

# Funding Instruments and Effort Choices in Higher Education \*

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## Abstract

This paper examines the effects of Free College policies on student enrollment and academic performance, with a focus on the 2016 Chilean reform that granted tuition-free higher education to students from the lowest five income deciles. Using a difference-in-differences approach, we find that Free College increased enrollment and persistence in higher education on the eligible but had modest effects on graduation and dropout rates. To disentangle the role of student effort from selection effects, we develop a structural model in which students choose effort levels in response to financial incentives. Our results highlight that Free College expanded access, in particular for low-achieving students. Despite the removal of academic progress requirements, we found no evidence of weakening performance.

**Keywords:** Financial aid, Effort choices, Higher Education

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# 1 Introduction

Access to Higher Education has become an increasingly heated topic. In the US, president Biden proposed to (i) increase the funding for merit-based scholarships, (ii) a debt forgiveness plan, and (iii) expand free community college across the country. Advocates of Free College point to the debt crisis, lack of mobility and increasing inequality. As of today, more than 40 countries around the world have an ongoing Free College policy, generally limited to public institutions.

Traditional funding schemes such as merit-based scholarships are out of reach for students who don't meet the performance requirements. Even popular means-based programs, such as the Pell Grants in the US, require beneficiaries to maintain satisfactory academic progress. Subsidized loans give credit to students at below-the-market interest rates, which helps expand access to higher education. However, the burden it places upon graduation leads to high default rates, and the bad financial credit history leaves long-lasting effects on borrowers. Free College is a particular case of a continuum of prices that students face (which could be negative, if they receive a stipend). A progressive system would opt for increasing the amount of benefits the lower the income. However, specially in the developing world, Free College is the solution that policy makers find under the impossibility of correctly determining households' incomes.

The effect that Free College can have on a student outcome is ex-ante ambiguous. From one point, it eases the budget constraint, increasing the disposable time a student can assign to his studies (no need for a part-time job), and making less likely to drop-out following a negative income shock. However, moving from a conditional (on performance) cash transfer to an unconditional scheme (Free College) potentially involves moral hazard concerns, where

students exert low effort (“good enough to pass”) and progression is less satisfactory.

When looking at the aggregate data, countries that have Free College policies tend to have lower graduation rates than similar countries in which students face out-of-pocket fees. While that might provide a pessimistic view, it is related to the fact that increases in enrollment come disproportionately from students of lower performance. In general, funding policies imply responses on both the extensive (enrollment) and intensive (performance) margins, and any aggregate measure is the result of (i) marginal students who self-select to higher education because of the policy, and (ii) infra-marginal students who would have enrolled nevertheless, but change their behavior because of the policy.

This paper studies the effects of Free College on both enrollment and performance. Our analysis centers in the Chilean case, where in 2016 a Free College program called *Gratuidad* allowed students in the first 5 deciles of the income distribution to enroll for free in almost half of the operating universities in the country, including some private institutions. In a first step, we exploit a difference-in-differences strategy and find that the policy increased enrollment and persistence (defined as time spent enrolled) for eligible individuals, while it had a modest effect on graduation and dropout. Building on the results obtained on the reduced-form analysis, we build a structural model to (i) decompose the relative influence of marginal and infra-marginal students for each aggregate outcome and (ii) estimate the effort responses to changes in the funding scheme. Estimation of the model allows for simulation of counterfactual funding policies.

We build on several strands of literature. First, we contribute to the research on higher education financing. There is substantial evidence showing that increasing funding for higher education has a positive effect on enrollment (Angrist et al., 2015, Denning, 2017, Londoño-

Vélez et al., 2020, Dobbin et al., 2022). The effect on performance is less clear, with studies tending to find a negative effect on graduation (Dynarski, 2003, Cohodes and Goodman, 2014). A typical problem when performing inference is the aforementioned composition effect. Denning (2019) studies a reform in the US that increased financial aid only for already enrolled students, finding that it increased graduation rate and decreased completion time. Free College releases students of accumulating student debt, which has been shown to affect major choice (Rothstein and Rouse, 2011), dropout (Stinebrickner and Stinebrickner, 2008) career decisions (Sieg and Wang, 2018) and home ownership (Black et al., 2023). This particular policy has been studied by Bucarey (2017). However, his paper focuses on the crowding out of low-income student from selective programs upon implementation of the policy. Our contribution is to study the effect of Free College in both enrollment and performance.

Secondly, we contribute to the literature that examines the roles of incentives and moral hazard of funding instruments in education. For instance, Montalbán (2022) exploit changes in performance requirements in Spanish university loans using a regression discontinuity approach. He finds that cash allowances are only effective when accompanied with relatively high performance requirements. In parallel, a growing structural and semi-structural literature investigates responses to changes in incentives by modeling effort as an exogenous stochastic process. It is the case for Arcidiacono (2005) for college admission probabilities completion under affirmative action, Beffy et al. (2012) for length of studies as a result of different returns to education induced by the French business cycle, or course credits in Sweden as in Joensen and Mattana (2024). In our paper, we endogenize and recover effort from a first-order condition as in De Groote (2025). We additionally exploit a large-scale reform in Chile that introduced variation in out-of-pocket fees and performance requirements to

identify both extensive and intensive margin responses to the policy. Other papers that endogenized effort decisions include Ferreyra et al. (2022) in Colombia by modeling the number of targeted classes in higher education and Tincani et al. (2023), who examines changes in efforts as a response to subjective beliefs in Chile. In her case, effort measures are directly observed from survey data and enter the scores’ production functions.

The rest of the paper is organized as follows: [section 2](#) describes the institutional details, the Free College policy and introduces the data; [section 3](#) reports results from differences-in-differences estimation; and [section 4](#) builds a model of enrollment and effort in higher education. [section 5](#) discussed the main results of the model and [section 6](#) performs counterfactual simulations. [section 7](#) provides a discussion and next steps.

## 2 Background

### 2.1 Institutional Setting

Chilean higher education is well developed, with enrollment rising steadily since the General Law of Universities (*Ley General de Universidades*) of 1981, which incentivized the creation of institutions by allowing entry without state dependence. In 2023, the system comprised a total of 138 institutes: 80 vocational schools (33 Professional Institutes and 47 Centers of Technical Formation) and 58 universities.<sup>1</sup>

Universities are divided between the so-called *traditional* universities (a mix of public and private institutions that receive direct support from the government) and the *private* universities, which constitute the rest. The latter were created after 1981 and are mainly financed through tuition. Traditional universities are officially known as the Universities of

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<sup>1</sup><https://www.ayudamineduc.cl/ficha/instituciones-vigentes-reconocidas-por-el-mineduc>.

the Rector’s Council (*Consejo de Rectores de las Universidades Chilenas, CRUCH*) and are responsible for coordinating the higher education system. They comprise 30 universities and account for around half of total enrollment.<sup>2</sup>

Access to higher education is through a Centralized Admission Platform, compulsory for traditional universities and some private institutions. Vocational program admission is also possible, although most are conducted off-platform. After high-school completion, students applying to higher education take the centralized admission exam (Prueba de Selección Universitaria, PSU). This standardized test, similar to the SAT in the United States, ranges from 150 to 850 points, with a mean of 500 and a standard deviation of 110. Students then submit rank-ordered lists of up to 10 degree programs through the centralized system and are assigned via a deferred-acceptance algorithm (Gale and Shapley, 1962). In practice, the platform imposes a minimum PSU score of 450 to apply.

Higher education is costly in Chile. In 2009, the average tuition fee for a university program was equivalent to 47% of the median family income (Solis, 2017). Costs vary across institution types but remain high even in public institutions. Different funding instruments coexist. Students rely mainly on loans and grants from the Ministry of Education. Eligibility criteria are strongly related to PSU scores. The State Guaranteed Loan program (SGL), introduced in 2006, finances 90% of reference tuition and requires a PSU score of 475 or higher, with no socioeconomic requirement since 2014. Several scholarships also exist, with the *Beca Bicentenario* and *Beca Excelencia* being the most popular. On average, scholarships finance 80% of reference tuition. Eligibility requires a PSU score between 510 and 550, depending on the scholarship, and excludes students in the top two or three income deciles. Both short-cycle

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<sup>2</sup>In 2018, the government established a clear set of rules for being considered a traditional university with the promulgation of Law 21091, allowing the first three non-traditional universities to enter.

programs (SCPs) and university degrees are covered by scholarships. Two-year programs typically require a high-school GPA above 5.0. Access to private loans is limited: in 2015, only 7.5% of student loans came from banks without a state guarantee.

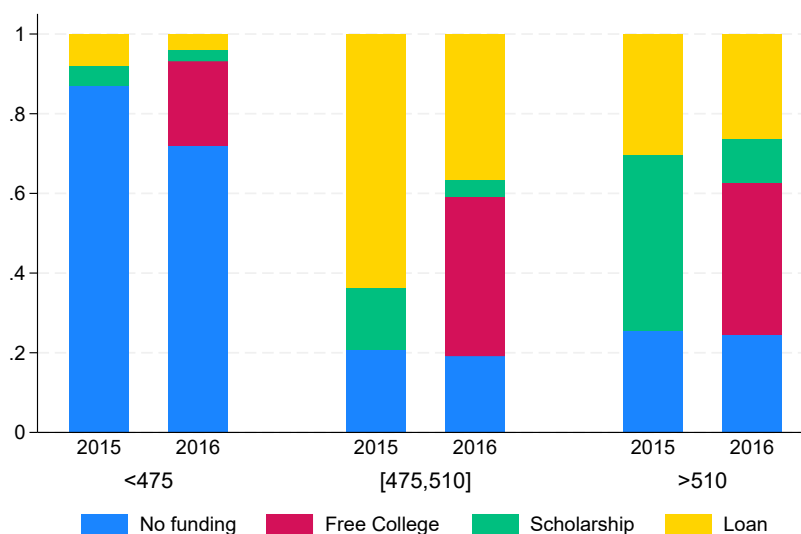
## 2.2 Tuition-Free Higher Education (TFHE) program

Since the implementation of SGL, student debt rose at a rate of 70% annually, and the number of recipients increased steadily from 15,800 in the first year to 652,000 students in 2016 (Bucarey et al., 2020). In 2011, students led mass protests demanding more affordable higher education. Michelle Bachelet was elected president in 2014 on a platform that included making college free by 2020. In 2015, the TFHE law was enacted, removing tuition fees for students in the bottom half of the income distribution. The policy was first implemented for the 2016 university cohort, expanded in 2017 to include vocational institutes, and further extended in 2018 to the sixth income decile. Applications for TFHE, loans, and scholarships are made through the Centralized Admission Platform by completing a short online form during the application period, regardless of whether the applicant applies to a platform degree or not. A schematic visualization of the policy is provided in [Figure A4](#).

