Questions

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DECSAI

DaSCI

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Data Science and **Computational Intelligence**

Correlation vs Convolution

Cross-correlation vs Convolution

Cross-correlation

Kernel H

1	2	3
4	5	6
7	8	9

$\otimes$

Image F

0	0	0
0	1	0
0	0	0

Cross-correlation vs Convolution

Cross-correlation

Kernel H

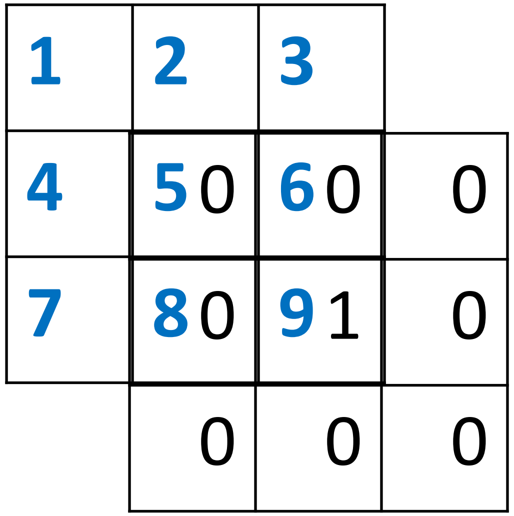
1	2	3
4	5	6
7	8	9

$\otimes$

Image F

0	0	0
0	1	0
0	0	0

$=$



$=$

9		

Cross-correlation vs Convolution

Cross-correlation

$$\underset{\textit{Kernel } H}{\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array}} \otimes \underset{\textit{Image } F}{\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}} = \begin{array}{|c|c|c|}\hline \color{blue}{1} & \color{blue}{2} & \color{blue}{3} \\ \hline \color{blue}{4}\;0 & \color{blue}{5}\;0 & \color{blue}{6}\;0 \\ \hline \color{blue}{7}\;0 & \color{blue}{8}\;1 & \color{blue}{9}\;0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 9 & 8 & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

Cross-correlation vs Convolution

Cross-correlation

Kernel H

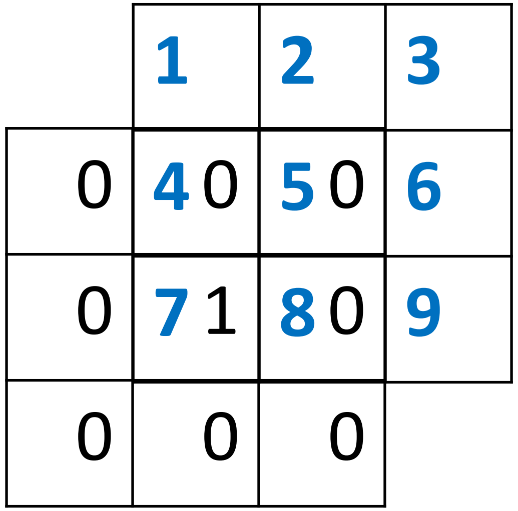
1	2	3
4	5	6
7	8	9

$\otimes$

Image F

0	0	0
0	1	0
0	0	0

$=$



$=$

9	8	7

Cross-correlation vs Convolution

Cross-correlation

Kernel H

1	2	3
4	5	6
7	8	9

$\otimes$

Image F

0	0	0
0	1	0
0	0	0

$=$

1	2 0	3 0	0
4	5 0	6 1	0
7	8 0	9 0	0

$=$

9	8	7
6		

Cross-correlation vs Convolution

Cross-correlation

$$\underset{\textit{Kernel } H}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}} \otimes \underset{\textit{Image } F}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} \mathbf{1}\,0 & \mathbf{2}\,0 & \mathbf{3}\,0 \\ \mathbf{4}\,0 & \mathbf{5}\,1 & \mathbf{6}\,0 \\ \mathbf{7}\,0 & \mathbf{8}\,0 & \mathbf{9}\,0 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & \\ & & \end{bmatrix}$$

Cross-correlation vs Convolution

Cross-correlation

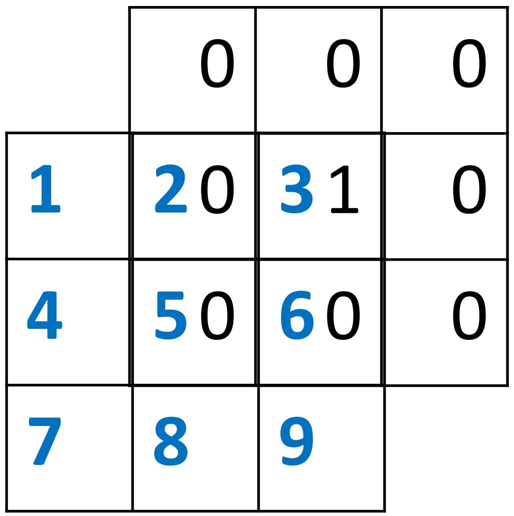
Kernel H *Image F*

$$
\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}
\otimes
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
=
\begin{bmatrix} 0 & \mathbf{1}\ 0 & \mathbf{2}\ 0 & \mathbf{3} \\ 0 & \mathbf{4}\ 1 & \mathbf{5}\ 0 & \mathbf{6} \\ 0 & \mathbf{7}\ 0 & \mathbf{8}\ 0 & \mathbf{9} \end{bmatrix}
=
\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ & & \end{bmatrix}
$$

