

W271-2 – Spring 2016 – HW 6

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Exercises

Exercise 1

- a. Discuss the mean and variance functions and how the similarities and differences from those we studied in classical linear model.

...

- b. Define strict and weak stationarity

...

Exercise 2

- a. Generate a zero-drift random walk model using 500 simulation.

```
set.seed(123)
N <- 500 # number of simulations / time periods
wn <- rnorm(n = N, mean = 0, sd = 1) # white noise (can use any mean and sd)
rw <- cumsum(wn)
```

- b. Provide the descriptive statistics of the simulated realizations. The descriptive statistics should include the mean, standard deviation, 25th, 50th, and 75th quantiles, minimum, and maximum.

```
# See the definition of the function in ## @knitr Libraries-Functions-Constants
desc_stat(rw, 'Random walk',
          'Descriptive statistics of the simulated random walk')
```

Table 1: Descriptive statistics of the simulated random walk

	Random walk
Mean	4.58
St. Dev	4.87
1st Quartile	0.97
Median	3.42
3rd Quartile	8.27
Min	-5.16
Max	19.35

- c. Plot the time-series plot of the simulated realizations.

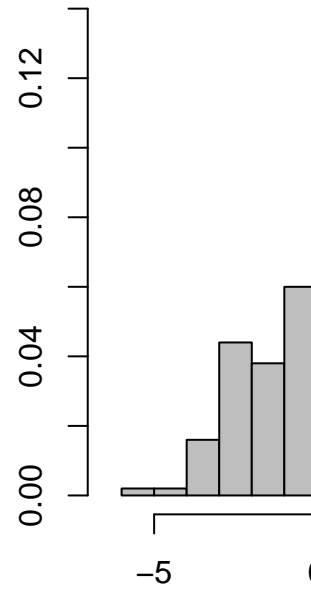
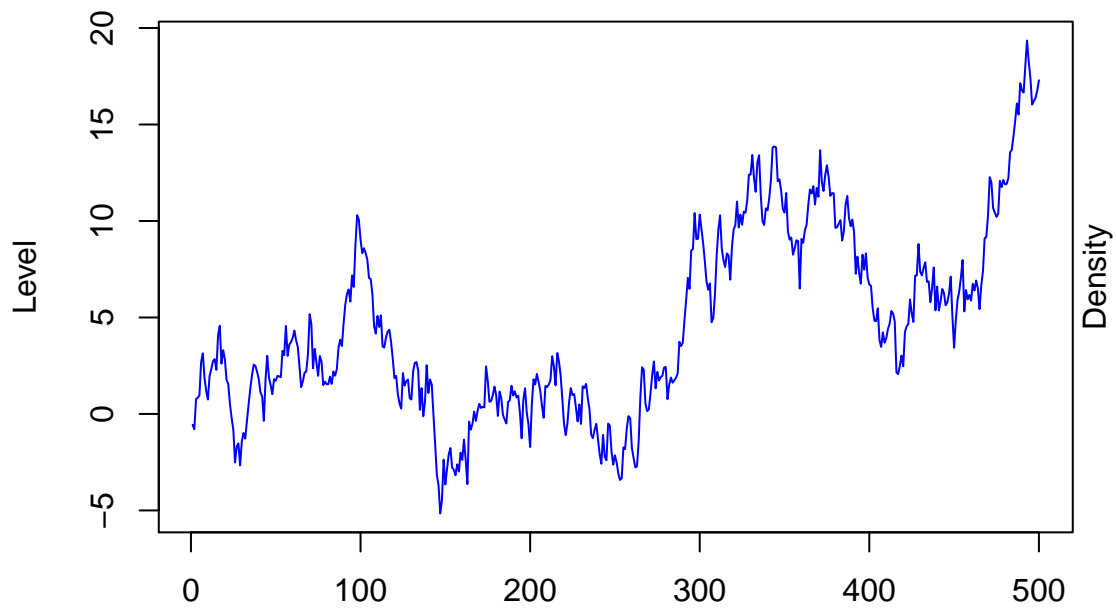
See the last part of this Exercise in the following page.

- d. Plot the autocorrelation graph.