The introduction of TFHE policy generated different substitution patterns depending on students' test scores. [Figure 1](#) shows the distribution of funding instruments before and after the policy, by test score segment. Students scoring below 475 were ineligible for subsidized loans or merit-based scholarships (except for a small scholarship targeting top students from low-SES schools). For this group, TFHE reduced the share of students with no funding. Students scoring between 475 and 510 qualified for subsidized loans but remained below the threshold for most scholarships; for them, substitution was mainly from loans to TFHE, relieving them of any debt upon graduation. Students scoring above 510 qualified for

merit-based scholarships (conditional on income eligibility). For this group, substitution was mainly from scholarships to TFHE, which had two advantages: first, scholarships typically covered only 80% of tuition and fees, whereas TFHE provided full funding; second, scholarships required students to meet satisfactory progression standards (validating about 70% of annual course credits), while TFHE imposed no such requirements. Before the policy, about 20% of scholarship beneficiaries lost eligibility; of these, around 40% switched to subsidized loans, while the rest continued without public funding. After the policy, most substituted to TFHE, while those above the income threshold continued either with loans or no funding, in roughly the same proportions as before.

Figure 1: Distribution of funding instruments, by test score



**Notes:** This figure shows the distribution of funding instruments by test score group, for 2015 and 2016. The Scholarship category includes any merit-based scholarship, while Loan includes both the SGL and CAE schemes.



## 2.3 Data

We use Chilean administrative records that provide detailed student and program data. We observe enrollment, admission test scores, scholarship assignment, socioeconomic information, and demographic characteristics. Institutional data include type, tuition, location, and program length. Between 170,000 and 180,000 students take the national exam each year, of whom about 60% enroll in higher education. Descriptive statistics are reported in [Table B1](#).

In addition, we use restricted data on credit completion from the Information Service of Higher Education (SIES) covering the universe of enrolled students from 2016 to 2023. These data report the number of courses registered and whether they were completed.

### Quasi-Experimental sample

We restrict the sample to first-time test takers for the 2012 to 2017 cohort. Since we observe the data until 2023 and the standard university program lasts five years, we allow seven years for students to complete their program. In 2012, scholarships were expanded to include the third income quintile, such that all eligible students for FTHE were already eligible for scholarships conditional on scoring a certain PSU score (Bucarey, [2017](#)).

### Model sample

We are constrained by the fact that credits are only available from 2016. Together with allowing students 7 years for graduation, we estimate the model on a random sample of 2016-2017 cohorts. CCPs are estimated using the *universe* of 2016-2017 cohorts.

### 3 Policy Effects

We are interested on causally estimating whether the TFHE policy had an impact on enrollment and educational outcomes. To do so, we exploit the exogenous variation in eligibility for free university prices introduced by TFHE to use differences-in-differences.

$$Y_{it} = \sum_{\substack{k=2013 \\ k \neq 2015}}^{2020} \beta_k (\mathbb{1}\{t = k\} \times \mathbb{1}\{dec_i < 6\}) + \gamma \mathbb{1}\{dec_i < 6\} + \delta_t + \mathbf{X}_i' \alpha_x + \epsilon_{it} \quad (1)$$

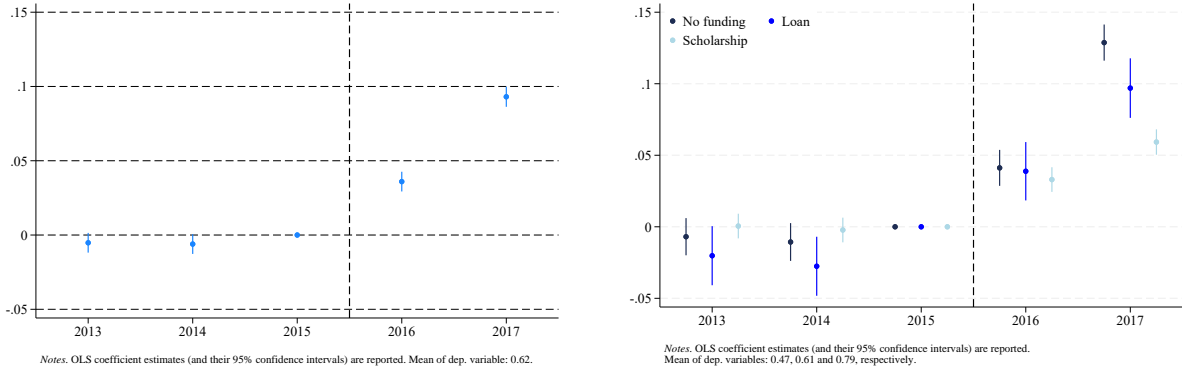
$Y_{it}$  is an outcome of interest including enrollment and educational outcomes such as dropout, graduation, and persistence, defined as the time (in years) spent in college.  $\mathbf{X}_i'$  includes a constant, gender, PSU score, degree type, mother education, and family income.  $\mathbb{1}\{dec_i < 6\}$  determines eligibility to TFHE, based on belonging to the lower half of the income distribution.  $\delta_t$  are cohort fixed effects. Using the year before the implementation as reference category,  $\beta_t$  can be interpreted as the effect of TFHE compared to 2015.

#### 3.1 Effects on enrollment

We first examine the impact of TFHE on the extensive margin, i.e. enrollment. The cohort of 2016 is the first for which it can affect the enrollment decision. [Figure 2](#) shows the results of estimating [Equation 1](#). As can be seen in panel a), enrollment increased by 3.5 p.p. in 2016 and 9.3 p.p. in 2017, compared to 2015. This increase in enrollment comes from students that either returns to education became positive or were credit constrained. The higher effect for year 2017 is consistent with the expansion of free higher education to SCPs. As explained in [section 2](#), the control group includes students that had either no (public) funding, or were already eligible to loans or scholarships. We expect these different groups

of students to react differently to the policy. In panel b), we observe that students of lower ability, which had no access to public funding before 2016, are the group which increases enrollment the most. Differences between groups are significant for 2017, where students of lower ability increase enrollment by 12 p.p. compared to 5.5 p.p. of students that had already access to merit scholarships.

Figure 2: Enrollment



a) Effect on enrollment

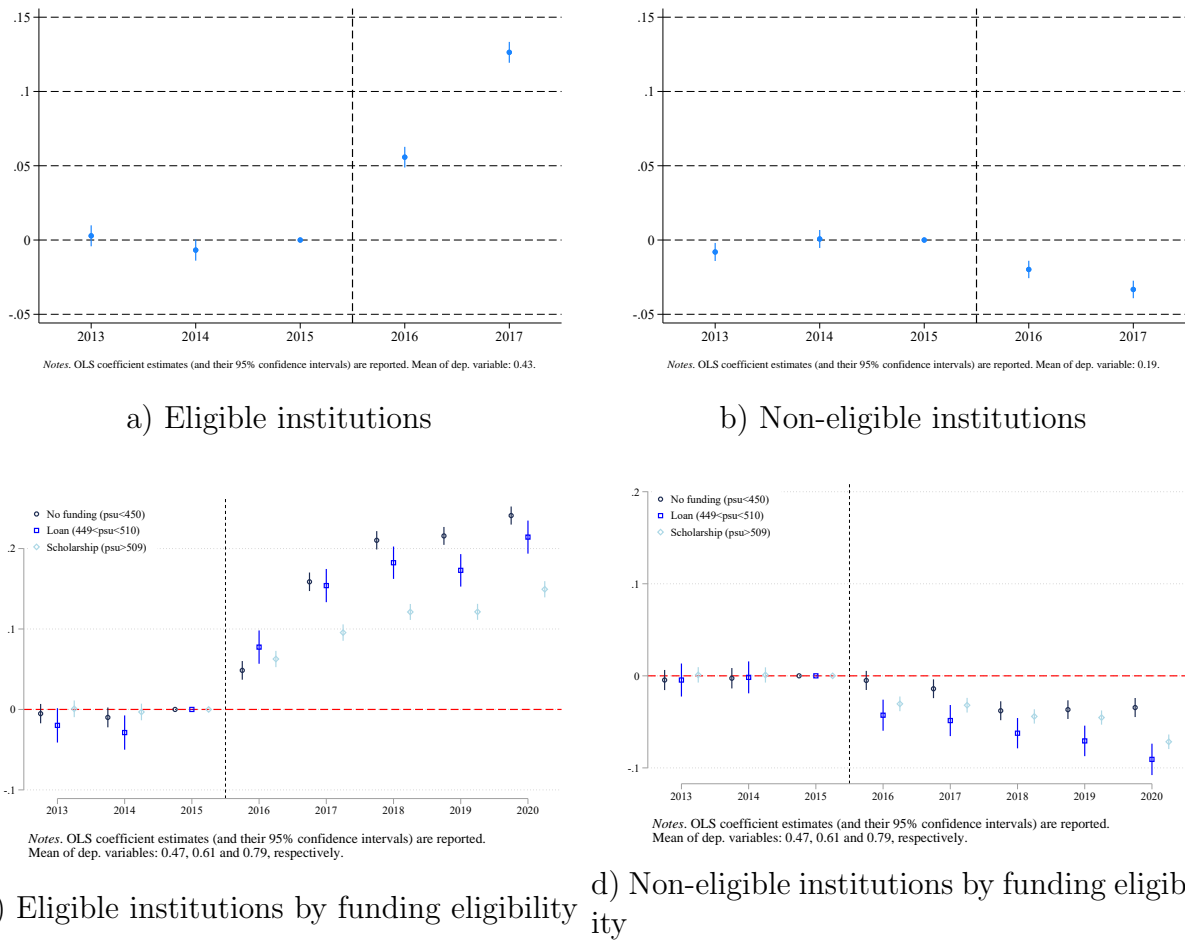
b) Effect on enrollment by funding eligibility

**Notes:** This figures show the results of estimating Equation 1. Panel (a) shows the aggregate results, while panel (b) estimates the equation for groups based on their test scores .

Since free higher education affected only around 40% of institutions, it is also important to examine whether there was substitution from non-eligible to eligible centers, or whether enrollment is driven completely by marginal students. Figure 3 reports the estimation on enrollment for eligible and non-eligible institutions. Results show a significant increase of eligible students to higher educations institutes that were eligible to the policy. In panel a), we observe increases of 5 and 12 p.p. for years 2016 and 2017, respectively. In contrast to Londoño-Vélez et al., 2020, panel b) suggests no crowding-out. In fact, we find a negative effect on non-eligible institutions of -2 and -3.5 p.p. When looking at panel c), we find for 2016 that loan-eligible students are the ones increasing enrollment the most in eligible institutions,

almost 10 p.p. Scholarship-eligible (high-ability) students are the ones increasing the least. This is consistent with the fact that institutions participating in the policy have on average students of higher ability than non-participating institutions ([Figure A1](#)). Panel d) show that loan-eligible students have the strongest negative coefficient, reaching -4 and -6 p.p. in 2016 and 2017, respectively.

