

Funding Instruments and Effort Choices in Higher Education ^{*}

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Abstract

This paper examines the effects of Free College policies on student enrollment and academic performance, with a focus on the 2016 Chilean reform that granted tuition-free higher education to students from the lowest five income deciles. Using a difference-in-differences approach, we find that Free College increased enrollment and persistence in higher education on the eligible but had modest effects on graduation and dropout rates. To disentangle the role of student effort from selection effects, we develop a structural model in which students choose effort levels in response to financial incentives. Our results highlight that Free College expanded access, in particular for low-achieving students. Despite the removal of academic progress requirements, we found no evidence of weakening performance.

Keywords: Financial aid, Effort choices, Higher Education

JEL Codes: I22, I23, I26

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1 Introduction

Access to Higher Education has become an increasingly heated topic. In the US, president Biden proposed to (i) increase the funding for merit-based scholarships, (ii) a debt forgiveness plan, and (iii) expand free community college across the country. Advocates of Free College point to the debt crisis, lack of mobility and increasing inequality. As of today, more than 40 countries around the world have an ongoing Free College policy, generally limited to public institutions.

Traditional funding schemes such as merit-based scholarships are out of reach for students who don't meet the performance requirements. Even popular means-based programs, such as the Pell Grants in the US, require beneficiaries to maintain satisfactory academic progress. Subsidized loans give credit to students at below-the-market interest rates, which helps expand access to higher education. However, the burden it places upon graduation leads to high default rates, and the bad financial credit history leaves long-lasting effects on borrowers. Free College is a particular case of a continuum of prices that students face (which could be negative, if they receive a stipend). A progressive system would opt for increasing the amount of benefits the lower the income. However, specially in the developing world, Free College is the solution that policy makers find under the impossibility of correctly determining households' incomes.

The effect that Free College can have on a student outcome is ex-ante ambiguous. From one point, it eases the budget constraint, increasing the disposable time a student can assign to his studies (no need for a part-time job), and making less likely to drop-out following a negative income shock. However, moving from a conditional (on performance) cash transfer to an unconditional scheme (Free College) potentially involves moral hazard concerns, where

students exert low effort (“good enough to pass”) and progression is less satisfactory.

When looking at the aggregate data, countries that have Free College policies tend to have lower graduation rates than similar countries in which students face out-of-pocket fees. While that might provide a pessimistic view, it is related to the fact that increases in enrollment come disproportionately from students of lower performance. In general, funding policies imply responses on both the extensive (enrollment) and intensive (performance) margins, and any aggregate measure is the result of (i) marginal students who self-select to higher education because of the policy, and (ii) infra-marginal students who would have enrolled nevertheless, but change their behavior because of the policy.

This paper studies the effects of a Tuition-Free Higher Education (TFHE) policy on both enrollment and performance. Our analysis centers in the Chilean case, where in 2016 a program called *Gratuidad* allowed students in the lower half of the income distribution to enroll for free in almost half of the operating universities and vocational centers in the country, including some private institutions. In a first step, we exploit a difference-in-differences strategy and find that the policy increased enrollment and persistence (defined as time spent enrolled) for eligible individuals, while it had a modest effect on graduation and dropout. Building on the results obtained on the reduced-form analysis, we build a structural model to (i) decompose the relative influence of marginal and infra-marginal students for each aggregate outcome and (ii) estimate the effort responses to changes in the funding scheme. Estimation of the model allows for simulation of counterfactual funding policies.

We build on several strands of literature. First, we contribute to the research on higher education financing and student outcomes. There is substantial evidence showing that increasing funding for higher education has a positive effect on enrollment (Eppele et al., 2006, Angrist

et al., 2015, Denning, 2017, Epple et al., 2017, Solis, 2017, Abbott et al., 2019, Londoño-Vélez et al., 2020, Dobbin et al., 2022, Falco and Reichlin, 2025). Conversely, research has mostly found a negative effect on performance (Dynarski, 2003, Cohodes and Goodman, 2014). A typical problem when performing inference is the aforementioned composition effect. Denning (2019) studies a reform in the US that increased financial aid only for already enrolled students, finding that it increased graduation rate and decreased completion time. TFHE releases students of accumulating student debt, which has been shown to affect major choice (Rothstein and Rouse, 2011), dropout (Stinebrickner and Stinebrickner, 2008) career decisions (Sieg and Wang, 2018) and homeownership (Black et al., 2023). This particular policy has been studied by Bucarey (2017). However, his paper focuses on the crowding out of low-income student from selective programs upon implementation of the policy. Our contribution is to study the effect of a large-scale tuition-free policy on both enrollment and performance in a setting where public financial aid system is already well-developped.

Second, we contribute to the literature on the incentive structures and moral hazard inherent in educational funding. Traditional subsidies act as a double-edged sword: while they lower the cost of entry, they may inadvertently reduce the marginal cost of failure, leading to a potential decrease in student effort. Garibaldi et al. (2012) and Beneito et al. (2018) show that “continuation tuition” (increasing fees for retaking modules) can significantly reduce late graduation and boost effort, suggesting that students respond to the shadow price of their academic progress. Searching for optimal requirement design, Montalbán (2022) exploits changes in performance requirements in Spanish university loans using a regression discontinuity approach. He finds that cash allowances are only effective when accompanied by relatively high performance requirements.

A growing structural and semi-structural literature investigates responses to changes in in-

centives by modeling effort as an exogenous stochastic process. It is the case for Arcidiacono (2005) for college admission probabilities under affirmative action, Beffy et al. (2012) for length of studies as a result of different returns to education induced by the French business cycle, or course credits in Sweden as in Joensen and Mattana (2024). In our paper, we endogenize and recover effort from a first-order condition as in De Groote (2025). We additionally exploit a large-scale reform in Chile that introduced variation in out-of-pocket fees and performance requirements to identify both extensive and intensive margin responses to the policy. Other papers that endogenized effort decisions include Ferreyra et al. (2022) in Colombia by modeling the number of targeted classes in higher education and Tincani et al. (2023), who examines changes in efforts as a response to subjective beliefs in Chile. In her case, effort measures are directly observed from survey data and enter the scores’ production functions. We contribute by coupling out-of-pocket fees to past performance. Students don’t just work hard for future wages; their effort today also affect short-term educational costs.

The rest of the paper is organized as follows: [section 2](#) describes the institutional details, the TFHE policy and introduces the data; [section 3](#) reports results from differences-in-differences estimation; and [section 4](#) builds a model of enrollment and effort in higher education. [section 5](#) discussed the main results of the model and [section 6](#) performs counterfactual simulations. [section 7](#) provides a discussion and next steps.

2 Background

2.1 Institutional Setting

Chilean higher education is well developed, with enrollment rising steadily since the General Law of Universities (*Ley General de Universidades*) of 1981, which incentivized the creation of institutions by allowing entry without state dependence. In 2023, the system comprised a total of 138 institutes: 80 vocational schools (33 Professional Institutes and 47 Centers of Technical Formation) and 58 universities.¹

Universities are divided between the so-called *traditional* universities (a mix of public and private institutions that receive direct support from the government) and *private* universities, which constitute the rest. The latter were created after 1981 and are mainly financed through tuition. Traditional universities are officially known as the Universities of the Rector’s Council (*Consejo de Rectores de las Universidades Chilenas, CRUCH*) and are responsible for coordinating the higher education system. They comprise 30 universities and account for around half of total enrollment.²

Access to higher education is through a Centralized Admission Platform, compulsory for traditional universities and some private institutions. Vocational program admission is also possible, although most are conducted off-platform. After high-school completion, students applying to higher education take the centralized admission exam (*Prueba de Selección Universitaria, PSU*). This standardized test, similar to the SAT in the United States, ranges from 150 to 850 points, with a mean of 500 and a standard deviation of 110. Students then submit rank-ordered lists of up to 10 degree programs through the centralized system and

¹<https://www.ayudamineduc.cl/ficha/instituciones-vigentes-reconocidas-por-el-mineduc>.

²In 2018, the government established a clear set of rules for being considered a traditional university with the promulgation of Law 21091, allowing the first three non-traditional universities to enter.

are assigned via a deferred-acceptance algorithm (Gale & Shapley, 1962). In practice, the platform imposes a minimum PSU score of 450 to apply.

Higher education is costly in Chile. In 2009, the average tuition fee for a university program was equivalent to 47% of the median family income (Solis, 2017). Costs vary across institution types but remain high even in public institutions. Different funding instruments coexist. Students rely mainly on loans and grants from the Ministry of Education. Eligibility criteria are strongly related to PSU scores. The State Guaranteed Loan program (SGL), introduced in 2006, finances 90% of reference tuition and requires a PSU score of 475 or higher, with no socioeconomic requirement since 2014. Several scholarships also exist, with the *Beca Bicentenario* and *Beca Excelencia* being the most popular. On average, scholarships finance 80% of reference tuition. Eligibility requires a PSU score between 510 and 550, depending on the scholarship, and excludes students in the top two or three income deciles. Both short-cycle programs (SCPs) and university degrees are covered by scholarships. Two-year programs typically require a high-school GPA above 5.0. Access to private loans is limited: in 2015, only 7.5% of student loans came from banks without a state guarantee.

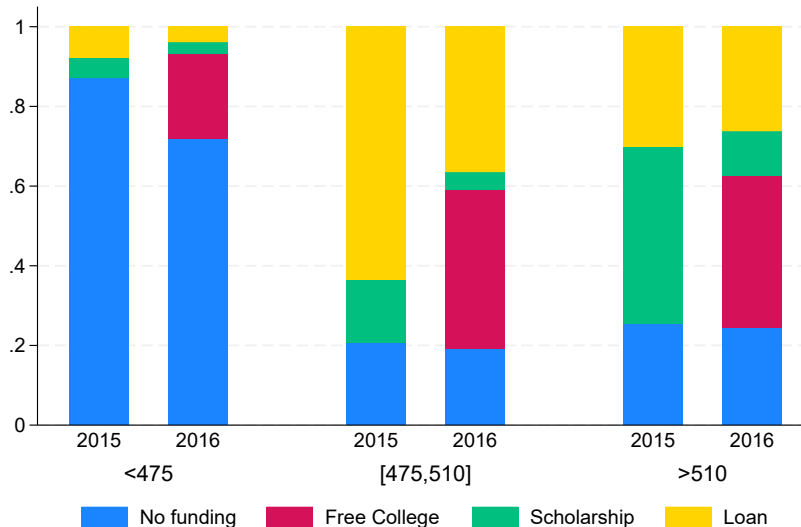
2.2 Tuition-Free Higher Education (TFHE) program

Since the implementation of SGL, student debt rose at a rate of 70% annually, and the number of recipients increased steadily from 15,800 in the first year to 652,000 students in 2016 (Bucarey et al., 2020). In 2011, students led mass protests demanding more affordable higher education. Michelle Bachelet was elected president in 2014 on a platform that included making college free by 2020. In 2015, the TFHE law was enacted, removing tuition fees for students in the bottom half of the income distribution. The policy was first implemented for the 2016 university cohort, expanded in 2017 to include vocational institutes, and further

extended in 2018 to the sixth income decile. Applications for TFHE, loans, and scholarships are made through the Centralized Admission Platform by completing a short online form during the application period, regardless of whether the applicant applies to a platform degree or not. A schematic visualization of the policy is provided in [Figure A4](#).

