ED

Relación de Estructura de Datos - Tema 1
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1.º Ejercicio 1

a) 17 es $O(1)$

$f(n) = 1$

$$\boxed{17 \leq 17 \cdot f(n) \ \forall n}$$

b) $\frac{n(n-1)}{2}$ es $O(n^2)$ y $\Omega(n^2)$ (es decir, $\Theta(n^2)$)

$$\boxed{\frac{n^2-n}{2} \in K \cdot n^2, K \in \mathbb{R}}$$

c) $\max(n^3, 10n^2)$ es $O(n^3)$

d) $\log_2 n$ es $\Theta(\log_3 n)$

2.° Ejercicio 2

1.) $f(n) = 13n^2 + 4n - 73$

$K = 2 \Rightarrow f(n)$ es $O(n^2)$

2.) $f(n) = 1/(n+1)$

$$\frac{1}{n+1} \leq n^{-1}$$

$$\frac{1}{n+1} \leq \frac{1}{n}$$

$$\frac{1}{n+1} - \frac{1}{n} \leq 0$$

$$\frac{n}{n^2+n} - \frac{n+1}{n^2+n} \leq 0$$

$$\frac{n-(n+1)}{n^2+n} \leq 0$$

$$\boxed{-\frac{1}{n^2+n} \leq 0} \quad \forall n > 0$$

3.) $f(n) = 1/(n-1)$

$$\frac{1}{n-1} \leq n^{-1}$$

$$\frac{1}{n-1} \leq \frac{1}{n}$$

$$\frac{1}{n-1} - \frac{1}{n} \leq 0$$

$$\frac{n}{n^2-n} - \frac{n-1}{n^2-n} \leq 0$$

$$\frac{n-(n-1)}{n^2-n} \leq 0$$

$$\boxed{\frac{1}{n^2-n} \leq 0} \quad \forall n > 1$$

4.) $f(n) = (n-1)^3$

$$(n-1)^3 = n^3 - (3n^2 \cdot 1) + (3n \cdot 1^2) - 1^3 =$$

$$= n^3 - 3n^2 + 3n - 1 \Rightarrow \text{es } O(n^3) \Rightarrow \boxed{K=3}$$

5.) $f(n) = (n^3 + 2n - 1) / (n+1)$

$K=2$

$$\frac{n^3+2n-1}{n+1} \leq n^2$$

$$\frac{n^3+2n-1}{n+1} - n^2 \leq 0$$

$$\frac{n^3+2n-1}{n+1} - \frac{n^3+n^2}{n+1} \leq 0$$

$$\Rightarrow \frac{n^3+2n-1-(n^3+n^2)}{n+1} \leq 0$$

$$\boxed{\frac{-n^2+2n-1}{n+1} \leq 0} \quad \forall n \geq 0$$

3.) Ejercicio 3

1 → 20000

2 → $\sqrt{n}$

3 → n

4 → $n+100$

5 → $n \log_2 \log_2 (n^2)$

6 → $n \log_2 (n^2)$

7 → n^2

8 → n^3+1

9 → 2^n

10 → $n2^n$

11 → $3^{\log_2(n)}$

12 → $2^n + 3^{n-1}$

13 → 3^n

4.- Ejercicio 4

a) $T_1(n) + T_2(n) \in O(f(n))$ Verdadero

$T_1(n) + T_2(n) \in \max(f(n), f(n)) \Rightarrow O(f(n))$ ✓

b) $T_1(n) \in O(f^2(n))$ Verdadero

$O(f(n)) \subseteq O(f^2(n)) \Rightarrow T_1(n) \in O(f^2(n))$ ✓

c) $T_1(n) / T_2(n) \in O(1)$ Verdadero.

$$\frac{T_1(n)}{T_2(n)} \in O\left(\frac{f(n)}{f(n)}\right) = 1 \Rightarrow \in O(1) ✓$$

9.- Ejercicio 9

```
for (i=0; i<n; i++) O(n)
  for (j=0; j<n; j++) O(n)
    C[i][j] = 0; O(1)
    for (K=0; K<n; K++) O(n)
      C[i][j] += A[i][K] * B[K][j]; O(1)
```

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \left(1 + \sum_{K=0}^{n-1} 1\right) \in O(n^3)$$

10.- Ejercicio 10

```
void ejemplo (int a) {
  int i, j, k;
  for( i = 1; i < n; i++)
    for ( j = i+1; j <= n; j++)
      for (k = 1; k <= j; k++)
        Global += k * i      O(1)
}
```

