14.07, Fall 2019: Formula Sheet for Midterm Exam 3
Final version: use this version only

- Q Theory of investment over two-periods (no borrowing constraints):

$$\frac{i_0}{k_0} = \frac{Q-1}{\phi}$$
$$Q = \frac{\overline{e}_1}{1+\overline{R}_k}$$
$$\overline{R}_k - R_f = \beta_k\left(\overline{R}_m - R_f\right)$$

- Holmstrom-Tirole model for banks' investment and payoff:

$$I = \frac{N}{1-\overline{\rho}} \text{ and } U = \left(1 + \frac{R}{1-\overline{\rho}}\right)N$$

- Kiyotaki-Moore credit cycles

  Procyclical leverage ($\overline{\rho}_1(R_1)$ increasing in R_1):

$$I_1 = \frac{1}{1-\overline{\rho}_1(R_1)}N_1$$

  Fire sales externalities (in terms of prices, prices are increasing in I_1)

$$P_1 = P_1^{\text{fair}} + \overbrace{P_0 G\left(l_1\right)}^{\text{fire-sale influence}}$$

  (in terms of returns, returns are increasing in I_1)

$$1 + R_1 = \frac{D_1 + P_1}{P_0} = \underbrace{1 + R_1^{\text{fair}}}_{\text{fair return}} + \overbrace{G\left(l_1\right)}^{\text{demand influence/fire sales}}$$

  Putting all together:

$$\begin{aligned} I_1 &= \frac{1}{1-\overline{\rho}_1}N_1 \\ &= \frac{1}{1-\overline{\rho}_1\left(R_1\right)}\left(\left(1+R_1\right)I_0 - B_0\right) \\ &= \frac{1}{1-\overline{\rho}_1\left(R_1\right)}\left(\left(1+R_1^{\text{fair}} + G\left(I_1\right)\right)I_0 - \overline{\rho}_0 I_0\right) \text{ where } I_0 = \frac{N_0}{1-\overline{\rho}_0} \end{aligned}$$

- Diamond-Dybvig bank runs. λ is the expected share of withdrawers, $\tilde{\lambda}$ is the actual share of withdrawers, $\underline{C}_1$ the promised payment to withdrawers, R the long-term return, L the liquidation value:

  1. When $\tilde{\lambda} = \lambda$:
$$C_1 = \underline{C}_1$$
$$C_2 = \frac{1}{1-\lambda}R(1-\lambda\underline{C}_1)$$
  2. When $\tilde{\lambda} > \lambda$ but not too high:
$$C_1(\tilde{\lambda}) = \underline{C}_1$$
$$C_2(\tilde{\lambda}) = \frac{1}{1-\tilde{\lambda}}R(1-\lambda\underline{C}_1 - \frac{\underline{C}_1}{L}(\tilde{\lambda}-\lambda))$$
  3. When $\tilde{\lambda}$ is higher than a threshold:
$$C_1(\tilde{\lambda}) = \frac{\lambda\underline{C}_1 + (1-\lambda\underline{C}_1)L}{\tilde{\lambda}} < \underline{C}_1$$
$$C_2(\tilde{\lambda}) = 0$$