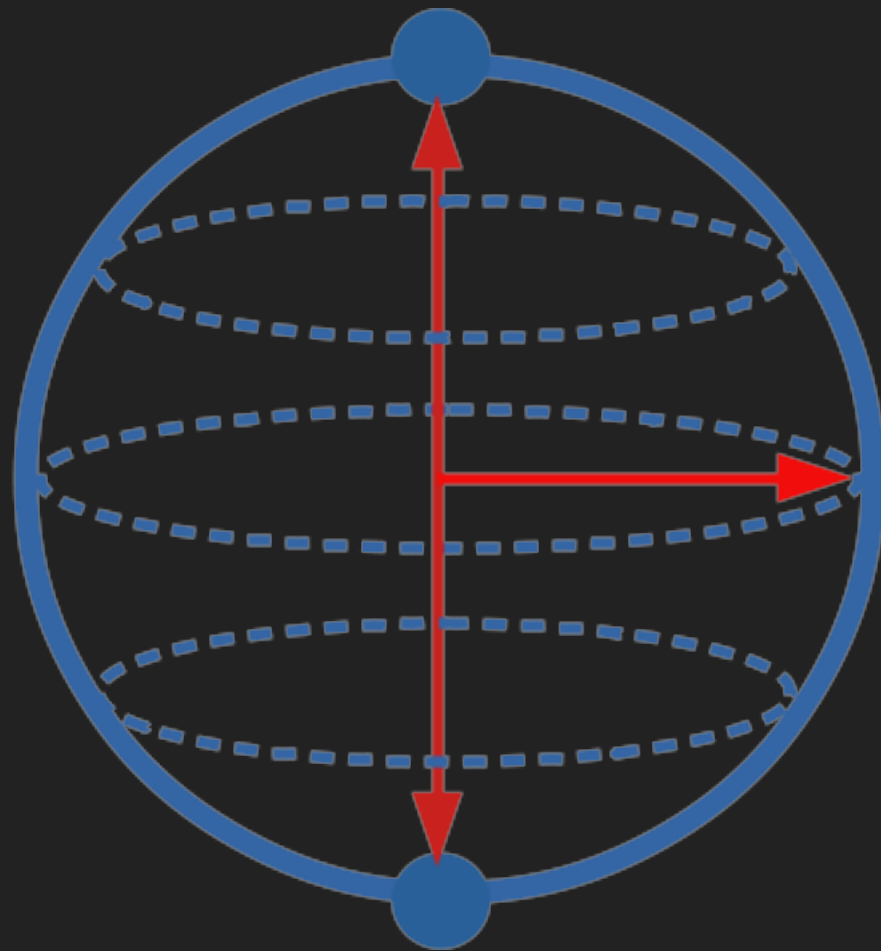


DISPLAY OF QUANTUM ENTANGLEMENT

MERMIN-GHZ GAME

PREREQUISITE FOR QUANTUM PSEUDO-TELEPATHY GAMES

- ▶ Qubit: analogous of bit in classical computer



THE GAME

- ▶ 3 players Alice, Bob, Charlie
- ▶ 3 bit inputs, 3 bit outputs
- ▶ Promise on the inputs: even parity
- ▶ Condition to win:

$$\sum_{i=1}^n a_i \equiv \frac{\sum_{i=1}^n x_i}{2^\ell} \pmod{2}.$$

$$a \oplus b \oplus c \equiv x \wedge y \wedge z$$

CLASSICAL STRATEGY

- ▶ Cannot win 100% of time
- ▶ optimal: 75%

$$a_0 + b_0 + c_0 \equiv 0$$

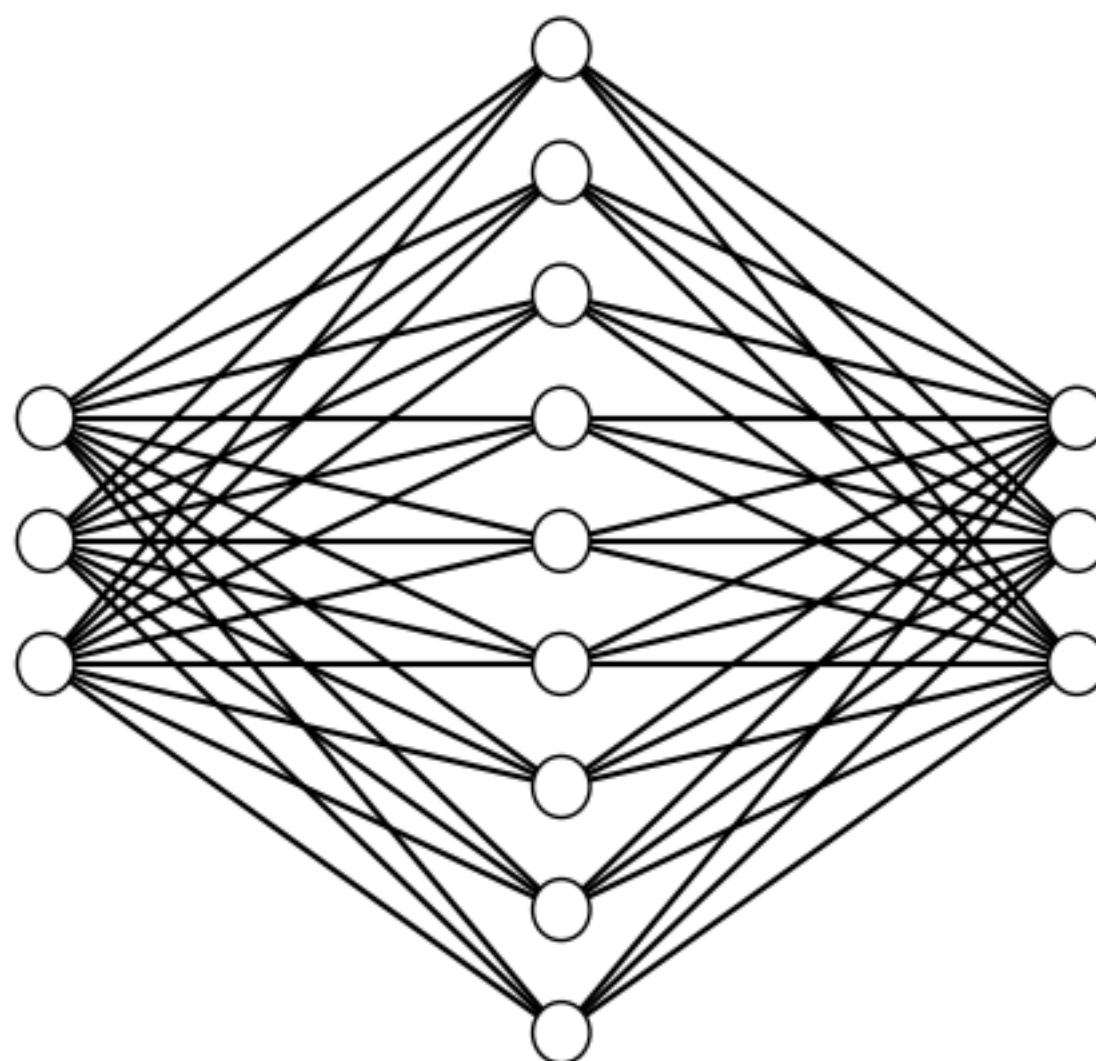
$$a_0 + b_1 + c_1 \equiv 1$$

$$a_1 + b_0 + c_1 \equiv 1$$

$$a_1 + b_1 + c_0 \equiv 1$$

1 HIDDEN LAYER NEURAL NETWORK

- ▶ By the Universal Approximation Theorem:
 - ▶ 1 hidden layer can approximate any binary function
 - ▶ Xor works!
 - ▶ But cannot learn our game
 - ▶ Reach the randomized strategy



QUANTUM STRATEGY

$$\frac{1}{\sqrt{2}}|0^n\rangle + \frac{1}{\sqrt{2}}|1^n\rangle$$

1. apply to his share of the entangled state the unitary transformation S defined by

$$|0\rangle \mapsto |0\rangle \quad (3.2)$$

$$|1\rangle \mapsto e^{\pi i x_i / 2^\ell} |1\rangle, \quad (3.3)$$

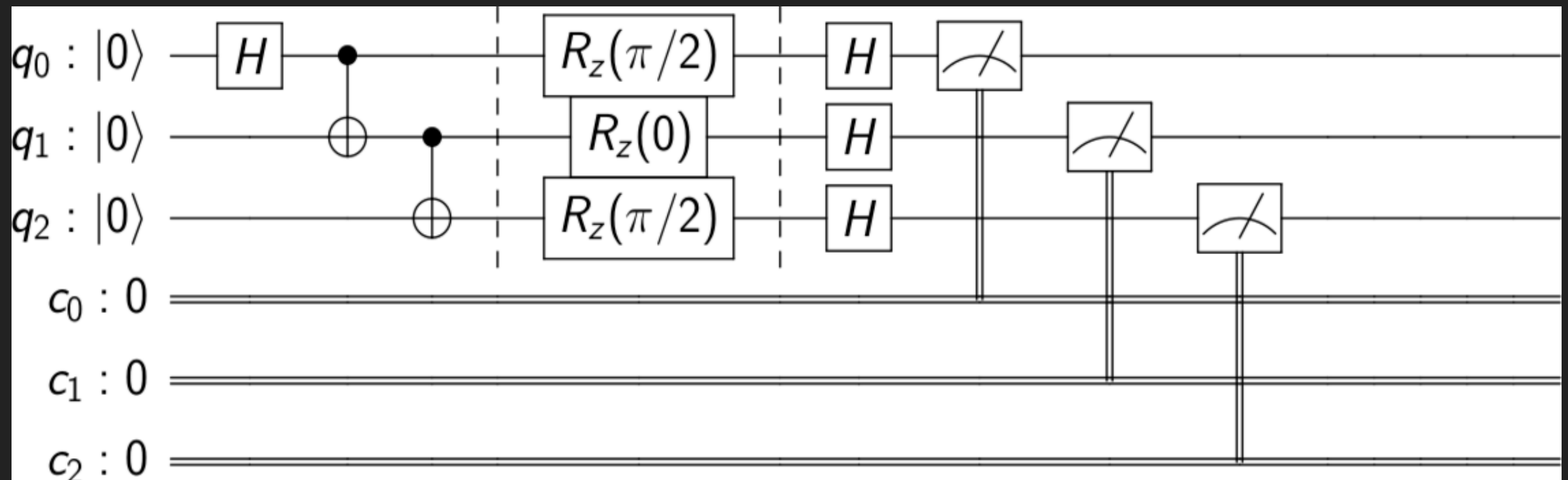
2. apply the Walsh–Hadamard transform, H , defined as usual by

$$|0\rangle \mapsto \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \quad (3.4)$$

$$|1\rangle \mapsto \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle; \quad (3.5)$$

3. measure the qubit in the computational basis to obtain a_i ;
4. output a_i .

QUANTUM CIRCUIT



CLASSICAL STRATEGY

- ▶ random: each output bit: 0/1 with prob $1/2$
- ▶ optimal: (1 instance) flip input bit
 - ▶ Wins $3/4$ of the time

DEMO

DEMO



Let's see for yourself!