Figure 3: Enrollment by institution eligibility



**Notes:** This figures show the results of estimating [Equation 1](#) on enrollment for eligible and non-eligible institutions to the Free College policy. Panel a) and b) show the aggregate results, while panels c) and d) disaggregate results by test score group.

### 3.2 Effects on educational outcomes

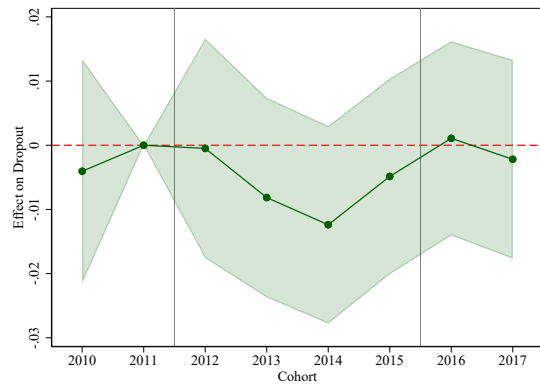
Recovering treatment effects for the intensive margin is not as straightforward. Estimation results from Equation 1 suffer from a composition effect: the composition of students after 2016 is not comparable to the one of 2015, given that a substantial fraction of lower-ability enrolled (marginal students) as can be seen in Figure A2. A particular feature of the policy is that students before 2016 had access to TFHE for the remaining years of the nominal length of their studies. For example, if they enrolled in 2015 in a five-year degree, they can be exempted of tuition for the remaining four years.

We can use this exogenous variation to partial out selection. 2012 to 2015 cohorts are potentially eligible to TFHE starting 2016 but cannot change their enrollment decision. We redefine Equation 1 as

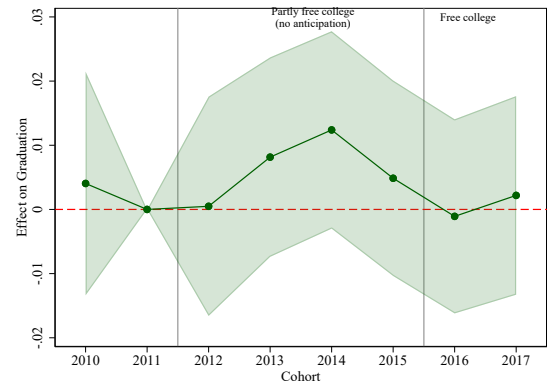
$$Y_{it} = \sum_{\substack{k=2010 \\ k \neq 2011}}^{2017} \beta_k^c (\mathbb{1}\{t = k\} \times \mathbb{1}\{dec_i < 6\}) + \gamma^c \mathbb{1}\{dec_i < 6\} + \delta_t^c + \mathbf{X}_i' \boldsymbol{\alpha}_x^c + \epsilon_{it}^c \quad (2)$$

We do not find any significant effect on dropout, graduation, persistence, or graduation on time. Figure 4 shows the results of estimating Equation 2. Panels a) to d) show the results for dropout, graduation, persistence, and graduation on time, respectively. We do not find any significant effect of TFHE on these outcomes. If anything, we find a small non-significant negative (-0.1p.p.) effect on dropout as student receive more years of TFHE.

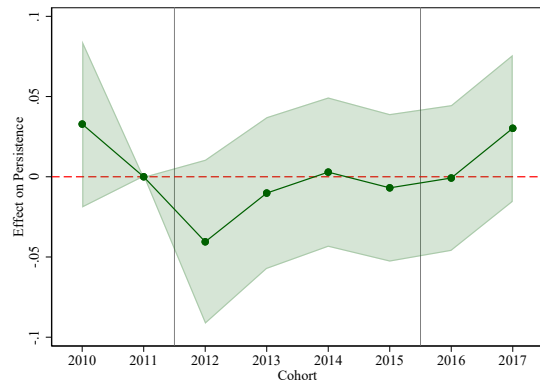
These results are best interpreted as the overall effect of the policy on those who already enrolled. TFHE lowers the cost of staying enrolled, which tends to reduce dropout and increase graduation. Conversely, removing performance requirement to students who previously benefited from scholarships could also entail a negative effect on dropout: out-of-pocket fees



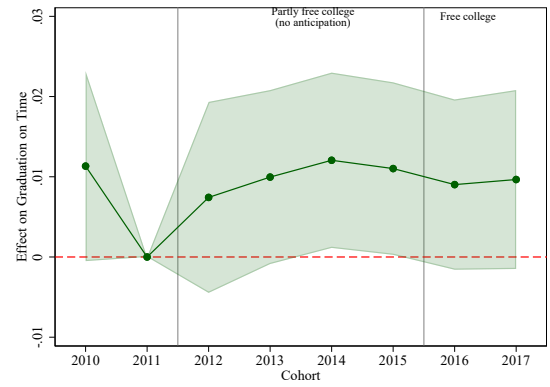
a) Dropout



b) Graduation



c) Persistence



d) Graduation on Time

Figure 4: DID outcomes.

do not longer depend on performance outcomes. What we found empirically is that the cost reduction of staying enrolled seems to dominate. To validate reduced-form results and quantify both effects, we next build a model of enrollment and major choice with endogenous effort.

## 4 Model

The decision process of an individual considering attending higher education can be depicted in two stages. In the first stage, she chooses whether to enroll, and if so, which field–institution combination to attend. To decide, she observes program characteristics and available funding instruments, and optimizes how much effort to exert in each program in her personalized choice set (Fack et al., 2019). Effort choice will determine her distribution of performance outcomes (credits). Conditional on enrollment, performance today endogenously determines out-of-pocket fees for the subsequent year. The student graduates when she cumulates the required number of credits of the enrolled program.

Direct measures of effort are not observed. Instead, we attempt to uncover *effective* effort through realized outcomes, i.e. course credits completion (De Groote, 2025).

### 4.1 Timing and choices

The model is discrete in time, with  $t \in \{0, 1, \dots, T\}$  representing academic years. The nominal length of the program  $j$  is  $\bar{t}_j$ . The student can drop out at any time until the terminal period is  $T = 7$  years, where she is forced to drop out if she did not yet graduate. The student’s type is denoted by  $Z_i$ , which includes sex, ability (psu and gpa), and socioeconomic status.

During the first period, the student decides whether to enroll in higher education. If she enrolls, she picks a field–institution combination  $d_{it} = j$ , with  $j \in \mathcal{J}(x_{it}) = \{1, \dots, J(x_{it})\}$  a program from her personalized choice set, after solving for optimal effort in each program  $e_j^*(x_{it})$ . The outside option  $j = 0$  corresponds to not enrolling. Conditional on enrollment, in each subsequent period she decides whether to continue in the program,  $d_{it} \in \{d_{i,t-1}, 0\}$ , and how much effort  $e_{it} \in [0, +\infty)$  to exert. By exerting effort, the student implicitly chooses the probabilities of performance outcomes  $g_{i,t+1} \in \mathcal{G} = \{0, 1, 2, 3, 4\}$ , which accumulates over time as  $G_{it} := \sum_{s=1}^t g_{is}$ .

This means that for periods prior to graduation ( $t = 1, \dots, \bar{t}_j$ ), effort affects both the probability of dropout and, for scholarship holders, out-of-pocket fees  $OP_j(x_{it})$ : failing to meet the performance requirement ( $g_{it} \geq 3$ ) implies paying the full tuition the following year. During the graduation period ( $t = \bar{t}_j + 1, \dots, T$ ), the student can eventually graduate with some probability and obtain the continuation value associated with graduation, or alternatively postpone graduation if  $t < T$ , or drop out. At terminal period  $T = 7$ , any student who has not yet graduated is assumed to drop out. Hence, the state space is defined as  $x_{it} = (t, d_{i,t-1}, g_{i,t-1}, G_{i,t-1}, Z_i)$ .

## 4.2 Utility and the dynamic program

We interpret flow utility as a negative cost, consisting of a fixed cost ( $FC_j$ ) that captures out-of-pocket fees and preferences for higher education, and a variable cost of effort:

$$u_j(x_{it}) + \varepsilon_{ijt} = -FC_j(x_{it}) - c_j(x_{it})e_{it} + \varepsilon_{ijt}, \quad (3)$$



with marginal cost  $c_j(x_{it}) > 0$ . The fixed cost term  $FC_j(x_{it})$  depends on  $x_{it}$  and out-of-pocket fees  $OP_j(x_{it})$ , defined as the fraction of program fees not covered by the State for student  $i$  in program  $j$ . These depend on personal characteristics  $Z_i$  and past performance  $g_{i,t-1}$ :

$$OP_j(x_{it}) = \begin{cases} (1 - \lambda(t, g_{i,t-1}, Z_i))P_j, & \text{if the student holds a scholarship,} \\ (1 - \lambda(t, Z_i))P_j, & \text{otherwise.} \end{cases}$$

Here,  $\lambda(\cdot) \in [0, 1]$  is the subsidy rate, i.e. the fraction of program fees  $P_j$  covered by the Government. For scholarship holders,  $\lambda$  depends on past performance  $g_{i,t-1}$  as well as on individual characteristics  $Z_i$ , since failure to meet credit requirements results in the loss of the subsidy. For loans and TFHE,  $\lambda$  depends only on observable characteristics and does not vary with performance. Finally, the distribution of the idiosyncratic shock  $\varepsilon_{ijt} \sim \text{EV1}$  is iid and common knowledge. It captures what the student learns at the beginning of the period.

The dynamics of the optimization problem are the following: On the one hand, by exerting more effort today, the student i) increases her probability of graduating in subsequent periods and raises her expected future wage, and ii) increases her probability to maintain public funding (passing the performance threshold) if eligible. On the other hand, exerting effort is costly and reduces the flow utility of attending higher education. The conditional value function  $v_j(x_{it}, e_{it})$  can be written as

$$v_j(x_{it}, e_{it}) + \varepsilon_{ijt} = u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) \bar{V}(x_{i,t+1}(\bar{g})) + \varepsilon_{ijt}, \quad (4)$$

where  $\bar{V}(x_{i,t+1}(\bar{g}))$  denotes the ex-ante value function conditional on choosing program  $j$ .

The first term corresponds to the current flow utility from enrolling in field–institution  $j$ . The second term captures the expected continuation value, discounted by  $\beta$ . Uncertainty in  $x_{i,t+1}(\bar{g})$  arises solely from the realization of the performance outcome  $\bar{g}$ .

Applying the log-sum expression to [Equation 4](#) yields

$$v_j(x_{it}, e_{it}) + \varepsilon_{ijt} = u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) \ln \left( \sum_{j \in \mathcal{J}} \exp(v_j(x_{i,t+1}, e_{i,t+1})) \right) + \beta\gamma + \varepsilon_{ijt}.$$

### 4.3 Performance outcomes

Performance outcomes  $g_{it}$  are indirectly observed through students' ability to retain a scholarship. In particular, scholarship holders must validate (i) at least 60% of registered credits in the first year and (ii) 70% from the second year onward in order to remain eligible. After the introduction of TFHE, however, students are no longer subject to performance thresholds to maintain funding.

