

PROBLEM SET 1

Problems

Problem 1

$$(3 + 2i) + (-7 - i) \quad (1)$$

Problem 2

$$[(5 + 3i) + (-1 + 2i)] - (7 - 5i) \quad (2)$$

Problem 3

$$(5 + 3i) + [(-1 + 2i)] - (7 - 5i) \quad (3)$$

Problem 4

$$(2 - 3i)(4 + 2i) \quad (4)$$

Problem 5

$$(4 + 2i)(2 - 3i) \quad (5)$$

Problem 6

$$(2 - i)[(-3 + 2i)(5 - 4i)] \quad (6)$$

Problem 7

$$[(2 - i)(-3 + 2i)](5 - 4i) \quad (7)$$

Problem 8

$$(-1 + 2i)[(7 - 5i) + (-3 + 4i)] \quad (8)$$

Problem 9

Let $z_1 = 2 + i$, and $z_2 = 3 - 2i$. Calculate the following.

$$|3z_1 - 4z_2| \quad (9)$$

Problem 10

Let $z = 2 + i$. Calculate the following.

$$z^3 - 3z^2 + 4z - 8 \quad (10)$$

Problem 11

Let $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$. Calculate the following.

$$z^4 \quad (11)$$

Problem 12

Find x and y if the following holds.

$$3x + 2yi - ix + 5y = 7 + 5i \quad (12)$$

Problem 13

Express the following complex number in polar form.

$$2 + 2\sqrt{3}i \quad (13)$$

Problem 14

Express the following complex number in polar form.

$$-5 + 5i \quad (14)$$

Problem 15

Express the following complex number in rectangular form

$$3e^{\frac{\pi}{2}i} \quad (15)$$

Solutions

Problem 1

```
In[ ]:= (3 + 2 i) + (-7 - i)
Out[ ]:= -4 + i
```

Problem 2

```
In[*]:= (5 + 3 i) + (-1 + 2 i) - (7 - 5 i)
Out[*]= -3 + 10 i
```

Problem 3

```
In[*]:= (5 + 3 i) + ((-1 + 2 i) - (7 - 5 i))
Out[*]= -3 + 10 i
```

Problem 4

```
In[*]:= (2 - 3 i) (4 + 2 i)
Out[*]= 14 - 8 i
```

Problem 5

```
In[*]:= (4 + 2 i) (2 - 3 i)
Out[*]= 14 - 8 i
```

Problem 6

```
In[*]:= (2 - i) ((-3 + 2 i) (5 - 4 i))
Out[*]= 8 + 51 i
```

Problem 7

```
In[*]:= ((2 - i) (-3 + 2 i)) (5 - 4 i)
Out[*]= 8 + 51 i
```

Problem 8

```
In[*]:= (-1 + 2 i) ((7 - 5 i) + (-3 + 4 i))
```

Problem 9

```
In[*]:= Arg[3 (2 + i) - 4 (3 - 2 i)]
Out[*]=  $\pi - \text{ArcTan}\left[\frac{11}{6}\right]$ 
```

Problem 10

`In[]:= (2 + I)^3 - 3 (2 + I)^2 + 4 (2 + I) - 8`
`Out[]:=`

$$-7 + 3i$$

Problem 11

`In[]:=`
$$\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^4$$

`Out[]:=`

$$\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^4$$

Problem 12

$$\begin{aligned} 3x + 2yi - ix + 5y &= 7 + 5i \\ (3x + 5y) + i(-x + 2y) &= 7 + 5i \\ 3x + 5y &= 7 \\ -x + 2y &= 5 \end{aligned}$$

`In[]:= MatrixForm@RowReduce[{{3, 5, 7}, {-1, 2, 5}}]`
`Out[]//MatrixForm=`

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

We have that $x = -1$ and $y = 2$.

Problem 13

`In[]:= Abs[2 + 2 Sqrt[3] I]`
`Out[]:=`

$$4$$

`In[]:= Arg[2 + 2 Sqrt[3] I]`
`Out[]:=`

$$\frac{\pi}{3}$$

$$2 + 2\sqrt{3}i = 4e^{i\left(\frac{\pi}{3} + 2\pi k\right)}, \quad k \in \mathbb{Z}$$

Problem 14

`In[]:= Abs[-5 + 5 I]`
`Out[]:=`

$$5\sqrt{2}$$

In[*]:= Arg[-5 + 5 i]

Out[*]=

$$\frac{3\pi}{4}$$

$$-5 + 5i = 5\sqrt{2} e^{i\left(\frac{3\pi}{4} + 2\pi k\right)}, \quad k \in \mathbb{Z}$$

Problem 15

$$3 e^{\frac{\pi}{2} i}$$

$$\theta = \frac{\pi}{2}$$

$$r = 3$$

$$x = r \cos(\theta) = 3 \cos\left(\frac{\pi}{2}\right) = 0$$

$$y = r \sin(\theta) = 3 \sin\left(\frac{\pi}{2}\right) = 3$$

$$3i$$

In[*]:= Arg[(3 + 4 i) (2 - i)] // N

Out[*]=

$$0.463648$$

In[*]:= Arg[3 + 4 i] Arg[2 - i] // N

Out[*]=

$$-0.429938$$