The introduction of TFHE policy generated different substitution patterns depending on students' test scores. [Figure 1](#) shows the distribution of funding instruments before and after the policy, by test score segment. Students scoring below 475 were ineligible for subsidized loans or merit-based scholarships (except for a small scholarship targeting top students from low-SES schools). For this group, TFHE reduced the share of students with no funding. Students scoring between 475 and 510 qualified for subsidized loans but remained below the threshold for most scholarships; for them, substitution was mainly from loans to TFHE, relieving them of any debt upon graduation. Students scoring above 510 qualified for merit-based scholarships (conditional on income eligibility). For this group, substitution was mainly from scholarships to TFHE, which had two advantages: first, scholarships typically covered only 80% of tuition and fees, whereas TFHE provided full funding; second, scholarships required students to meet satisfactory progression standards (validating about 70% of annual course credits), while TFHE imposed no such requirements. Before the policy, about 20% of scholarship beneficiaries lost eligibility; of these, around 40% switched to subsidized loans, while the rest continued without public funding. After the policy, most substituted to TFHE, while those above the income threshold continued either with loans or no funding, in roughly the same proportions as before.

Figure 1: Distribution of funding instruments, by test score



Notes: This figure shows the distribution of funding instruments by test score group, for 2015 and 2016. The Scholarship category includes any merit-based scholarship, while Loan includes both the SGL and CAE schemes.

2.3 Data

We use Chilean administrative records that provide detailed student and program data. We observe enrollment, admission test scores, scholarship assignment, socioeconomic information, and demographic characteristics. Institutional data include type, tuition, location, and program length. Between 170,000 and 180,000 students take the national exam each year, of whom about 60% enroll in higher education. Descriptive statistics are reported in [Table B1](#).

In addition, we use restricted data on credit completion from the Information Service of Higher Education (SIES) covering the universe of enrolled students from 2016 to 2023. These data report the number of courses registered and whether they were completed.

Quasi-Experimental sample

We restrict the sample to first-time test takers for the 2012 to 2017 cohort. Since we observe the data until 2023 and the standard university program lasts five years, we allow seven years for students to complete their program. In 2012, scholarships were expanded to include the third income quintile, such that all eligible students for TFHE were already eligible for scholarships conditional on scoring a certain PSU score (Bucarey, 2017).

Model sample

We are constrained by the fact that credits are only available from 2016. Together with allowing students 7 years for graduation, we estimate the model on a random sample of 2016-2017 cohorts. CCPs are estimated using the *universe* of 2016-2017 cohorts.

3 Policy Effects

We are interested in causally estimating whether the TFHE policy had an impact on enrollment and educational outcomes. To do so, we exploit the exogenous variation in eligibility introduced by the policy and use differences-in-differences. We estimate the following regression model by OLS:

$$Y_{it} = \sum_{\substack{k=2011 \\ k \neq 2015}}^{2020} \beta_k (\mathbb{1}\{t = k\} \times \mathbb{1}\{dec_i < 6\}) + \gamma \mathbb{1}\{dec_i < 6\} + \delta_t + \mathbf{X}_i' \alpha_x + \epsilon_{it} \quad (1)$$

Y_{it} is an outcome of interest including enrollment and educational outcomes such as dropout, graduation, on-time graduation and persistence, defined as the time (in years) spent in college. \mathbf{X}_i' includes a constant, gender, PSU score, high school GPA, degree type, and mother education. $\mathbb{1}\{dec_i < 6\}$ determines eligibility to TFHE, defined as belonging to

the lower half of the income distribution. δ_t are cohort fixed effects. We cluster standard errors at the income decile level using wild bootstrap (Cameron et al., 2008). We use the year before the implementation (2015) as the reference category. β_t is the differences-in-differences estimator.

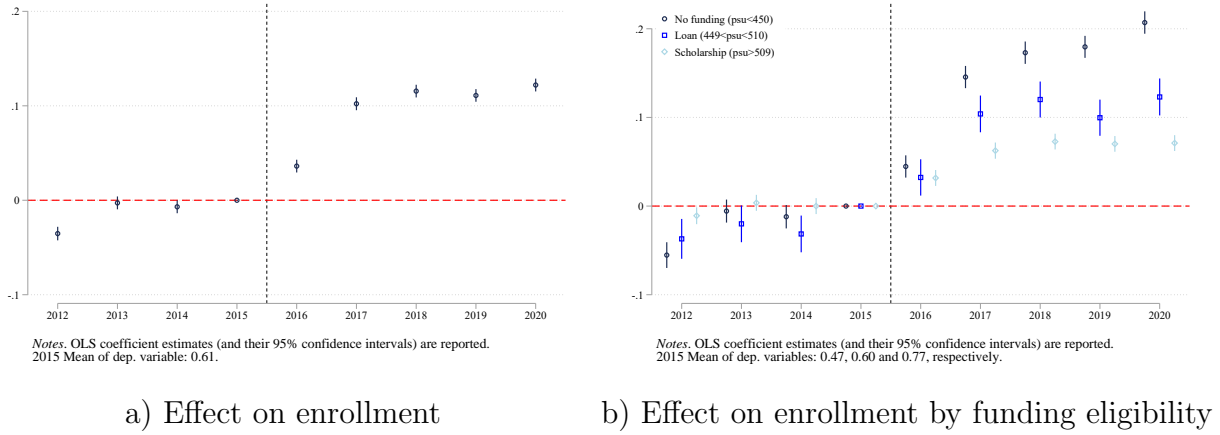
The identifying assumption is that, in the absence of treatment, eligible and non-eligible students would have experienced similar trends in the outcomes of interest. We test for the validity of this assumption by examining pre-treatment trends. Additionally, we perform several robustness checks, including controlling for differential trends by income decile and restricting the sample to adjacent income deciles. The differences-in-differences estimator can be interpreted as the intent-to-treat effect (ITT) of the policy, given that not all eligible students actually receive TFHE (some do not enroll, while others enroll in non-eligible institutions).

3.1 Effects on enrollment

We first examine the impact of TFHE on the extensive margin, i.e. enrollment. The cohort of 2016 is the first for which it can affect the enrollment decision. [Figure 2](#) shows the results of estimating [Equation 1](#). As can be seen in panel a), enrollment of eligible students increased by 3.5 p.p. in 2016 and around 10 p.p. from 2017 onwards, compared to 2015. This increase in enrollment comes from students that either returns to education became positive or were credit constrained. The higher effect starting year 2017 is consistent with the expansion of free higher education to SCPs. As explained in [section 2](#), the control group includes students that had either no (public) funding, or were already eligible to loans or scholarships. We expect these different groups of students to react differently to the policy. In panel b), we observe that students of lower ability, which had limited access to public funding before

2016, are the group which increases enrollment the most. Differences between groups are significant for 2017, where students of lower ability increase enrollment by 14 p.p. compared to 5.5 p.p. of students that had already access to merit scholarships.

Figure 2: Enrollment



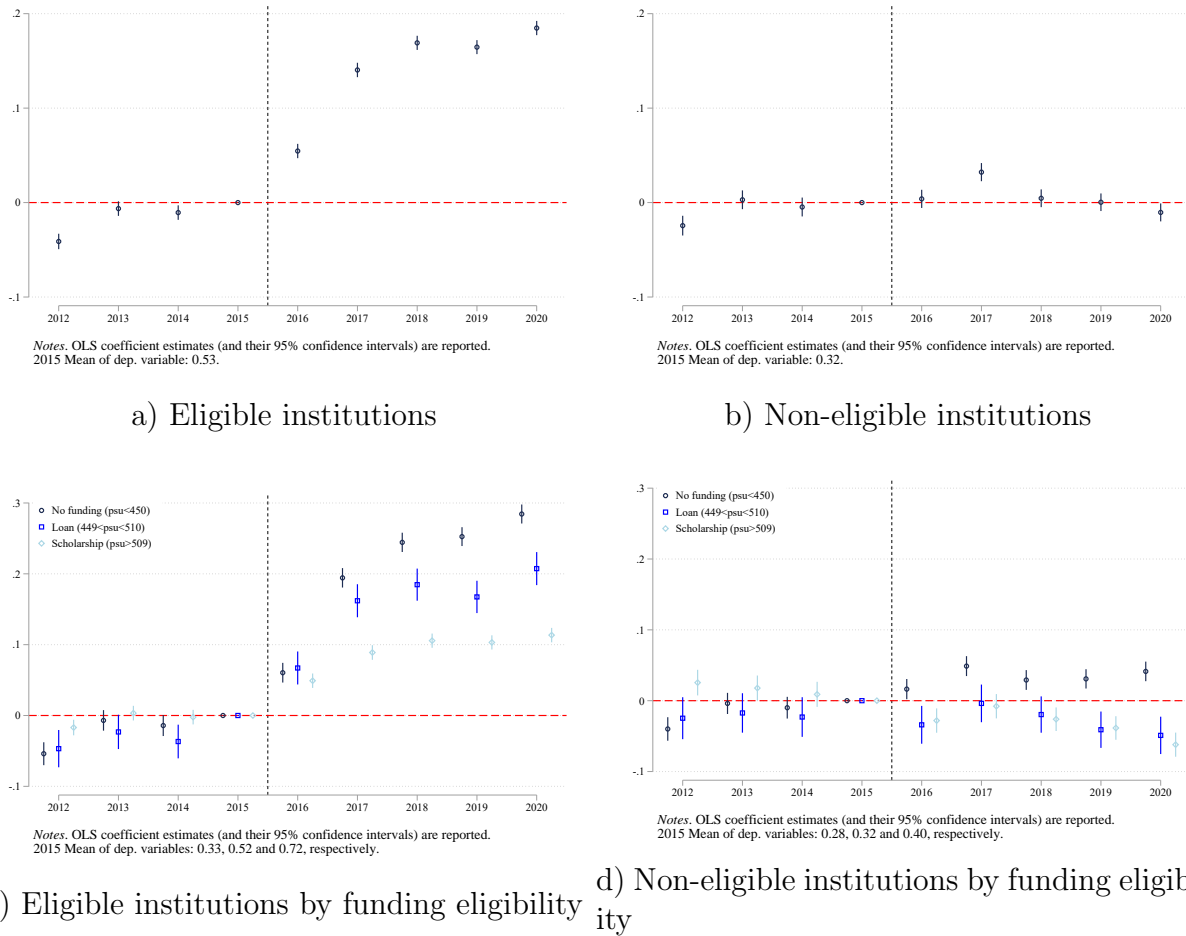
Notes: These figures show the results of estimating Equation 1. Sample is the universe of first time test takers. Panel (a) shows the aggregate results, while panel (b) estimates the equation for groups based on their test scores .