We model the performance outcome  $g_{i,t+1}$  as the result of effort  $e_{it}$  and a logistically distributed shock  $\eta_{it}$ :

$$g_{i,t+1} = \kappa \quad \text{if } \bar{g}_{jt}^{\kappa} < \ln(e_{it}) + \eta_{i,t+1} \leq \bar{g}_{jt}^{\kappa+1}, \quad (5)$$

where the cutoff values  $\bar{g}_{jt}^{\kappa}$  and  $\bar{g}_{jt}^{\kappa+1}$  are field–institution–year specific. In practice, performance is discretized into five categories,  $\kappa \in \{0, 1, 2, 3, 4\}$ . Students observe the realization of the shock  $\eta_{i,t+1}$  only after choosing their effort  $e_{it}$ , but since they know the distribution of  $\eta_{i,t+1}$  (standard logistic), they can compute the probability of each outcome conditional

on effort:

$$\Pr(g_{i,t+1} = \kappa \mid e_{it}, d_{it}) = F(\ln(e_{it}) - \bar{g}_{jt}^\kappa) - F(\ln(e_{it}) - \bar{g}_{jt}^{\kappa+1}), \quad (6)$$

where  $F(\cdot)$  denotes the logistic CDF.

Following De Groote (2025), effective effort can be interpreted as the odds of avoiding the lowest performance outcome (failing all registered credits):

$$e_{it} = \frac{1 - \Pr(g_{i,t+1} = 0 \mid x_{it}, d_{it})}{\Pr(g_{i,t+1} = 0 \mid x_{it}, d_{it})}. \quad (7)$$

Higher effort reduces the likelihood of obtaining the lowest performance outcome.

#### 4.4 Solution of the model

The model is solved by backward induction. Higher education is no longer feasible after seven years, or once the student completes all program course credits. Formally,

$$\bar{V}(x_{i,t+1}(\bar{g})) = \alpha_w \text{wage}_j(x_{i,t+1}) \quad \text{if } t+1 = T = 7 \text{ or } G_{i,t+1} \geq G_j^{\text{req}}. \quad (8)$$

and is used as an input in earlier periods. Here,  $\text{wage}_j(x_{i,t+1})$  denotes lifetime expected wage for program  $j$  or dropout wage if  $d_{i,t+1} = 0$ .

At each period  $t$ , the student first chooses an effort level  $e_{it}$  for every available program  $j \in \mathcal{J}(x_{it})$ . The decision trades off a loss in flow utility with higher future expected gains. Assuming she is rational and behaving optimally, she is solving the following first-order condition for each program:

$$\frac{\partial v_j(x_{it}, e_{it})}{\partial e_{it}} = \frac{\partial u_j(x_{it}, e_{it})}{\partial e_{it}} + \beta \sum_{g \in \mathcal{G}} \frac{\partial \phi^{\bar{g}}(e_{it})}{\partial e_{it}} \bar{V}(x_{it+1}(\bar{g})) = 0 \quad \text{if } e_{it} = e_j^*(x_{it}).$$

Given the set of optimal effort levels  $\{e_j^*(x_{it})\}_{j \in \mathcal{J}(x_{it})}$ , the student chooses the program with the highest value  $v_j(x_{it}, e_j^*(x_{it}))$ . The resulting choice probabilities take the familiar logit form:

$$p_{jt} = \frac{\exp(v_j(x_{it}, e_j^*(x_{it})))}{\sum_{j' \in \mathcal{J}(x_{it})} \exp(v_{j'}(x_{it}, e_{j'}^*(x_{it})))}.$$

The ex-ante value function in period  $t$  can then be expressed, using the logsum formula, as

$$\bar{V}(x_{it}) = \gamma + \ln \left( \sum_{j \in \mathcal{J}(x_{it})} \exp(v_j(x_{it}, e_j^*(x_{it}))) \right), \quad (9)$$

and the model is solved recursively by iterating backward to  $t = 1$ .

## 4.5 Identification

### 4.5.1 Fixed and Marginal costs

$$\begin{cases} u_j(x_{it}, e_{it}) = -FC_j(x_{it}) - c_j(x_{it})e_{it}, & \text{if } j \neq 0, \\ u_0(x_{it}) = \alpha_w wage_0(x_{it}), & \text{if } j = 0. \end{cases} \quad (10)$$

We assume that not enrolling in higher education (and dropping out) is a terminal action. This means we do not allow for individuals to resit the exam one year after or re-enter higher

education after dropping out.

$$\frac{\partial v_j(x_{it}, e_{it})}{\partial e_{it}} = \underbrace{\frac{\partial u_j(x_{it}, e_{it})}{\partial e_{it}}}_{=-c_j(x_{it})} + \beta \sum_{g \in \mathcal{G}} \frac{\partial \phi^{\bar{g}}(e_{it})}{\partial e_{it}} \bar{V}(x_{it+1}(\bar{g})) = 0 \quad \text{if } e_{it} = e_j^*(x_{it}).$$

Making use of the FOC and rearranging, we get an expression for marginal costs:

$$c_j^*(x_{it}) = \beta \sum_{\bar{g} \in \mathcal{G}} \frac{\partial \phi^{\bar{g}}(e_{it})}{\partial e_{it}} \bar{V}(x_{it+1}(\bar{g})) \quad \text{if } e_{it} = e_j^*(x_{it}), \quad (11)$$

where  $\bar{V}(x_{it+1}(\bar{g}))$  is estimated using the CCPs, and  $\frac{\partial \phi^{\bar{g}}(e_{it})}{\partial e_{it}}$  can be computed given the logistic assumption about  $\eta_{t+1}$  in [Equation 5](#). A sufficient condition for an interior solution is that the student always faces a strictly positive probability of the lowest outcome being realized; otherwise the optimal effort would collapse to  $e_{it}^* = 0$ . Positive marginal costs ensures that the support is bounded.

Fixed costs then rationalize preference heterogeneity across higher education programs.

#### 4.5.2 Policy variation

We exploit the variation induced by the policy in two dimensions: (i) the exogenous variation in out-of-pocket fees and (ii) the exogenous variation in performance requirements from switching between funding instruments.

TFHE induced marginal students to enroll either because expected returns to education became positive or because credit constraints were relaxed. We can identify marginal students from those who change their enrollment decision, i.e.  $d_{it}(FC_j(x_{it})) \neq d_{it}(FC'_j(x_{it}))$ . Changes in performance outcomes can be directly identified from infra-marginal students, and treatment effects can be computed pre- and post-policy, i.e.  $\sum_{i=IM} n_{IM}^{-1} (Y_{it}(FC_j(x_{it}), c_j(x_{it})) -$

$$Y_{it}(FC'_j(x_{it}), c'_j(x_{it})).$$

## 4.6 Estimation

Given the Type-1 extreme value assumption, CCPs are of logit type.

$$\Pr(d_{it} = j | x_{it}) = p_{jt} = \frac{\exp(v_j(x_{it}, e_j^*(x_{it})))}{\sum_{j \in \mathcal{J}} \exp(v_j(x_{it}, e_j^*(x_{it})))}$$

Hotz and Miller (1993) show that the future value term can be written as the conditional value function and a non-negative term that depends on the empirical choice probabilities. Without loss of generality, we can write choice probabilities in terms of a base category  $j'$ .

$$p_{j't} = \frac{1}{1 + \exp(v_1(x_t))} \quad p_{jt} = \frac{\exp(v_j(x_t) - v_{j'}(x_t))}{\sum_{j \in \mathcal{J}} \exp(v_j(x_t) - v_{j'}(x_t))}$$

It is therefore convenient to use  $j' = 0$  as an arbitrary choice and write

$$\begin{aligned} & v_j(x_{it}, e_j^*(x_{it})) + \varepsilon_{ijt} \\ &= u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) \ln \left( \exp(v_0(x_{i,t+1}, e_j^*(x_{i,t+1}))) \sum_{j \in \mathcal{J}} \exp(v_j(x_{i,t+1}, e_j^*(x_{i,t+1})) - v_0(x_{i,t+1}, e_j^*(x_{i,t+1}))) \right) \\ &+ \beta \gamma + \varepsilon_{ijt} \\ &= u_j(x_{it}, e_{it}) + \beta \sum_{g \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (v_0(x_{t+1}, e_j^*(x_{i,t+1})) - \ln p_{0,t+1}(x_{i,t+1}(\bar{g}))) + \beta \gamma + \varepsilon_{ijt} \end{aligned}$$

Recall the continuation value of dropping from higher education can be written as the expected wage of the degree minus a log correction term that depends on the dropout proba-

bility. Since  $v_0(x_{t+1}, e_j^*(x_{it})) = \alpha_w wage_0(x_{i,t+1})$ , we can rewrite the expression as

$$v_j(x_{it}, e_{it}) + \varepsilon_{ijt} = u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (\alpha_w wage_0(x_{it+1}) - \ln p_{0,t+1}(x_{i,t+1}(\bar{g}))) + \beta\gamma + \varepsilon_{ijt} \quad (12)$$

Estimation can be performed following three sequential steps.

1. We recover  $\Pr(d_{it+1} = 0|x_{it+1})$  from a logistic regression and  $\ln e_j^*(x_{it})$  the underlying index of an ordered logit model of credit completion (0-4).
2. The FOC equates marginal cost to marginal benefits, with an expression that can be directly estimated from the data. We can substitute the expression in  $v_j(x_{it})$  and estimate  $FC_j(x_{it})$  by maximum likelihood.
3. Finally, we use the estimates and the FOC to compute  $c_j(x_{it})$ .

We set the discount factor  $\beta$  to 0.95 and normalize the utility of the outside option to the dropout wage (Magnac and Thesmar, 2002).

