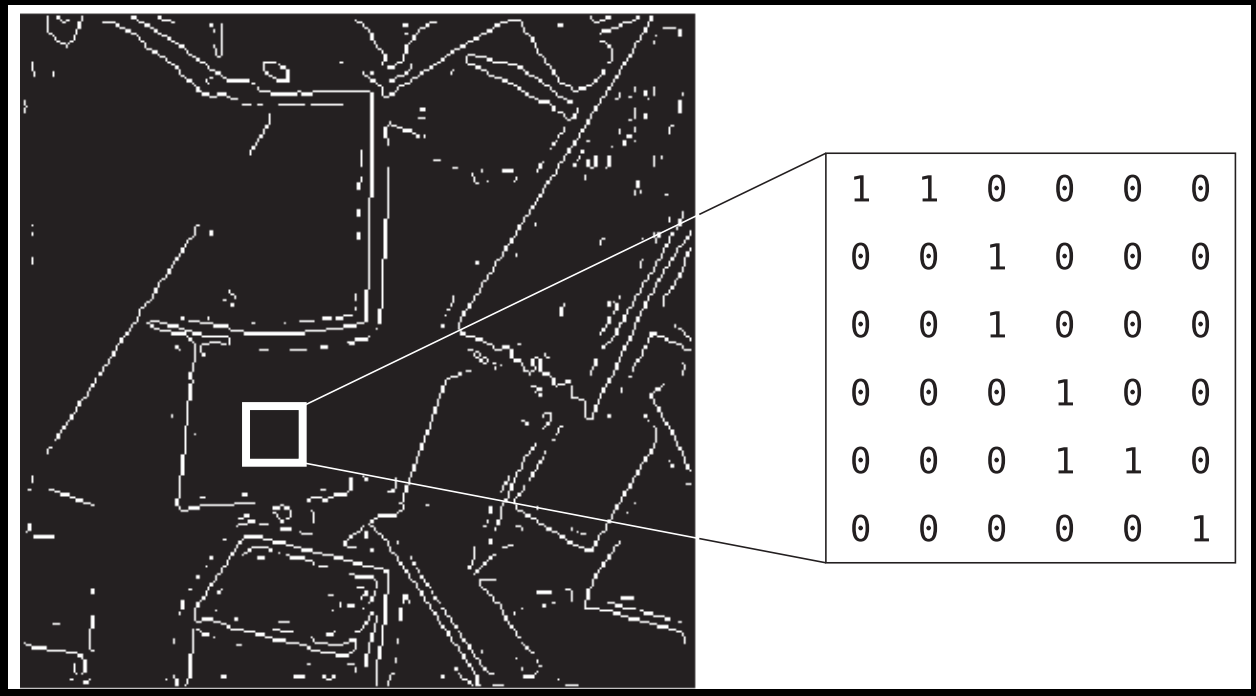
CS 470

Thresholding

Binary Image

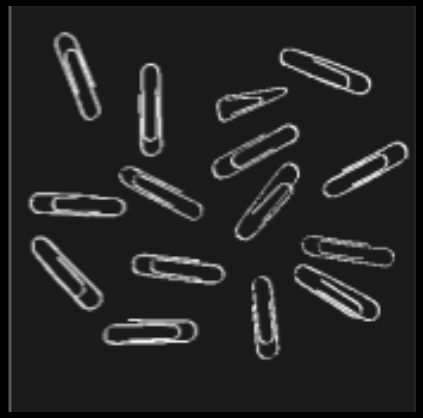


1. Draw a function y = f(x) that will take image pixels (x) and output a binary image (y)

Goal of Thresholding

- To isolate objects from the background.



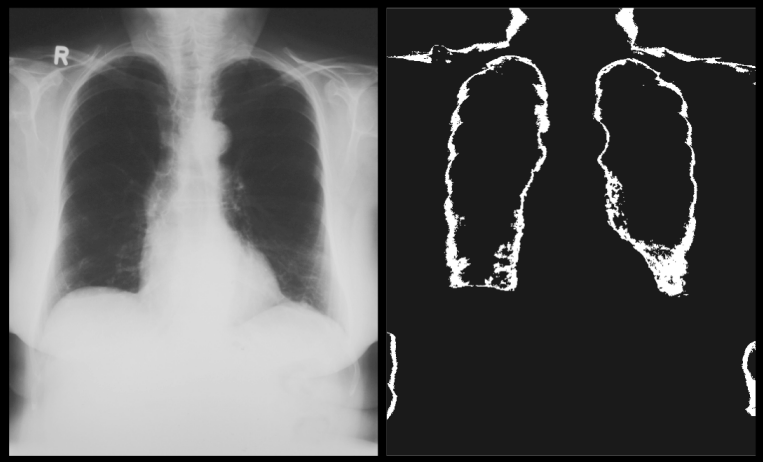



Single Thresholding

- Gray-scale image is turned into a binary (black and white) image
  - Choose a gray level T in original image
  - Turn every pixel black or white depending on whether its gray value is greater than or less than T
  - MATLAB: im2bw(image, T)

