Parametric equations *

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1 Introduction

Parametric equations in $\mathbb{R}^2$ express a single variable function in terms of a parameter (another variable), such as t. As t varies over an interval, so does the position on the curve (single variable function) originally given.

2 Plane curves and parametric equations

Definition 2.1. *Given two functions $x\left(t\right)$ and $y\left(t\right)$ on the same domain D the equations $x = \left(t\right)$ and $y = \left(t\right)$ are termed **parametric equations**, such that any point on the plane is specified by $\left(x, y\right) = \left[x\left(t\right), y\left(t\right)\right]$.*

Explanation 2.1. *To parametrize a single variable polynomial $y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, set $x = t$ and replace x with t in the polynomial.*

Definition 2.2. *The parametric equations for a **straight line segment** are $x = a + bt$ and $y = c + dt$ on the domain of some parameter t, where $a, b \in \mathbb{R}$.*

Explanation 2.2. *To calculate the parametric equations for a line segment $x = a + bt$ and $y = c + dt$ from some points $\left(p, q\right)$ and $\left(r, s\right)$.*

1. *Set $t = 0$ and express $x = p$ and $y = q$*
2. *Set $t = 1$ and solve $p + b = r$ for b and $q + d = s$ for d*

Explanation 2.3. *To find the parametric equations of a function between two points $\left(a, b\right)$ and $\left(c, d\right)$.*

1. *Set $t = 0$ and find an expression for $x\left(t\right)$ such that $x\left(0\right) = a$*
2. *Substitute this expression into $y = f\left(x\right)$*

*Vector Calculus

3. *Given the parametrized version of x above, find a value for t such that $x(t)$ is equal to the second point's x value*

Explanation 2.4. *To find the intersection of two parametric curves.*

1. *Set the values for x equal to each other, as well as the values for y*
2. *Solve the two equations in two unknowns by isolation, substitution, and back substitution*