## 5 Results

### 5.1 Dropout probabilities

Table 1 reports the results of a logistic regression model of dropout. Overall, dropout probabilities depend strongly on the year of enrollment. Progressing through the program reduces the likelihood of dropping out (relative to period 1), although the probability rises again when students reach the nominal length of the degree. Being female reduces the probability of dropping out, with the effect becoming stronger as students progress in their

Table 1: Dropout probabilities

	dropout	
OP fees	0.083***	(0.003)
Cumulated credits (t-1)	-0.113***	(0.002)
Credits (t-1)	-0.651***	(0.005)
Delay (t-1)	1.200***	(0.019)
female	-0.439***	(0.036)
Middle SES	-0.190***	(0.042)
High SES	-0.550***	(0.047)
PSU	-0.017	(0.026)
GPA	-0.164***	(0.022)
OP $\times$ female	-0.003	(0.003)
OP $\times$ middle SES	0.023***	(0.004)
OP $\times$ high SES	0.070***	(0.005)
OP $\times$ psu	-0.028***	(0.002)
OP $\times$ gpa	-0.004**	(0.002)
Cumul (t-1) $\times$ female	-0.033***	(0.001)
Cumul (t-1) $\times$ middle SES	0.002	(0.001)
Cumul (t-1) $\times$ high SES	-0.005***	(0.001)
Cumul (t-1) $\times$ psu	0.022***	(0.001)
Cumul (t-1) $\times$ gpa	0.001*	(0.001)
Credits (t-1) $\times$ female	0.133***	(0.005)
Credits (t-1) $\times$ middle SES	0.029***	(0.006)
Credits (t-1) $\times$ high SES	0.073***	(0.007)
Credits (t-1) $\times$ psu	-0.031***	(0.003)
Credits (t-1) $\times$ gpa	0.042***	(0.003)
Delay (t-1) $\times$ female	0.508***	(0.020)
Delay (t-1) $\times$ middle SES	0.022	(0.024)
Delay (t-1) $\times$ high SES	0.040	(0.026)
Delay (t-1) $\times$ psu	-0.221***	(0.013)
Delay (t-1) $\times$ gpa	0.164***	(0.013)
t= 2	-0.340***	(0.011)
t= 3	-0.570***	(0.015)
t= 4	-0.612***	(0.020)
t= 5	0.117***	(0.024)
t= 6	1.654***	(0.028)
t= 7	3.862***	(0.036)
Observations	1,417,543	
Mean of Dep. Variable	0.180	

Standard errors in parentheses

Logistic regression of choosing the outside option (dropping out) in period t. Controls include vector of characteristics, Institution and major FE, year enrolled FE and cumulated performance. PSU and GPU are standardised. Base category is low SES and t=1.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$



studies.

Ability is strongly correlated with persistence in higher education, but conditional on the student's progress (accumulated credits), its estimated coefficient diminishes. Both cumulative credits and the number of credits completed in the previous year are strong determinants of student progression, especially for low-income students. Being delayed in the program increases the probability of dropping out, with a stronger magnitude for females.

## 5.2 Performance

[Table B2](#) presents the results of the ordered logit model of performance with five categories (0–4). Coefficients follow a similar pattern as in the logistic dropout model. Ability is a strong predictor of performance, as is income. Out-of-pocket fees have a negative effect on performance, which is stronger for low-income and low-ability students.

Being delayed in the program reduces the probability of achieving the maximum number of credits, although the effect is milder for females. Finally, past progression—measured through cumulative and previous-year credits—is a strong predictor of current performance, particularly for low-income students.

## 5.3 Fixed Costs

Estimates of the conditional value function are presented in [Table 2](#). Students' fixed costs increase with out-of-pocket fees, particularly at the program choice stage. Once enrolled, this effect is substantially attenuated in the continuation decision. Females are more sensitive to price than males. Students also face higher fixed costs when the program is located in a different region.

Table 2: Fixed costs

	cvf	
OP fees	-0.588***	(0.037)
Distance	-1.397***	(0.031)
Delay (t-1)	-2.744***	(0.127)
OP x female	-0.106***	(0.034)
OP x middle SES	0.026	(0.040)
OP x high SES	-0.056	(0.045)
OP x psu	0.014	(0.022)
OP x gpa	0.015	(0.020)
OP (during HE)	0.440***	(0.042)
Delay (t-1) x female	-0.280**	(0.136)
Delay (t-1) x middle SES	-0.150	(0.160)
Delay (t-1) x high SES	-0.291	(0.181)
Delay (t-1) x psu	0.377***	(0.075)
Delay (t-1) x gpa	-0.131	(0.086)
EMAX	0.950	(.)
Expected wage	0.001***	(0.000)
Field FE	Yes	
Institution FE	Yes	

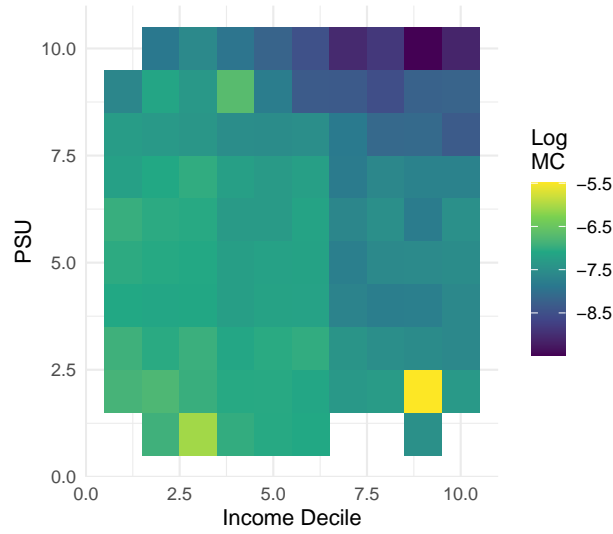
Standard errors in parentheses

Conditional Value function Estimation

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Past progression significantly affects fixed costs, as reflected in the coefficient of the delay variable. Again, females face higher fixed costs than males when delayed, and students who performed well on the national exam appear to experience smaller penalties from lagging behind. Expected wage has a positive effect on the conditional value function, indicating that students value their future expected earnings.

Figure 5: Distribution of funding instruments, by test score



**Notes:** Heatmap of log marginal costs by PSU (y-axis) and income (x-axis) decile.

Figure 5 shows the distribution of log marginal costs by income and PSU score. The heatmap exhibits a gradient in marginal costs in both ability and socioeconomic status dimensions.

Table 3 presents, for interpretational purposes, the OLS regression of log marginal costs on state variables. Marginal costs decrease with out-of-pocket fees, in particular for low-income students. Females tend to have higher marginal costs compared to males, especially at lower income levels.

Table 3: Marginal costs

	log(mc)	
female	-0.226**	(0.105)
Middle SES	-0.005	(0.108)
High SES	0.242**	(0.115)
Standardized values of psu_avg	-0.322***	(0.120)
Standardized values of gpa_hs	-0.130**	(0.058)
OP fees	-0.038**	(0.019)
Distance	0.064***	(0.025)
Delay (t-1)	-0.801***	(0.170)
t= 2	-0.221***	(0.075)
t= 3	-0.107	(0.079)
t= 4	-0.039	(0.097)
t= 5	2.393***	(0.089)
t= 6	2.932***	(0.180)
t= 7	4.966***	(0.177)
OP x female	0.032	(0.020)
OP x middle SES	-0.017	(0.018)
OP x high SES	-0.018	(0.026)
OP x psu	-0.009	(0.010)
OP x gpa	0.007	(0.012)
Delay (t-1) x female	0.158	(0.127)
Delay (t-1) x middle SES	0.421***	(0.151)
Delay (t-1) x high SES	0.307*	(0.166)
Delay (t-1) x psu	0.034	(0.074)
Delay (t-1) x gpa	-0.011	(0.070)
Observations	10,811	
ymean		

Standard errors in parentheses

MC heterogeneity

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

## 6 Counterfactuals

We use the estimated model to simulate counterfactual funding policies, evaluating impacts on both enrollment and intensive (effort, credit completion, time-to-degree, graduation) margins. The exercises vary (i) the standardized exam threshold and (ii) the performance requirements.

### 6.1 PSU and performance requirements

We saw that TFHE substitute most of the preexisting aid programs, which were conditional on performance. The removal of performance requirements may have weakened incentives for effort and progression. Preexisting aid programs had performed well in attracting students, but it was not generous enough to attract low-income students. From the Government perspective, the current policy is very costly. From 2011 to 2019, the spending on higher education increased by 160%, representing 5.4% of GDP, in comparison with a 2.9% in the rest of OECD countries (OECD, 2019). Counterfactuals proposed build on expanding the conditions of previous aid programs. They are budget improving, while preserving the equity target of the TFHE policy. We therefore explore the following three scenarios:

1. **Performance requirement.** Impose an annual performance requirement (70% of registered credits) to *retain* free tuition. This restores the performance-contingent component removed by TFHE.

Model prediction: First period fixed and marginal costs are not affected. However, starting from the second year, students might want to increase effort to maintain the scholarship. Those that should increase it the most are the ones with high marginal and fixed costs, so the probability of dropout should increase. If the utility from

attending a first year of higher education does not compensate enough, it might deter from enrolling at all in higher education.

2. **PSU requirement.** Impose  $PSU \geq 510$  for eligibility (harmonizing with merit scholarships). Mechanically reduces Government expenditure.

Model prediction: Attracts higher ability students, with lower fixed and marginal costs.

Persistance and graduation should increase.

3. **Joint requirement.** Combine (1) and (2):  $PSU \geq 510$  and maintaining the performance threshold to retain free tuition.

Model prediction: Combines the selection and incentive approach. Reduces beneficiaries, but those that remain should have lower fixed and marginal costs, and stronger incentives to progress.

Mechanically, (1) affects  $OP(x_{it})$  via  $g_{t-1}$  for *all* recipients, tightening incentives within a cohort; (2) affects the selection into TFHE; and (3) compounds both. Because the model allows  $c_j(x_{it})$  to depend on OP, both  $FC_j(x_{it})$  and  $c_j(x_{it})$  increase for less performant students.

## 6.2 Additional scenarios

We outline additional scenarios for future work:

- **“New Zealand”-style.** Free Higher Education only in year 1; reversion to baseline thereafter. Tests whether early liquidity relief plus a return to progress-contingent support improves completion cost-effectively. Students learn their fix and marginal cost during year 1, and may adjust effort and dropout accordingly.
- **Targeted to academic/vocational education.** Make different ability thresholds

for academic and vocational programs.

- **Eligibility index combining merit and need.** Replace hard cutoffs with an index (e.g., weighted PSU and income) for a smoother assignment that may mitigate bunching and cliffs.
- **Remove merit requirements for loans/scholarships.** Harmonize loans and scholarships with the unconditional nature of Free Higher Education to isolate access from performance and compare purely through *OP*.

For each scenario, we report changes in enrollment, persistence, effort, credit accumulation, graduation.

## 7 Conclusion

This paper investigated the impacts of Free College policies on student enrollment, persistence, and academic performance, using the 2016 Chilean reform as a natural experiment. Our difference-in-differences analysis reveals that the policy significantly boosted enrollment and persistence among eligible low-income students, particularly those with lower academic achievement, while exerting only modest effects on graduation and dropout rates. Through a structural model that endogenizes effort choices, we disentangle selection effects from behavioral responses, finding that the removal of performance-based requirements did not lead to reduced effort or weakened outcomes overall. Instead, Free College effectively expanded access without compromising educational quality, highlighting its role in mitigating financial barriers and moral hazard concerns. These findings have important implications for higher education financing worldwide, suggesting that unconditional aid can promote equity and mobility, especially in contexts with high tuition costs and imperfect income verification.

Counterfactual simulations underscore the trade-offs between enrollment gains and potential fiscal burdens, informing policymakers on optimizing funding instruments. Future research could extend this framework to long-term labor market outcomes or comparative analyses across countries to further refine these insights.