Cross-correlation vs Convolution

Cross-correlation

$$\underset{\textit{Kernel } H}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}} \otimes \underset{\textit{Image } F}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}} =$$



$$= \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & & \end{bmatrix}$$

Cross-correlation vs Convolution

Cross-correlation

Kernel H

1	2	3
4	5	6
7	8	9

$\otimes$

Image F

0	0	0
0	1	0
0	0	0

$=$

0	0	0
1 0	2 1	3 0
4 0	5 0	6 0
7	8	9

$=$

9	8	7
6	5	4
3	2	

Cross-correlation vs Convolution

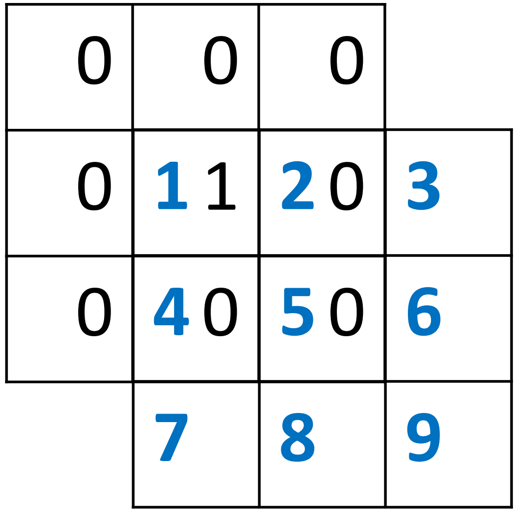
Cross-correlation

Kernel H		
1	2	3
4	5	6
7	8	9

$\otimes$

Image F		
0	0	0
0	1	0
0	0	0

=



=

9	8	7
6	5	4
3	2	1

Cross-correlation vs Convolution

Cross-correlation

Kernel H

0	0	0
0	1	0
0	0	0

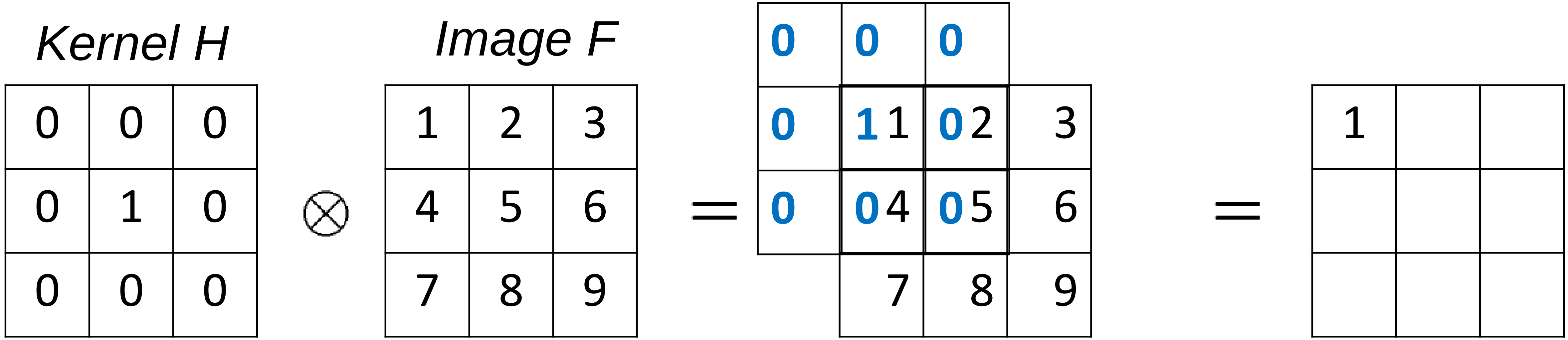
$\otimes$

Image F

1	2	3
4	5	6
7	8	9

Cross-correlation vs Convolution

Cross-correlation



Cross-correlation vs Convolution

Cross-correlation

Kernel H

0	0	0
0	1	0
0	0	0

$\otimes$

Image F

1	2	3
4	5	6
7	8	9

$=$

0	**0**	**0**
0 1	**1** 2	**0** 3
0 4	**0** 5	**0** 6
7	8	9

$=$

1	2	

Cross-correlation vs Convolution

Cross-correlation

Kernel H

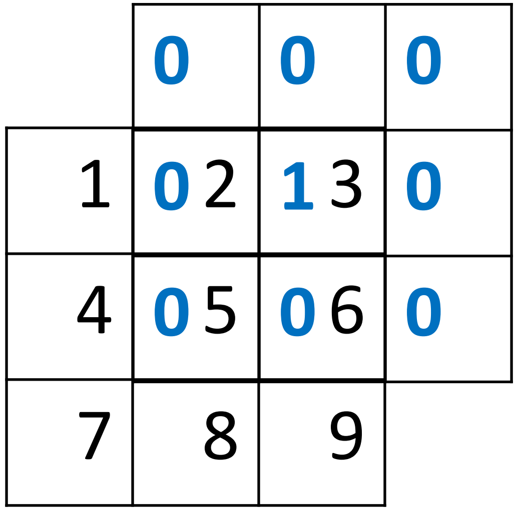
0	0	0
0	1	0
0	0	0

$\otimes$

Image F

1	2	3
4	5	6
7	8	9

=



=

1	2	3

Cross-correlation vs Convolution

Cross-correlation

Kernel H

0	0	0
0	1	0
0	0	0

$\otimes$

Image F

1	2	3
4	5	6
7	8	9

=

0	0 1	0 2	3
0	1 4	0 5	6
0	0 7	0 8	9

=

1	2	3
4		

Cross-correlation vs Convolution

Cross-correlation

$$\underset{\textit{Kernel } H}{\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}} \otimes \underset{\textit{Image } F}{\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array}} = \begin{array}{|c|c|c|}\hline \mathbf{0}\ 1 & \mathbf{0}\ 2 & \mathbf{0}\ 3 \\ \hline \mathbf{0}\ 4 & \mathbf{1}\ 5 & \mathbf{0}\ 6 \\ \hline \mathbf{0}\ 7 & \mathbf{0}\ 8 & \mathbf{0}\ 9 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & \\ \hline & & \\ \hline \end{array}$$

