Calculus of parametric equations example problems*

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Problem 1.6. *Find the second derivative of the parametric equations* $x = t^3 + t$ *and* $y = t^7 + t + 1$*.*

Problem 1.7. *Find the first derivative of the parametric equations* $x = x(t)$ *and* $y = y(t)$ *using time derivative dot notation.*

Problem 1.8. *Find the second derivative of the parametric equations* $x = x(t)$ *and* $y = y(t)$ *using time derivative dot notation.*

Problem 1.9. *Find the angle at witch the cycloid* $(x, y) = (at - a \sin t, a - a \cos t)$ *meets the* x *axis at the origin.*

Hint: Use the limit as $t \to 0^+$*.*

Problem 1.10. *Find the slope of the parametric curve* $(x, y) = \left(t^5 + \sin 2\pi t, t + e^t\right)$

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Problem 1.11. *Find the slope of the curve* $(x, y) = \left(t^2 + e^t, t + e^t\right)$ *at the point* $(1, 1)$.

Hint: Find the value of t *at the point.*

Problem 1.12. *Find the point(s) on the parametric curve* $(x, y) = (2\cos t + \sin 2t, 2\sin t + \cos 2t)$ *where there is a horizontal tangent line.*

Hint: This is the point where the numerator of the first derivative is 0*, but the denominator is not.*

Problem 1.13. *Find the point(s) on the parametric curve* $(x, y) = (2\cos t + \sin 2t, 2\sin t + \cos 2t)$ *where there is a vertical tangent line.*

Hint: A vertical tangent line to a parametric curve is where the numerator of the first derivative not 0*, while the denominator is* 0.

Problem 1.14. *Find the points on the parametric curve* $(x, y) = (\cos 2t, \sin 3t)$*, where there are horizontal tangent lines.*

Hint: This is the point where the numerator of the first derivative is 0*, but the denominator is not.*

Problem 1.15. *Find the points on the parametric curve* $(x, y) = (\cos 2t, \sin 3t)$*, where there are vertical tangent lines.*

Hint: A vertical tangent line to a parametric curve is where the numerator of the first derivative not 0*, while the denominator is* 0.

2 Parametric curve integration

Problem 2.1. *Consider a single variable function,* f *, continuous on an interval* $a \leq x \leq b$*. Derive an equation for the area enclosed by the parametric curve of this function, given* $c \leq t \leq d$ *(with no intersection of the curve on this interval, except perhaps for a coincident start and end position and a clockwise traversal on the interval).*

Problem 2.2. *Find the area enclosed by the parametric curve* $(x, y) = (2\cos t + \sin 2t, 2\sin t + \cos 2t)$ *for* $0 \leq t \leq 2\pi$.

Hint: Use $\int_c^d y'(t)\, x(t)\ dt$ *or* $-\int_c^d y(t)\, x'(t)\ dt$

Problem 2.3. *Find the area enclosed by the ellipse in (3).*

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1}$$

Hint: Use a trigonometric identity.

Problem 2.4. *Find the area enclosed by the the parametric equations given in (4).*

$$(x, y) = \left(\frac{1}{2}\cos t - \frac{1}{4}\cos 2t, \frac{1}{2}\sin t - \frac{1}{4}\sin 2t\right) \tag{2}$$

Problem 2.5. *Find the area enclosed by the parametric curve* $(x, y) = (\cos t, \sin 2t)$ *on the interval* $\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$

Problem 2.6. *Find the area enclosed by the parametric equation* $(x, y) = \left(t^3 - 4t, t^4 - 1\right)$ *on the interval* $-2 \leq t \leq 2$.