Calculus of parametric equations *

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Contents

1 Arc length

Problem 1.1. *Derive an equation for the arc length of a parametric curve,* $(x, y) = (x(t), y(t))$*, that is continuous in* $a \leq t \leq b$*, has continuous first derivatives in the interval, and has at most a finite number of intersections in the interval.*

Problem 1.2. *Calculate the length of the parametric curve* $(x, y) = (a\cos\theta, a\sin\theta)$ *on the interval* $0 \leq \theta\, 2 \leq \pi$*.*

Problem 1.3. *Calculate the arc length of the parametric curve in (1) below.*

$$(x, y) = \left(\frac{1}{2}t^2, \frac{1}{9}(6t+9)^2\right), 0 \leq t \leq 4 \tag{1}$$

Problem 1.4. *Calculate the arc length of the parametric curve in (2) below.*

$$(x, y) = (\cos t + t\sin t, \sin t - t\cos t), t \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right] \tag{2}$$

Problem 1.5. *Calculate the arc length of the parametric curve in (3) below.*

$$(x, y) = \left(\frac{1}{2}\ln 1 + t^2, \tan^{-1}(t)\right), 0 \leq t \leq 1 \tag{3}$$

*A course in vector calculus