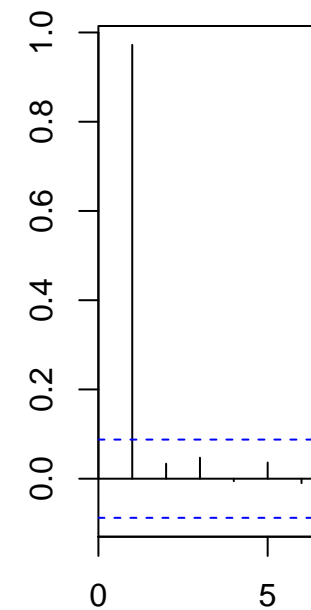
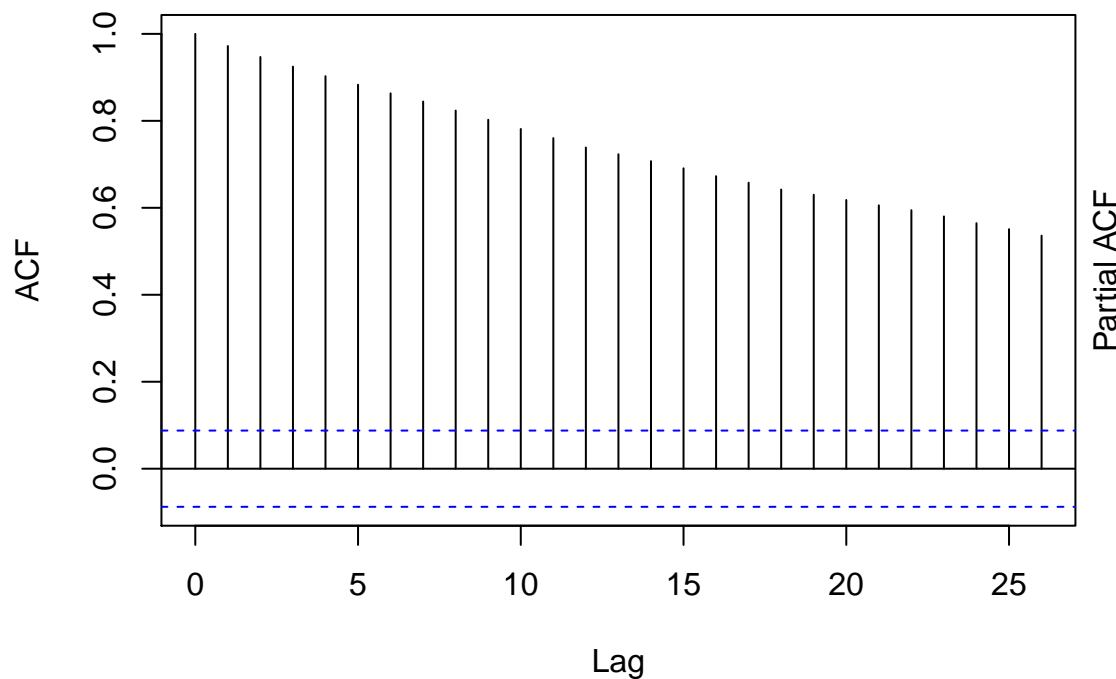
See the last part of this Exercise in the following page.

e. Plot the partial autocorrelation graph.

**Time plot of a 500 simulation
of a zero-drift random walk**



**ACF of a 500 simulation
of a zero-drift random walk**



Exercise 3

- a. Generate a random walk with drift model using 500 simulation, with the drift = 0.5.

```
drift <- 0.5 # drift
# Use the same GWN that generate the prev. zero-drift RW
# set.seed(123); wn <- rnorm(n = N, mean = 0, sd = 1)
rw_drift <- cumsum(wn + drift)
```

- b. Provide the descriptive statistics of the simulated realizations. The descriptive statistics should include the mean, standard deviation, 25th, 50th, and 75th quantiles, minimum, and maximum.

```
# See the definition of the function in ## @knitr Libraries-Functions-Constants
desc_stat(cbind(rw, rw_drift), c('Random walk', 'Random walk with 0.5 drift'),
          'Descriptive statistics of the two simulated random walks')
```

Table 2: Descriptive statistics of the two simulated random walks

	Random walk	Random walk with 0.5 drift
Mean	4.58	129.83
St. Dev	4.87	75.43
1st Quartile	0.97	64.58
Median	3.42	122.91
3rd Quartile	8.27	199.68
Min	-5.16	-0.06
Max	19.35	267.30

- c. Plot the time-series plot of the simulated realizations.

See the last part of this Exercise in the following page.

- d. Plot the autocorrelation graph.

See the last part of this Exercise in the following page.

e. Plot the partial autocorrelation graph.

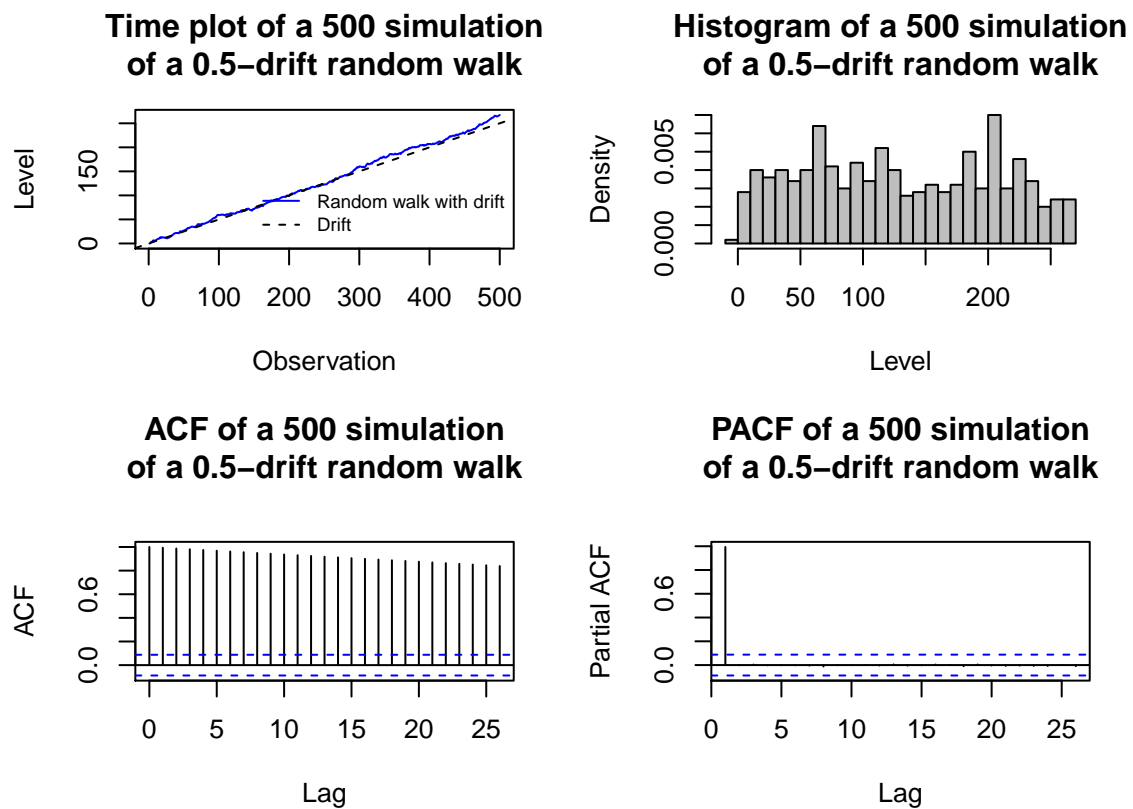


Figure 1: Time-series plot, histogram, correlogram, and partial autocorrelogram of the 500 simulation of a 0.5-drift random walk

Exercise 4

Use the series from `INJCJC.csv`.