## References

- Angrist, J. et al. (Jan. 2015). “Leveling Up: Early Results from a Randomized Evaluation of Post-Secondary Aid”.
- Arcidiacono, P. (Sept. 2005). “Affirmative Action in Higher Education: How Do Admission and Financial Aid Rules Affect Future Earnings?” In: *Econometrica* 73 (5), pp. 1477–1524.
- Beffy, M., D. Fougère, and A. Maurel (Feb. 2012). “Choosing the Field of Study in Post-secondary Education: Do Expected Earnings Matter?” In: *The Review of Economics and Statistics* 94 (1), pp. 334–347.
- Black, S. E. et al. (2023). “Taking it to the Limit: Effects of Increased Student Loan Availability on Attainment, Earnings, and Financial Well-Being”. In: *SSRN Electronic Journal*.
- Bucarey, A. (2017). “Who Pays for Free College? Crowding Out on Campus”. In: *Job Market Paper*, pp. 1–71.
- Bucarey, A., D. Contreras, and P. Muñoz (2020). “Labor market returns to student loans for university: Evidence from Chile”. In: *Journal of Labor Economics* 38 (4), pp. 959–1007.
- Cohodes, S. R. and J. S. Goodman (2014). “Merit Aid, College Quality, and College Completion: Massachusetts Adams Scholarship as an In-Kind Subsidy”. In: *American Economic Journal: Applied Economics* 6 (4), pp. 251–85.
- De Groote, O. (Apr. 2025). “Dynamic Effort Choice in High School: Costs and Benefits of an Academic Track”. In: *Journal of Labor Economics*.



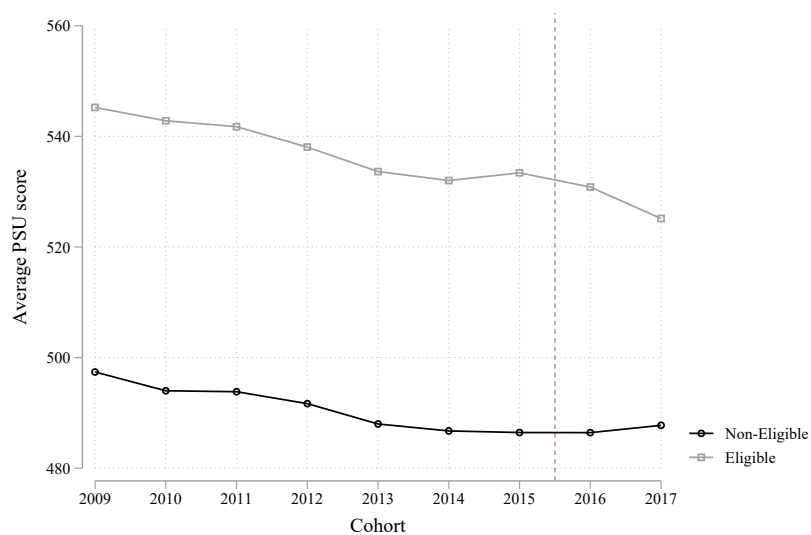
- Denning, J. T. (May 2017). “College on the cheap: Consequences of community college tuition reductions”. In: *American Economic Journal: Economic Policy* 9 (2), pp. 155–188.
- (2019). “Born under a Lucky Star”. In: *Journal of Human Resources* 54 (3), pp. 760–784.
- Dobbin, C., N. Barahona, and S. Otero (2022). “The Equilibrium Effects of Subsidized Student Loans”.
- Dynarski, S. M. (Mar. 2003). “Does Aid Matter? Measuring the Effect of Student Aid on College Attendance and Completion”. In: *American Economic Review* 93 (1), pp. 279–288.
- Fack, G., J. Grenet, and Y. He (Apr. 2019). “Beyond Truth-Telling: Preference Estimation with Centralized School Choice and College Admissions”. In: *American Economic Review* 109 (4), pp. 1486–1529.
- Ferreyra, M. et al. (2022). “Cows Don’t Give Milk: An Effort Model of College Graduation”. In.
- Gale, D. and L. S. Shapley (Jan. 1962). “College Admissions and the Stability of Marriage”. In: *The American Mathematical Monthly* 69 (1), p. 9.
- Hotz, V. J. and R. A. Miller (1993). “Conditional choice probabilities and the estimation of dynamic models”. In: *Review of Economic Studies* 60 (3), pp. 497–529.
- Joensen, J. S. and E. Mattana (June 2024). “Student Aid Design, Academic Achievement, and Labor Market Behavior: Grants or Loans?” In: *SSRN Electronic Journal*.
- Londoño-Vélez, J., C. Rodríguez, and F. Sánchez (May 2020). “Upstream and downstream impacts of college merit-based financial aid for low-income students: Ser pilo paga in Colombia”. In: *American Economic Journal: Economic Policy* 12 (2), pp. 193–227.
- Magnac, T. and D. Thesmar (Mar. 2002). “Identifying Dynamic Discrete Decision Processes”. In: *Econometrica* 70 (2), pp. 801–816.

- Montalbán, J. (Dec. 2022). “Countering Moral Hazard in Higher Education: The Role of Performance Incentives in Need-Based Grants\*”. In: *The Economic Journal* 133 (649), pp. 355–389.
- OECD (2019). *Education at a Glance*. Tech. rep.
- Rothstein, J. and C. E. Rouse (Feb. 2011). “Constrained after college: Student loans and early-career occupational choices”. In: *Journal of Public Economics* 95 (1-2), pp. 149–163.
- Sieg, H. and Y. Wang (Oct. 2018). “The impact of student debt on education, career, and marriage choices of female lawyers”. In: *European Economic Review* 109, pp. 124–147.
- Solis, A. (2017). “Credit access and college enrollment”. In: *Journal of Political Economy* 125 (2), pp. 562–622.
- Stinebrickner, R. and T. Stinebrickner (Dec. 2008). “The Effect of Credit Constraints on the College Drop-Out Decision: A Direct Approach Using a New Panel Study”. In: *American Economic Review* 98 (5), pp. 2163–84.
- Tincani, M. M. et al. (2023). “College Access When Preparedness Matters : New Evidence from Large Advantages in College Admissions”. In.

# Appendices

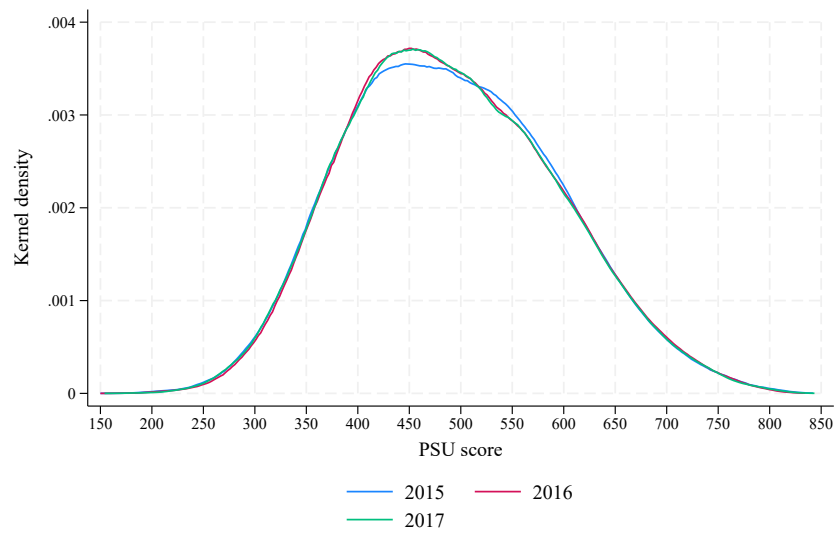
## A Figures

Figure A1: Average PSU score over time for participating and non-participating institutions



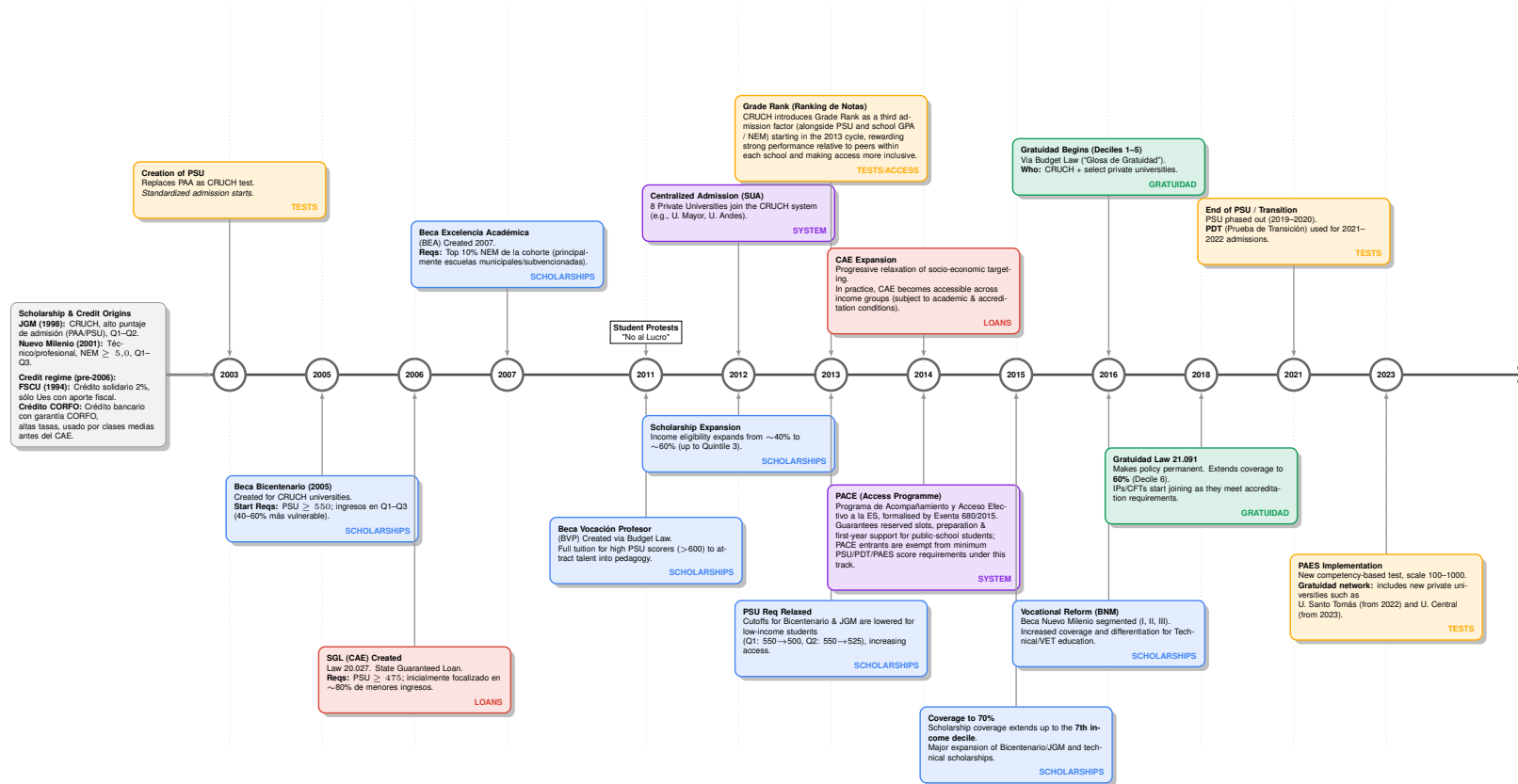
**Notes:** Annual average PSU score by groups of eligible and non-eligible institutions to free higher education.