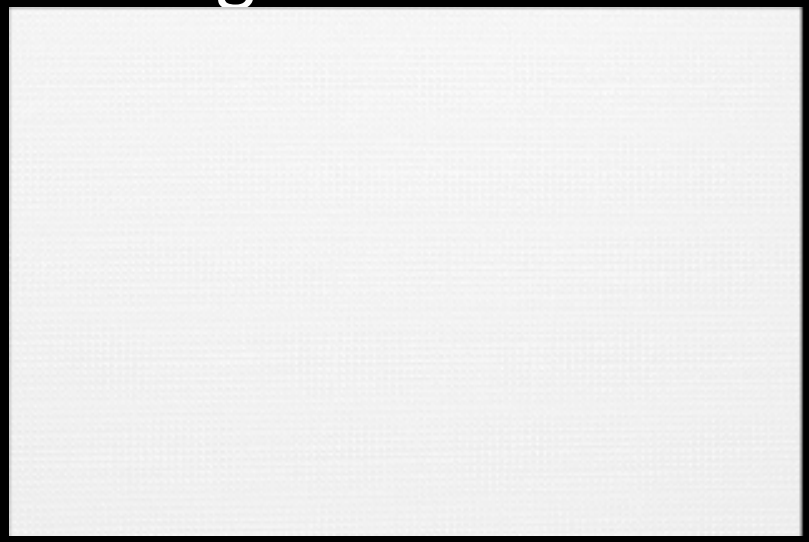
Double Thresholding

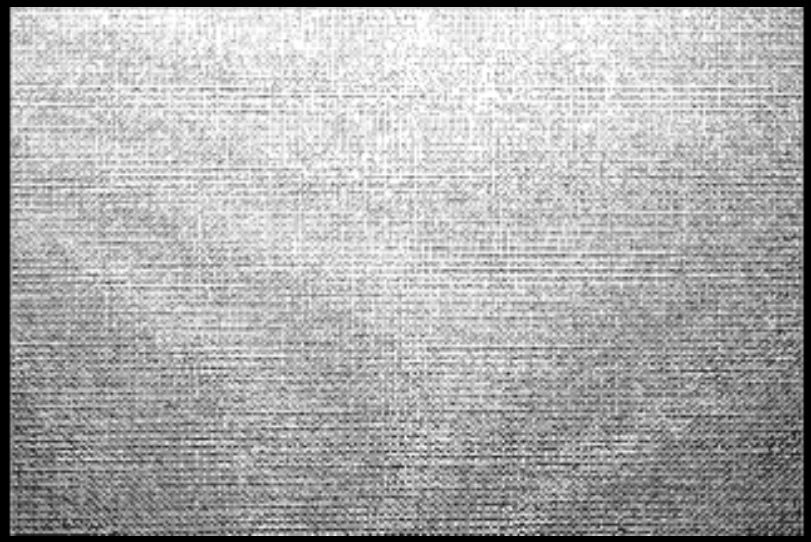
- Choose two values T_1 and T_2 and apply a thresholding operation as:
  - Pixel becomes white, if its gray level is between T_1 and T_2
  - Black, if its gray level is otherwise



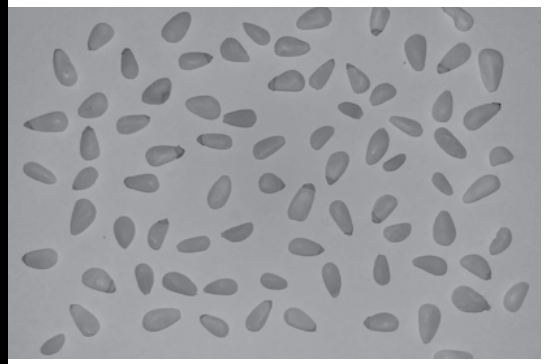
Applications of Thresholding

- Remove unnecessary detail from an image
  - Helps in investigating sizes, shapes or numbers of objects
- To bring out hidden detail



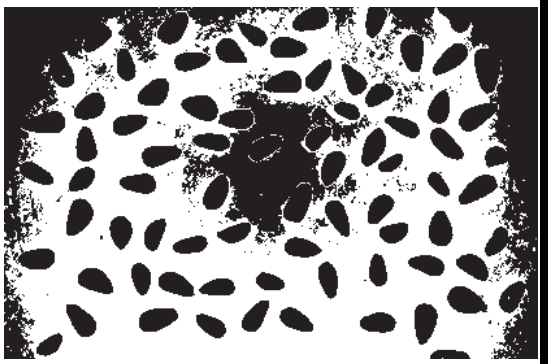


How to choose T?