Cross-correlation vs Convolution

Cross-correlation

Kernel H

0	0	0
0	1	0
0	0	0

$\otimes$

Image F

1	2	3
4	5	6
7	8	9

$=$

1	0 2	0 3	0
4	0 5	1 6	0
7	0 8	0 9	0

$=$

1	2	3
4	5	6

Cross-correlation vs Convolution

Cross-correlation

Kernel H

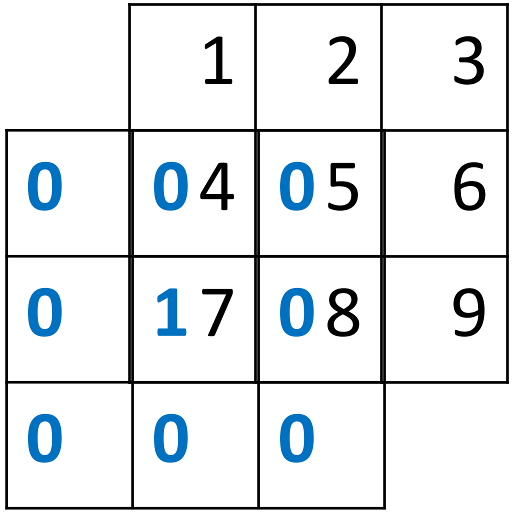
0	0	0
0	1	0
0	0	0

$\otimes$

Image F

1	2	3
4	5	6
7	8	9

$=$



$=$

1	2	3
4	5	6
7		

Cross-correlation vs Convolution

Cross-correlation

Kernel H

0	0	0
0	1	0
0	0	0

$\otimes$

Image F

1	2	3
4	5	6
7	8	9

$=$

1	2	3
0 4	**0** 5	**0** 6
0 7	**1** 8	**0** 9
0	**0**	**0**

$=$

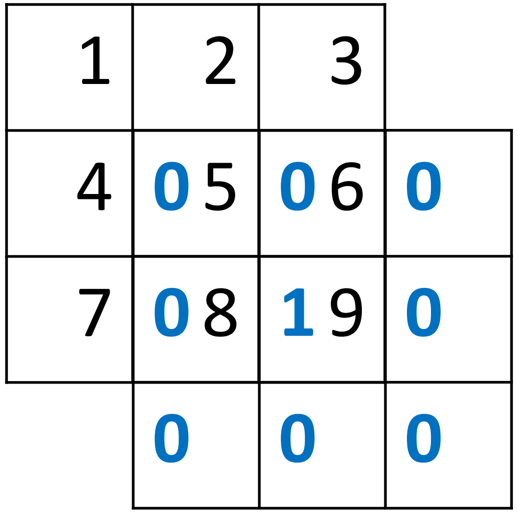
1	2	3
4	5	6
7	8	

Cross-correlation vs Convolution

Cross-correlation

Kernel H   *Image F*

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} =$$



$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Cross-correlation vs Convolution

Cross-correlation

Kernel H *Image F*

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array} \otimes \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 9 & 8 & 7 \\ \hline 6 & 5 & 4 \\ \hline 3 & 2 & 1 \\ \hline \end{array}$$

Kernel H *Image F*

$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \otimes \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array}$$

No commutative!!!!

Cross-correlation vs Convolution

Convolution

Kernel H $\quad\quad$ *Image F*

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array} \star \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}$$