Figure A2: Average PSU score over time for participating and non-participating institutions



**Notes:** Kernel density of PSU average scores enrolled in higher education for years 2015, 2016 and 2017.

Figure A3: Education reforms 2003-2024



Notes: .

## B Tables

Table B1: Descriptive statistics, students

	2012	2013	2014	2015	2016	2017
<b>Enrollment</b>						
N Students	165762	169257	169415	176027	180128	180302
Enrolled in platform	.260	.273	.280	.275	.280	.276
Enrolled out of platform	.338	.351	.346	.341	.330	.329
Not enrolled	.400	.374	.372	.382	.388	.394
<b>Demographics</b>						
Family Income	3.5	3.7	3.9	4	4.1	4.5
Private School	.111	.111	.111	.109	.106	.105
Private Health	.268	.267	.268	.263	.262	.235
Father With College	.168	.169	.170	.171	.169	.184
Mother Employed	.414	.436	.460	.460	.461	.461
Test Score	490	491	491	492	492	491
Free College	0	0	0	0	.145	.243
Subsidized Loan	.222	.213	.215	.187	.145	.105
Merit-based Scholarship	.136	.191	.200	.249	.158	.101
<b>Field</b>						
Business	.136	.136	.139	.148	.155	.160
Farming	.021	.022	.021	.023	.024	.025
Art and Architecture	.051	.050	.051	.052	.053	.055
Basic Sciences	.032	.033	.035	.033	.034	.031
Social Sciences	.081	.077	.076	.078	.083	.083
Law	.038	.035	.037	.037	.040	.040
Education	.110	.096	.091	.091	.095	.095
Humanities	.010	.010	.010	.010	.010	.009
Health	.214	.201	.200	.196	.196	.196
Technology	.284	.318	.318	.312	.290	.289

**Notes:** This table shows descriptive statistics on every student who enrolled and took the college entrance exam. Family income is categorized in 1-10 brackets, and field classification is performed following the ISCED-UNESCO guidelines.

	PSU $\geq$ 510	$475 \leq$ PSU < 510	PSU < 475
Deciles 1-5	Scholarship TFHE (univ) TFHE (univ+SCP)	SGL loan TFHE (univ) TFHE (univ+SCP)	No public aid TFHE (univ) TFHE (univ+SCP)
Deciles 6-7	Scholarship	SGL loan	No public aid
Deciles 8-10	SGL loan	SGL loan	No public aid

PSU = admissions test; SGL = State-Guaranteed Loan; SCP = short-cycle programs; TFHE = Tuition-Free Higher Education

Years: 2015 2016 2017

Figure A4: Changes in funding instruments after TFHE implementation

Table B2: Performance probabilities

	index	
female	0.457***	(0.017)
Middle SES	0.123***	(0.019)
High SES	0.082***	(0.022)
PSU	0.527***	(0.013)
GPA	0.350***	(0.010)
OP fees	-0.029***	(0.002)
Cum. credits completed (t-1)	0.095***	(0.002)
Credits completed (t-1)	0.154***	(0.004)
Delay (t-1)	-1.627***	(0.010)
1 cred. left (t-1)	-1.711***	(0.038)
2 cred. left (t-1)	-0.810***	(0.026)
3 cred. left (t-1)	-0.531***	(0.032)
OP $\times$ female	0.011***	(0.002)
OP $\times$ middle SES	0.010***	(0.002)
OP $\times$ high SES	0.014***	(0.003)
OP $\times$ psu	0.008***	(0.001)
OP $\times$ gpa	0.005***	(0.001)
1 left (t-1) $\times$ female	0.200***	(0.041)
1 left (t-1) $\times$ middle SES	0.069	(0.048)
1 left (t-1) $\times$ high SES	0.029	(0.054)
1 left (t-1) $\times$ psu	-0.358***	(0.027)
1 left (t-1) $\times$ gpa	-0.022	(0.026)
2 left (t-1) $\times$ female	0.154***	(0.029)
2 left (t-1) $\times$ middle SES	-0.058*	(0.033)
2 left (t-1) $\times$ high SES	-0.090**	(0.037)
2 left (t-1) $\times$ psu	-0.140***	(0.019)
2 left (t-1) $\times$ gpa	0.006	(0.018)
3 left (t-1) $\times$ female	0.165***	(0.035)
3 left (t-1) $\times$ middle SES	0.002	(0.040)
3 left (t-1) $\times$ high SES	-0.048	(0.044)
3 left (t-1) $\times$ psu	-0.003	(0.024)
3 left (t-1) $\times$ gpa	0.026	(0.022)
t=2 $\times$ SCP	-0.270***	(0.013)
t=3 $\times$ SCP	-1.465***	(0.014)
t=4 $\times$ SCP	-1.437***	(0.017)
t=5 $\times$ SCP	-1.964***	(0.023)
t=6 $\times$ SCP	-1.160***	(0.034)
t=7 $\times$ SCP	-0.721***	(0.067)
Credits (t-1) $\times$ female	0.006**	(0.003)
Credits (t-1) $\times$ middle SES	-0.016***	(0.003)
Credits (t-1) $\times$ high SES	-0.004	(0.004)
Credits (t-1) $\times$ psu	-0.034***	(0.002)
Credits (t-1) $\times$ gpa	0.004**	(0.002)
Cumul (t-1) $\times$ female	-0.040***	(0.001)
Cumul (t-1) $\times$ middle SES	-0.006***	(0.001)
Cumul (t-1) $\times$ high SES	-0.010***	(0.001)
Cumul (t-1) $\times$ psu	-0.021***	(0.001)
Cumul (t-1) $\times$ gpa	-0.017***	(0.001)
Delay (t-1) $\times$ female	-0.035***	(0.009)
Delay (t-1) $\times$ middle SES	-0.004	(0.010)
Delay (t-1) $\times$ high SES	0.091***	(0.012)
Delay (t-1) $\times$ psu	0.022***	(0.006)
Delay (t-1) $\times$ gpa	0.064***	(0.005)
t= 2	-0.092***	(0.012)
t= 3	0.291***	(0.015)
t= 4	0.432***	(0.019)
t= 5	0.126***	(0.023)
t= 6	-0.661***	(0.027)
t= 7	-1.294***	(0.031)
Observations	1416806.000	
Mean of Dep. Variable	3.096	

Standard errors in parentheses

Ordered logistic regression of discretised credit achievement (0-4). Controls include vector of characteristics, Institution and major FE, year enrolled FE and cumulated performance. PSU and GPU are standardised. Base category is low SES and t=1.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$



## C Estimation

### C.1 Functional forms

#### Performance outcomes

For  $g \in \mathcal{G} = \{0, 1, 2, 3, 4\}$ , denote  $\tilde{g}$  the underlying index:

$$\begin{aligned} \tilde{g}_j(x_{it}) = & \sum_m \sum_n \alpha_{mn}^g Z_{im} Z_{jn} + \sum_m \sum_n \alpha_{gm}^g Z_{gm} Z_{in} + \mathbf{Z}_g' \alpha_{\mathbf{G}}^g \\ & + \alpha_{field(j)}^g + \alpha_{inst(j)}^g + \lambda_t^g + \lambda_{tscp}^g \cdot \text{SCP}_{ij} + \varepsilon_{ijt}, \end{aligned}$$

where  $Z_i = (\text{psu}, \text{gpa}, \text{income}, \text{female})$ ,  $Z_j = (\text{field1}, \dots, \text{field10})$ ,  $Z_g = (g_{it-1}, G_{it-1}, \text{delay}_{it-1}, \text{1left}_{it-1}, \text{2left}_{it-1}, \text{3left}_{it-1})$ .  $g_{it-1}$  and  $G_{it-1}$  denote previous-period and cumulative credit completion, respectively.  $\text{Xleft}$  indicates whether the student left the program with  $X$  or more credits remaining.  $\alpha_{field(j)}^g$  are field fixed effects,  $\alpha_{inst(j)}^g$  are institution fixed effects, and  $\lambda_t^g$  are period fixed effects. Period fixed effects are allowed to differ between SCPs and university programs.  $\varepsilon_{ijt}$  is a standard logistic performance shock.

$\text{Xleft}$  variables take into account the fact that passing all the credits in a period where less credits are needed to graduate might be different. An alternative modelling approach would estimate separate models for each possible credit left in a year (1,2,3,4). However, estimating it together gives us more precise estimates.

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#### Dropout probability

We estimate the following logistic regression by maximum likelihood:

$$\begin{aligned} \tilde{drop}_j(x_{it}) = & \sum_m \sum_n \alpha_{mn}^d Z_{im} Z_{jn} + \sum_m \sum_n \alpha_{gm}^d Z_{gm} Z_{in} + \mathbf{Z}_g' \alpha_G^d \\ & + \alpha_{field(j)}^d + \alpha_{inst(j)}^d + \lambda_t^d + \lambda_{tscp}^d \cdot SCP_{ij} + \epsilon_{ijt}, \end{aligned}$$

where  $Z_i = (\text{psu}, \text{gpa}, \text{income}, \text{female})$ ,  $Z_j = (\text{field1}, \dots, \text{field10})$ ,  $Z_g = (g_{it-1}, G_{it-1}, \text{delay}_{it-1}, \text{1left}_{it-1}, \text{2left}_{it-1}, \text{3left}_{it-1})$ .  $g_{it-1}$  and  $G_{it-1}$  denote previous-period and cumulative credit completion, respectively.  $X$  indicates whether the student left the program with  $X$  or more credits remaining.  $\alpha_{field(j)}^g$  are field fixed effects,  $\alpha_{inst(j)}^g$  are institution fixed effects, and  $\lambda_t^g$  are period fixed effects. Period fixed effects are allowed to differ between SCPs and university programs.  $\epsilon_{ijt}$  is a standard logistic dropout shock.

## Labour market

We estimate the following wage equation,

$$\log(w_j(x_{it})) = \alpha_0^w + \alpha_f^w \text{female}_i + \alpha_e^w \text{exper}_{it} + \alpha_{e2}^w \text{exper}_{it}^2 + \alpha_j^w + \lambda_R^w \quad (13)$$

$$+ \sum_{r=1}^{16} \sum_j \alpha_{rj}^w R_{ir} \cdot 1\{j\} + \varepsilon_{ijt}, \quad (14)$$

where  $R_{ir}$  is a dummy for region  $r$ . The region–field interactions  $\{R_{ir} \cdot 1\{j\}\}$  are crucial for identification of the wage coefficient in the structural model, as they serve as an exclusion restriction: regional variation affects utility only through wages.