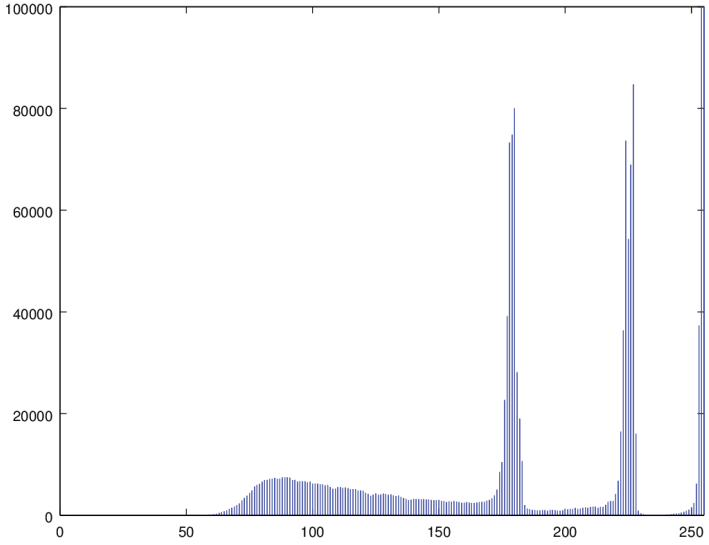
`n`: Original image

`n1`: Threshold too low

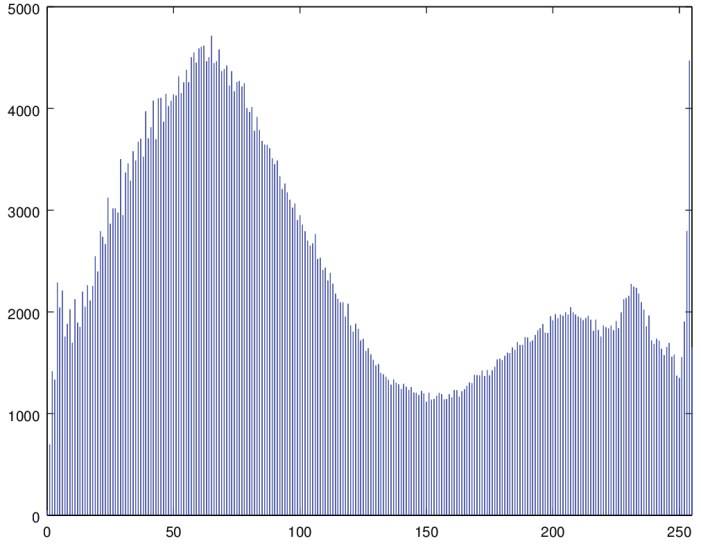


`n2`: Threshold too high

Check histograms!

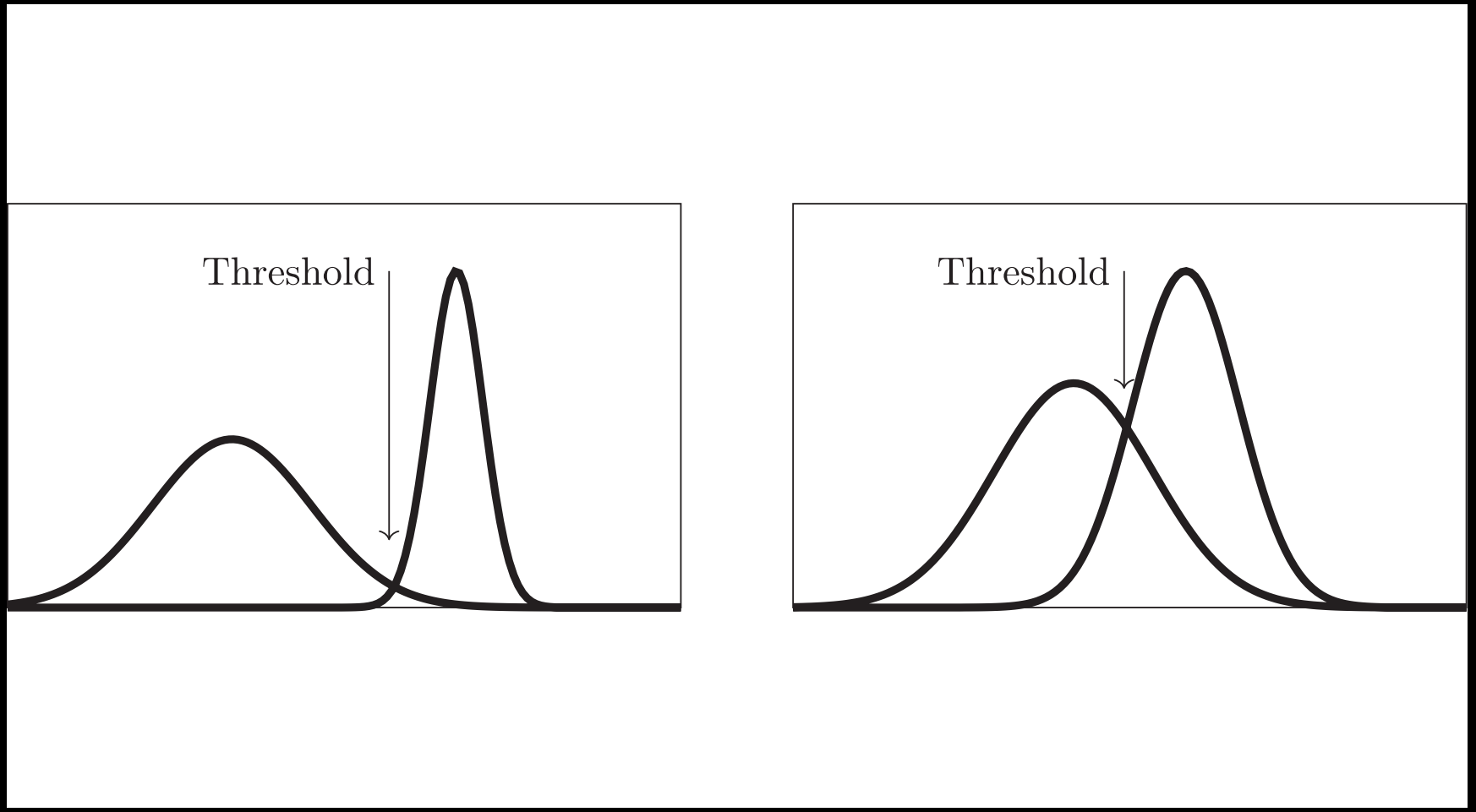


Pine nuts image: `pinenuts.png`



Daisies image: `daisies.png`

Splitting up histogram



Finding threshold automatically: Otsu Method

- Maximize the *"inter-class variance":* $w_f w_b (\mu_f - \mu_b)^2$
- p_i : probability of a pixel having gray value i
- t : threshold value (to determine)