Substitution between eligible and non-eligible institutions

Since TFHE affected around 50% of institutions and 70% of enrollment, it is also important to examine whether there was substitution from non-eligible to eligible centers, or whether enrollment is driven completely by marginal students. Figure 3 reports the estimation on enrollment for eligible and non-eligible institutions. Results show a significant increase of eligible students to higher educations institutes that were eligible to the policy. In panel a), we observe increases of 5 and 13 p.p. for years 2016 and 2017, respectively. In contrast to Londoño-Vélez et al., 2020, panel b) suggests no crowding-out. In fact, we find mostly a null effect on enrollment (except for 2017). When looking at panel c), we observe that the increase in enrollment in eligible institutions is driven by low-ability students. Panel d) decomposes

the null effect on noneligible institutions in a decrease in enrollment coming from high-ability students while low-ability students increase enrollment around 2 to 4 p.p. A possible explanation is that the policy generated awareness about higher education opportunities and low-ability students that had enough resources opted for higher education even if not eligible to TFHE.

Figure 3: Enrollment by institution eligibility



Notes: These figures show the results of estimating Equation 1 on enrollment for eligible and non-eligible institutions to the TFHE policy. Sample is the universe of first time test takers. Panel a) and b) show the aggregate results, while panels c) and d) disaggregate results by test score group.

Preferences

We next look at whether students’ preferences changed as a result of the policy. We use students’ rank-ordered list and construct the dependent variable as the share of a field in student’s list. We restrict the sample to eligible institutions and estimate [Equation 1](#) for each field. Results are presented in [Figure A5](#). Panel a) shows that TFHE increased preferences for Health by 1.5p.p. in 2016 and 2p.p. in 2017, while decreasing preferences Arts and Humanities by around 2p.p. STEM majors also lose popularity starting in 2018 with respect to 2015 by 2 p.p. Panel b) decomposes the results into finer categories. The decreases in Arts and Humanities are driven by Education, which decreases by 1p.p., 10% with respect to 2015 average. These results match the evidence found by Castro-Zarzur et al. (2022), which finds that TFHE eclipsed an incentive program to attract talent in Education majors ³. The increase in Health preferences can be explained by the fact that they are the most expensive programs, along with law majors. Finally, the the 2p.p. decrease in STEM majors is driven by Technology.

3.2 Effects on educational outcomes

Recovering treatment effects for the intensive margin is not as straightforward. If we were to estimate educational outcomes using [Equation 1](#), composition and moral hazard could difficult interpretation: the distribution of students’ test scores shifted to the left starting 2016, given that a substantial fraction of lower-ability enrolled (marginal students) as can be seen in [Figure A2](#). On top of that, 2016 cohort onwards eligible students were not subject to performance requirements anymore. To keep the composition of students fixed, we exploit a particularity of the policy: cohorts prior to 2016 were eligible to TFHE for the remaining

³see Pal (2025) for a comprehensive analysis of this policy.

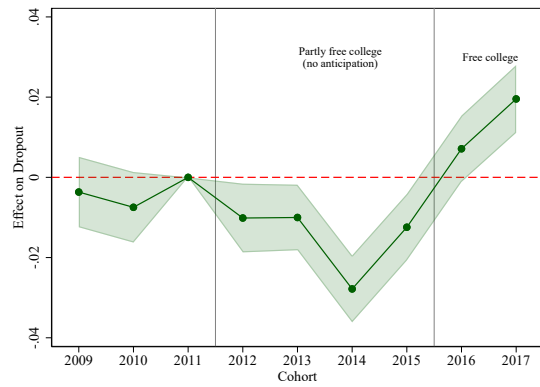
years of the nominal length of their degrees. For example, if they enrolled in 2015 in a five-year degree, they can be exempted of tuition for the remaining four years, two years for the 2015 cohorts, etc.

We can use this exogenous variation to partial out selection. 2012 to 2015 cohorts are potentially eligible to TFHE starting 2016 but cannot change their initial enrollment decision by construction. We adapt [Equation 1](#) as

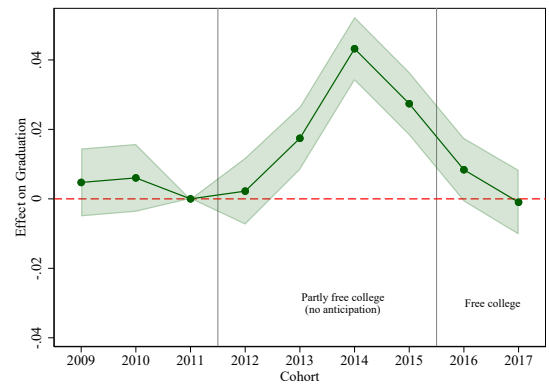
$$Y_{it} = \sum_{\substack{k=2010 \\ k \neq 2011}}^{2017} \beta_k^c (\mathbb{1}\{t = k\} \times \mathbb{1}\{dec_i < 6\}) + \gamma^c \mathbb{1}\{dec_i < 6\} + \delta_t^c + \mathbf{X}_i' \boldsymbol{\alpha}_x^c + \epsilon_{it}^c \quad (2)$$

The above regression remains a standard TWFE differences-in-differences regression, as the time dimension are cohorts. Treatment varies in intensity by cohort, but effects are always estimated with respect to a never treated cohort, so there are no issues of negative weighting (Callaway et al., [2024](#); de Chaisemartin & D’Haultfœuille, [2024](#)). The variation exploited is purely cross-sectional.

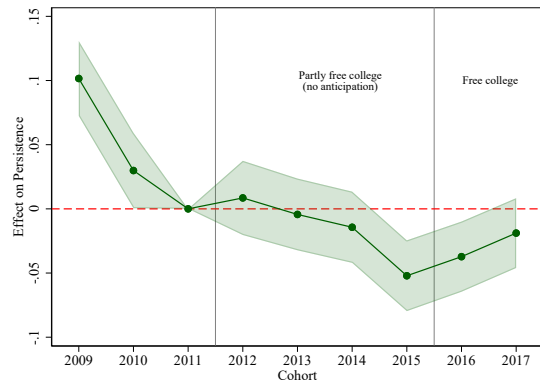
[Figure 4](#) shows the results of estimating [Equation 2](#). Panels a) to d) show the results for dropout, graduation, persistence, and graduation on time, respectively. We find that dropout decrease up to 2.5 p.p. for infra-marginal students, and the effect reverses for 2017 cohort. Graduation consequently also rises for students that receive the subsidy after their enrollment decision, while cohorts starting 2016 graduate at a similar rate as never-treated students. Persistence results are hard to interpret as the pre-treatment period exhibits a pronounced negative trend, which the effect of policy seems to moderate. Finally, as the subsidy gets larger (2015 cohorts), students the proportion of students who graduates on time rose by 1.8 p.p., while there are null effects of TFHE starting 2016.



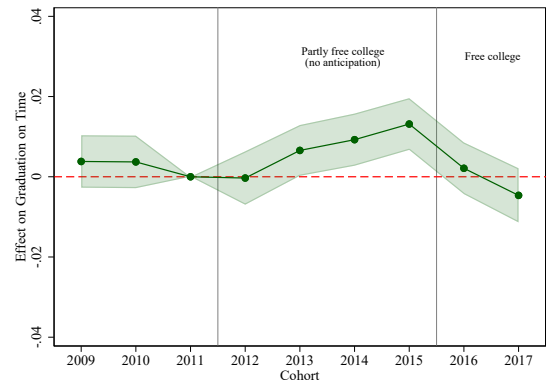
a) Dropout



b) Graduation



c) Persistence



d) Graduation on Time

Figure 4: DID outcomes.

These results are best interpreted as the overall effect of the policy on those who already enrolled. TFHE lowers the cost of staying enrolled, which tends to reduce dropout and increase graduation. Conversely, removing performance requirement to students who previously benefited from scholarships could also entail a negative effect on dropout: out-of-pocket fees do not longer depend on performance outcomes. Empirically, quasi-experimental findings suggest. To validate reduced-form results and quantify both effects, we next build a model of enrollment and major choice with endogenous effort.

3.3 Discussion and robustness

Our quasi-experimental analysis suggests that TFHE policy had a positive effect on enrollment on eligible students, especially for low-ability students that previously had limited access to public funding. We find no evidence of crowding-out of non-eligible institutions, suggesting that most of the increase in enrollment comes from marginal students. Eligible institutions were able to absorb the increase in enrollment, especially lower-quality institutions. Regarding educational outcomes, we find modest effects on dropout and graduation for infra-marginal students, while there are no significant effects for students that enrolled after the implementation of the policy, despite being of lower observed ability on average.

There are some limitations to our empirical strategy. We used eligibility to TFHE as a proxy for treatment, and compared eligible to non-eligible students as we do not have a continuous measure of income.

Crowd-out of average, high-income students? If high-income students were to reduce their enrollment as a result of the policy, our estimates would be biased upwards. However, spillovers are unlikely as cut-offs scores were mostly unchanged, suggesting that the policy

did not put additional stress on the capacity of institutions. As a robustness check, we estimate [Equation 1](#) restricting the sample to income deciles 1, 2 and 9, 10, as crowd-out is more likely to occur around the eligibility threshold, and find similar results.

Increase in test-takers? If the policy induced more low-income students to take the PSU test, our control group would be positively selected, biasing our results upwards. However, we find no evidence of an increase in test-takers as a result of the policy. This is because take-up rates were already high (around 85% of students in their last year of high school take the test). In fact, what we see is an increase in higher education applications from low income students.

Income manipulation? Income is self-reported and verified by the government. Misreporting could lead to students that are actually ineligible being classified as eligible. If this is done on purpose, it could bias our estimates upwards.

4 Model

The decision process of an individual considering attending higher education can be depicted in two stages. In the first stage, she chooses whether to enroll, and if so, which field-institution combination to attend. To decide, she observes program characteristics and available funding instruments, and optimizes how much effort to exert in each program in her personalized choice set, determined by her score in the national exam (Fack et al., [2019](#)). Effort choice will determine her distribution of performance outcomes (credits). Conditional on enrollment, performance today endogenously determines out-of-pocket fees for the subsequent year. The student graduates when she cumulates the required number of credits of the enrolled program.

Direct measures of effort are not observed. Instead, we attempt to uncover *effective* effort through realized outcomes, i.e. course credits completion (De Groote, 2025).