- Load the data and examine the basic structure of the data using `str()`, `dim()`, `head()`, and `tail()` functions.

```
INJCJC_df <- read.csv('INJCJC.csv', header = TRUE)
str(INJCJC_df)
```

```
## 'data.frame':    1300 obs. of  3 variables:
## $ Date      : Factor w/ 1300 levels "10-Apr-09","10-Apr-92",...: 1102 101 400 741 907 1271 270 611 922 ...
## $ INJCJC    : int   355 369 375 345 368 367 348 350 351 349 ...
## $ INJCJC4   : num   362 366 364 361 364 ...
```

```
dim(INJCJC_df); obs <- dim(INJCJC_df)[1]
```

```
## [1] 1300    3
```

```
head(INJCJC_df)
```

```
##           Date INJCJC INJCJC4
## 1  5-Jan-90    355   362.25
## 2 12-Jan-90    369   365.75
## 3 19-Jan-90    375   364.25
## 4 26-Jan-90    345   361.00
## 5  2-Feb-90    368   364.25
## 6  9-Feb-90    367   363.75
```

```
tail(INJCJC_df)
```

```
##           Date INJCJC INJCJC4
## 1295 24-Oct-14    288   281.25
## 1296 31-Oct-14    278   279.00
## 1297  7-Nov-14    293   285.75
## 1298 14-Nov-14    292   294.25
## 1299 21-Nov-14    314   294.25
## 1300 28-Nov-14    297   299.00
```

Table 3: Descriptive statistics of the INJCJC variables

	INJCJC	INJCJC4
Mean	371.14	371.24
St. Dev	67.38	66.30
1st Quartile	324.00	324.69
Median	353.50	352.12
3rd Quartile	406.00	405.75
Min	259.00	266.25
Max	665.00	659.25

The 1300 observations (of two variables, INJCJC and INJCJC4) correspond to $1,300/52 = 25$ periods of 52 weeks, i.e., almost 25 years from January 5, 1990, until November 28, 2014. All observations correspond to Fridays.

Since years are slightly longer (by 1 or 2 days) than 52 weeks, some years have 53 Fridays instead, and hence the last year in the sample (2014) is not complete (there were 1,304 Fridays in that 25-year period from 1990 to 2014).

```
INJCJC_df %>%
  mutate(Date = as.Date(as.character(Date), '%d-%b-%y')) %>%
  mutate(Year = year(Date)) %>%
  group_by(Year) %>%
  summarise(obs = n(), start_date = min(Date), end_date = max(Date)) %>%
  print(n=Inf)
```

```
## Source: local data frame [25 x 4]
##
##   Year   obs start_date   end_date
##   (dbl) (int)   (date)     (date)
## 1  1990    52 1990-01-05 1990-12-28
## 2  1991    52 1991-01-04 1991-12-27
## 3  1992    52 1992-01-03 1992-12-25
## 4  1993    53 1993-01-01 1993-12-31
## 5  1994    52 1994-01-07 1994-12-30
## 6  1995    52 1995-01-06 1995-12-29
## 7  1996    52 1996-01-05 1996-12-27
## 8  1997    52 1997-01-03 1997-12-26
## 9  1998    52 1998-01-02 1998-12-25
## 10 1999    53 1999-01-01 1999-12-31
## 11 2000    52 2000-01-07 2000-12-29
## 12 2001    52 2001-01-05 2001-12-28
## 13 2002    52 2002-01-04 2002-12-27
## 14 2003    52 2003-01-03 2003-12-26
## 15 2004    53 2004-01-02 2004-12-31
## 16 2005    52 2005-01-07 2005-12-30
## 17 2006    52 2006-01-06 2006-12-29
## 18 2007    52 2007-01-05 2007-12-28
## 19 2008    52 2008-01-04 2008-12-26
## 20 2009    52 2009-01-02 2009-12-25
## 21 2010    53 2010-01-01 2010-12-31
## 22 2011    52 2011-01-07 2011-12-30
## 23 2012    52 2012-01-06 2012-12-28
## 24 2013    52 2013-01-04 2013-12-27
## 25 2014    48 2014-01-03 2014-11-28
```

```
levels(as.factor(weekdays(as.Date(as.character(INJCJC_df$Date), '%d-%b-%y'))))
```

```
## [1] "Friday"
```

```
INJCJC_df %>%
  mutate(week_num = 1:obs,
         week = ifelse(week_num %% 52 == 0, 52, week_num %% 52)) %>%
  filter(week %% 52 < 2)
```