$$w_b = \sum_{k=0}^{t-1} p_k,$$
$$w_f = \sum_{k=t}^{L-1} p_k.$$

$$\mu_b = \frac{1}{w_b} \sum_{k=0}^{t-1} k p_k,$$
$$\mu_f = \frac{1}{w_f} \sum_{k=t}^{L-1} k p_k.$$

- What is $w_b + w_f$?
- What is $w_b \mu_b + w_f \mu_f$?

Otsu example

i	n_i	p_i
0	4	0.062500
1	8	0.125000
2	10	0.156250
3	9	0.140625
4	4	0.062500
5	8	0.125000
6	12	0.187500
7	9	0.140625

Otsu example

k	n_k	p_k	w_b	w_f	kp_k	$\sum kp_k$	μ_b	μ_f	vars
0	4	0.06250	0.06250	0.93750	0.00000	0.00000	0.00000	4.10000	0.98496
1	8	0.12500	0.18750	0.81250	0.12500	0.12500	0.66667	4.57692	2.32935
2	10	0.15625	0.34375	0.65625	0.31250	0.43750	1.27273	5.19048	3.46246
3	9	0.14062	0.48438	0.51562	0.42188	0.85938	1.77419	5.78788	4.02348
4	4	0.06250	0.54688	0.45312	0.25000	1.10938	2.02857	6.03448	3.97657
5	8	0.12500	0.67188	0.32812	0.62500	1.73438	2.58140	6.42857	3.26296
6	12	0.18750	0.85938	0.14062	1.12500	2.85938	3.32727	7.00000	1.63013
7	9	0.14062	1.00000	0.00000	0.98438	3.84375	3.84375	–	–

ISODATA: Iterative Self-Organizing Data Analysis Technique A

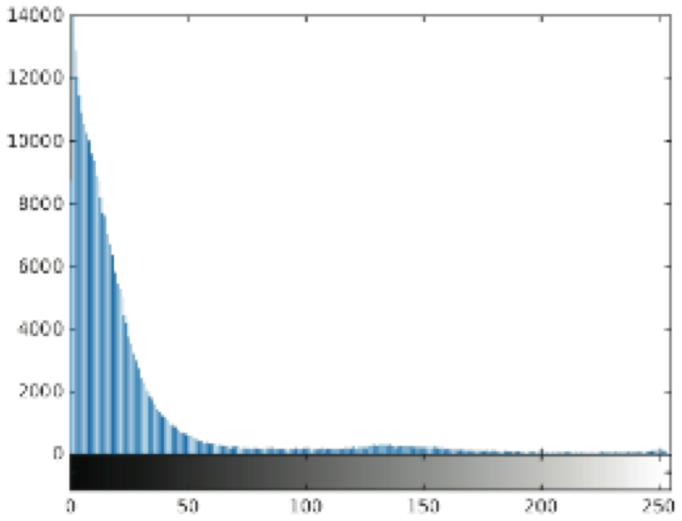
Section 9.4: Reading

1. Pick a precision value ε and a starting threshold value t. Often $t = L/2$ is used.
2. Compute μ_f and μ_b for this threshold value.
3. Compute $t' = (\mu_b + \mu_f)/2$.
4. If $|t - t'| < \varepsilon$ the stop and return t, else put $t = t'$ and go back to step 2.

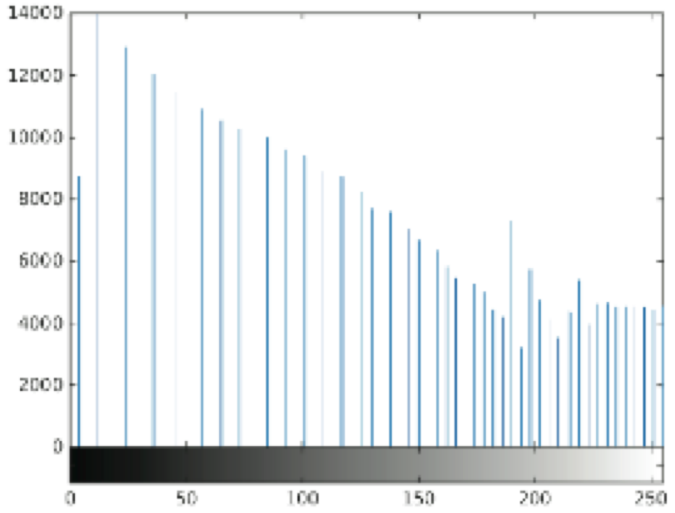
Thresholding in Matlab

- Matlab:
  - graythresh,
  - otsuthresh,
  - imbinarize

Histogram: Poor Contrast



Histogram: Good Contrast



Histogram Equalization

- Suppose:
  - image has *L different gray levels 0, 1, 2, … L − 1,*
  - *gray level i occurs n_i times in the image.*
  - $n_0 + n_1 + n_2 + \cdots + n_{L-1} = n$.
- Change gray level i to: $\left(\frac{n_0 + n_1 + \cdots + n_i}{n}\right)(L-1)$.
- Round the result to nearest integer

Choose the correct answer

- The cumulative sum for ith gray-level is given by

$$c_i = n_0 + n_1 + n_2 + \cdots + n_i$$

As i increases, c_i must increase too.

