d) $f(x) \approx P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$

$$\int_a^b f(x)\,dx = \int_a^b P_1(x)\,dx \quad : I$$

$$I = \int_a^b \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)\,dx = \int_a^b \frac{x-b}{a-b} f(a)\,dx + \int_a^b \frac{x-a}{b-a} f(b)\,dx$$

$$= f(a)\left(\int_a^b \frac{x}{a-b}\,dx - \int_a^b \frac{b}{a-b}\,dx\right) + f(b)\left(\int_a^b \frac{x}{b-a}\,dx - \int_a^b \frac{a}{b-a}\,dx\right)$$

$$= f(a)\left(\frac{1}{a-b}\left(\frac{x^2}{2}\Big|_a^b\right) - \frac{b}{a-b}x\Big|_a^b\right) + f(b)\left(\frac{1}{b-a}\left(\frac{x^2}{2}\Big|_a^b\right) - \frac{a}{b-a}x\Big|_a^b\right)$$

$$= f(a)\left(\frac{1}{a-b}\left(\frac{b^2-a^2}{2}\right) - \left(\frac{b^2-ab}{a-b}\right)\right) + f(b)\left(\frac{1}{b-a}\left(\frac{b^2-a^2}{2}\right) - \left(\frac{ab-a^2}{b-a}\right)\right)$$

$$= f(a)\left(\frac{b^2-a^2}{2a-2b} - \left(\frac{b^2-ab}{a-b}\right)\right) + f(b)\left(\frac{b^2-a^2}{2b-2a} - \left(\frac{ab-a^2}{b-a}\right)\right)$$

$$= f(a)\left(\frac{-b^2+2ab-a^2}{2a-2b}\right) + f(b)\left(\frac{b^2-2ab+2a^2}{2b-2a}\right)$$

$$= f(a)\,\frac{-(b-a)^2}{-2(b-a)} + f(b)\left(\frac{(b-a)^2}{2(b-a)}\right)$$

$$= f(a)\,\frac{b-a}{2} + f(b)\,\frac{b-a}{2}$$

$$\boxed{= \frac{b-a}{2}\left(f(a)+f(b)\right)}$$

2) $\epsilon(x) = \frac{f''(\xi)}{2}(x-a)(x-b)$

$$E = \int_a^b \epsilon(x)\,dx$$

$$E = \int_a^b \frac{f''(\xi)}{2}(x-a)(x-b)\,dx = \frac{f''(\xi)}{2}\int_a^b (x-a)(x-b)\,dx$$

$$= \frac{f''(\xi)}{2}\int_a^b x^2 - bx - ax + ab\,dx = \frac{f''(\xi)}{2}\left(\int_a^b x^2\,dx - \int_a^b bx\,dx - \int_a^b ax\,dx + \int_a^b ab\,dx\right)$$

$$= \frac{f''(\xi)}{2}\left(\frac{x^3}{3}\Big|_a^b - \frac{bx^2}{2}\Big|_a^b + \frac{ax^2}{2}\Big|_a^b + abx\Big|_a^b\right)$$

$$= \frac{f''(\xi)}{2}\left(\left(\frac{b^3-a^3}{3}\right) - \left(\frac{b^3-a^2b}{2}\right) - \left(\frac{ab^2-a^3}{2}\right) + \left(ab^2 - a^2b\right)\right)$$

$$= \frac{f''(\xi)}{2}\left(\left(\frac{2b^3-2a^3}{6}\right) - \left(\frac{3b^3-3a^2b}{6}\right) - \left(\frac{3ab^2+3a^3}{6}\right) + \left(\frac{6ab^2-6a^2b}{6}\right)\right)$$

$$= \frac{f''(\xi)}{2}\left(\frac{-b^3+a^3+3a^2b+3ab^2}{6}\right) = \frac{f''(\xi)}{2}\left(\frac{-(b-a)^3}{6}\right)$$

$$= -\frac{h^3}{12}f''(\xi)$$