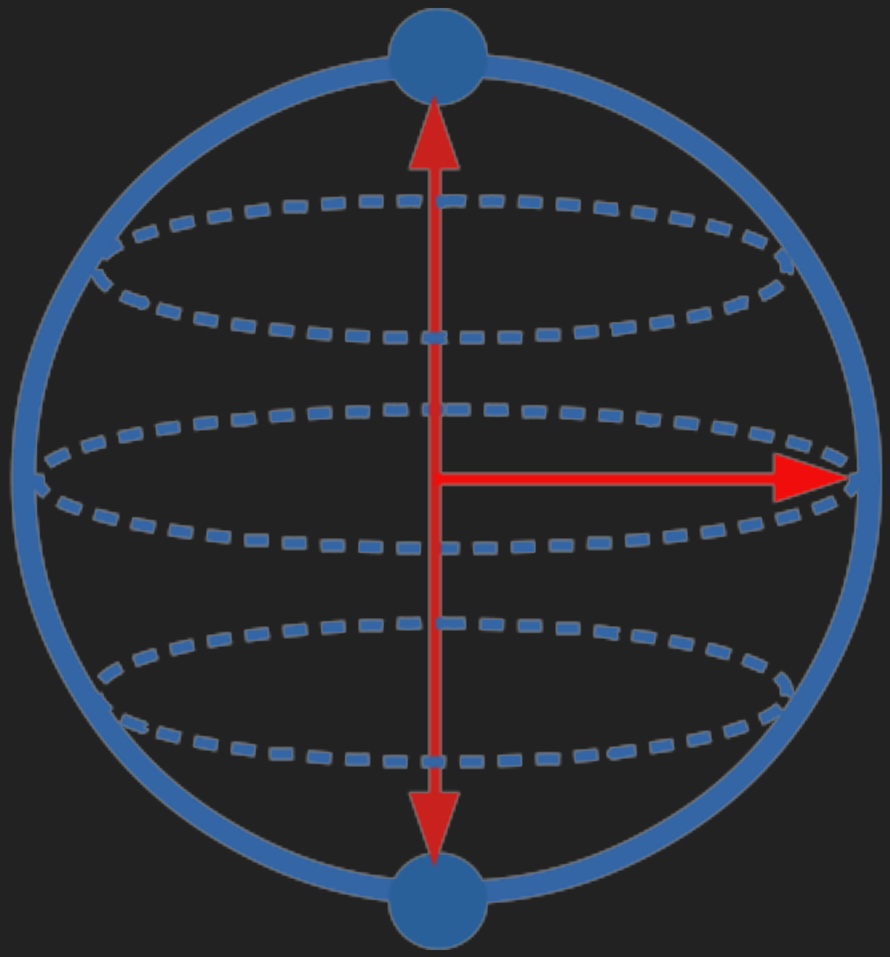
DISPLAY OF QUANTUM ENTANGLEMENT

MERMIN-GHZ GAME

PREREQUISITE FOR QUANTUM PSEUDO-TELEPATHY GAMES

- Qubit: analogous of bit in classical computer



THE GAME

- 3 players Alice, Bob, Charlie
- 3 bit inputs, 3 bit outputs
- Promise on the inputs: even parity
- Condition to win:
- $a + b + c = x \vee y \vee z \pmod 2$