Cross-correlation vs Convolution

Convolution

Kernel H

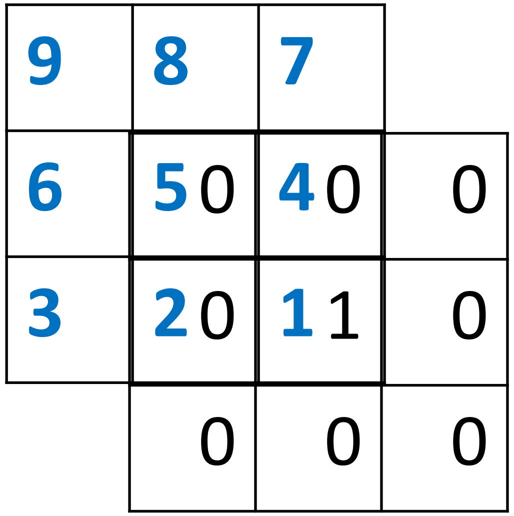
1	2	3
4	5	6
7	8	9

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$



$=$

1		

Cross-correlation vs Convolution

Convolution

Kernel H

1	2	3
4	5	6
7	8	9

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$

9	8	7
6 0	5 0	4 0
3 0	2 1	1 0
0	0	0

$=$

1	2	

Cross-correlation vs Convolution

Convolution

Kernel H

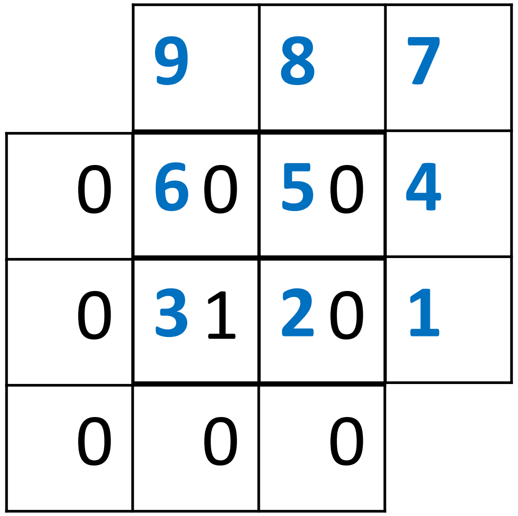
1	2	3
4	5	6
7	8	9

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$



$=$

1	2	3

Cross-correlation vs Convolution

Convolution

Kernel H

1	2	3
4	5	6
7	8	9

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$

9	8 0	7 0	0
6	5 0	4 1	0
3	2 0	1 0	0

$=$

1	2	3
4		

Cross-correlation vs Convolution

Convolution

Kernel H   *Image F*

$$
\begin{array}{|c|c|c|}
\hline
1 & 2 & 3 \\ \hline
4 & 5 & 6 \\ \hline
7 & 8 & 9 \\ \hline
\end{array}
\star
\begin{array}{|c|c|c|}
\hline
0 & 0 & 0 \\ \hline
0 & 1 & 0 \\ \hline
0 & 0 & 0 \\ \hline
\end{array}
=
\begin{array}{|c|c|c|}
\hline
9\ 0 & 8\ 0 & 7\ 0 \\ \hline
6\ 0 & 5\ 1 & 4\ 0 \\ \hline
3\ 0 & 2\ 0 & 1\ 0 \\ \hline
\end{array}
=
\begin{array}{|c|c|c|}
\hline
1 & 2 & 3 \\ \hline
4 & 5 & \\ \hline
 & & \\ \hline
\end{array}
$$

Cross-correlation vs Convolution

Convolution

Kernel H

1	2	3
4	5	6
7	8	9

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$

0	**9** 0	**8** 0	**7**
0	**6** 1	**5** 0	**4**
0	**3** 0	**2** 0	**1**

$=$

1	2	3
4	5	6

Cross-correlation vs Convolution

Convolution

Kernel H

1	2	3
4	5	6
7	8	9

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$

	0	0	0
9	8 0	7 1	0
6	5 0	4 0	0
3	2	1	

$=$

1	2	3
4	5	6
7		

Cross-correlation vs Convolution

Convolution

Kernel H

1	2	3
4	5	6
7	8	9

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$

0	0	0
9 0	8 1	7 0
6 0	5 0	4 0
3	2	1

$=$

1	2	3
4	5	6
7	8	

Cross-correlation vs Convolution

Convolution

Kernel H

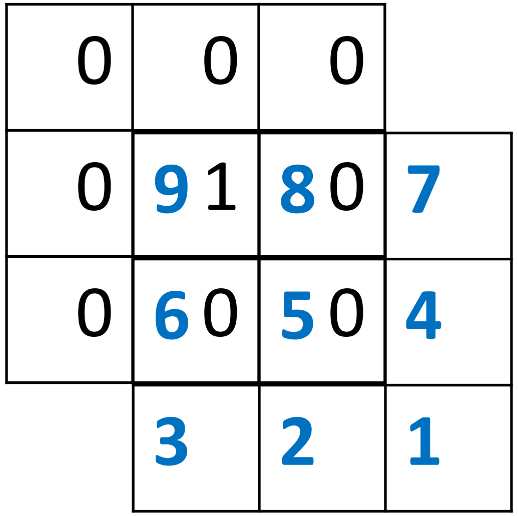
1	2	3
4	5	6
7	8	9

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$



$=$

1	2	3
4	5	6
7	8	9

Cross-correlation vs Convolution

Convolution

Kernel H

0	0	0
0	1	0
0	0	0

$\star$

Image F

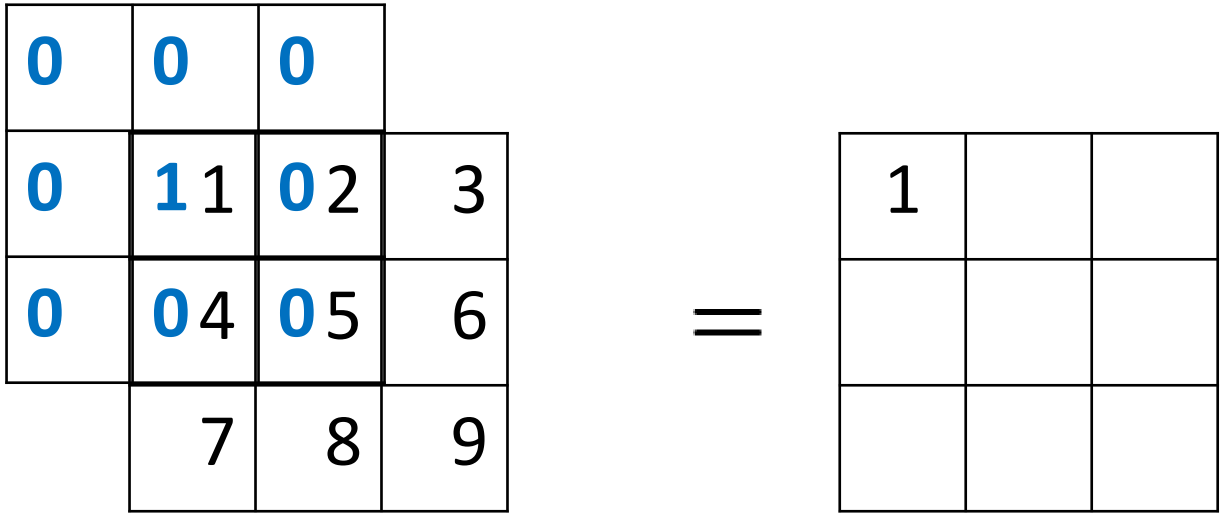
1	2	3
4	5	6
7	8	9

Cross-correlation vs Convolution

Convolution

Kernel H — *Image F*

$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \star \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array} =$$



