Juan Marín. DSAA PS2 Q3 – Skip Lists Theory:

3a. Prove that the (worst case) number of nodes looked at to access the i th element is Θ(k). [10 marks]

Proof:

Base Case: For k=1
We have that $n = 2^k$, therefore n = 2, so the number of nodes transversed to find an element is 2. You can check the first node(head), reject it and then check the node which that leads to, see if that is the element you are looking for, and if it isn't then you reject both nodes. This evaluates to a total of 2 operations = 2^k, therefore we have Θ(k) as the number of nodes to look at for k = 1

Assumption:
We assume that this holds for $n = 2^k$ for any other k, so the most nodes which can be looked at to access the ith element is Θ(k).

Induction:
Using k+1 as k we would have a skip list of size 2^{k+1}, this is the same as 2^k*2 by the laws of indices. If we take log to the base 2 of these we have k and 1, giving us the complexities Θ(k) and Θ(1) respectively. When we add the two we get Θ(k+1) which in this case is equal to Θ(k) as required.

Conclusion:
As the statement is true for n=1 and true for k implies truth for k+1 we have proved that the statement saying that the number of nodes looked at to access an element i in a skip list is Θ(k) in the worst case is true.

3b and 3c not attempted