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{k=1}^{j} 1 = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} j = \sum_{i=1}^{n-1} \frac{(n+i+1)(n-i)}{2} =$$

$$= \sum_{i=1}^{n-1} \frac{n^2 - i^2 + n - i}{2} =$$

$$= \frac{n^2}{2}\sum_{i=1}^{n-1} 1 - \frac{1}{2}\sum_{i=1}^{n-1} i^2 + \frac{n}{2}\sum_{i=1}^{n-1} 1 - \frac{1}{2}\sum_{i=1}^{n-1} i$$

$$\frac{n^2}{2}\sum_{i=1}^{n-1} 1 = \frac{n^2}{2} \cdot (n-1) = \frac{n^2}{2} \cdot \frac{2(n-1)}{2} =$$

$$= \frac{n^2}{2} \cdot \frac{2n-2}{2} = \frac{2n^3 - 2n^2}{2} = \boxed{n^3 - n^2} \in O(n^3)$$

11.- Ejercicio 11

```
for(i=0; i<n; i++)
  if(i%2){  O(1)
    for(j=i; j<n; j++)
      x *= i;  O(1)
    for(j=1; j<i; j++)
      y *= j;  O(1)
  }
```

$$\sum_{i=0}^{n-1}\left(1+\sum_{j=i}^{n-1}1+\sum_{j=1}^{i-1}1\right)=\sum_{i=0}^{n-1}\left(1+(n-\cancel{i})+(\cancel{i}-1)\right)=$$

$$=\sum_{i=0}^{n-1}n = n\sum_{i=0}^{n-1}1 = \boxed{n^2 \in O(n^2)}$$

12.- Ejercicio 12

$cont = \log_2(n) - 1$

13.- Ejercicio 13

```
                 T(n)
int recursiva(int n)
{
  if(n<=1)  O(1)
    return 1;  O(1)
  else
    return (recursiva(n-1) + recursiva(n-1));  1 + 2T(n-1)
}
```

$$T(n) = \begin{cases} 1 & n \leq 1 \\ 1 + 2(T(n-1)) & n \geq 2 \end{cases}$$

$T(n) = 2T(n-1) + 1$

$\quad \rightarrow 2(2T(n-2) + 1) + 1$

$T(n) = 4T(n-2) + 3$

$\quad \rightarrow 4(2T(n-3) + 1) + 3$

$T(n) = 8T(n-3) + 7$

$K < n$

$T(n) = 2^K T(n-K) + 2^K - 1$

$K = n-1$

$T(n) = 2^{n-1} \underbrace{T(n-n+1)}_{1} + 2^{n-1} - 1 =$

$= 2^{n-1} + 2^{n-1} - 1 = 2^1(2^{n-1}) - 1 =$

$= 2^n - 1 \Rightarrow \in \boxed{O(2^n)}$

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(14.-) Ejercicio 14

```
int E(int n)
{
  if(n == 1)
     return n;
  else
     return (E(n/2)+1);
}
```

n return

1) 2 → 2
3 → 2 2^1
4 → 3
5 → 3
2^2 6 → 3
7 → 3
8 → 4
9 → 4
10 → 4
11 → 4
2^3 12 → 4
13 → 4
14 → 4
15 → 4
16 → 5

Asumiendo que $\log_2(n)$ solo utiliza la parte entera

$$\boxed{\text{resultado} = \log_2(n)+1}$$

b)

$$T(n) \begin{cases} T(n/2) + 1 & \geq 2 \\ 1 & \leq 1 \end{cases}$$

$T(n) = T(n/2) + 1$

$n = 2^m$

$T(2^m) = T\left(\frac{2^m}{2}\right) + 1$

$T(2^m) = T(2^{m-1}) + 1$

$\quad \hookrightarrow T(2^{m-2}) + 1$

$T(2^m) = T(2^{m-2}) + 2$

$\quad \hookrightarrow T(2^{m-3}) + 1$

$T(2^m) = T(2^{m-3}) + 3 = T(2^{m-k}) + k$

$k = m$

$T(2^m) = 1 + m$

$\boxed{T(n) = 1 + \log_2 n}$

$\left\{ \begin{array}{l} n = 2^m \\ \log_2 n = m \end{array} \right.$

$\Downarrow$

$\underline{\in O(\log_2 n)}$

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