Cross-correlation vs Convolution

Convolution

Kernel H		
0	0	0
0	1	0
0	0	0

$\star$

Image F		
1	2	3
4	5	6
7	8	9

$=$

0	0	0
0 1	1 2	0 3
0 4	0 5	0 6
7	8	9

$=$

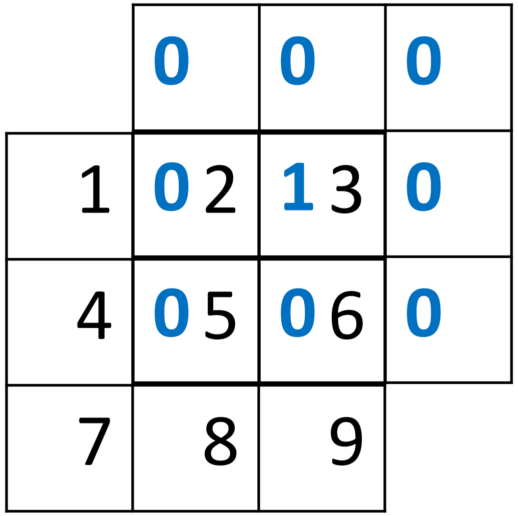
1	2	

Cross-correlation vs Convolution

Convolution

Kernel H				*Image F*		
0	0	0		1	2	3
0	1	0	$\star$	4	5	6
0	0	0		7	8	9

$=$



$=$

1	2	3

Cross-correlation vs Convolution

Convolution

$$\underset{\textit{Kernel } H}{\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}} \star \underset{\textit{Image } F}{\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array}} = \begin{array}{|c|c|c|c|}\hline \color{blue}{0} & \color{blue}{0}\,1 & \color{blue}{0}\,2 & 3 \\ \hline \color{blue}{0} & \color{blue}{1}\,4 & \color{blue}{0}\,5 & 6 \\ \hline \color{blue}{0} & \color{blue}{0}\,7 & \color{blue}{0}\,8 & 9 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & & \\ \hline & & \\ \hline \end{array}$$

Cross-correlation vs Convolution

Convolution

Kernel H

0	0	0
0	1	0
0	0	0

$\star$

Image F

1	2	3
4	5	6
7	8	9

$=$

0 1	**0** 2	**0** 3
0 4	**1** 5	**0** 6
0 7	**0** 8	**0** 9

$=$

1	2	3
4	5	

Cross-correlation vs Convolution

Convolution

Kernel H and *Image F*

$$
\begin{array}{|c|c|c|}
\hline 0 & 0 & 0 \\
\hline 0 & 1 & 0 \\
\hline 0 & 0 & 0 \\
\hline
\end{array}
\star
\begin{array}{|c|c|c|}
\hline 1 & 2 & 3 \\
\hline 4 & 5 & 6 \\
\hline 7 & 8 & 9 \\
\hline
\end{array}
=
\begin{array}{|r|r|r|l|}
\hline 1 & 0\;2 & 0\;3 & 0 \\
\hline 4 & 0\;5 & 1\;6 & 0 \\
\hline 7 & 0\;8 & 0\;9 & 0 \\
\hline
\end{array}
=
\begin{array}{|c|c|c|}
\hline 1 & 2 & 3 \\
\hline 4 & 5 & 6 \\
\hline & & \\
\hline
\end{array}
$$