##	Date	INJCJC	INJCJC4	week_num	week
## 1	5-Jan-90	355	362.25	1	1
## 2	28-Dec-90	454	456.00	52	52
## 3	4-Jan-91	415	447.50	53	1
## 4	27-Dec-91	441	456.75	104	52
## 5	3-Jan-92	432	446.00	105	1
## 6	25-Dec-92	313	339.00	156	52
## 7	1-Jan-93	341	336.75	157	1
## 8	24-Dec-93	290	323.00	208	52
## 9	31-Dec-93	341	324.00	209	1
## 10	23-Dec-94	314	324.25	260	52
## 11	30-Dec-94	319	323.00	261	1
## 12	22-Dec-95	374	366.50	312	52
## 13	29-Dec-95	359	363.00	313	1
## 14	20-Dec-96	350	347.25	364	52
## 15	27-Dec-96	357	353.50	365	1
## 16	19-Dec-97	310	317.00	416	52
## 17	26-Dec-97	303	313.25	417	1
## 18	18-Dec-98	297	309.50	468	52
## 19	25-Dec-98	336	316.00	469	1
## 20	17-Dec-99	287	283.50	520	52
## 21	24-Dec-99	268	278.50	521	1
## 22	15-Dec-00	354	342.25	572	52
## 23	22-Dec-00	364	344.25	573	1
## 24	14-Dec-01	389	434.50	624	52
## 25	21-Dec-01	416	415.75	625	1
## 26	13-Dec-02	429	405.25	676	52
## 27	20-Dec-02	394	406.25	677	1
## 28	12-Dec-03	363	360.25	728	52
## 29	19-Dec-03	354	360.25	729	1
## 30	10-Dec-04	316	326.75	780	52
## 31	17-Dec-04	322	329.00	781	1
## 32	9-Dec-05	327	320.75	832	52
## 33	16-Dec-05	312	317.75	833	1
## 34	8-Dec-06	311	328.25	884	52
## 35	15-Dec-06	318	326.25	885	1
## 36	7-Dec-07	332	340.00	936	52
## 37	14-Dec-07	350	344.50	937	1
## 38	5-Dec-08	570	541.75	988	52
## 39	12-Dec-08	566	549.25	989	1
## 40	4-Dec-09	497	490.25	1040	52
## 41	11-Dec-09	498	488.00	1041	1
## 42	3-Dec-10	431	427.50	1092	52
## 43	10-Dec-10	428	426.00	1093	1
## 44	2-Dec-11	390	390.25	1144	52
## 45	9-Dec-11	369	386.25	1145	1
## 46	30-Nov-12	379	406.25	1196	52
## 47	7-Dec-12	343	380.50	1197	1
## 48	29-Nov-13	317	328.75	1248	52
## 49	6-Dec-13	358	332.75	1249	1
## 50	28-Nov-14	297	299.00	1300	52

```
# Count weeks
```

```
sum(sapply(1990:2014, function(y)
```

```

    ifelse(((y %% 4 == 0) & (y %% 100 != 0)) | (y %% 400 == 0), 366, 365))) / 7

## [1] 1304.429

# Count Fridays in that period
ceiling(as.numeric(as.Date('2014-12-31')) + 1 - 5 + 4) / 7) -
  ceiling(as.numeric(as.Date('1990-01-01')) - 5 + 4) / 7)

## [1] 1304

```

- b. Convert the variables INJCJC into a time series object `frequency=52`, `start=c(1990,1,1)`, `end=c(2014,11,28)`. Examine the converted data series.

The above parameters may be wrong: according to the [ts documentation](#), `start` and `end` have to be “*Either a single number or a vector of two integers, which specify a natural time unit and a (1-based) number of samples into the time unit,*” i.e., and not a date in `y-m-d` format. If we use the parameters in the instructions, R discards 28 and takes 11 observations from 2014, not the 52 that there really are.

```

INJCJC_wrong <- ts(INJCJC_df$INJCJC, frequency = 52, start = c(1990, 1, 1),
                  end = c(2014, 11, 28))
length(INJCJC_wrong) # 1300 - (52 - 11)

## [1] 1259

tail(INJCJC_wrong, 10)

## [1] 368 339 344 333 329 334 345 328 343 330

tail(INJCJC_df$INJCJC[1:length(INJCJC_wrong)], 10)

## [1] 368 339 344 333 329 334 345 328 343 330

```

We can plot both the original vector and the time series object to confirm this.