4.1 Timing and choices

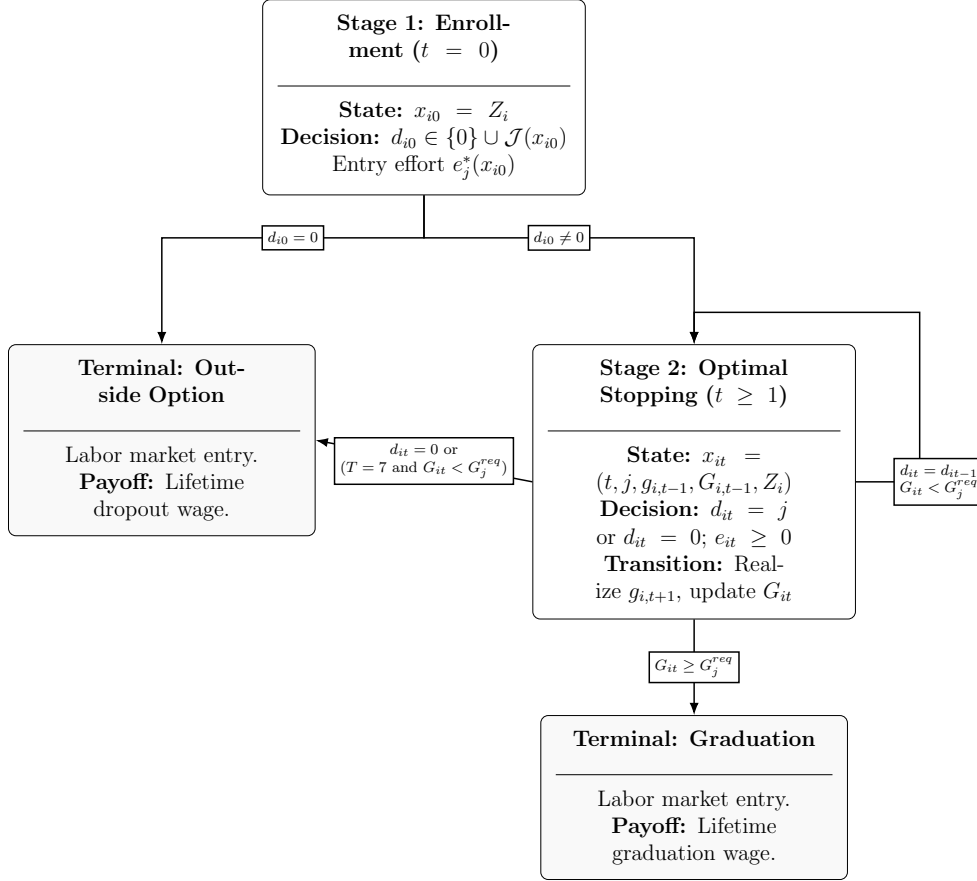


Figure 5: Structural model layout.

In stage 1, the student decides whether to enroll in higher education. She observes her type Z_i , her personalized choice set $\mathcal{J}(x_{i0})$ and the funding instruments available. She decides between a field–institution combination $d_{i0} = j$, with $j \in \mathcal{J}(x_{i0}) = \{1, \dots, J(x_{i0})\}$ a program from her personalized choice set, and the outside option $j = 0$. The student solves for the optimal “entry effort” $e_j^*(x_{i0})$ for every potential program and chooses the one yielding the

higher lifetime expected utility.

Stage 2 is conditional on enrollment. It is an optimal stopping problem. The student decides in each subsequent period whether to continue in the program, $d_{it} = \{d_{i,t-1}, 0\}$, and how much effort $e_{it} \in [0, +\infty)$ to exert. By exerting effort, the student implicitly chooses the probabilities of performance outcomes $g_{i,t+1}$, which accumulates over time as $G_{it} := \sum_{s=1}^t g_{is}$.

The model is discrete in time, with $t \in \{0, 1, \dots, T\}$ representing academic years. The nominal length of the program j is \bar{t}_j . The student can drop out at any time until the terminal period is $T = 7$ years, where she is forced to drop out if she did not yet graduate. The student's type Z_i includes sex, ability (psu and gpa), and socioeconomic status.

During periods before graduation ($t = 1, \dots, \bar{t}_j$), effort affects both the probability of dropout and, for scholarship holders, out-of-pocket fees $OP_j(x_{it})$: failing to meet the performance requirement ($g_{it} \geq 3$) implies paying the full tuition the following year. During the graduation period ($t = \bar{t}_j + 1, \dots, T$), the student can eventually graduate with some probability and obtain the continuation value associated with graduation, or alternatively postpone graduation if $t < T$, or drop out. At terminal period $T = 7$, any student who has not yet graduated is assumed to drop out. Hence, the state space is defined as $x_{it} = (t, d_{i,t-1}, g_{i,t-1}, G_{i,t-1}, Z_i)$.

4.2 Utility and the dynamic program

Flow utility from being enrolled in program j during period t includes individual preferences for different programs, out-of-pocket fees sensitivity, and the psychological or physical "disutility" of academic work. We decompose it into a fixed cost (FC_j) that captures preferences

for higher education, and a variable cost of effort:

$$u_j(x_{it}) + \varepsilon_{ijt} = -FC_j(x_{it}) - c_j(x_{it})e_{it} + \varepsilon_{ijt}, \quad (3)$$

with marginal cost $c_j(x_{it}) > 0$. Fixed costs $FC_j(x_{it})$ summarize all non-effort related preferences for programs including out-of-pocket fees $OP(Z_i, g_{i,t-1}, P_{j,t})$, defined as the fraction of program fees not covered by the state for student i in program j . These depend on tuition $P_{j,t}$, personal characteristics Z_i and past performance $g_{i,t-1}$:

$$OP(Z_i, g_{i,t-1}, P_{j,t}) = \begin{cases} (1 - \lambda(Z_i, g_{i,t-1}))P_{j,t} & \text{if the student holds a scholarship,} \\ (1 - \lambda(Z_i))P_{j,t} & \text{otherwise.} \end{cases}$$

where $\lambda(\cdot) \in [0, 1]$ is the subsidy rate, i.e. the fraction of program fees $P_{j,t}$ covered by the Government. For scholarship holders, $\lambda(\cdot)$ depends on past performance $g_{i,t-1}$ as well as on individual characteristics Z_i , since failure to meet credit requirements results in the loss of the subsidy. For loans and TFHE, $\lambda(\cdot)$ depends only on observable characteristics and does not vary with performance. Finally, the distribution of the idiosyncratic shock $\varepsilon_{ijt} \sim \text{EV1}$ is iid and common knowledge. It captures what the student learns at the beginning of the period ⁴.

The dynamics of the optimization problem are the following: On the one hand, by exerting more effort today, the student i) increases her probability of graduating in subsequent periods and raises her expected future wage, and ii) increases her probability to maintain public funding (passing the performance threshold) if eligible. On the other hand, exerting effort is costly and reduces the flow utility of attending higher education. The conditional value

4

function $v_j(x_{it}, e_{it})$ can be written as

$$v_j(x_{it}, e_{it}) + \varepsilon_{ijt} = u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) \bar{V}(x_{i,t+1}(\bar{g})) + \varepsilon_{ijt}, \quad (4)$$

where $\bar{V}(x_{i,t+1}(\bar{g}))$ denotes the ex-ante value function conditional on choosing program j . The first term corresponds to the current flow utility from enrolling in field-institution j . The second term captures the expected continuation value, discounted by β . Uncertainty in $x_{i,t+1}(\bar{g})$ arises solely from the realization of the performance outcome \bar{g} .

Applying the log-sum expression to [Equation 4](#) yields

$$v_j(x_{it}, e_{it}) + \varepsilon_{ijt} = u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) \ln \left(\sum_{j \in \mathcal{J}} \exp(v_j(x_{i,t+1}, e_{i,t+1})) \right) + \beta \gamma + \varepsilon_{ijt}.$$

4.3 *Effective* effort and performance

Effective effort e_{it} is unobserved, while performance outcomes g_{it} are directly recovered from students' records. We model performance outcome $g_{i,t+1}$ as the result of effort e_{it} and a logistically distributed shock η_{it} :

$$g_{i,t+1} = \kappa \quad \text{if } \bar{g}_{jt}^{\kappa} < \ln(e_{it}) + \eta_{i,t+1} \leq \bar{g}_{jt}^{\kappa+1}, \quad (5)$$

where the cutoff values \bar{g}_{jt}^{κ} and $\bar{g}_{jt}^{\kappa+1}$ are field-institution-year specific. In practice, performance is discretized into five categories, $\kappa \in \{0, 1, 2, 3, 4\}$. Students observe the realization of the shock $\eta_{i,t+1}$ only after choosing their effort e_{it} , but since they know the distribution of $\eta_{i,t+1}$ (standard logistic), they can compute the probability of each outcome conditional

on effort:

$$\Pr(g_{i,t+1} = \kappa \mid e_{it}, x_{it}) = F(\ln(e_{it}) - \bar{g}_{jt}^\kappa) - F(\ln(e_{it}) - \bar{g}_{jt}^{\kappa+1}), \quad (6)$$

where $F(\cdot)$ denotes the logistic CDF. Making use of the logistic assumption of the performance shock $\eta_{i,t+1}$, for $\kappa = 0$ we can rewrite Equation 6 as:

$$e_{it} = \frac{1 - \Pr(g_{i,t+1} = 0 \mid e_{it}, x_{it})}{\Pr(g_{i,t+1} = 0 \mid e_{it}, x_{it})}. \quad (7)$$

The higher effort, the higher the expected performance. Effective effort can thus be interpreted as the odds of avoiding the lowest performance outcome (failing all registered credits).

4.4 Solution of the model

The model is solved by backward induction. Higher education is no longer feasible after seven years, or once the student completes all program course credits. Formally,

$$\bar{V}(x_{i,t+1}(\bar{g})) = \alpha_w \text{wage}_j(x_{i,t+1}) \quad \text{if } t + 1 = T = 7 \text{ or } G_{i,t+1} \geq G_j^{\text{req}}. \quad (8)$$

and is used as an input in earlier periods. Here, $\text{wage}_j(x_{i,t+1})$ denotes lifetime expected wage for program j or dropout wage if $d_{i,t+1} = 0$.

At each period t , the student first chooses an effort level e_{it} in her program j . The decision trades off a loss in flow utility with higher future expected gains. Assuming she is rational and

behaving optimally, she is solving the following first-order condition for each program:

$$\frac{\partial v_j(x_{it}, e_{it})}{\partial e_{it}} = \frac{\partial u_j(x_{it}, e_{it})}{\partial e_{it}} + \beta \sum_{g \in \mathcal{G}} \frac{\partial \phi^{\bar{g}}(e_{it})}{\partial e_{it}} \bar{V}(x_{it+1}(\bar{g})) = 0 \quad \text{if } e_{it} = e_j^*(x_{it}).$$

Given the set of optimal effort levels $\{e_j^*(x_{it})\}_{j \in \mathcal{J}(x_{it})}$, the student chooses the program with the highest value $v_j(x_{it}, e_j^*(x_{it}))$. The resulting choice probabilities take the familiar logit form:

$$p_{jt} = \frac{\exp(v_j(x_{it}, e_j^*(x_{it})))}{\sum_{j' \in \mathcal{J}(x_{it})} \exp(v_{j'}(x_{it}, e_{j'}^*(x_{it})))}.$$

The ex-ante value function in period t can then be expressed, using the logsum formula, as

$$\bar{V}(x_{it}) = \gamma + \ln \left(\sum_{j \in \mathcal{J}(x_{it})} \exp(v_j(x_{it}, e_j^*(x_{it}))) \right), \quad (9)$$

and the model is solved recursively by iterating backward to $t = 1$.

