CME 108/MATH 114
Introduction to Scientific Computing
Summer 2017

Problem Set 4

Due: Thursday, July 27 at 9:30 AM

Total number of points: 100

1. (15 points) Explain why the Midpoint Rule can integrate a straight line exactly and the Simpson's Rule can integrate a cubic exactly.

2. (25 points) Consider the integral over one panel of width h:

$$I = \int_a^{a+h} f(x)\, dx.$$

Utilize the error analysis of the midpoint formula presented in class to find the error term for each of the following quadrature rules. For each case, determine the corresponding general quadrature rule for the entire interval $[a, b]$ if the division is uniform, i.e. $x_j = a + jh$ for $j = 0, 1, 2, \ldots N$, where $h = (b - a)/N$.

Hint: Expand $f'(a)$ in a Taylor series expansion around midpoint ξ:

$$f'(a) = f'(\xi) - \frac{h}{2} f''(\xi) + \frac{h^2}{8} f'''(\xi) - \cdots$$

(a) $I \approx hf(a + h)$

(b) $I \approx hf(a + h) - \dfrac{h^2}{2} f'(a)$.

3. (20 points) Use the Newton-Raphson method to solve for the roots of

$$f(x) = (x - 1)^m,$$

for $m = 2, 4, 8$. Use the starting point $x_0 = 1.1$, and set the tolerance to 10^{-8}. Plot $f(x)$ for the three values of m to verify your solutions. Reconcile your observations with theory.

4. (25 points) Use the secant method to solve for the root(s) of

$$f(x) = \cos(x^2)(x - 1)^3$$

for x between 3 and 5. Model the secant script in Matlab after the script for Newton-Raphson in problem (3) above.

5. (15 points, *Bradie* 6.3.8) In lecture, you learned how to use Richardson extrapolation to improve the accuracy of numerical integration. Now use the same extrapolation method to improve the order of accuracy of numerical differentiation without increasing the number of points in the stencil. Let D denote the true derivative of a function, and let D_h denote the first-order backward difference approximation to the derivative at x_0; that is,

$$D_h = \frac{f(x_0) - f(x_0 - h)}{h}.$$

This is first order backward difference. It has been shown that

$$D = D_h + k_1 h + k_2 h^2 + k_3 h^3 + o(h^3),$$

where k_1, k_2, and k_3 are constants independent of h. Let $f(x) = \ln(x^2 + 1)$ and $x_0 = 1$.

(a) Starting from $h = 1$, approximate the value of the first derivative of f at x_0 by applying extrapolation to D_h. Follow Section 4.3 of the lecture notes to obtain the formula for $D_{k,m}$, where k and m are row and column numbers of the Richardson extrapolation table. Calculate $D_{k,m}$ up to 4 rows.

(b) What is the error in the final approximation?