U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

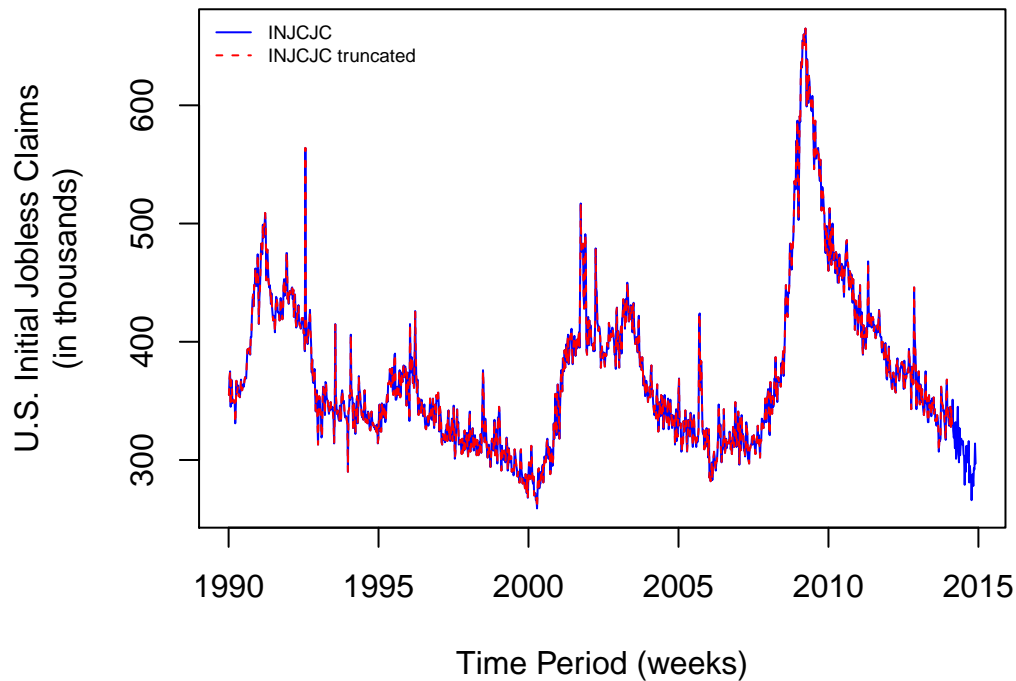


Figure 2: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014

Hence, we slightly change the R command to get the proper time series object:

```
INJCJC <- ts(INJCJC_df$INJCJC, frequency = 52, start = c(1990, 1),
             end = c(2014, 52))
length(INJCJC)
```

```
## [1] 1300
```

c. Define a variable using the command `INJCJC.time<-time(INJCJC)`.

```
INJCJC.time <- time(INJCJC)
head(INJCJC.time)
```

```
## [1] 1990.000 1990.019 1990.038 1990.058 1990.077 1990.096
```

```
tail(INJCJC.time)
```

```
## [1] 2014.885 2014.904 2014.923 2014.942 2014.962 2014.981
```

```
tail(time(INJCJC_wrong))
```

```
## [1] 2014.096 2014.115 2014.135 2014.154 2014.173 2014.192
```

- d. Using the following command to examine the first 10 rows of the data. Change the parameter to examine different number of rows of data.

```
head(cbind(INJCJC.time, INJCJC),10)
```

```
head(cbind(INJCJC.time, INJCJC), 10)
```

```
##      INJCJC.time INJCJC
## [1,]    1990.000    355
## [2,]    1990.019    369
## [3,]    1990.038    375
## [4,]    1990.058    345
## [5,]    1990.077    368
## [6,]    1990.096    367
## [7,]    1990.115    348
## [8,]    1990.135    350
## [9,]    1990.154    351
## [10,]   1990.173    349
```

```
head(cbind(INJCJC.time, INJCJC)) # default: 6
```

```
##      INJCJC.time INJCJC
## [1,]    1990.000    355
## [2,]    1990.019    369
## [3,]    1990.038    375
## [4,]    1990.058    345
## [5,]    1990.077    368
## [6,]    1990.096    367
```

```
head(cbind(INJCJC.time, INJCJC), -(length(INJCJC)-6)) # -1294: equivalent
```

```
##      INJCJC.time INJCJC
## [1,]    1990.000    355
## [2,]    1990.019    369
## [3,]    1990.038    375
## [4,]    1990.058    345
## [5,]    1990.077    368
## [6,]    1990.096    367
```

```
head(cbind(INJCJC.time, INJCJC), 13) # approximately 3 months (1 quarter)
```

```
##      INJCJC.time INJCJC
## [1,]    1990.000    355
## [2,]    1990.019    369
## [3,]    1990.038    375
## [4,]    1990.058    345
## [5,]    1990.077    368
## [6,]    1990.096    367
## [7,]    1990.115    348
## [8,]    1990.135    350
## [9,]    1990.154    351
```

```
## [10,] 1990.173 349
## [11,] 1990.192 349
## [12,] 1990.212 331
## [13,] 1990.231 346
```

e.

1. Plot the time series plot of INJCJC. Remember that the graph must be well labelled.

```
plot.ts(INJCJC, col = 'blue', xlab = "Time Period (weeks)",
        ylab = "U.S. Initial Jobless Claims\n(in thousands)",
        main = "U.S. Initial Jobless Claims\nfrom 5 Jan 1990 to 28 Nov 2014")
```