Cross-correlation vs Convolution

Convolution

Kernel H

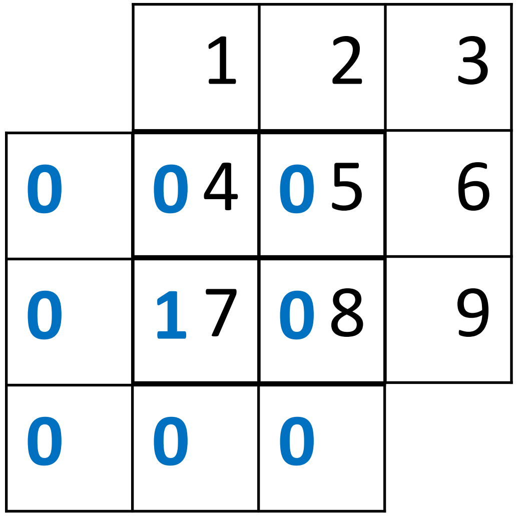
0	0	0
0	1	0
0	0	0

$\star$

Image F

1	2	3
4	5	6
7	8	9

$=$



$=$

1	2	3
4	5	6
7		

Cross-correlation vs Convolution

Convolution

Kernel H and *Image F*

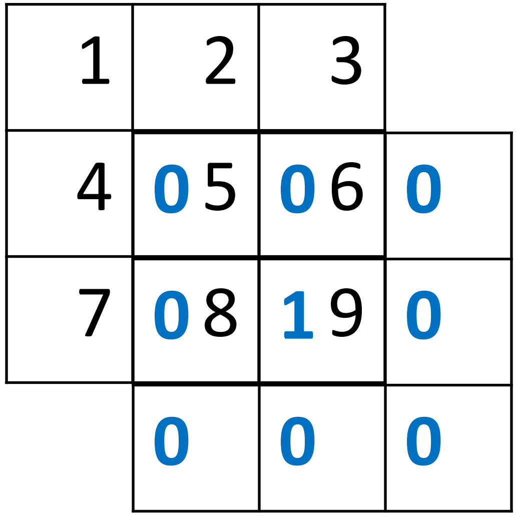
$$
\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}
\star
\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array}
=
\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 0\;4 & 0\;5 & 0\;6 \\ \hline 0\;7 & 1\;8 & 0\;9 \\ \hline 0 & 0 & 0 \\ \hline \end{array}
=
\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & \\ \hline \end{array}
$$

Cross-correlation vs Convolution

Convolution

Kernel H $\quad\quad$ *Image F*

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \star \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} =$$



$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Cross-correlation vs Convolution

Convolution

$$\underset{\textit{Kernel } H}{\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline\end{array}} \star \underset{\textit{Image } F}{\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline\end{array}} = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline\end{array}$$

Commutative!!!!

$$\underset{\textit{Kernel } H}{\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline\end{array}} \star \underset{\textit{Image } F}{\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline\end{array}} = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline\end{array}$$

Cross-correlation vs Convolution

Cross-correlation

Kernel H

$\otimes$

Image F

0	0	0
0	1	0
0	0	0

$=$?

Convolution

Kernel H

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$

Kernel H (rotated 180˚)

$\otimes$

Image F

0	0	0
0	1	0
0	0	0

$=$?

Cross-correlation vs Convolution

Cross-correlation

Kernel H

$\otimes$

Image F

0	0	0
0	1	0
0	0	0

$=$

Convolution

Kernel H

$\star$

Image F

0	0	0
0	1	0
0	0	0

$=$

Kernel H (rotated 180˚)

$\otimes$

Image F

0	0	0
0	1	0
0	0	0

$=$

Linear filters: examples

Original

$\otimes$

0	0	0
0	0	1
0	0	0

$= \; ?$

Slide 46 of 2.1.ImageFiltering actually shows correlation not convolution!!!!!

Linear filters: examples

Image

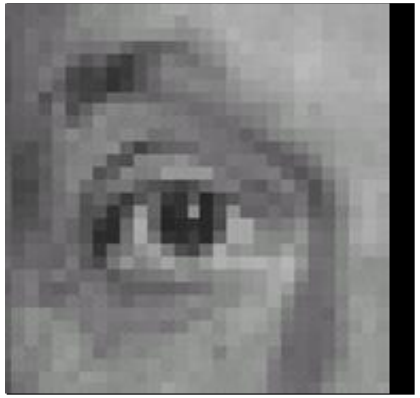
Original

$\otimes$

Kernel

0	0	0
0	0	1
0	0	0

=



Shifted left by 1 pixel
(we lose the first column of pixels on the left)

Slide 46 of 2.1.ImageFiltering actually shows correlation not convolution!!!!!