4.5 Identification

We set the discount factor β to 0.95 and normalize the utility of the outside option to the dropout wage (Magnac & Thesmar, 2002).

$$\begin{cases} u_j(x_{it}, e_{it}) = -FC_j(x_{it}) - c_j(x_{it})e_{it}, & \text{if } j \neq 0, \\ u_0(x_{it}) = \alpha_w \text{wage}_0(x_{it}), & \text{if } j = 0. \end{cases} \quad (10)$$

We assume that not enrolling in higher education (and dropping out) is a terminal action. This means we do not allow for individuals to resit the exam one year after or re-enter higher education after dropping out.

4.5.1 Fixed and Marginal costs

Fixed costs are identified by the observed enrollment shares across programs. The sensitivity to out-of-pocket fees is pinned down by the variation in public funding eligibility, which serves as a price shifter. Marginal costs c_j are identified via the First-Order Condition (FOC) (de Groote, 2025). The model attributes higher performance levels, conditional on the same continuation value, to lower marginal costs of effort.

Wage elasticity α_w is identified through an exclusion restriction. We use local labor market field-region variation. The underlying assumption is that conditional on region and field fixed effects, the interaction provides idiosyncratic local demand shocks that shift utility only through the wage.

4.5.2 CCPs

Dropout CCPs are identified using both cross-sectional and longitudinal variation in dropout decisions. The key identifying assumption is that, conditional on observed characteristics, the idiosyncratic dropout shock is independent of the state variables. Performance CCPs are also identified using both cross-sectional and longitudinal variation in performance outcomes. The key identifying assumption is that, conditional on observed characteristics, the idiosyncratic performance shock is independent of the state variables.

4.5.3 Decomposing adverse selection and moral hazard

TFHE induced marginal students to enroll either because expected returns to education became positive or because credit constraints were relaxed. We can identify marginal students from those who change their enrollment decision, i.e. $d_{it}(FC_j(x_{it})) \neq d_{it}(FC'_j(x_{it}))$. Changes in performance outcomes can be directly identified from infra-marginal students, and treatment effects can be computed pre- and post-policy, i.e. $\sum_{i=IM} n_{IM}^{-1} (Y_{it}(FC_j(x_{it}), c_j(x_{it})) - Y_{it}(FC'_j(x_{it}), c'_j(x_{it})))$.

4.6 Estimation

We specify parametric functional forms for CCPs, Fixed Costs and wage regression. Details on the estimation of these components are provided in [Appendix C](#).

Given the Type-1 extreme value assumption, CCPs are of logit type.

$$\Pr(d_{it} = j | x_{it}) = p_{jt} = \frac{\exp(v_j(x_{it}, e_j^*(x_{it})))}{\sum_{j \in \mathcal{J}} \exp(v_j(x_{it}, e_j^*(x_{it})))}$$

Hotz and Miller (1993) show that the future value term can be written as the conditional value function and a non-negative term that depends on the empirical choice probabilities. Without loss of generality, we can write choice probabilities in terms of a base category j' .

$$p_{j't} = \frac{1}{1 + \exp(v_1(x_t))} \quad p_{jt} = \frac{\exp(v_j(x_t) - v_{j'}(x_t))}{\sum_{j \in \mathcal{J}} \exp(v_j(x_t) - v_{j'}(x_t))}$$

It is therefore convenient to use $j' = 0$ as an arbitrary choice and write

$$\begin{aligned}
& v_j(x_{it}, e_j^*(x_{it})) + \varepsilon_{ijt} \\
&= u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) \ln \left(\exp(v_0(x_{i,t+1}, e_j^*(x_{i,t+1}))) \sum_{j \in \mathcal{J}} \exp(v_j(x_{i,t+1}, e_j^*(x_{i,t+1})) - v_0(x_{i,t+1}, e_j^*(x_{i,t+1}))) \right) \\
&+ \beta\gamma + \varepsilon_{ijt} \\
&= u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (v_0(x_{i,t+1}, e_j^*(x_{i,t+1})) - \ln p_{0,t+1}(x_{i,t+1}(\bar{g}))) + \beta\gamma + \varepsilon_{ijt}
\end{aligned}$$

Recall the continuation value of dropping from higher education can be written as the expected wage of the degree minus a log correction term that depends on the dropout probability. Since $v_0(x_{i,t+1}, e_j^*(x_{i,t+1})) = \alpha_w wage_0(x_{i,t+1})$, we can rewrite the expression as

$$v_j(x_{it}, e_{it}) + \varepsilon_{ijt} = u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (\alpha_w wage_0(x_{i,t+1}) - \ln p_{0,t+1}(x_{i,t+1}(\bar{g}))) + \beta\gamma + \varepsilon_{ijt} \tag{11}$$

Estimation can be performed following three sequential steps.

1. We recover $p_{0,t+1}$ from a logistic regression and $\ln e_j^*(x_{it})$ the underlying index of an ordered logit model of credit completion (0-4).
2. The FOC equates marginal cost to marginal benefits, with an expression that can be directly estimated from the data. We can substitute the expression in $v_j(x_{it})$ and estimate $FC_j(x_{it})$ by maximum likelihood.
3. Finally, we use the estimates and the FOC to compute $c_j(x_{it})$.

$$\frac{\partial v_j(x_{it}, e_{it})}{\partial e_{it}} = \underbrace{\frac{\partial u_j(x_{it}, e_{it})}{\partial e_{it}}}_{=-c_j(x_{it})} + \beta \sum_{g \in \mathcal{G}} \frac{\partial \phi^{\bar{g}}(e_{it})}{\partial e_{it}} \bar{V}(x_{it+1}(\bar{g})) = 0 \quad \text{if } e_{it} = e_j^*(x_{it}).$$

Making use of the FOC and rearranging, we get an expression for marginal costs:

$$c_j^*(x_{it}) = \beta \sum_{\bar{g} \in \mathcal{G}} \frac{\partial \phi^{\bar{g}}(e_{it})}{\partial e_{it}} \bar{V}(x_{it+1}(\bar{g})) \quad \text{if } e_{it} = e_j^*(x_{it}), \quad (12)$$

where $\bar{V}(x_{it+1}(\bar{g}))$ is estimated using the CCPs, and $\frac{\partial \phi^{\bar{g}}(e_{it})}{\partial e_{it}}$ can be computed given the logistic assumption about η_{t+1} in [Equation 5](#). A sufficient condition for an interior solution is that the student always faces a strictly positive probability of the lowest outcome being realized; otherwise the optimal effort would collapse to $e_{it}^* = 0$. Positive marginal costs ensures that the support is bounded.

5 Results

5.1 Dropout probabilities

[Table 1](#) reports the results of a logistic regression model of dropout. Overall, dropout probabilities depend strongly on the year of enrollment. Progressing through the program reduces the likelihood of dropping out (relative to period 1), although the probability rises again when students reach the nominal length of the degree. Being female reduces the probability of dropping out, with the effect becoming stronger as students progress in their studies.

Ability is strongly correlated with persistence in higher education, but conditional on the stu-

Table 1: Dropout probabilities

	dropout	
OP fees	0.083***	(0.003)
Cumulated credits (t-1)	-0.113***	(0.002)
Credits (t-1)	-0.651***	(0.005)
Delay (t-1)	1.200***	(0.019)
female	-0.439***	(0.036)
Middle SES	-0.190***	(0.042)
High SES	-0.550***	(0.047)
PSU	-0.017	(0.026)
GPA	-0.164***	(0.022)
OP \times female	-0.003	(0.003)
OP \times middle SES	0.023***	(0.004)
OP \times high SES	0.070***	(0.005)
OP \times psu	-0.028***	(0.002)
OP \times gpa	-0.004**	(0.002)
Cumul (t-1) \times female	-0.033***	(0.001)
Cumul (t-1) \times middle SES	0.002	(0.001)
Cumul (t-1) \times high SES	-0.005***	(0.001)
Cumul (t-1) \times psu	0.022***	(0.001)
Cumul (t-1) \times gpa	0.001*	(0.001)
Credits (t-1) \times female	0.133***	(0.005)
Credits (t-1) \times middle SES	0.029***	(0.006)
Credits (t-1) \times high SES	0.073***	(0.007)
Credits (t-1) \times psu	-0.031***	(0.003)
Credits (t-1) \times gpa	0.042***	(0.003)
Delay (t-1) \times female	0.508***	(0.020)
Delay (t-1) \times middle SES	0.022	(0.024)
Delay (t-1) \times high SES	0.040	(0.026)
Delay (t-1) \times psu	-0.221***	(0.013)
Delay (t-1) \times gpa	0.164***	(0.013)
t= 2	-0.340***	(0.011)
t= 3	-0.570***	(0.015)
t= 4	-0.612***	(0.020)
t= 5	0.117***	(0.024)
t= 6	1.654***	(0.028)
t= 7	3.862***	(0.036)
Observations	1,417,543	
Mean of Dep. Variable	0.180	

Standard errors in parentheses

Logistic regression of choosing the outside option (dropping out) in period t. Controls include vector of characteristics, Institution and major FE, year enrolled FE and cumulated performance. PSU and GPU are standardised. Base category is low SES and t=1.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

dent’s progress (accumulated credits), its estimated coefficient diminishes. Both cumulative credits and the number of credits completed in the previous year are strong determinants of student progression, especially for low-income students. Being delayed in the program increases the probability of dropping out, with a stronger magnitude for females.

5.2 Performance

[Table B2](#) presents the results of the ordered logit model of performance with five categories (0–4). Coefficients follow a similar pattern as in the logistic dropout model. Ability is a strong predictor of performance, as is income. Out-of-pocket fees have a negative effect on performance, which is stronger for low-income and low-ability students.

Being delayed in the program reduces the probability of achieving the maximum number of credits, although the effect is milder for females. Finally, past progression—measured through cumulative and previous-year credits—is a strong predictor of current performance, particularly for low-income students.

5.3 Fixed Costs

Estimates of the conditional value function are presented in [Table 2](#). Students’ fixed costs increase with out-of-pocket fees, particularly at the program choice stage. Once enrolled, this effect is substantially attenuated in the continuation decision. Females are more sensitive to price than males. Students also face higher fixed costs when the program is located in a different region.

