Cálculo - Entrega 1ª (25 de Septiembre de 2017)

Juan Carlos Ruiz García
José Antonio Ruiz Millán

1 a) $\frac{-6x+1}{2x^2-8} + \frac{1}{2x-4} + \frac{5}{3x+6} = \frac{-6x+1}{2(x^2-4)} + \frac{1}{2(x-2)}$

$+ \frac{5}{3(x+2)} = \frac{-6x+1}{2(x+2)(x-2)} + \frac{1}{2(x-2)} + \frac{5}{3(x+2)} =$

$= \frac{-18x+3+3(x+2)+10(x-2)}{6(x+2)(x-2)} = \boxed{\frac{-5x-11}{6(x+2)(x-2)}}$

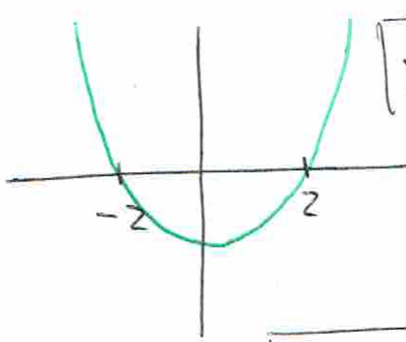
b)

$\frac{-5x-11}{6(x+2)(x-2)} < 0 \quad x \neq \pm 2$

$-5x-11 > 0 \iff -11 > 5x \iff \boxed{x < -\frac{11}{5}}$ NO

(y)

$(x+2)(x-2) < 0$ $\boxed{x \in \,]-2, 2[}$



(ó)

$-5x-11 < 0 \iff -11 < 5x \iff \boxed{x > -\frac{11}{5}}$

(y) (y)

$(x+2)(x-2) > 0 \iff \boxed{x \in \,]\leftarrow, -2[\, \cup \,]2, \rightarrow[}$

Sol. final $\Rightarrow$ $x \in \,]-\frac{11}{5}, -2[\, \cup \,]2, \rightarrow[$

2.

$$\left|\frac{2x-5}{x-4}\right| \leq 3 \iff -3 \leq \frac{2x-5}{x-4} \leq 3 \quad \left\{ \begin{array}{l} \dfrac{2x-5}{x-4} \leq 3 \quad (1) \\ \text{(y)} \\ \dfrac{2x-5}{x-4} \geq -3 \quad (2) \end{array} \right.$$

(1) $$\frac{2x-5}{x-4} \leq 3 \iff \frac{2x-5}{x-4} - 3 \leq 0 \iff \frac{2x-5-3x+12}{x-4} \leq 0 \iff$$

$$\iff \frac{-x+7}{x-4} \leq 0 \iff \left\{ \begin{array}{l} -x+7 \leq 0 \iff x \geq 7 \\ \text{(y)} \\ x-4 > 0 \iff x > 4 \end{array} \right\} \boxed{x \geq 7}$$

(ó)

$$\left\{ \begin{array}{l} -x+7 \geq 0 \iff x \leq 7 \\ \text{(y)} \\ x-4 < 0 \iff x < 4 \end{array} \right\} \boxed{x < 4}$$

Solución: (1)

$$\boxed{x \in]\leftarrow, 4[\cup [7, \rightarrow[}$$

(2) $$\frac{2x-5}{x-4} \geq -3 \iff \frac{2x-5}{x-4} + 3 \geq 0 \iff \frac{2x-5+3x-12}{x-4} \geq 0 \iff$$

$$\iff \frac{5x-17}{x-4} \geq 0 \quad \left\{ \begin{array}{l} 5x-17 \geq 0 \iff x \geq \dfrac{17}{5} \\ \text{(y)} \\ x-4 > 0 \iff x > 4 \end{array} \right\} \boxed{x > 4}$$

(o)

$$\left\{ \begin{array}{l} 5x-17 \leq 0 \iff x \leq \dfrac{17}{5} \\ \text{(y)} \\ x-4 < 0 \iff x < 4 \end{array} \right\} \boxed{x \leq \frac{17}{5}}$$