A. True
B. False
C. Depends on n_i
D. I don't know

Choose the correct answer

- The cumulative sum for i^{th} gray-level is given by

$$c_i = n_0 + n_1 + n_2 + \cdots + n_i$$

What is the best way to compute c_{i+1}?

A. Compute the sum $n_0 + n_1 + n_2 + \cdots + n_i$
B. Use the previous sum c_i
C. There is no shortcut to success and in computing c_{i+1}

Choose the correct answer

- The cumulative sum for L-1th gray-level is given by

$$c_{L-1} = n_0 + n_1 + n_2 + \cdots + n_{L-1} = n$$

If the size of image is [h, w] then c_{L-1} =

A. hw
B. Depends on n_{L-1}
C. Does not depend on h or w
D. I don't know

Perform Histogram Equalization

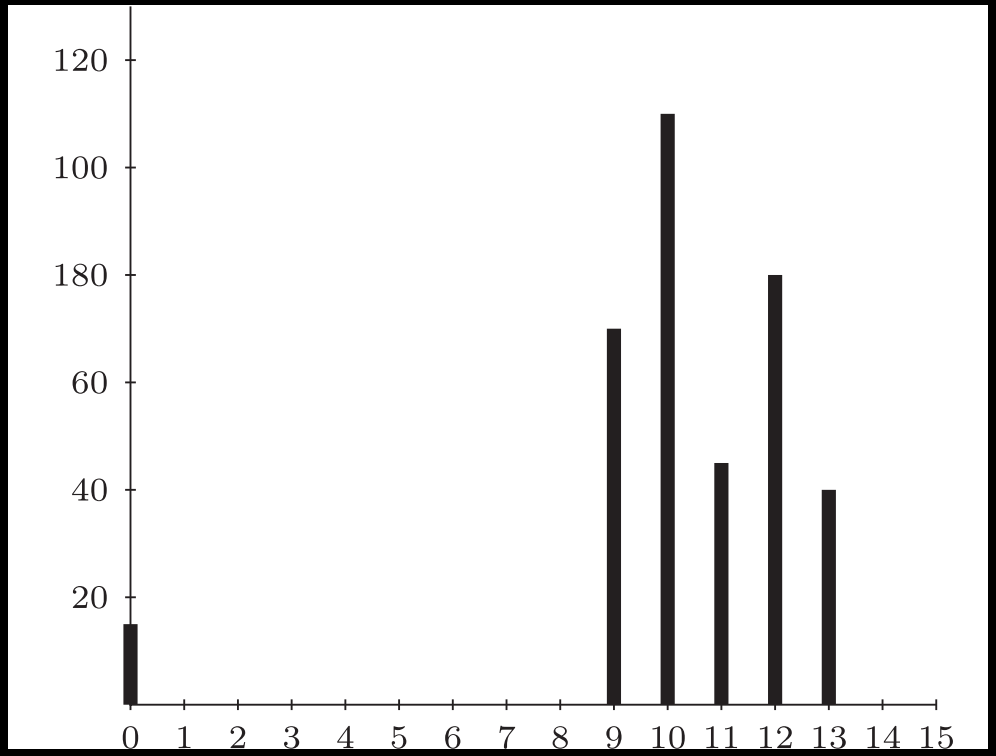
Given are number of pixels at each of the gray levels 0–15.

1. Compute the histogram n_i
2. Then, compute the cumulative sum for ith gray-level: $c_i = n_0 + n_1 + n_2 + \cdots + n_i$
3. Apply formula to compute the transformation
   f(i) = c_i*(L-1)/n

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	40	80	45	110	70	0	0	0	0	0	0	0	0	15

