it is so high that a low of modes, just due to sampling, are shown). Hence, the best representation in the previous Figure would be the one in the midle.

3. Plot the autocorrelation graph of INJCJC series.

We plot the first 52 correlations (apart from ρ_0) just to check all possible seasonality components (incl. annual).

```
acf(INJCJC, lag.max = 52,
    main="ACF of U.S. Initial Jobless Claims\nfrom 5 Jan 1990 to 28 Nov 2014")
```

ACF of U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

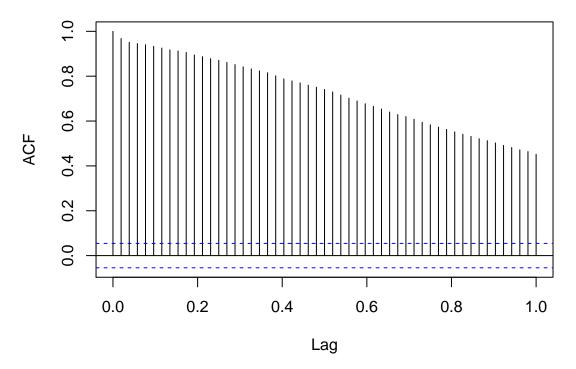


Figure 7: Partial autocorrelation graph of U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

The autocorrleation decreases exponentially, but very slowly (this is typical of an AR(1) model with a coefficient very close to 1). Nonetheless, we must note that the time series has neither been seasonally nor detrended.

4. Plot the partial autocorrelation graph of INJCJC series.

```
pacf(INJCJC, lag.max = 52,
    main="PACF of U.S. Initial Jobless Claims\nfrom 5 Jan 1990 to 28 Nov 2014")
```

PACF of U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

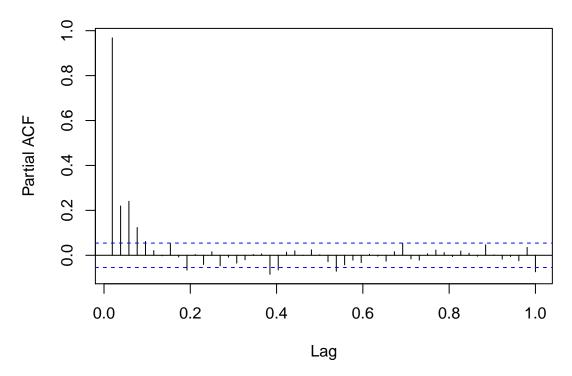


Figure 8: Partial autocorrelation graph of U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

The partial autocorrelation plot makes it evident that this time series was not generated by an AR(1) model. After controlling for the effect of the process at lags 1 and 2, the partial correlation at lags 2 and 3, respectively, is still significant.

5. Plot a 3x3 Scatterplot Matrix of correlation against lag values.

Autocorr. between the INJCJC time series and its Own Lags

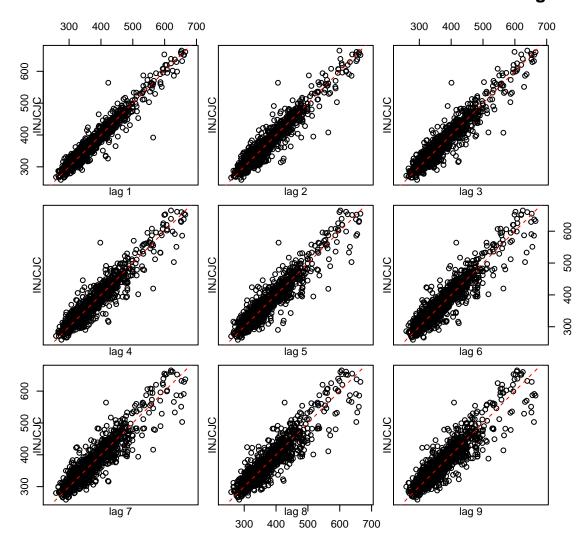


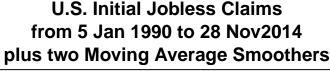
Figure 9: Scatterplot matrix of the correlation of the U.S. Initial Jobless Claims time series against its first 9 own lags

As the correlogram in Figue 7 suggested, the correlation between the time series and its lagged version is very high, and that correlation decreases very slowly with the lag.

f.

1. Generate two symmetric Moving Average Smoothers. Choose the number of moving average terms such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

```
INJCJC.1 = stats::filter(INJCJC, sides=2, rep(1, 5)/5)
INJCJC.2 = stats::filter(INJCJC, sides=2, rep(1, 53)/53)
```



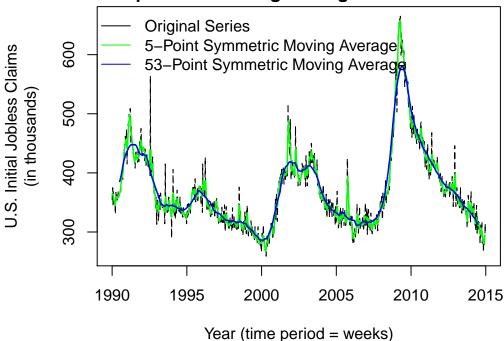


Figure 10: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014 (original series plus two Moving Average Smoothers)