Linear filters: examples

Image

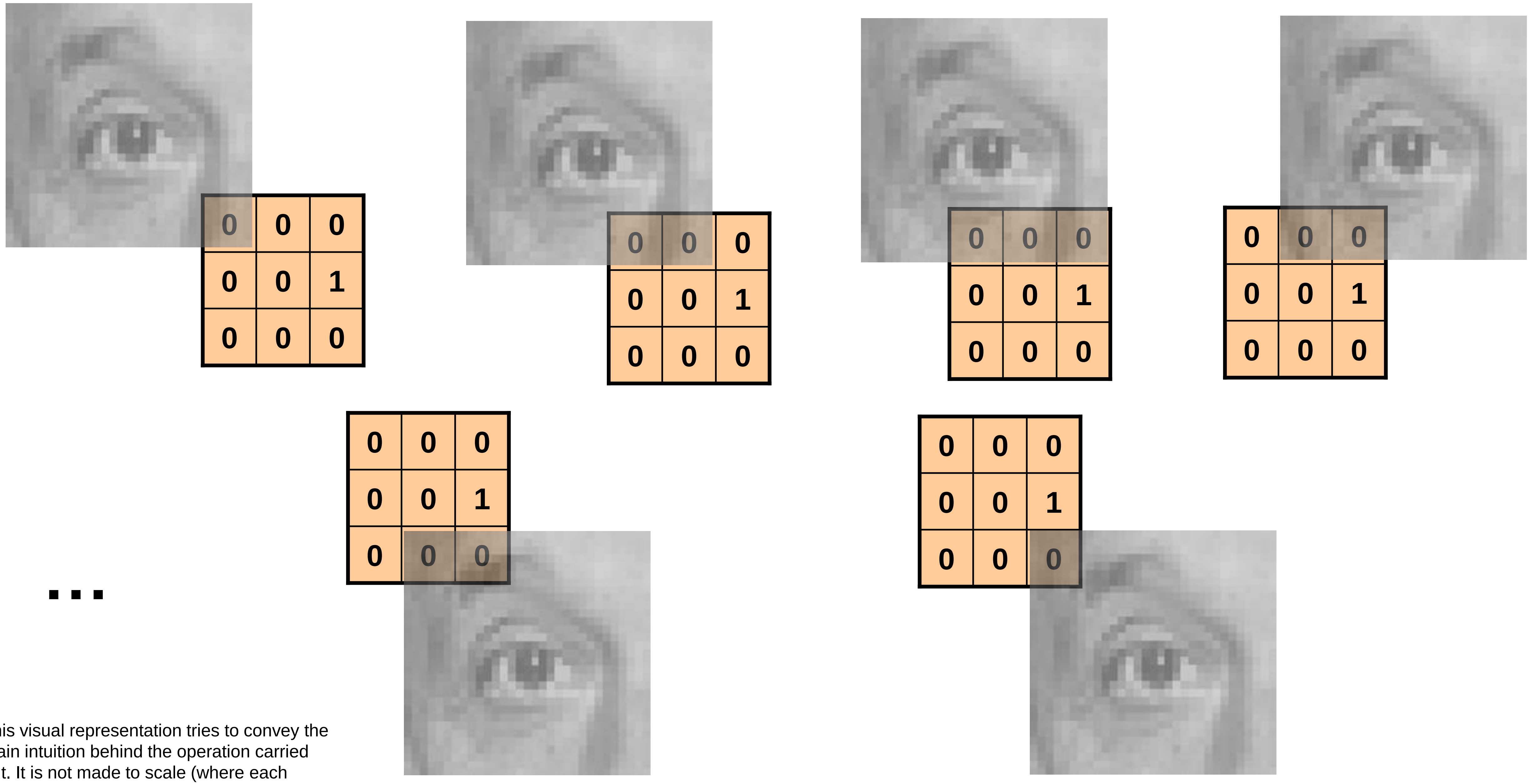
0	0	0
0	0	1
0	0	0

$\otimes$

Kernel

$=$ **?**

mode='full'



This visual representation tries to convey the main intuition behind the operation carried out. It is not made to scale (where each position of the kernel (Einstein's eye) should coincide with a single pixel of the image)

mode='full'

Linear filters: examples

Image

0	0	0
0	0	1
0	0	0

$\otimes$

Kernel

$=$

mode='full'

Linear filters: examples

Image

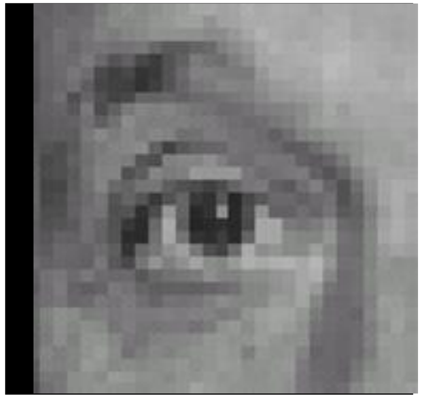
Original

$\otimes$

Kernel

0	0	0
1	0	0
0	0	0

$=$



Shifted right by 1 pixel
(we lose the first column of pixels on the right)

Linear filters: examples

Image

0	0	0
1	0	0
0	0	0

$\otimes$

Kernel

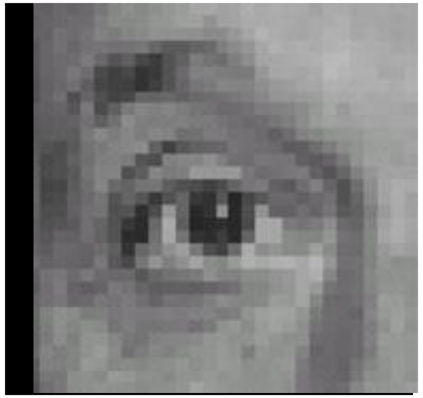
$=$

mode='full'

Linear filters: examples

Original

$$\star \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} =$$



Shifted right by 1 pixel
(we lose the first column of pixels on the right)

$$=$$

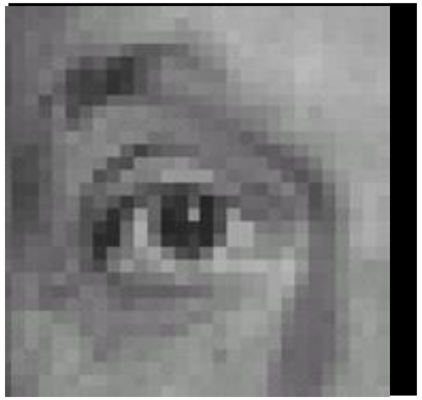
Original

$$\otimes \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Linear filters: examples

Original

$$\star \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$$



Shifted left by 1 pixel
(we lose the first column of pixels on the left)

$$=$$

Original

$$\otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Cross-correlation vs Convolution (technical note)

In case you want to check those results:

```
import scipy.signal as sp
import cv2
```

```
sp.convolve2d()
```
https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.convolve2d.html

```
sp.correlate2d()
```
https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.correlate2d.html

```
cv2.filter2D()
```
https://docs.opencv.org/4.x/d4/d86/group__imgproc__filter.html#ga27c049795ce870216ddfb366086b5a04

Gaussian kernels and separability

Separability

First derivative of 2D Gaussian is separable:

$$
\begin{aligned}
G(x,y) &= \frac{1}{2\pi\sigma^2}\exp\left(\frac{-(x^2+y^2)}{2\sigma^2}\right) \\
&= \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\cdot\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{y^2}{2\sigma^2}\right) \\
&= G_h(x)\cdot G_v(y) \\
\frac{\partial G(x,y)}{\partial x} &= -\frac{x}{\sigma^2}\cdot\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right)\exp\left(-\frac{y^2}{2\sigma^2}\right) \\
&= G'_h(x)\cdot G_v(y)
\end{aligned}
$$

horizontal 1D Gaussian derivative kernel

vertical 1D Gaussian kernel

How are the product and convolution related? Why do we assume that this product is equivalent to the convolution? 🤔

smooth in one direction, differentiate in the other

This allows us to know **which 1D kernels to apply to calculate the derivatives of an image.**
In the previous example, the first derivative on X.