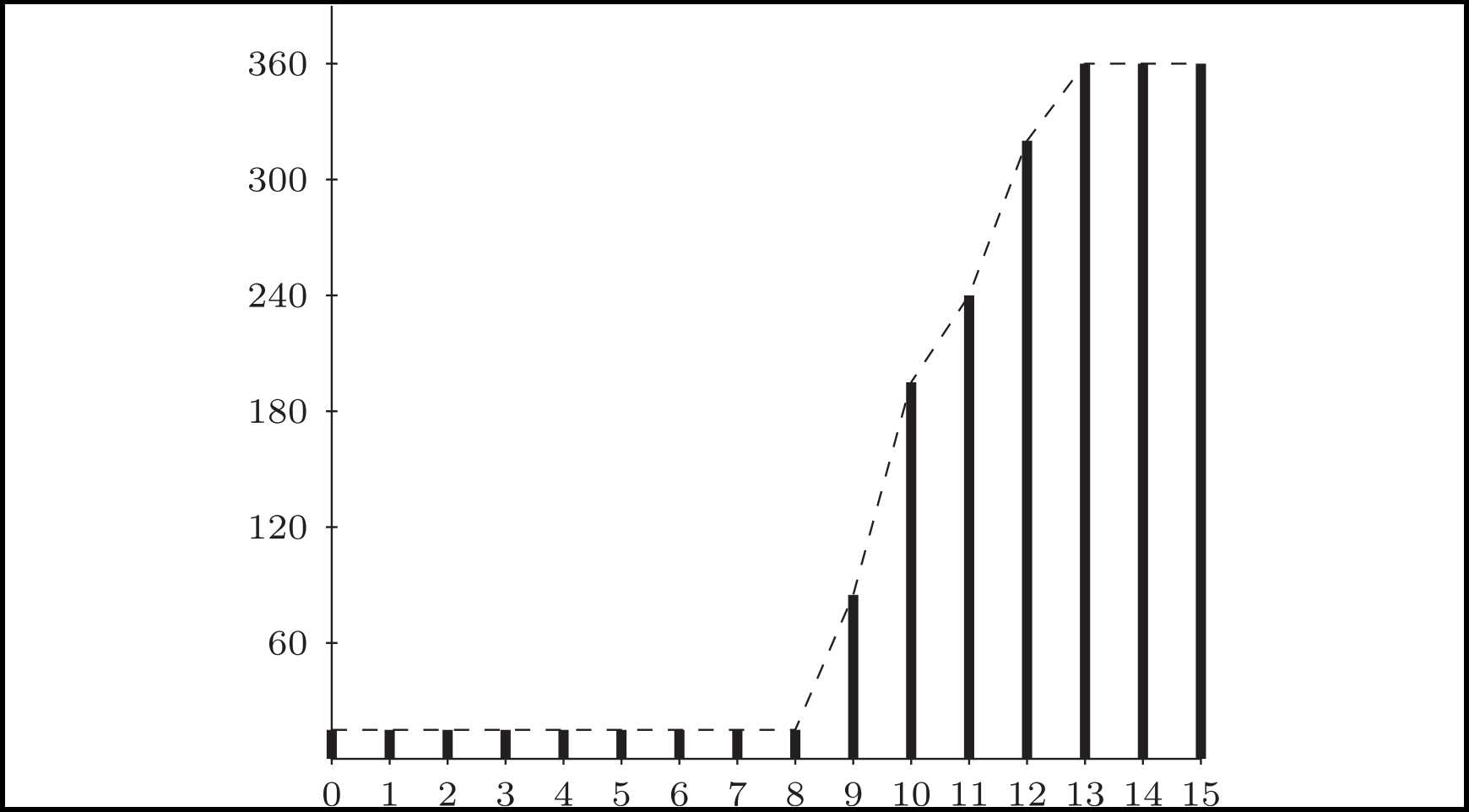
Answer:
0, 0, 2, 5, 7, 11, 14, 14, 14, 14, 14, 14, 14, 14, 14, 15

Why it works?

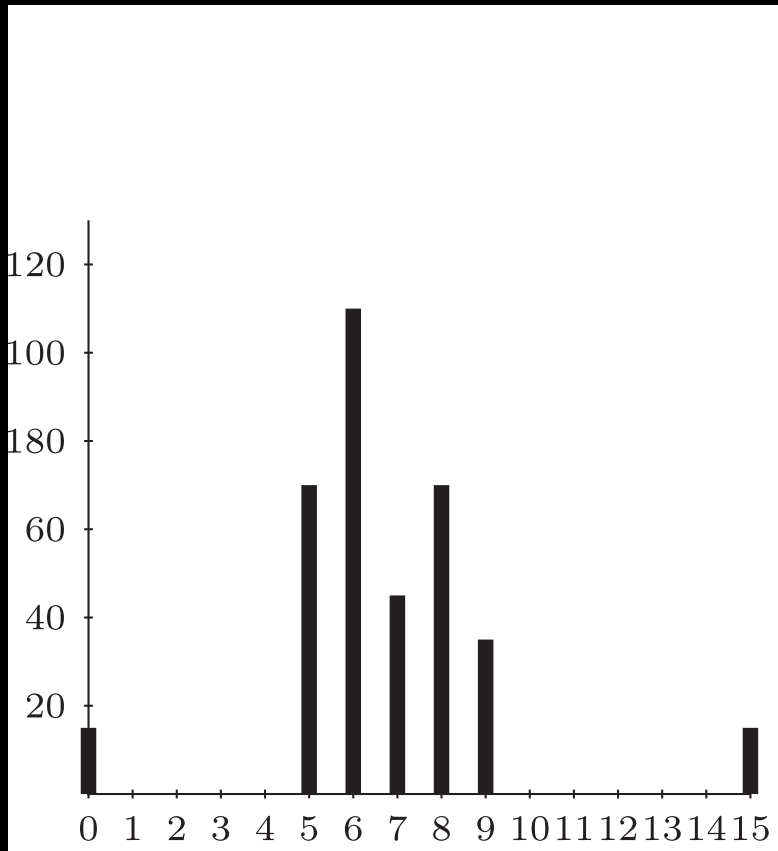


Gray level i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_i	15	0	0	0	0	0	0	0	0	70	110	45	80	40	0	0