Past progression significantly affects fixed costs, as reflected in the coefficient of the delay variable. Again, females face higher fixed costs than males when delayed, and students who

Table 2: Fixed costs

	cvf	
OP fees	-0.588***	(0.037)
Distance	-1.397***	(0.031)
Delay (t-1)	-2.744***	(0.127)
OP x female	-0.106***	(0.034)
OP x middle SES	0.026	(0.040)
OP x high SES	-0.056	(0.045)
OP x psu	0.014	(0.022)
OP x gpa	0.015	(0.020)
OP (during HE)	0.440***	(0.042)
Delay (t-1) x female	-0.280**	(0.136)
Delay (t-1) x middle SES	-0.150	(0.160)
Delay (t-1) x high SES	-0.291	(0.181)
Delay (t-1) x psu	0.377***	(0.075)
Delay (t-1) x gpa	-0.131	(0.086)
EMAX	0.950	(.)
Expected wage	0.001***	(0.000)
Field FE	Yes	
Institution FE	Yes	

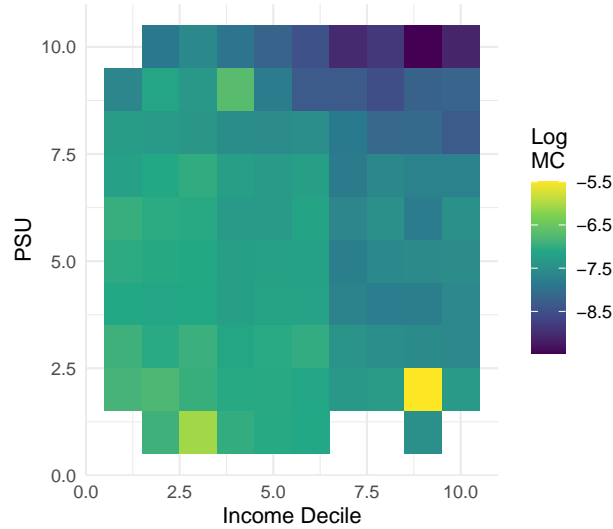
Standard errors in parentheses

Conditional Value function Estimation

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

performed well on the national exam appear to experience smaller penalties from lagging behind. Expected wage has a positive effect on the conditional value function, indicating that students value their future expected earnings.

Figure 6: Distribution of funding instruments, by test score



Notes: Heatmap of log marginal costs by PSU (y-axis) and income (x-axis) decile.

Figure 6 shows the distribution of log marginal costs by income and PSU score. The heatmap exhibits a gradient in marginal costs in both ability and socioeconomic status dimensions.

Table 3 presents, for interpretational purposes, the OLS regression of log marginal costs on state variables. Marginal costs decrease with out-of-pocket fees, in particular for low-income students. Females tend to have higher marginal costs compared to males, especially at lower income levels.

Table 3: Marginal costs

	log(mc)	
female	-0.226**	(0.105)
Middle SES	-0.005	(0.108)
High SES	0.242**	(0.115)
Standardized values of psu_avg	-0.322***	(0.120)
Standardized values of gpa_hs	-0.130**	(0.058)
OP fees	-0.038**	(0.019)
Distance	0.064***	(0.025)
Delay (t-1)	-0.801***	(0.170)
t= 2	-0.221***	(0.075)
t= 3	-0.107	(0.079)
t= 4	-0.039	(0.097)
t= 5	2.393***	(0.089)
t= 6	2.932***	(0.180)
t= 7	4.966***	(0.177)
OP x female	0.032	(0.020)
OP x middle SES	-0.017	(0.018)
OP x high SES	-0.018	(0.026)
OP x psu	-0.009	(0.010)
OP x gpa	0.007	(0.012)
Delay (t-1) x female	0.158	(0.127)
Delay (t-1) x middle SES	0.421***	(0.151)
Delay (t-1) x high SES	0.307*	(0.166)
Delay (t-1) x psu	0.034	(0.074)
Delay (t-1) x gpa	-0.011	(0.070)
Observations	10,811	
ymean		

Standard errors in parentheses

MC heterogeneity

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

6 Counterfactuals

We use the estimated model to simulate counterfactual funding policies, evaluating impacts on both enrollment and intensive (effort, credit completion, time-to-degree, graduation) margins. The exercises vary (i) the standardized exam threshold and (ii) the performance requirements.

6.1 PSU and performance requirements

We saw that TFHE substitute most of the preexisting aid programs, which were conditional on performance. The removal of performance requirements may have weakened incentives for effort and progression. Preexisting aid programs had performed well in attracting students, but it was not generous enough to attract low-income students. From the Government perspective, the current policy is very costly. From 2011 to 2019, the spending on higher education increased by 160%, representing 5.4% of GDP, in comparison with a 2.9% in the rest of OECD countries (OECD, 2019). Counterfactuals proposed build on expanding the conditions of previous aid programs. They are budget improving, while preserving the equity target of the TFHE policy. We therefore explore the following three scenarios:

1. **Performance requirement.** Impose an annual performance requirement (70% of registered credits) to *retain* free tuition. This restores the performance-contingent component removed by TFHE.

Model prediction: First period fixed and marginal costs are not affected. However, starting from the second year, students might want to increase effort to maintain the scholarship. Those that should increase it the most are the ones with high marginal and fixed costs, so the probability of dropout should increase. If the utility from

attending a first year of higher education does not compensate enough, it might deter from enrolling at all in higher education.

2. **PSU requirement.** Impose $PSU \geq 510$ for eligibility (harmonizing with merit scholarships). Mechanically reduces Government expenditure.

Model prediction: Attracts higher ability students, with lower fixed and marginal costs.

Persistance and graduation should increase.

3. **Joint requirement.** Combine (1) and (2): $PSU \geq 510$ and maintaining the performance threshold to retain free tuition.

Model prediction: Combines the selection and incentive approach. Reduces beneficiaries, but those that remain should have lower fixed and marginal costs, and stronger incentives to progress.

Mechanically, (1) affects $OP(x_{it})$ via g_{t-1} for *all* recipients, tightening incentives within a cohort; (2) affects the selection into TFHE; and (3) compounds both. Because the model allows $c_j(x_{it})$ to depend on OP, both $FC_j(x_{it})$ and $c_j(x_{it})$ increase for less performant students.

6.2 Additional scenarios

We outline additional scenarios for future work:

- **“New Zealand”-style.** Free Higher Education only in year 1; reversion to baseline thereafter. Tests whether early liquidity relief plus a return to progress-contingent support improves completion cost-effectively. Students learn their fix and marginal cost during year 1, and may adjust effort and dropout accordingly.
- **Targeted to academic/vocational education.** Make different ability thresholds

for academic and vocational programs.

- **Eligibility index combining merit and need.** Replace hard cutoffs with an index (e.g., weighted PSU and income) for a smoother assignment that may mitigate bunching and cliffs.
- **Remove merit requirements for loans/scholarships.** Harmonize loans and scholarships with the unconditional nature of Free Higher Education to isolate access from performance and compare purely through *OP*.

For each scenario, we report changes in enrollment, persistence, effort, credit accumulation, graduation.

7 Conclusion

This paper investigated the impacts of Tuition-Free Higher Education policies on student enrollment, persistence, and academic performance, using the 2016 Chilean reform as a natural experiment. Our difference-in-differences analysis reveals that the policy significantly boosted enrollment amongst eligible students by 10p.p., driven by lower ability students. Effects on educational outcomes were estimated for infr-marginal students, with a decrease in dropout of 2 p.p. and an increase in on-time graduation of 1.8 p.p.

We built a structural model that endogenizes effort choices, capable of disentangling selection effects from behavioral responses, finding that the removal of performance-based requirements did not lead to reduced effort or weakened outcomes overall. Instead, TFHE effectively expanded access without compromising educational quality, highlighting its role in mitigating financial barriers and moral hazard concerns.

These findings have important implications for higher education financing worldwide, suggesting that unconditional aid can promote equity and mobility, especially in contexts with high tuition costs and imperfect income verification.

References

- Abbott, B., Gallipoli, G., Meghir, C., & Violante, G. L. (2019). Education policy and intergenerational transfers in equilibrium. *Journal of Political Economy*, 127, 2569–2624.
- Angrist, J., Autor, D., Hudson, S., Pallais, A., Cantoni, E., Caldwell, S., Kim, O., Malone, B., Oyewole, K., & Scott, K. (2015, January). *Leveling up: Early results from a randomized evaluation of post-secondary aid*, National Bureau of Economic Research.
- Arcidiacono, P. (2005). Affirmative action in higher education: How do admission and financial aid rules affect future earnings? *Econometrica*, 73, 1477–1524.
- Beffy, M., Fougère, D., & Maurel, A. (2012). Choosing the field of study in postsecondary education: Do expected earnings matter? *The Review of Economics and Statistics*, 94, 334–347.
- Beneito, P., Boscá, J. E., & Ferri, J. (2018). Tuition fees and student effort at university. *Economics of Education Review*, 64, 114–128.
- Black, S. E., Denning, J., Dettling, L. J., Goodman, S., & Turner, L. J. (2023). Taking it to the limit: Effects of increased student loan availability on attainment, earnings, and financial well-being. *SSRN Electronic Journal*.
- Bucarey, A. (2017). *Who pays for free college? crowding out on campus*.
- Bucarey, A., Contreras, D., & Muñoz, P. (2020). Labor market returns to student loans for university: Evidence from Chile. *Journal of Labor Economics*, 38, 959–1007.

- Callaway, B., Goodman-Bacon, A., & Sant’anna, P. (2024). *Difference-in-differences with a continuous treatment*.
- Cameron, A. C., Gelbach, J. B., & Miller, D. L. (2008). Bootstrap-based improvements for inference with clustered errors. *The Review of Economics and Statistics*, 90, 414–427.
- Castro-Zarzur, R., Espinoza, R., & Sarzosa, M. (2022). Unintended consequences of free college: Self-selection into the teaching profession. *Economics of Education Review*, 89, 102260.
- Cohodes, S. R., & Goodman, J. S. (2014). Merit aid, college quality, and college completion: Massachusetts adams scholarship as an in-kind subsidy. *American Economic Journal: Applied Economics*, 6, 251–85.
- de Chaisemartin, C., & D’Haultfoeuille, X. (2024). Difference-in-differences estimators of intertemporal treatment effects. *Review of Economics and Statistics*, 1–45.
- de Groote, O. (2025). Dynamic effort choice in high school: Costs and benefits of an academic track. *Journal of Labor Economics*.
- Denning, J. T. (2017). College on the cheap: Consequences of community college tuition reductions. *American Economic Journal: Economic Policy*, 9, 155–188.
- Denning, J. T. (2019). Born under a lucky star. *Journal of Human Resources*, 54, 760–784.
- Dobbin, C., Barahona, N., & Otero, S. (2022). *The equilibrium effects of subsidized student loans*.
- Dynarski, S. M. (2003). Does aid matter? measuring the effect of student aid on college attendance and completion. *American Economic Review*, 93, 279–288.
- Epple, D., Romano, R., Sarpça, S., & Sieg, H. (2017). A general equilibrium analysis of state and private colleges and access to higher education in the u.s. *Journal of Public Economics*, 155, 164–178.

