Calculus of polar coordinate functions *

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Problem 1.4. *Find a coordinate for the polar coordinate function* $r = \sin 3\theta$, *where* $|r|$ *is at a maximum.*

Problem 1.5. *Find the slope of a tangent line to the polar coordinate function* $r = \sin 3\theta$, *where* $|r|$ *is at a maximum.*

Problem 1.6. *Find the maximum* y*-value of the polar coordinate curve* $r = 2(1 + \cos\theta)$.

2 Integrals of polar coordinate functions

Problem 2.1. *Derive an equation for the area of a polar coordinate curve on the arc from* θ_{start} *to* θ_{end}, *where* $r = f(\theta)$ *is continuous and positive.*

Problem 2.2. *Calculate the area enclosed by the polar coordinate curve* $r = \sin 3\theta$ *on the interval* $\theta = \left[0, \frac{\pi}{3}\right]$.

Problem 2.3. *Calculate the area enclosed by the cardioid* $r = 1 + \cos\theta$.

Hint: The interval is $\theta = [0, 2\pi]$ *and* $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

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Problem 2.4. *Calculate the area enclosed by the inner loop of the limaçon* $r = 1 + 2\cos\theta$.

Hint: The interval is $\theta = \left[\frac{5\pi}{6}, \frac{7\pi}{6}\right]$.

Problem 2.5. *Calculate the area enclosed by the inner loop of the lemniscate* $r^2 = \sin 2\theta$.

Hint: The interval is $\theta = \left[0, \frac{\pi}{4}\right]$ *and the area is twice this interval.*

Problem 2.6. *Calculate the area of one petal of the polar coordinate curve* $r = \sin 2\theta$.

Hint: The interval is $\theta = \left[0, \frac{\pi}{2}\right]$ *and* $\sin 2\theta = 2\sin\theta\cos\theta$

Problem 2.7. *calculate the area inside* $r = 2 + \cos 2\theta$ *on the interval* $\theta = [0, 2\pi]$.

Problem 2.8. *Calculate the area inside* $r = \cos^2\left(\frac{\theta}{2}\right)$

Problem 2.9. *Calculate the area swept out by* $r = \tan\theta$ *on the interval* $\theta = \left[0, \frac{\pi}{4}\right]$.

Problem 2.10. *Calculate the area inside the cardioid* $r = 1 + \cos\theta$ *and outside the circle* $r = 1$.

Hint: Consider only the first quadrant $\theta = \left[0, \frac{\pi}{4}\right]$. *The solution subtracts the area of the circle from the area of the cardioid.*