Solución (2)

$$\boxed{x \in]\leftarrow, \frac{17}{5}] \cup]4, \rightarrow[}$$

Solución final

$$\boxed{x \in]\leftarrow, 4] \cup]7, \rightarrow[}$$

3.

a) $|x^2-x-2| = |x-2| \iff \dfrac{|x^2-x-2|}{|x-2|} = 1 \iff$

$\boxed{x \neq 2}$

$\iff \left| \dfrac{x^2-x-2}{x-2} \right| = 1 \iff \left| \dfrac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}} \right| = 1 \iff$

$\iff |(x+1)| = 1 \quad \begin{cases} x+1 = 1 \iff \boxed{x=0} \\ (\text{ó}) \\ x+1 = -1 \iff \boxed{x=-2} \end{cases}$

$\boxed{\text{Soluciones: } \quad x=0\ ;\ x=2\ ;\ x=-2}$

b) ~~xxx~~

$|x+1| \leq 1 \quad \Big\{ -1 \leq x+1 \leq 1 \iff -2 \leq x \leq 0$

$\Updownarrow$

$\boxed{x \in [-2, 0]}$

4.

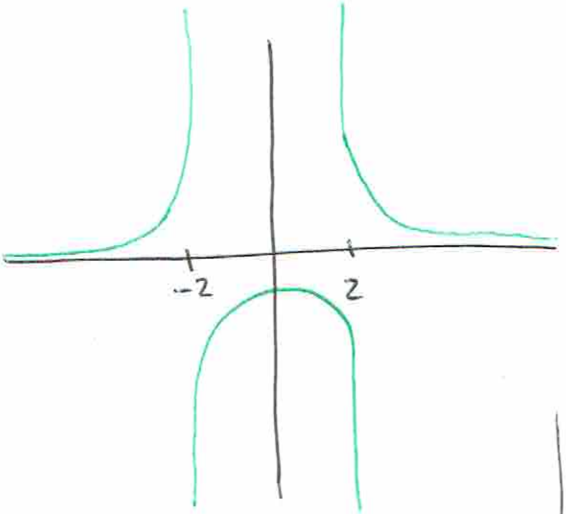
Paridad

$f(-x) = f(x)$ par, $\forall x \in A$

$f(-x) = -f(x)$ impar, $\forall x \in$

a) $f(x) = \dfrac{1}{x^2-4}$

$\mathrm{Dom}(f) = \mathbb{R} \setminus \{2, -2\}$



$$\boxed{\begin{array}{c} f(-1) = f(1) \\ \Updownarrow \\ -\dfrac{1}{3} = -\dfrac{1}{3} \end{array}}$$

Es par

$$\boxed{\begin{array}{c} f(-1) = -f(1) \\ \Updownarrow \\ -\dfrac{1}{3} \neq \dfrac{1}{3} \end{array}}$$

No es par

$$\boxed{\lim_{x \to \infty} \frac{1}{x^2-4} = 0}$$

$\exists$ asintotas horizontales en 0

$$\boxed{\lim_{x \to 2} \frac{1}{x^2-4} = +\infty}$$

$\exists$ asintotas verticales en ± 2

Asintotas

H: $\lim\limits_{x \to \pm\infty} f(x) = a$

V: $\lim\limits_{x \to a \notin \mathrm{Dom}} = \infty$

b) $g(x) = \log\left(\dfrac{1}{x^2-4}\right)$

$\mathrm{Dom}(g) = \,]\leftarrow, -2[\, \cup \,]2, \rightarrow[$

$g(-x) = g(x) \Leftrightarrow g(-3) = g(3) \Leftrightarrow \boxed{\log\left(\frac{1}{5}\right) = \log\left(\frac{1}{5}\right)}$ Es par

$g(-x) = -g(x) \Leftrightarrow g(-3) = -g(3) \Leftrightarrow \boxed{\log\left(\frac{1}{5}\right) \neq -\log\left(\frac{1}{5}\right)}$ No es impar

$$\boxed{\lim_{x \to \infty} \log\left(\frac{1}{x^2-4}\right) = +\infty}$$

$\nexists$ asintotas horizontales

$$\boxed{\lim_{x \to 2} \log\left(\frac{1}{x^2-4}\right) = \infty}$$

$\exists$ asintotas verticales en ± 2