2. Generate two regression smoothers, one being a cubic trend regression and the other being a periodic regression. Plot the smoothers and the original series in one graph.

```
wk = INJCJC.time - mean(INJCJC.time)
wk2 = wk^2
wk3 = wk^3
cs = cos(2 * pi * wk)
sn = sin(2 * pi * wk)
reg1 = lm(INJCJC ~ wk + wk2 + wk3, na.action = NULL)
reg2 = lm(INJCJC ~ wk + wk2 + wk3 + cs + sn, na.action = NULL)
```

U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov2014 plus two Regression Smoothers

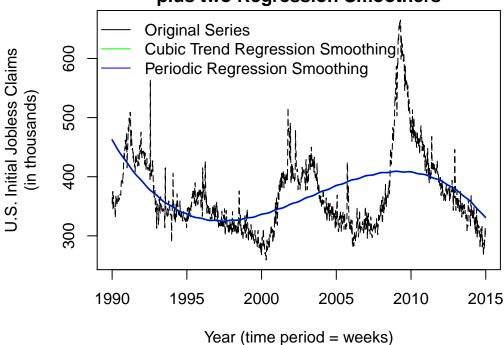


Figure 11: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014 (original series plus two Regression Smoothers)

3. Generate kernel smoothers. Choose the smoothing parametrs such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

```
INJCJC.1 <- ksmooth(INJCJC.time, INJCJC, "normal", bandwidth = 5/52)
INJCJC.2 <- ksmooth(INJCJC.time, INJCJC, "normal", bandwidth = 2)</pre>
```

U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov2014 plus two Kernel Smoothers

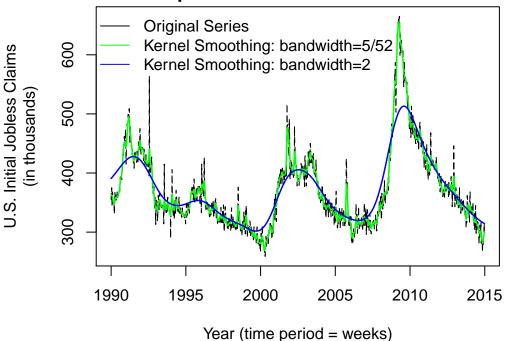


Figure 12: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014 (original series plus two Regression Smoothers)

4. Generate two nearest neighborhood smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

```
INJCJC.1 <- supsmu(INJCJC.time, INJCJC, span = .01)
INJCJC.2 <- supsmu(INJCJC.time, INJCJC, span = .1)</pre>
```

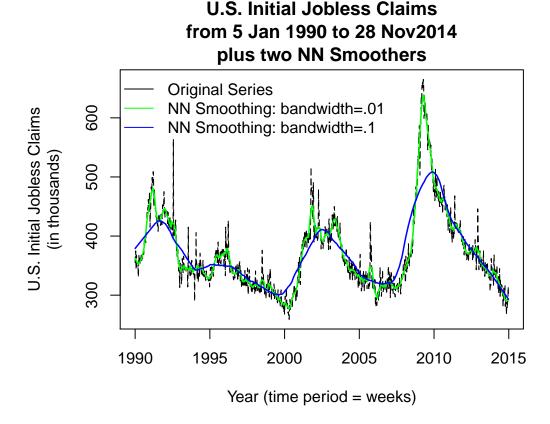


Figure 13: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014 (original series plus two Nearest Neighborhood Smoothers)

5. Generate two LOWESS smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

```
INJCJC.1 <- lowess(INJCJC, f = .02)
INJCJC.2 <- lowess(INJCJC, f = .2)</pre>
```

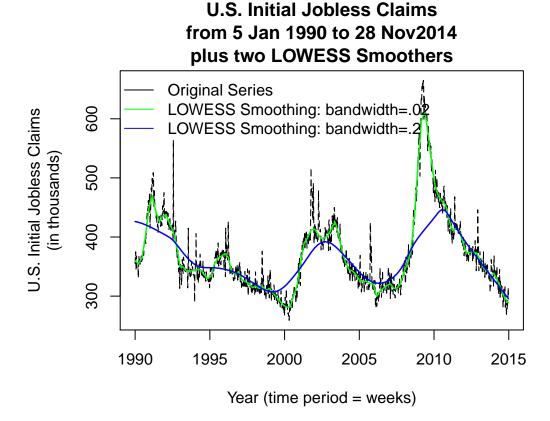


Figure 14: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014 (original series plus two LOWESS Smoothers)

6. Generate two spline smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

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```
INJCJC.1 <- smooth.spline(INJCJC.time, INJCJC, spar = 0.05)
INJCJC.2 <- smooth.spline(INJCJC.time, INJCJC, spar = 0.8)</pre>
```

U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov2014 plus two Spline Smoothers

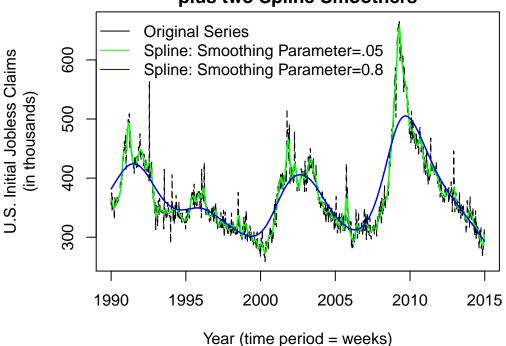


Figure 15: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014 (original series plus two Spline Smoothers)