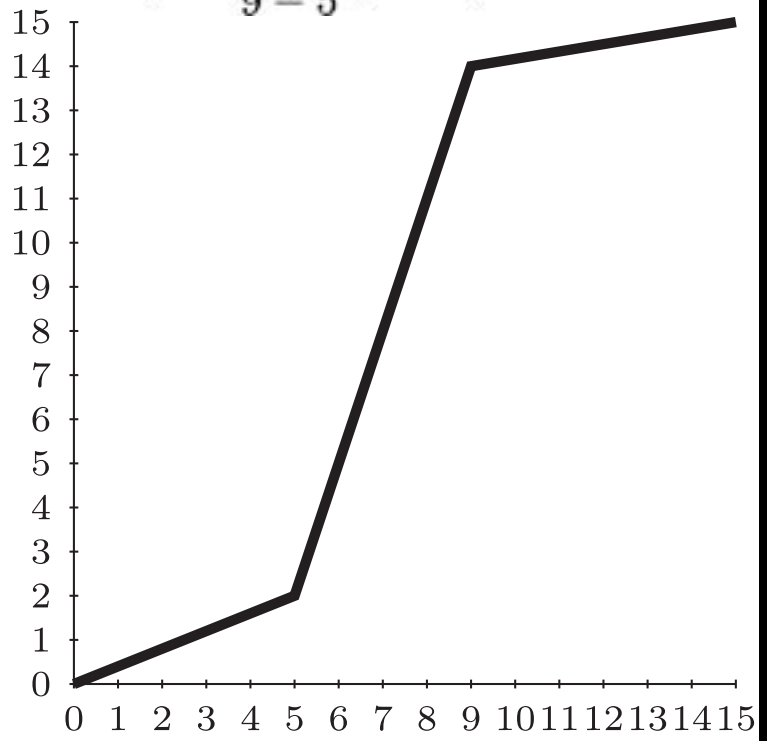
Why it works?:
Plotting the cumulative sum c_i



