3) $$P_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)$$

$$\int_a^b P_2(x)\,dx = \frac{f(a)}{(a-b)(a-x_m)}\left(\int_a^b (x-b)(x-x_m)\,dx\right) + \frac{f(x_m)}{(x_m-a)(x_m-b)}\left(\int_a^b (x-a)(x-b)\,dx\right) + \frac{f(b)}{(b-a)(b-x_m)}\left(\int_a^b (x-a)(x-x_m)\,d\right.$$

$$= \frac{f(a)}{(a-b)(a-x_m)}\left(\int_a^b x^2 - x_m x - bx + bx_m\,dx\right) + \frac{f(x_m)}{(x_m-a)(x_m-b)}\left(\int_a^b x^2 - ax - bx + ab\,dx\right)$$

$$+ \frac{f(b)}{(b-a)(b-x_m)}\left(\int_a^b x^2 - ax - x_m x + ax_m\,dx\right)$$

4)
$$\int_a^b E(x)\,dx = \int_a^b \frac{f^{iv}(\xi)}{4!}(x-a)(x-b)\left(x-\frac{a+b}{2}\right)dx$$

$$= \frac{f^{iv}(\xi)}{4!}\left(\int_a^b (x^2-ax-bx+ab)\left(x-\frac{a+b}{2}\right)dx\right)$$

$$= \frac{f^{iv}(\xi)}{4!}\left(\int_a^b x^3 - \frac{ax^2+bx^2}{2} - ax^2 + \frac{a^2+ab}{2}x - bx^2 + \frac{ab+b^2}{2}x + abx - \frac{a^2b+ab^2}{2}\,dx\right)$$

$$= \underbrace{\frac{f^{iv}(\xi)}{4!}}_{u}\left(\frac{x^4}{4}\Big|_a^b - \frac{a+b}{2}\left(\frac{x^2}{2}\right)\Big|_a^b - \frac{a}{3}x^3\Big|_a^b + \frac{a^2+ab}{4}x^2\Big|_a^b - \frac{b}{3}x^3\Big|_a^b + \frac{ab+b^2}{4}x^2\Big|_a^b + \frac{ab}{2}x^2\Big|_a^b - \frac{a^2b+b^2a}{2}x\Big|_a^b\right)$$

$$= u\left(\frac{\cancel{b^4}-\cancel{a^4}}{4} - \left(\frac{ab^3+\cancel{b^4}-\cancel{a^4}-\cancel{a^3b}}{4}\right) - \left(\frac{\cancel{ab^3}-\cancel{a^4}}{3}\right) + \left(\frac{\cancel{a^2b^2}+\cancel{ab^3}-\cancel{a^4}-\cancel{a^3b}}{4}\right)\right.$$
$$- \left(\frac{\cancel{b^4}-\cancel{ba^3}}{3}\right) + \left(\frac{\cancel{ab^3}+\cancel{b^4}-\cancel{a^3b}-\cancel{a^2b^2}}{4}\right)$$
$$\left. + \left(\frac{\cancel{ab^3}-\cancel{a^3b}}{2}\right) - \left(\frac{\cancel{a^2b^2}+\cancel{b^3a}-\cancel{a^3b}-\cancel{a^2b^2}}{2}\right)\right)$$

$$= u(0)$$

$$= 0$$