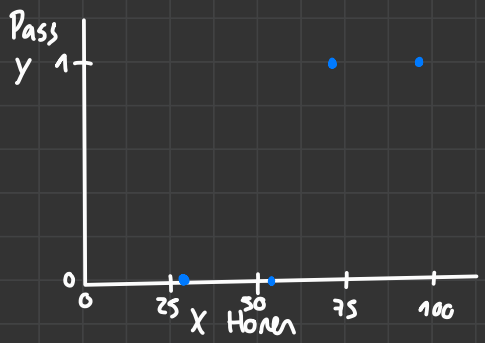
Problema 2

Hores	26'8	57'8	71'8	95'2
Pass	0	0	1	1

Exercici A

a) Logistic Regression, LR=1, 2 iters



1- LogOdds

$z_i = w_0 + w_1 \cdot Hores$ { Intercept: $w_0 = 0$; Weights: $w_1 = 0$

$z_1 = w_0 + w_1 \cdot x_1 = 0 + 0 \cdot 26'8 = 0$
$z_2 = w_0 + w_1 \cdot x_2 = 0 + 0 \cdot 57'8 = 0$
$z_3 = w_0 + w_1 \cdot x_3 = 0 + 0 \cdot 71'8 = 0$
$z_4 = w_0 + w_1 \cdot x_4 = 0 + 0 \cdot 95'2 = 0$

2- Probs

$$f(x;w) = \frac{1}{1+e^{-(w_0 + w_1 x_1 + w_2 x_2 + \dots)}}$$

$$f(x_1;w) = \frac{1}{1+e^{-(w_0 + w_1 \cdot x_1)}} = \frac{1}{1+e^0} = 0'5$$

$$f(x_2;w) = \frac{1}{1+e^{-(w_0 + w_1 \cdot x_2)}} = \frac{1}{1+e^0} = 0'5$$

$$f(x_3;w) = \frac{1}{1+e^0} = 0'5$$

$$f(x_4;w) = \frac{1}{1+e^0} = 0'5$$

3- Cost 1a iteració

$$J(w) = \frac{1}{m}\sum_{i=1}^{m} Cost(f(x_i;w), y_i) \rightarrow J(w) = -\frac{1}{m}\left[\sum_{i=1}^{m} y_i \log(f(x_i;w)) + (1-y_i)\log(1-f(x_i;w))\right]$$

$$J(w) = -\frac{1}{4}\left[0 \cdot \log(0'5) + (1-0)\log(1-0'5) + 0 \cdot \log(0'5) + (1-0)\log(1-0'5) + \right.$$

$$\left. + 1 \cdot \log(0'5) + (1-1)\log(1-0'5) + 1 \cdot \log(0'5) + (1-1)\log(1-0'5)\right] =$$

$$J(w) = -0'25 \cdot (-2'76) = 0'69$$

4- Neuronas

$$w^* = \text{argmin}_w J(w) \quad ; \quad w_j = w_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (g(x_i; w) - y_i) \cdot x_j^i$$

Intercept: $w_0 = w_0 - \alpha \frac{1}{4} \left[[(g(x_1; w) - y_1) \cdot 1] + [(g(x^2; w) - y^2) \cdot 1] + [(g(x^3; w) - y^3) \cdot 1] + \right.$

$\left. [(g(x^4; w) - y^4) \cdot 1] \right] = 0 - 1 \cdot \frac{1}{4} \cdot (0'5 + 0'5 - 0'5 - 0'5) = 0$

Weights: $w_1 = w_1 - \alpha \frac{1}{4} \left([g(x_1, w) - y_1) \cdot x_1] + [g(x^2, w) - y^2) \cdot x_2] + [g(x^3, w) - y_3) \cdot x_3] \right.$

$+ [g(x_4, w) - y_4) \cdot x_4]$ =

$= 0 - 1 \cdot \frac{1}{4} \left([0'5 - 0] \cdot 26'8] + [(0'5 - 0) \cdot 57'8] + [(0'5 - 1) \cdot 71'8] + \right.$

$[(0'5 - 1) \cdot 95'2] = 0 - 1 \cdot 0'25 \cdot (13'4 + 28'9 - 35'9 - 47'6) = 10'3$

5-

a) Log Odds

$z_1 = w_0 + w_1 \cdot x_1 = 0 + 10'3 \cdot 26'8 = 276'04$

$z_2 = w_0 + w_1 \cdot x_2 = 0 + 10'3 \cdot 57'8 = 595'34$

$z_3 = w_0 + w_1 \cdot x_3 = 0 + 10'3 \cdot 71'8 = 739'54$

$z_4 = w_0 + w_1 \cdot x_4 = 0 + 10'3 \cdot 95'2 = 980'56$

b) Probs

$$g(x_1; w) = \frac{1}{1 + e^{-(w_0 + w_1 \cdot x_1)}} = \frac{1}{1 + e^{-276'04}} = 1$$

$$g(x_2; w) = \frac{1}{1 + e^{-(w_0 + w_1 \cdot x_2)}} = \frac{1}{1 + e^{-595'34}} = 1$$

$$g(x_3; w) = \frac{1}{1 + e^{-739'54}} = 1$$

$$g(x_4; w) = \frac{1}{1 + e^{-980'56}} = 1$$

c) Cost amb els nous pesos

$$J(W) = -\frac{1}{4}\left[0 \cdot \log 1 + (1-0)\log(1-1) + 0 \cdot \log 1 + (1-0)\log(1-1) + \right.$$

$$\left. + 1 \cdot \log(1) + (1-1)\log(1-1) + 1 \cdot \log(1) + (1-1)\log(1-1)\right]$$