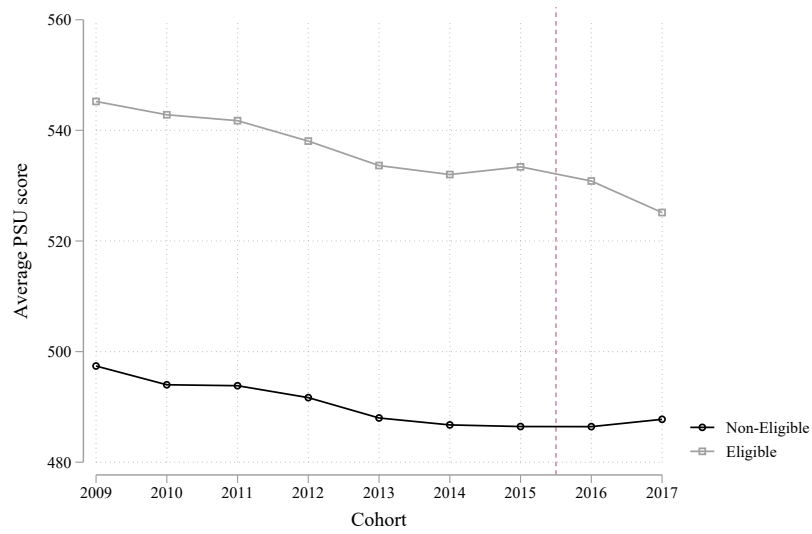
- Epple, D., Romano, R., & Sieg, H. (2006). Admission, tuition, and financial aid policies in the market for higher education. *Econometrica*, 74, 885–928.
- Fack, G., Grenet, J., & He, Y. (2019). Beyond truth-telling: Preference estimation with centralized school choice and college admissions. *American Economic Review*, 109, 1486–1529.
- Falco, A. D., & Reichlin, Y. (2025). Grants vs. loans: The role of financial aid in college major choice. *The Economic Journal*.
- Ferreyra, M., Garriga, C., Martin-Ocampo, J. D., & Diaz, A. M. S. (2022). Cows don’t give milk: An effort model of college graduation.
- Gale, D., & Shapley, L. S. (1962). College admissions and the stability of marriage. *The American Mathematical Monthly*, 69, 9.
- Garibaldi, P., Giavazzi, F., Ichino, A., & Rettore, E. (2012). College cost and time to complete a degree: Evidence from tuition discontinuities. *The Review of Economics and Statistics*, 94, 699–711.
- Hotz, V. J., & Miller, R. A. (1993). Conditional choice probabilities and the estimation of dynamic models. *The Review of Economic Studies*, 60, 497–529.
- Joensen, J. S., & Mattana, E. (2024, April). *Student aid design, academic achievement, and labor market behavior: Grants or loans?*
- Londoño-Vélez, J., Rodríguez, C., & Sánchez, F. (2020). Upstream and downstream impacts of college merit-based financial aid for low-income students: Ser pilo paga in colombia. *American Economic Journal: Economic Policy*, 12, 193–227.
- Magnac, T., & Thesmar, D. (2002). Identifying dynamic discrete decision processes. *Econometrica*, 70, 801–816.

- Montalbán, J. (2022). Countering moral hazard in higher education: The role of performance incentives in need-based grants*. *The Economic Journal*, 133, 355–389.
- OECD. (2019). *Education at a glance* (tech. rep.).
- Pal, J. (2025). Education policy and the quality of public servants.
- Rothstein, J., & Rouse, C. E. (2011). Constrained after college: Student loans and early-career occupational choices. *Journal of Public Economics*, 95, 149–163.
- Sieg, H., & Wang, Y. (2018). The impact of student debt on education, career, and marriage choices of female lawyers. *European Economic Review*, 109, 124–147.
- Solis, A. (2017). Credit access and college enrollment. *Journal of Political Economy*, 125, 562–622.
- Stinebrickner, R., & Stinebrickner, T. (2008). The effect of credit constraints on the college drop-out decision: A direct approach using a new panel study. *American Economic Review*, 98, 2163–84.
- Tincani, M. M., Kosse, F., Miglino, E., & Fern, P. (2023). College access when preparedness matters : New evidence from large advantages in college admissions.

Appendices

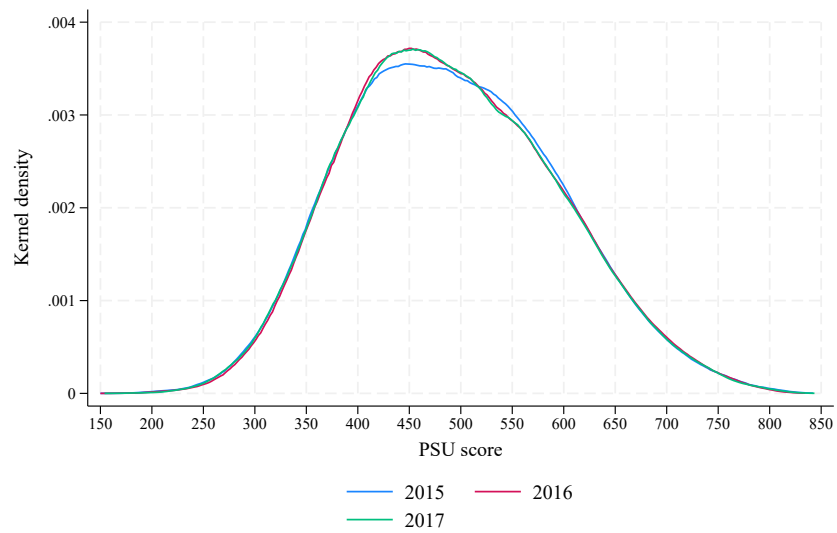
A Figures

Figure A1: Average PSU score over time for participating and non-participating institutions



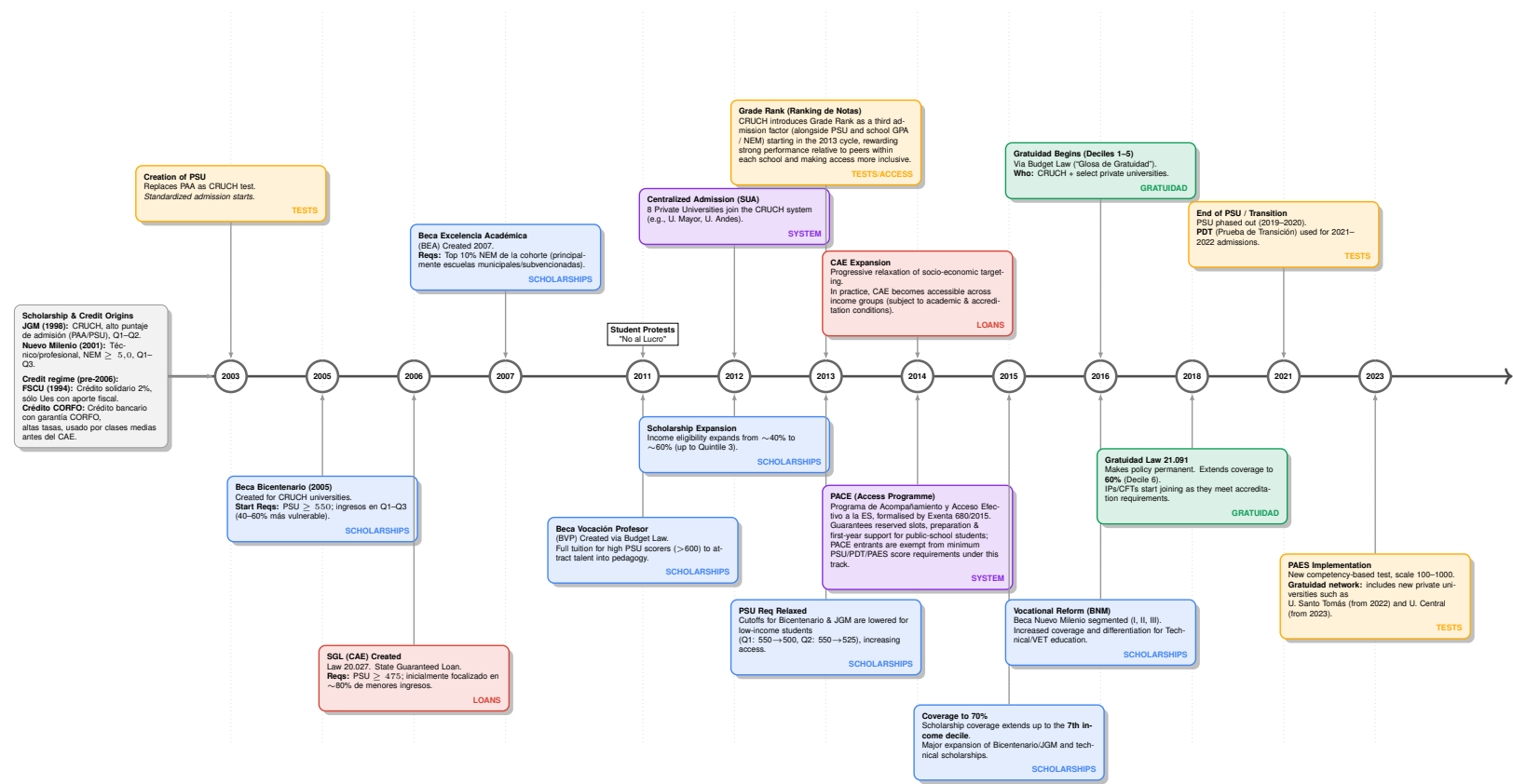
Notes: Annual average PSU score by groups of eligible and non-eligible institutions to free higher education.

Figure A2: Average PSU score over time for participating and non-participating institutions



Notes: Kernel density of PSU average scores enrolled in higher education for years 2015, 2016 and 2017.

Figure A3: Education reforms 2003-2024



Notes: .