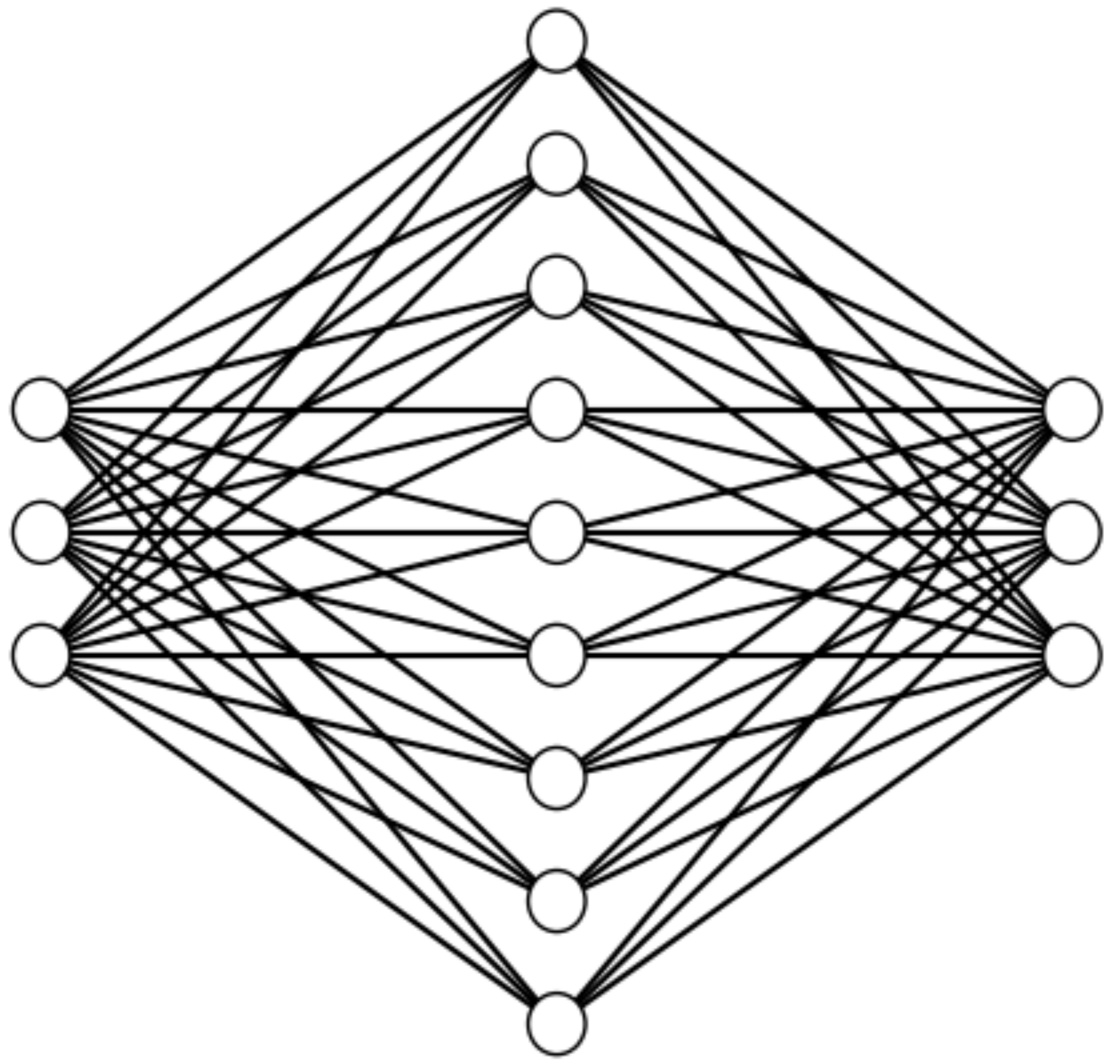
CLASSICAL STRATEGY

- Cannot win 100% of time
- optimal: 75%

$$
\begin{aligned}
a_0 + b_0 + c_0 &\equiv 0 \\
a_0 + b_1 + c_1 &\equiv 1 \\
a_1 + b_0 + c_1 &\equiv 1 \\
a_1 + b_1 + c_0 &\equiv 1
\end{aligned}
$$

1 HIDDEN LAYER NEURAL NETWORK

- By the Universal Approximation Theorem:
  - 1 hidden layer can approximate any binary function
  - Xor works!
  - But cannot learn our game
  - Reach the randomized strategy



QUANTUM STRATEGY

$$\frac{1}{\sqrt{2}}|0^n\rangle + \frac{1}{\sqrt{2}}|1^n\rangle$$

1. apply to his share of the entangled state the unitary transformation S defined by

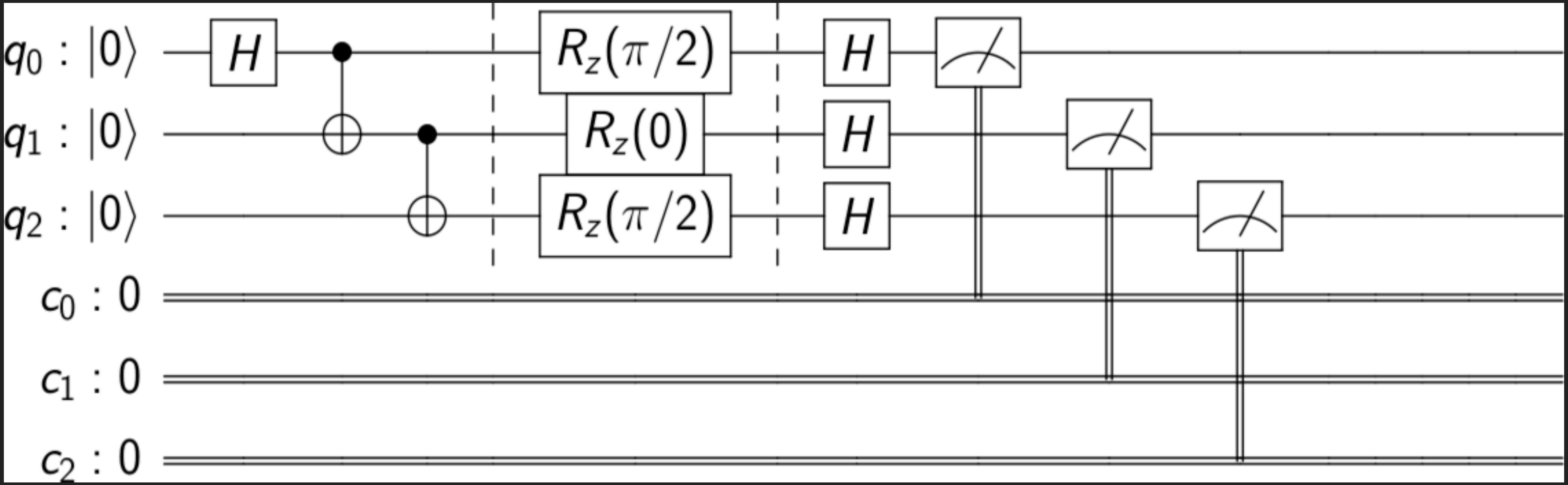
$$|0\rangle \mapsto |0\rangle \tag{3.2}$$
$$|1\rangle \mapsto e^{\pi \imath x_i/2^\ell}|1\rangle\,, \tag{3.3}$$

2. apply the Walsh–Hadamard transform, H, defined as usual by

$$|0\rangle \mapsto \tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle \tag{3.4}$$
$$|1\rangle \mapsto \tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle\,; \tag{3.5}$$

3. measure the qubit in the computational basis to obtain a_i;
4. output a_i .

QUANTUM CIRCUIT



CLASSICAL STRATEGY

- random: each output bit: 0/1 with prob 1/2
- optimal: (1 instance) flip input bit
  - Wins 3/4 of the time

DEMO

- Let’s see for yourself!