Separability

1. We know that the 1st derivative of a 2D Gaussian is separable.

$$\begin{aligned}\frac{\partial G(x,y)}{\partial x} &= -\frac{x}{\sigma^2}\cdot\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right)\exp\left(-\frac{y^2}{2\sigma^2}\right)\\ &= G'_h(x)\cdot G_v(y)\end{aligned}$$

The partial derivative of a Gaussian is composed of the derivative in that direction and the Gaussian in the other.

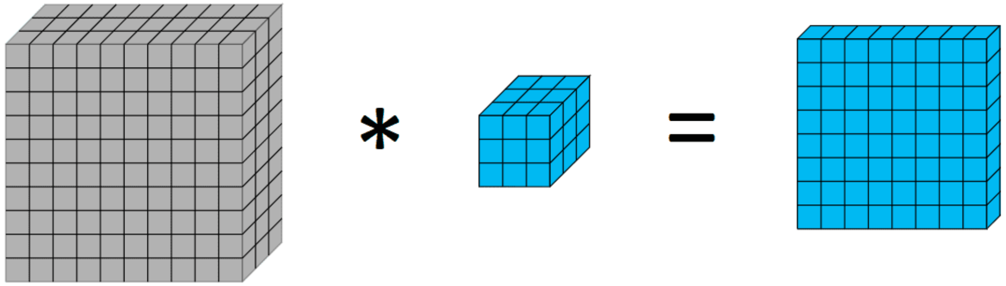
2. From a practical point of view, we are interested in implicitly **smoothing the signal to avoid magnifying the noise**. So, we smooth in one direction and differentiate in the other.

3. Mathematically, **a separable kernel function leads to a separable convolution**. See http://www.songho.ca/dsp/convolution/convolution2d_separable.html

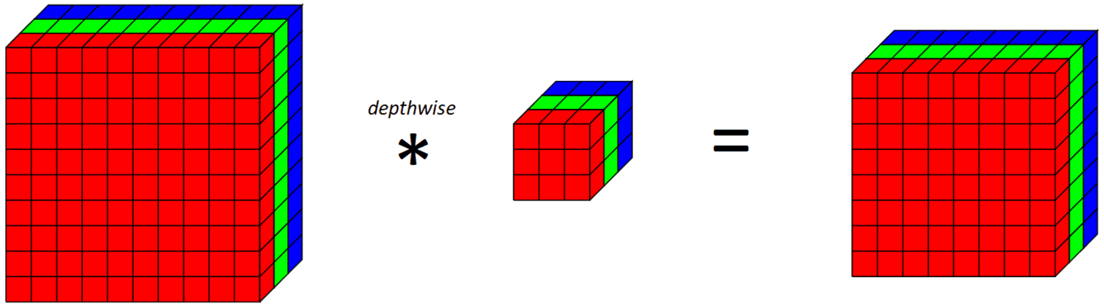
Convolutional Neural Networks and Separable Filters

ConvNets and Separable Convolutions

- Depthwise Separable Convolutions

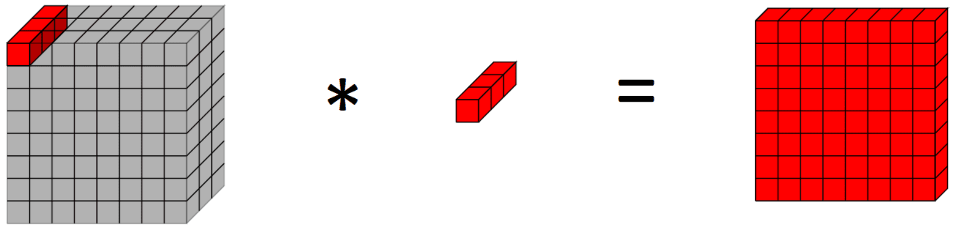


vs



We apply a kernel (3x3x3) over the whole input volume (10x10x3)

We apply one kernel (3x3x1) per channel in the input volume (10x10x3). We then apply a pointwise convolutional layer (1x1x3) on the resulting volume (8x8x3):



<u>Number of parameters (if we want to apply 64 convolutional filters to our RGB image)</u>:

3x3x3x64 + 64 = 1792 parameters

(3x3x1x3 + 3) + (1x1x3x64 + 64) = 286 parameters

https://machinelearningmastery.com/using-depthwise-separable-convolutions-in-tensorflow/

ConvNets and Separable Convolutions

- Depthwise Separable Convolutions
  - They have been shown to yield similar performance while using less parameters and less floating point operations.
- SVD is sometimes used to decompose/approximate the weight matrices and reduce the number of parameters in the model (regularization; light-weight DL models deployment)
  - Note: it is costly to use SVD on every training step
- Apart from this, I don’t have the impression that ConvNets generally use any filter factorization (e.g., SVD) to apply the separable convolution of smaller kernels…

Ghimire, Deepak, Dayoung Kil, and Seong-heum Kim. "A survey on efficient convolutional neural networks and hardware acceleration." *Electronics* 11.6 (2022): 945.

Questions

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