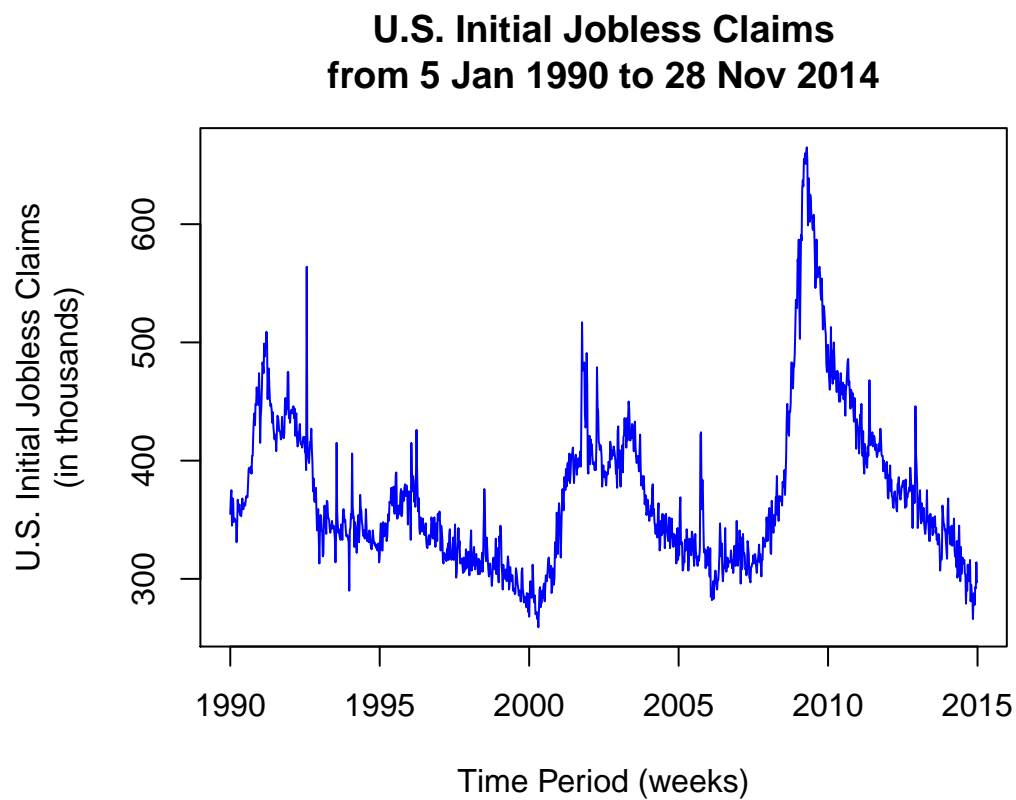
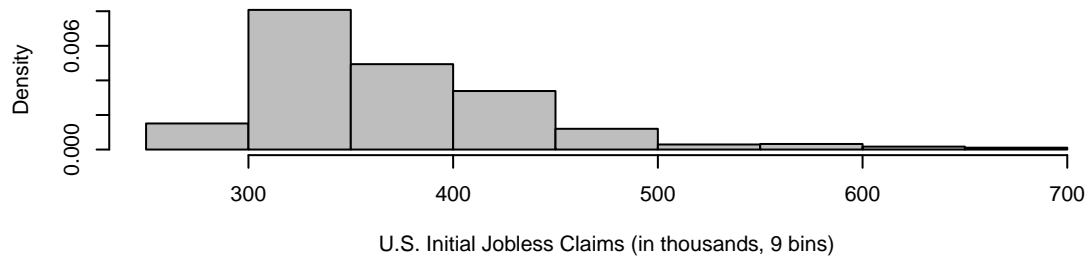


Figure 3: U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014

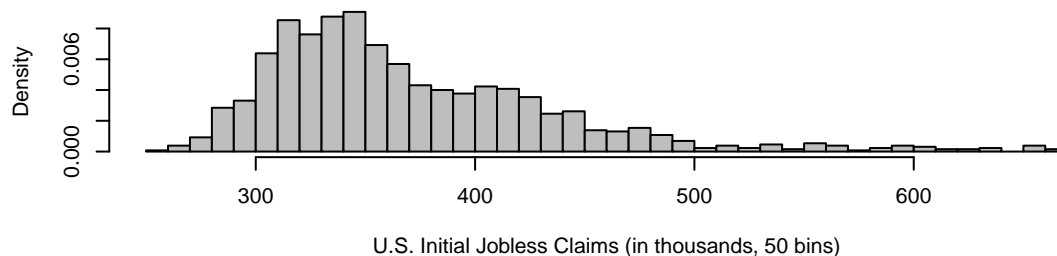
2. Plot the histogram of INJCJC. What is shown and not shown in a histogram?
How do you decide the number of bins used?

```
hist(INJCJC, breaks = 50, col="gray", freq = FALSE,
     xlab = "U.S. Initial Jobless Claims (in thousands, 50 bins)",
     main = paste0("Histogram of the U.S. Initial Jobless Claims",
                    "from 5 Jan 1990 to 28 Nov 2014"))
```

Histogram of the U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014



Histogram of the U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014



Histogram of the U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

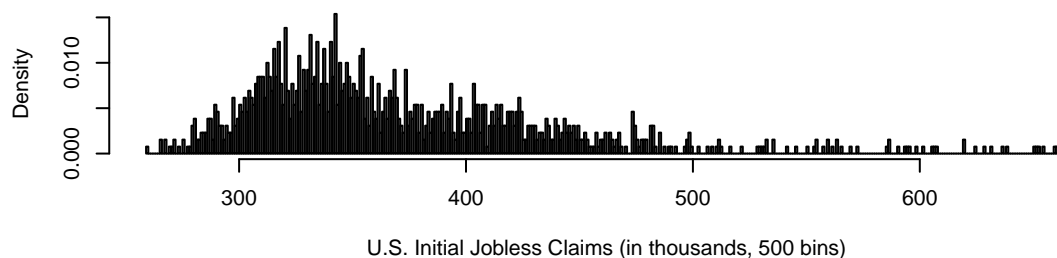


Figure 4: Histogram of the U.S. Initial Jobless Claims (in thousands) from 5 Jan 1990 to 28 Nov 2014, using different bins

The histograms tell us nothing about the dynamics of the series; e.g., it lets us know that there were a few time periods where the number of Initial Jobless Claims was greater than 600 (thousand), but not when that happened (in the 1st half of 2009).

A good number of bins is one that neither *oversimplifies* the (sample) distribution (i.e., it is not so low that some modes may be hidden) nor shows too much detail that is due to sampling and makes the underlying (real) distribution much more complex than it really is (i.e., it is so high that a lot of modes, just due to sampling, are shown). Hence, the best representation in the previous Figure would be the one in the middle.

