Computation and Discrete Structures III

Group 1 Semester 2026-2 12 Feb 2026

Quiz #1 – Solutions

A.

P.1. (50%)

Given. $\Sigma = \{0, 1, 2\}$ and $L = \{w \in \Sigma^* \mid |w| = 2 \text{ and } w \text{ ends with } 2\}$.

a) List all strings in L:

$$L = \{02, 12, 22\}.$$

b) Five strings in Σ^* not in L and whether $\varepsilon \in L$:
Examples not in L: ε (length 0), 0 (length 1), 120 (length 3), 10 (does not end with 2), 21 (does not end with 2).
$\varepsilon \notin L$ because $|\varepsilon| = 0 \neq 2$.

c) Is L finite or infinite?
L is finite (it has exactly 3 strings), while Σ^* is infinite.

Final. $L = \{02, 12, 22\}$, $\varepsilon \notin L$, and L is finite.

P.2. (50%)

Given. $\Sigma = \{a, b\}$ and $L = \{ab, ba\}$.

a) Compute L^2 and L^3 (no repetitions):

$$L^2 = \{abab,\ abba,\ baab,\ baba\}$$
$$L^3 = \{ababab,\ ababba,\ abbaab,\ abbaba,\ baabab,\ baabba,\ babaab,\ bababa\}.$$

b) Classify each string:

$$\begin{aligned} abab &\in L^2 && \text{since } abab = (ab)(ab).\\ abba &\in L^2 && \text{since } abba = (ab)(ba).\\ baab &\in L^2 && \text{since } baab = (ba)(ab).\\ ababab &\in L^3 && \text{since } ababab = (ab)(ab)(ab).\\ baabba &\in L^3 && \text{since } baabba = (ba)(ab)(ba). \end{aligned}$$

c) Can $w = (ab)^4$ belong to L^3?
No. Every string in L^3 has length 6, but $|(ab)^4| = 8$.

Final. $abab, abba, baab \in L^2$; $ababab, baabba \in L^3$; and $(ab)^4 \notin L^3$.

B.

P.1. **(50%)**

Given. $\Sigma = \{a, b, c\}$ and $L = \{w \in \Sigma^* \mid w \text{ starts with } a \text{ and } |w| \leq 3\}$.

a) List all strings in L:

$$L = \{a,\ aa, ab, ac,\ aaa, aab, aac, aba, abb, abc, aca, acb, acc\}.$$

b) Three strings in Σ^* not in L and whether $\varepsilon \in L$:
Examples: ε (does not start with a), b (does not start with a), $abca$ (length 4).
$\varepsilon \notin L$.

c) Membership of ε:
$\varepsilon \notin L$, but $\varepsilon \in \Sigma^*$.

Final. L has 13 strings, $\varepsilon \notin L$, and $\varepsilon \in \Sigma^*$.

P.2. **(50%)**

Given. $\Sigma = \{0, 1\}$ and $L = \{01,\ 10,\ 11\}$.

a) Compute L^2:

$$L^2 = \{0101, 0110, 0111, 1001, 1010, 1011, 1101, 1110, 1111\}.$$

b) Classify each string:

$$\begin{aligned}
0110 &\in L^2 && \text{since } 0110 = (01)(10). \\
1110 &\in L^2 && \text{since } 1110 = (11)(10). \\
1011 &\in L^2 && \text{since } 1011 = (10)(11). \\
010110 &\in L^3 && \text{since } 010110 = (01)(01)(10). \\
111111 &\in L^3 && \text{since } 111111 = (11)(11)(11).
\end{aligned}$$

c) Can an odd-length string be in L^4?
No. Every string in L has length 2, so every string in L^4 has length 8.

Final. $0110, 1110, 1011 \in L^2$; $010110, 111111 \in L^3$; and odd length $\notin L^4$.