From scraped data on wages from the <sup>3</sup>Chilean Ministry of Education, we recover institution-

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<sup>3</sup><https://www.mifuturo.cl>

specific wage premiums by regressing log wages on field and institution fixed effects. We then predict Equation 14 for each student in the sample, adding the institution premium.

—

### Fixed costs

$$\begin{aligned}
 FC_j(x_{it}) = & \alpha_p OP_{ij} + \sum_m \alpha_{pm} OP_{ij} Z_{im} + \sum_m \sum_n \alpha_{mn} Z_{im} Z_{jn} + \sum_m \sum_n \alpha_{gm} Z_{gm} Z_{in} \\
 & + \mathbf{Z}_g' \alpha_{\mathbf{G}} + \sum_{\mathbf{m}} \sum_{\mathbf{k}} \alpha_{\mathbf{k}\mathbf{m}} \mathbf{M}_{ij,\mathbf{k}} \mathbf{Z}_{i\mathbf{m}} + \mathbf{M}_{ij}' \alpha_{\mathbf{m}} + \alpha_{\text{field}(j)} + \alpha_{\text{inst}(j)},
 \end{aligned}$$

where  $D_{ij}$  is a dummy for the institution being in a different region (distance proxy), and  $A_{ij}$  is the student's relative ability in program  $j$ .  $M_{ij} = (D_{ij}, A_{ij})$  is a vector of student-program matched variables.

## C.2 Conditional Value Function

Conditional value function making use of a terminal action (dropping out) when log effort is the underlying index of an ordered logit model.

$$v_j(x_{it}, e_{it}) = u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (\gamma + v_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g})))$$

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) - c(x_{it})e(x_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (\gamma + v_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g})))$$

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) - \beta \sum_{\bar{g} \in \mathcal{G}} \frac{\partial \phi^{\bar{g}_{ijt}}(e_{it})}{\partial e_{it}} (\gamma + v_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) e(x_{it}) \\ + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (\gamma + v_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g})))$$

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) + \beta \gamma \\ + \beta \sum_{\bar{g} \in \mathcal{G}} \left( (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \left( \phi^{\bar{g}}(e_j^*(x_{it})) - \frac{\partial \phi^{\bar{g}_{ijt}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right) \right)$$

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) + \beta \gamma \\ + \beta \sum_{\bar{g} \in \mathcal{G}} ((\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \\ \times (\Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it}))) \\ - (\Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it}))) - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})))) \\ \times e_j^*(x_{it})))$$

where  $\Lambda(\cdot)$  denotes the standard logistic CDF. The general expression for ordered logit

probabilities are, denoting by  $\alpha_{\bar{g}}$  the estimated cutoff points,

$$\begin{aligned}\phi_{ijt}^{\bar{g}}(e_j^*(x_{it})) &= \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})) \\ &= \frac{1}{1 + \exp(e_j^*(x_{it}) - \alpha_{\bar{g}})} - \frac{1}{1 + \exp(e_j^*(x_{it}) - \alpha_{\bar{g}-1})}\end{aligned}$$

The derivative wrt  $e_j^*(x_{it})$  writes

$$\begin{aligned}\frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} &= -\Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it}))) \\ &\quad - (-\Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})))) \\ \frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} &= \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it}))) \\ &\quad - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})))\end{aligned}$$

Computing the marginal cost and EMAX expression

$$\sum_{\bar{g} \in \mathcal{G}} (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \left( \phi_{ijt}^{\bar{g}}(e_j^*(x_{it})) - \frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right)$$

For  $g = 1$

$$\begin{aligned}(\Lambda(\alpha_1 - e_j^*(x_{it})) + (\Lambda(\alpha_1 - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_1 - e_j^*(x_{it})))) \times e_j^*(x_{it}) \\ (\Lambda(\alpha_1 - e_j^*(x_{it})) + (\Lambda(\alpha_1 - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_1 - e_j^*(x_{it})))) \times e_j^*(x_{it})\end{aligned}$$

$$\begin{aligned}
& (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(1))) \times \\
& \left( \Lambda(\alpha_1 - index_j(x_{it})) - \frac{\partial \phi_{ijt}^{\bar{g}}(e_j(x_{it}))}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} \times e_j^*(x_{it}) \right)
\end{aligned}$$

We defined effort as

$$e_j(x_{it}) = \frac{1 - \Lambda(\alpha_1 - index_j(x_{it}))}{\Lambda(\alpha_1 - index_j(x_{it}))} \quad (15)$$

$$\Lambda(\alpha_1 - index_j(x_{it})) = \frac{1}{1 + e_j(x_{it})} \quad (16)$$

$$\alpha_1 - index_j(x_{it}) = \ln \left( \frac{\frac{1}{1+e_j(x_{it})}}{1 - \frac{1}{1+e_j(x_{it})}} \right) \quad (17)$$

$$\alpha_1 - index_j(x_{it}) = \ln \left( \frac{1}{e_j(x_{it})} \right) = -\ln(e_j(x_{it})) \quad (18)$$

$$\alpha_1 - \ln(e_j(x_{it})) = index_j(x_{it}) \quad (19)$$

then the derivative wrt effort looks like

$$\begin{aligned}
& \left( \Lambda(\alpha_1 - index_j(x_{it})) - \left( \lambda(\alpha_1 - index_j(x_{it})) \times \frac{-1}{e_j(x_{it})} \right) \times e_j(x_{it}) \right) \\
& = \frac{1}{1 + e_j(x_{it})} + \frac{e_j(x_{it})}{(1 + e_j(x_{it}))^2}
\end{aligned}$$

For  $g \in (2, 4)$

$$\begin{aligned} & \left( \alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g})) \right) \times \\ & (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) - \\ & (\Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it}))) - \Lambda(\alpha_{\bar{g}} - index_j(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}} - index_j(x_{it})))) : \end{aligned}$$

Recall that rearranging the effort measure in [Equation 15](#), we can express  $index_j(x_{it}) = \alpha_1 - \ln(e_j(x_{it}))$

$$\begin{aligned}
& (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \\
& (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) - \\
& \left. \frac{\partial \phi_{ijt}^{\bar{g}}(e_j(x_{it}))}{\partial e_{it}} \right|_{e_{it}=e_j^*(x_{it})} (\Lambda(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it}))) - \Lambda(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it}))) \times e_j^*(x_{it})) \Bigg) \\
& = (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \\
& (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) - \\
& \left( \lambda(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it}))) \times \left( \frac{-1}{e_j^*(x_{it})} \right) - \lambda(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it}))) \times \left( \frac{-1}{e_j^*(x_{it})} \right) \right) \times e_j^*(x_{it})) \\
& = (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \\
& (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) - \\
& \left( \frac{-e_j^*(x_{it})}{e_j^*(x_{it})} \right) \times \left( \frac{\exp(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it})))}{(1 + \exp(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it}))))^2} - \frac{\exp(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it})))}{(1 + \exp(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it}))))^2} \right)) \\
& = (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \\
& (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) + \\
& \left( \frac{\exp(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it})))}{(1 + \exp(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it}))))^2} - \frac{\exp(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it})))}{(1 + \exp(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it}))))^2} \right))
\end{aligned}$$

For  $g = 5$

$$\begin{aligned}
& (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(5))) \times \\
& (1 - \Lambda(\alpha_4 - e_j^*(x_{it})) - (\Lambda(\alpha_4 - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_4 - e_j^*(x_{it})))) \times e_j^*(x_{it}))
\end{aligned}$$



$$\sum_{\bar{g} \in \mathcal{G}} \left( \phi_{ijt}^{\bar{g}}(e_j^*(x_{it})) - \frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right)$$

Notice from the above expression that

$$\sum_{\bar{g} \in \mathcal{G}} (\phi_{ijt}^{\bar{g}}(e_j^*(x_{it}))) = 1$$

Since we are integrating over all possible states and that

$$\begin{aligned} & \sum_{\bar{g} \in \mathcal{G}} \left( \frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right) \\ &= \lambda(\alpha_1 - index(x_{it})) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) + \\ & (\lambda(\alpha_2 - index(x_{it})) - \lambda(\alpha_1 - index(x_{it}))) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) + \\ & (\lambda(\alpha_3 - index(x_{it})) - \lambda(\alpha_2 - index(x_{it}))) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) + \\ & (\lambda(\alpha_4 - index(x_{it})) - \lambda(\alpha_3 - index(x_{it}))) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) + \\ & (-\lambda(\alpha_4 - index(x_{it}))) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) \end{aligned}$$

since  $index = \ln(e_j^*(x_{it})) + \alpha_1$  and that  $\frac{\partial \lambda(\alpha_g - index(x_{it}))}{\partial e_{it}} = \lambda(\alpha_g - index(x_{it})) \times (-1/e_j^*(x_{it}))$ .

This can be rewritten as

$$\begin{aligned}
& \sum_{\bar{g} \in \mathcal{G}} \left( \frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right) \\
&= -\lambda(\alpha_1 - index(x_{it})) + \\
&\quad - (\lambda(\alpha_2 - index(x_{it})) - \lambda(\alpha_1 - index(x_{it}))) + \\
&\quad - (\lambda(\alpha_3 - index(x_{it})) - \lambda(\alpha_2 - index(x_{it}))) + \\
&\quad - (\lambda(\alpha_4 - index(x_{it})) - \lambda(\alpha_3 - index(x_{it}))) + \\
&\quad - (-\lambda(\alpha_1 - index(x_{it}))) \\
&= 0
\end{aligned}$$

Conditional value function depending on  $t$  and  $j$

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) + \beta\gamma + \beta \sum_{\bar{g} \in \mathcal{G}} \left( (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \left( \phi^{\bar{g}}(e_j^*(x_{it})) - \frac{\partial \phi^{\bar{g}_{ijt}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right) \right)$$

- if  $t = 1$  &  $j = 0$ :  $v_0(x_{it}, e_{it}) = \beta \alpha_w wage_0(x_{it})$
- if  $t > 1$  &  $j = 0$ :  $v_0(x_{it}, e_{it}) = \beta \alpha_w wage_0(x_{it})$
- if  $t = last$  &  $t \neq T$  &  $d_{it} = d_{it-1}$ :  $v_j(x_{it}, e_{it}) = \beta \alpha_w wage_1(x_{it})$
- if  $t = T$  &  $d_{it} = d_{it-1}$ :  $v_j(x_{it}, e_{it}) = \beta \alpha_w (wage_0(x_{it}) \text{ or } wage_1(x_{it}))$  depending on graduation (um accumulation variable).

In the last period ( $T = 7$ )

$$v_0(x_{it}, e_{it}) = \alpha_w wage_0$$

$$v_1(x_{it}, e_{it}) = \alpha_w wage_1$$