3. Plot the autocorrelation graph of INJCJC series.

We plot the first 52 correlations (apart from ρ_0) just to check all possible seasonality components (incl. annual).

```
acf(INJCJC, lag.max = 52,  
    main="ACF of U.S. Initial Jobless Claims\nfrom 5 Jan 1990 to 28 Nov 2014")
```

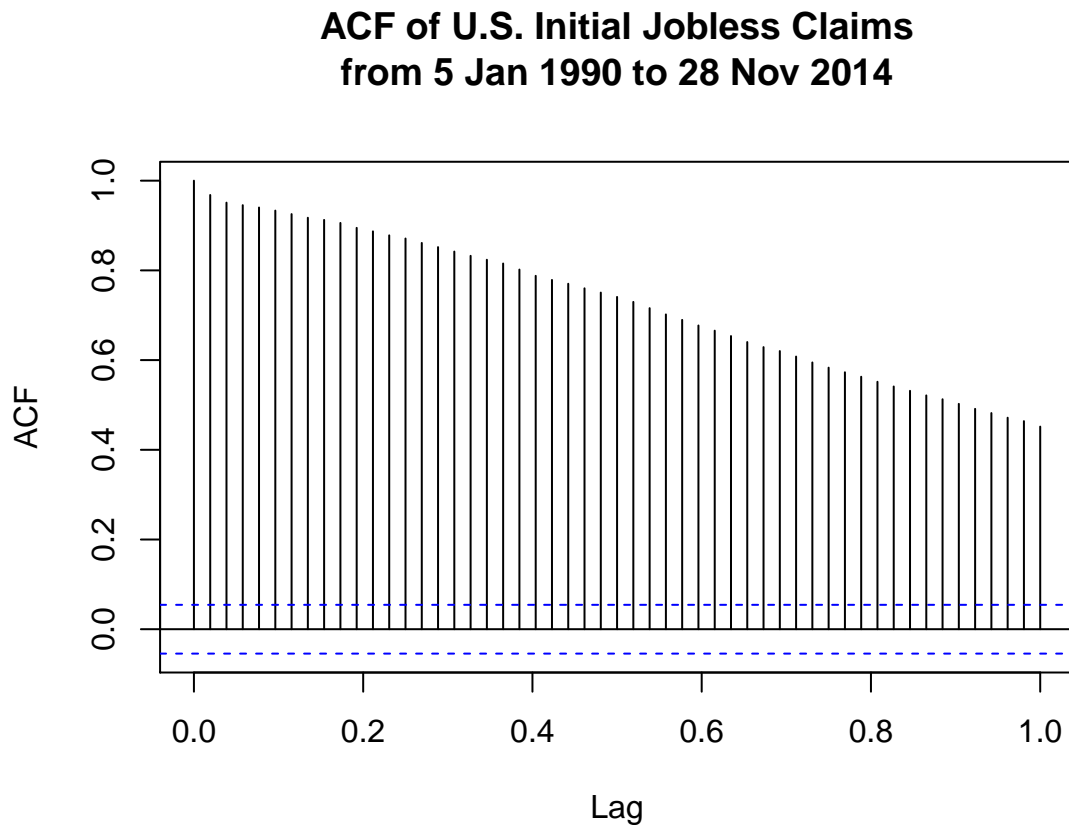


Figure 5: Partial autocorrelation graph of U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

The autocorrelation decreases exponentially, but very slowly (this is typical of an AR(1) model with a coefficient very close to 1). Nonetheless, we must note that the time series has neither been seasonally nor detrended.

4. Plot the partial autocorrelation graph of INJCJC series.

```
pacf(INJCJC, lag.max = 52,  
     main="PACF of U.S. Initial Jobless Claims\nfrom 5 Jan 1990 to 28 Nov 2014")
```

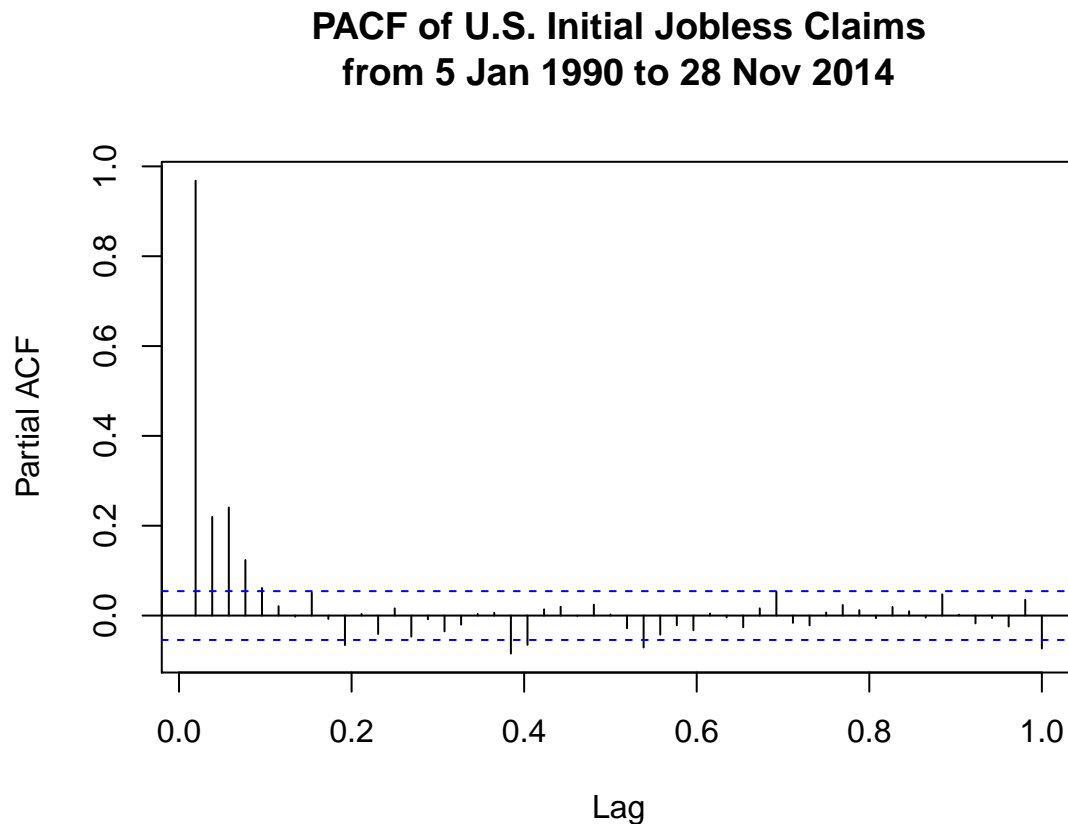


Figure 6: Partial autocorrelation graph of U.S. Initial Jobless Claims from 5 Jan 1990 to 28 Nov 2014