$J(W) = 0$ ⟵ Valen tant alts de Odds i Probabilitats no donen resultats concrets

d) Pesos

Intercept: $W_0 = 0 - 1 \cdot 0'25 \cdot (1+1+0+0) = -0'5$

Weights: $W_1 = 10'3 - 1 \cdot 0'25 \cdot (26'8 + 57'8 + 0 + 0) = -10'85$

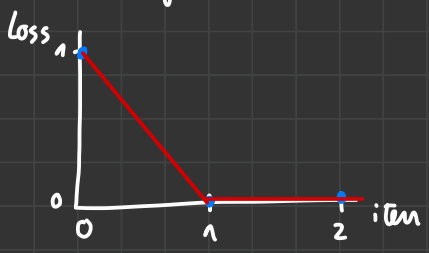
e)

Log Odds $z = -0'5 - 10'85x$

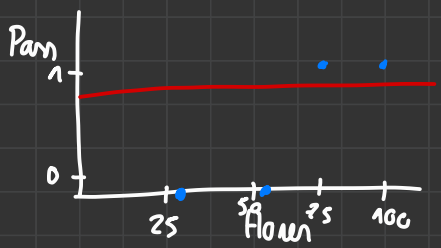
Odds $e^{-0'5 - 10'85x}$

Probs $P = \dfrac{1}{1 + e^{-(-0'5 - 10'85x)}}$

b) Training loss



c) Logistic Regression



d) Decision Boundary $\geq 50\%$

THR: 0'5

Hours	26'8	57'8	71'8	95'2
Pass	0	0	1	1
LogOdds	-291'28	-627'63	-779'53	-1033'42
Probs	1	1	1	1
Class:0'5	Pass	Pass	Pass	Pass

Confusion Matrix		
thr:0'5	TP=2	FP=2
	FN=0	TN=0

$$ACC = \frac{TP+TN}{TP+TN+FP+FN} = \frac{1}{2}$$

e) F1-Score, PR (THR=0'5)

$$F1 = \frac{2}{recall^{-1} + precision^{-1}} = 2 \cdot \frac{precision \cdot recall}{precision + recall} = \frac{0'5 \cdot 1}{0'5 + 1} = \frac{0'5}{1'5} \cdot 2 = \frac{1}{1'5} = 0'66$$

$$Precision = \frac{TP}{TP+FP} = \frac{2}{4} = 0'5$$

$$Recall = \frac{TP}{TP+FN} = \frac{2}{2} = 1$$

g) ROC (THR=0'5)

$$Recall = 1$$

$$FPR = \frac{FP}{FP+TN} = \frac{2}{2} = 1$$

g) Accuracy, PR i ROC (THR=0'6)

Hours	26'8	57'8	71'8	95'2
Pass	0	0	1	1
LogOdds	-291'28	-627'63	-779'53	-1033'42
Probs	1	1	1	1
Class:0'6	Pass	Pass	Pass	Pass

THR(0'6) TP=2 FP=2 FN=0 TN=0

$$ACC = \frac{TP+TN}{TP+TN+FP+FN} = \frac{1}{2}$$

$$Recall = \frac{TP}{TP+FN} = \frac{2}{2} = 1$$

$$Precision = \frac{TP}{TP+FP} = \frac{2}{4} = 0'5$$

$$FPR = \frac{FP}{FP+TN} = \frac{2}{2} = 1$$

h) Accuracy, PR, ROC (THR=0'3)

Hours	26'8	57'8	71'8	95'2
Pass	0	0	1	1
LogOdds	-291'28	-627'63	-779'53	-1033'42
Prob	1	1	1	1
Class:0'3	Pass	Pass	Pass	Pass

THR(0'3) TP=2 FP=2
FN=0 TN=0

$$ACC = \frac{TP+TN}{TP+TN+FP+FN} = \frac{1}{2}$$

$$Recall = \frac{TP}{TP+FN} = \frac{2}{2} = 1$$

$$Precision = \frac{TP}{TP+FP} = \frac{2}{4} = 0'5$$

$$FPR = \frac{FP}{FP+TN} = \frac{2}{2} = 1$$

i) Accuracy, PR, ROC (THR=0'9)

Hours	26'8	57'8	71'8	95'2
Pass	0	0	1	1
LogOdds	-291'28	-627'63	-779'53	-1033'42
Prob	1	1	1	1
Class:0'9	Pass	Pass	Pass	Pass

THR(0'9) TP=2 FP=2
FN=0 TN=0

$$ACC = \frac{TP+TN}{TP+TN+FP+FN} = \frac{1}{2}$$

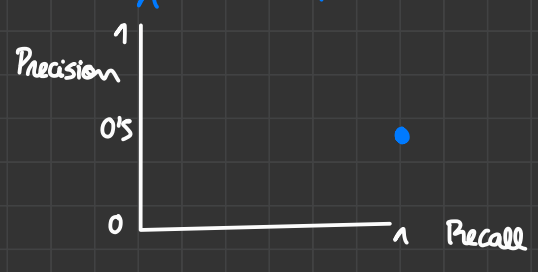
$$Recall = \frac{TP}{TP+FN} = \frac{2}{2} = 1$$

$$Precision = \frac{TP}{TP+FP} = \frac{2}{4} = 0'5$$

$$FPR = \frac{FP}{FP+TN} = \frac{2}{2} = 1$$

j) Curves ROC+PR

Error Metrica	THR=0'3	THR=0'5	THR=0'6	THR=0'7	THR=0'9
Acc	0'5	0'5	0'5	0'5	0'5
Precision	0'5	0'5	0'5	0'5	0'5
Recall	1	1	1	1	1
FPR	1	1	1	1	1



Valor Constante porque els valors són massa grans i els logodds no són útils.

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