Recall: Histogram Stretching

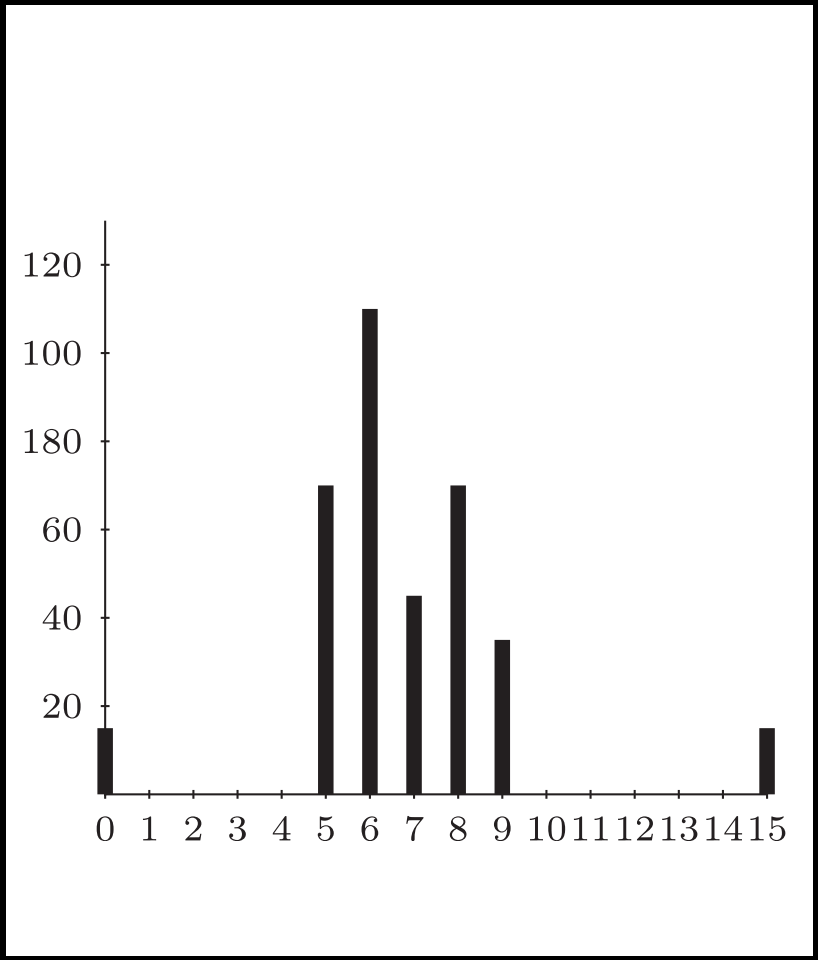


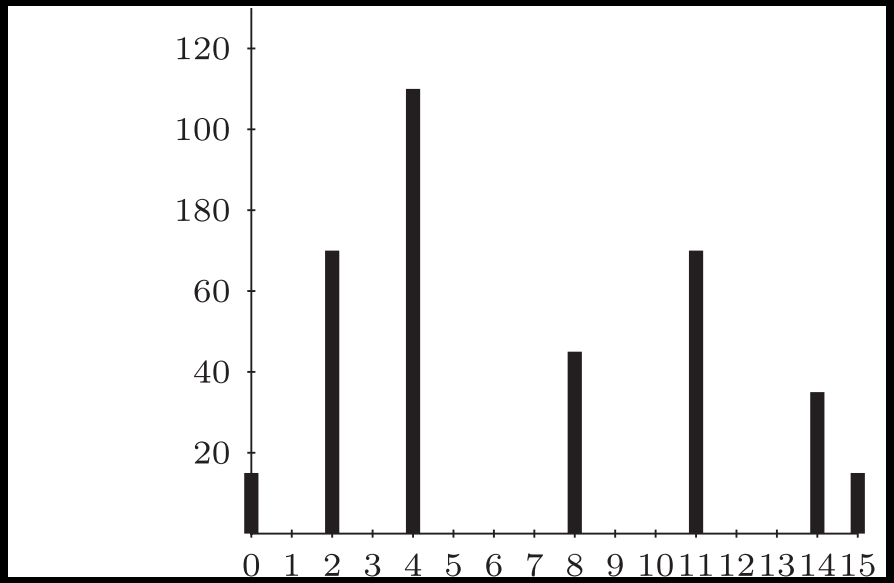
$$j = \frac{14-2}{9-5}(i-5)+2$$



Gray level i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_i	15	0	0	0	0	70	110	45	70	35	0	0	0	0	0	15

Histogram Stretching: After

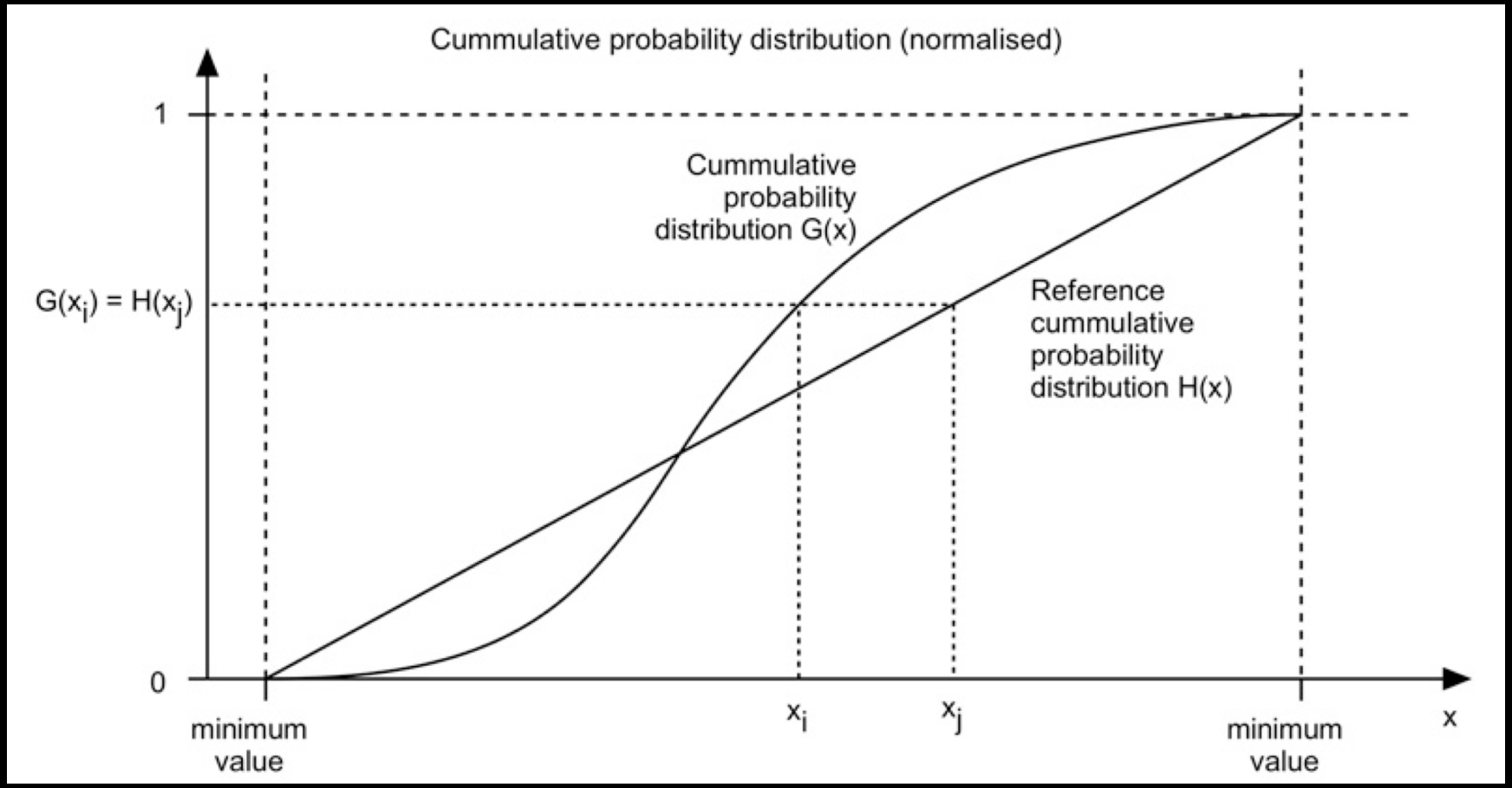




Why it works?

- Consider image of size w *x* h
- Area under histogram curve is constant
  - equal to number of pixels in image n = wh
- An equalized histogram has a uniform probability density function
  - with a constant function value of: (assume L gray levels)
  - n/(L-1)
- Some math that requires to compute cumulative sums.

Alternate way of thinking about histogram equalization



Source: http://paulbourke.net/miscellaneous/equalisation/

Histogram Matching