The partial autocorrelation plot makes it evident that this time series was not generated by an AR(1) model. After controlling for the effect of the process at lags 1 and 2, the partial correlation at lags 2 and 3, respectively, is still significant.

5. Plot a 3x3 Scatterplot Matrix of correlation against lag values.

```
lag.plot(INJCJC, lags = 9, layout = c(3, 3), diag=TRUE, diag.col="red",
        main = paste("Autocorr. between the INJCJC time series and its",
                      "Own Lags"))
```

Autocorr. between the INJCJC time series and its Own Lags

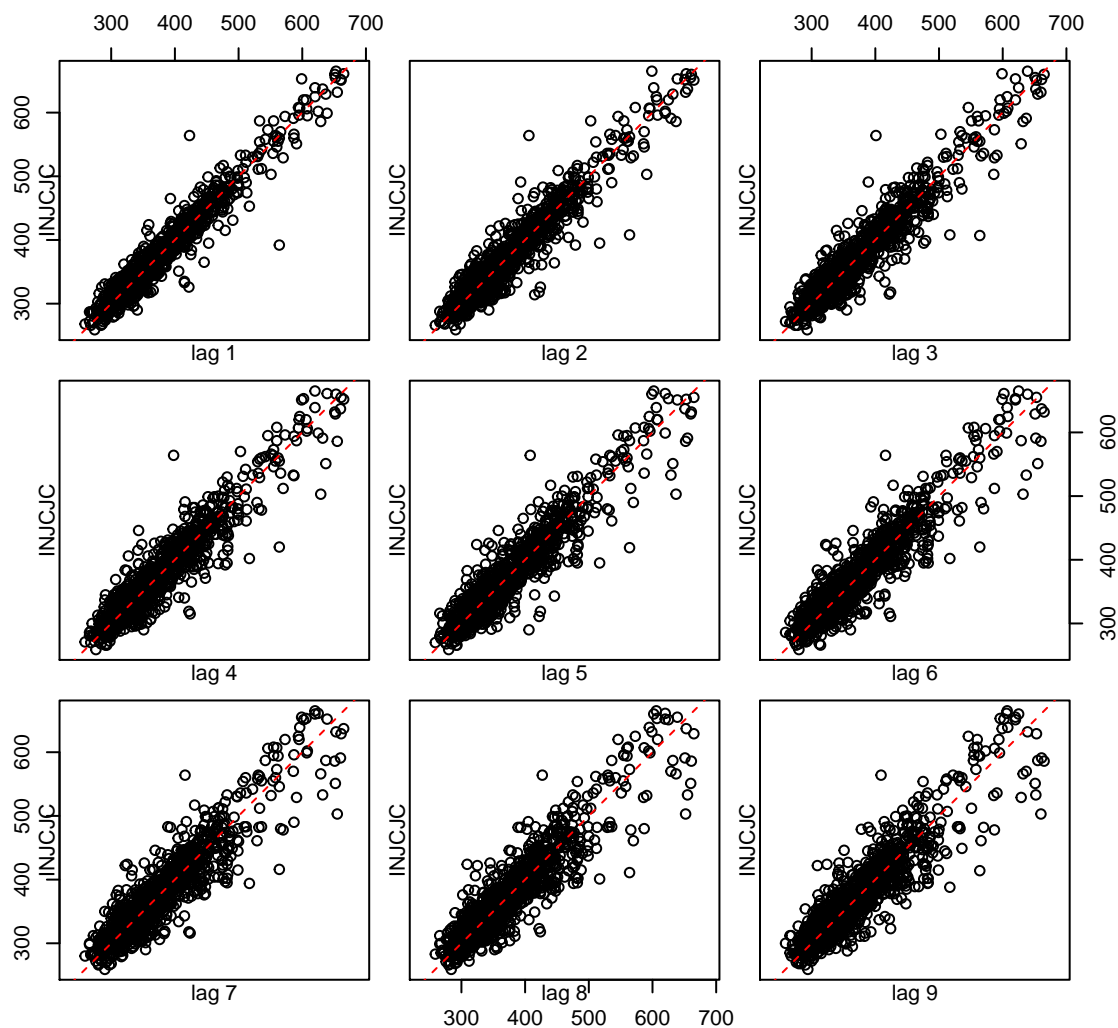


Figure 7: Scatterplot matrix of the correlation of the U.S. Initial Jobless Claims time series against its first 9 own lags

f.

1. Generate two symmetric Moving Average Smoothers. Choose the number of moving average terms such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

...

2. Generate two regression smoothers, one being a cubic trend regression and the other being a periodic regression. Plot the smoothers and the original series in one graph.

...

3. Generate kernel smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

...

4. Generate two nearest neighborhood smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

...

5. Generate two LOWESS smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

...

6. Generate two spline smoothers. Choose the smoothing parameters such that one of the smoothers is very smoother and the other one can trace through the dynamics of the series. Plot the smoothers and the original series in one graph.

...
