

Calculus of polar coordinate functions *

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1 Derivatives of polar coordinate functions

Problem 1.1. Derive an equation for the first derivative of a polar coordinate function $r = f(\theta)$.

Problem 1.2. Calculate the slope of the polar coordinate function $r = \sin 3\theta$.

Problem 1.3. Find the coordinates where the tangent line to the polar coordinate functions $r = \sin 3\theta$ is horizontal.

Problem 1.4. Find a coordinate for the polar coordinate function $r = \sin 3\theta$, where $|r|$ is at a maximum.

Problem 1.5. Find the slope of a tangent line to the polar coordinate function $r = \sin 3\theta$, where $|r|$ is at a maximum.

Problem 1.6. Find the maximum y -value of the polar coordinate curve $r = 2(1 + \cos \theta)$.

2 Integrals of polar coordinate functions

Problem 2.1. Derive an equation for the area of a polar coordinate curve on the arc from θ_{start} to θ_{end} , where $r = f(\theta)$ is continuous and positive.

Problem 2.2. Calculate the area enclosed by the polar coordinate curve $r = \sin 3\theta$, where $\theta \in [0, \frac{\pi}{3}]$.

Problem 2.3. Calculate the area enclosed by the cardioid $r = 1 + \cos \theta$.

Hint: The interval is $\theta \in [0, 2\pi]$ and $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$.

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Problem 2.4. Calculate the area enclosed by the inner loop of the limaçon $r = 1 + 2 \cos \theta$.

Hint: The interval is $\theta \in \left[\frac{5\pi}{6}, \frac{7\pi}{6}\right]$.

Problem 2.5. Calculate the area enclosed by the inner loop of the lemniscate $r^2 = \sin 2\theta$.

Hint: The interval is $\theta \in \left[0, \frac{\pi}{4}\right]$ and the area is twice this interval.

Problem 2.6. Calculate the area of one petal of the polar coordinate curve $r = \sin 2\theta$.

Hint: The interval is $\theta \in \left[0, \frac{\pi}{2}\right]$ and $\sin 2\theta = 2 \sin \theta \cos \theta$

Problem 2.7. Calculate the area inside the polar coordinate function $r = 2 + \cos 2\theta$, where $\theta \in [0, 2\pi]$.

Problem 2.8. Calculate the area inside the polar coordinate function $r = \cos^2\left(\frac{\theta}{2}\right)$.

Problem 2.9. Calculate the area swept out by by the polar coordinate function $r = \tan \theta$, where $\theta \in \left[0, \frac{\pi}{4}\right]$.

Problem 2.10. Calculate the area inside the cardioid $r = 1 + \cos \theta$ and outside the circle $r = 1$.

Hint: Consider only the first quadrant $\theta \in \left[0, \frac{\pi}{4}\right]$. The solution subtracts the area of the circle from the area of the cardioid.