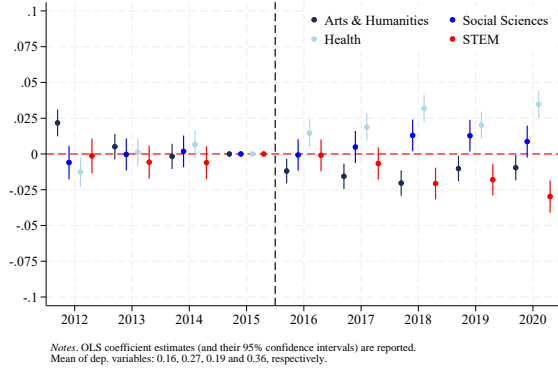
	PSU \geq 510	$475 \leq$ PSU $<$ 510	PSU $<$ 475
Deciles 1-5	Scholarship TFHE (univ) TFHE (univ+SCP)	SGL loan TFHE (univ) TFHE (univ+SCP)	No public aid TFHE (univ) TFHE (univ+SCP)
Deciles 6-7	Scholarship	SGL loan	No public aid
Deciles 8-10	SGL loan	SGL loan	No public aid

PSU = admissions test; SGL = State-Guaranteed Loan; SCP = short-cycle programs; TFHE = Tuition-Free Higher Education

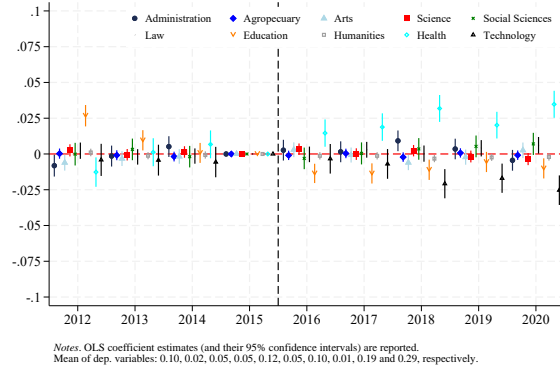
Years: 2015 2016 2017

Figure A4: Changes in funding instruments after TFHE implementation

Figure A5: DID: Preferences



a) 4 fields



b) 10 fields

Notes: These figures show the results of estimating Equation 1. Panel (a) groups the results in four fields, while panel (b) uses the original ten-field classification. STEM includes Agropecuaria, Science and Technology. Social Sciences includes Administration, Social Sciences and Law. Arts and Humanities includes Education, Arts and Humanities. Health includes Health Sciences.

B Tables

Table B1: Descriptive statistics, students

	2012	2013	2014	2015	2016	2017
Enrollment						
N Students	165762	169257	169415	176027	180128	180302
Enrolled in platform	.260	.273	.280	.275	.280	.276
Enrolled out of platform	.338	.351	.346	.341	.330	.329
Not enrolled	.400	.374	.372	.382	.388	.394
Demographics						
Family Income	3.5	3.7	3.9	4	4.1	4.5
Private School	.111	.111	.111	.109	.106	.105
Private Health	.268	.267	.268	.263	.262	.235
Father With College	.168	.169	.170	.171	.169	.184
Mother Employed	.414	.436	.460	.460	.461	.461
Test Score	490	491	491	492	492	491
Free College	0	0	0	0	.145	.243
Subsidized Loan	.222	.213	.215	.187	.145	.105
Merit-based Scholarship	.136	.191	.200	.249	.158	.101
Field						
Business	.136	.136	.139	.148	.155	.160
Farming	.021	.022	.021	.023	.024	.025
Art and Architecture	.051	.050	.051	.052	.053	.055
Basic Sciences	.032	.033	.035	.033	.034	.031
Social Sciences	.081	.077	.076	.078	.083	.083
Law	.038	.035	.037	.037	.040	.040
Education	.110	.096	.091	.091	.095	.095
Humanities	.010	.010	.010	.010	.010	.009
Health	.214	.201	.200	.196	.196	.196
Technology	.284	.318	.318	.312	.290	.289

Notes: This table shows descriptive statistics on every student who enrolled and took the college entrance exam. Family income is categorized in 1-10 brackets, and field classification is performed following the ISCED-UNESCO guidelines.

Table B2: Performance probabilities

	index	
female	0.457***	(0.017)
Middle SES	0.123***	(0.019)
High SES	0.082***	(0.022)
PSU	0.527***	(0.013)
GPA	0.350***	(0.010)
OP fees	-0.029***	(0.002)
Cum. credits completed (t-1)	0.095***	(0.002)
Credits completed (t-1)	0.154***	(0.004)
Delay (t-1)	-1.627***	(0.010)
1 cred. left (t-1)	-1.711***	(0.038)
2 cred. left (t-1)	-0.810***	(0.026)
3 cred. left (t-1)	-0.531***	(0.032)
OP \times female	0.011***	(0.002)
OP \times middle SES	0.010***	(0.002)
OP \times high SES	0.014***	(0.003)
OP \times psu	0.008***	(0.001)
OP \times gpa	0.005***	(0.001)
1 left (t-1) \times female	0.200***	(0.041)
1 left (t-1) \times middle SES	0.069	(0.048)
1 left (t-1) \times high SES	0.029	(0.054)
1 left (t-1) \times psu	-0.358***	(0.027)
1 left (t-1) \times gpa	-0.022	(0.026)
2 left (t-1) \times female	0.154***	(0.029)
2 left (t-1) \times middle SES	-0.058*	(0.033)
2 left (t-1) \times high SES	-0.090**	(0.037)
2 left (t-1) \times psu	-0.140***	(0.019)
2 left (t-1) \times gpa	0.006	(0.018)
3 left (t-1) \times female	0.165***	(0.035)
3 left (t-1) \times middle SES	0.002	(0.040)
3 left (t-1) \times high SES	-0.048	(0.044)
3 left (t-1) \times psu	-0.003	(0.024)
3 left (t-1) \times gpa	0.026	(0.022)
t=2 \times SCP	-0.270***	(0.013)
t=3 \times SCP	-1.465***	(0.014)
t=4 \times SCP	-1.437***	(0.017)
t=5 \times SCP	-1.964***	(0.023)
t=6 \times SCP	-1.160***	(0.034)
t=7 \times SCP	-0.721***	(0.067)
Credits (t-1) \times female	0.006**	(0.003)
Credits (t-1) \times middle SES	-0.016***	(0.003)
Credits (t-1) \times high SES	-0.004	(0.004)
Credits (t-1) \times psu	-0.034***	(0.002)
Credits (t-1) \times gpa	0.004**	(0.002)
Cumul (t-1) \times female	-0.040***	(0.001)
Cumul (t-1) \times middle SES	-0.006***	(0.001)
Cumul (t-1) \times high SES	-0.010***	(0.001)
Cumul (t-1) \times psu	-0.021***	(0.001)
Cumul (t-1) \times gpa	-0.017***	(0.001)
Delay (t-1) \times female	-0.035***	(0.009)
Delay (t-1) \times middle SES	-0.004	(0.010)
Delay (t-1) \times high SES	0.091***	(0.012)
Delay (t-1) \times psu	0.022***	(0.006)
Delay (t-1) \times gpa	0.064***	(0.005)
t= 2	-0.092***	(0.012)
t= 3	0.291***	(0.015)
t= 4	0.432***	(0.019)
t= 5	0.126***	(0.023)
t= 6	-0.661***	(0.027)
t= 7	-1.294***	(0.031)
Observations	1416806.000	
Mean of Dep. Variable	3.096	

Standard errors in parentheses

Ordered logistic regression of discretised credit achievement (0-4). Controls include vector of characteristics, Institution and major FE, year enrolled FE and cumulated performance. PSU and GPU are standardised. Base category is low SES and t=1.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

C Estimation

C.1 Functional forms

Performance outcomes

For $g \in \mathcal{G} = \{0, 1, 2, 3, 4\}$, denote \tilde{g} the underlying index:

$$\begin{aligned} \tilde{g}_j(x_{it}) = & \sum_m \sum_n \alpha_{mn}^g Z_{im} Z_{jn} + \sum_m \sum_n \alpha_{gm}^g Z_{gm} Z_{in} + \mathbf{Z}_g' \alpha_G^g \\ & + \alpha_{field(j)}^g + \alpha_{inst(j)}^g + \lambda_t^g + \lambda_{tscp}^g \cdot \text{SCP}_{ij}, \end{aligned} \quad (13)$$

where $Z_i = (\text{psu}, \text{gpa}, \text{income}, \text{female})$, $Z_j = (\text{field1}, \dots, \text{field10})$, $Z_g = (g_{it-1}, G_{it-1}, \text{delay}_{it-1}, \text{1left}_{it-1}, \text{2left}_{it-1}, \text{3left}_{it-1})$. g_{it-1} and G_{it-1} denote previous-period and cumulative credit completion, respectively. Xleft indicates whether the student left the program with X or more credits remaining. $\alpha_{field(j)}^g$ are field fixed effects, $\alpha_{inst(j)}^g$ are institution fixed effects, and λ_t^g are period fixed effects. Period fixed effects are allowed to differ between SCPs and university programs.

Xleft variables take into account the fact that passing all the credits in a period where less credits are needed to graduate might be different. An alternative modelling approach would estimate separate models for each possible credit left in a year (1,2,3,4). However, estimating it together gives us more precise estimates.

—

Dropout probability

We estimate the following logistic regression by maximum likelihood:

$$\begin{aligned} \tilde{drop}_j(x_{it}) = & \sum_m \sum_n \alpha_{mn}^d Z_{im} Z_{jn} + \sum_m \sum_n \alpha_{gm}^d Z_{gm} Z_{in} + \mathbf{Z}_g' \alpha_G^d \\ & + \alpha_{field(j)}^d + \alpha_{inst(j)}^d + \lambda_t^d + \lambda_{tscp}^d \cdot SCP_{ij}, \end{aligned} \quad (14)$$

where $Z_i = (\text{psu}, \text{gpa}, \text{income}, \text{female})$, $Z_j = (\text{field1}, \dots, \text{field10})$, $Z_g = (g_{it-1}, G_{it-1}, \text{delay}_{it-1}, \text{1left}_{it-1}, \text{2left}_{it-1}, \text{3left}_{it-1})$. g_{it-1} and G_{it-1} denote previous-period and cumulative credit completion, respectively. X_{left} indicates whether the student left the program with X or more credits remaining. $\alpha_{field(j)}^g$ are field fixed effects, $\alpha_{inst(j)}^g$ are institution fixed effects, and λ_t^g are period fixed effects. Period fixed effects are allowed to differ between SCPs and university programs.

—

Labour market

We estimate the following wage equation,

$$\begin{aligned} \log(w_j(x_{it})) = & \alpha_0^w + \alpha_f^w \text{female}_i + \alpha_e^w \text{exper}_{it} + \alpha_{e2}^w \text{exper}_{it}^2 + \alpha_j^w + \lambda_R^w \\ & + \sum_{r=1}^{16} \sum_j \alpha_{rj}^w R_{ir} \cdot 1\{\text{field}(j)\} + \varepsilon_{ijt}, \end{aligned} \quad (15)$$

where R_{ir} is a dummy for region r . The region–field interactions $\{R_{ir} \cdot 1\{\text{field}(j)\}\}$ are crucial for identification of the wage coefficient in the structural model, as they serve as an exclusion restriction: regional-field variation affects utility only through wages.

From scraped data on wages from the ⁵Chilean Ministry of Education, we recover institution-specific wage premiums by regressing log wages on field and institution fixed effects. We then

⁵<https://www.mifuturo.cl>

predict Equation 15 for each student in the sample, adding the institution premium.

—

Fixed costs

$$\begin{aligned}
 FC_j(x_{it}) = & \alpha_p OP_{ij} + \sum_m \alpha_{pm} OP_{ij} Z_{im} + \sum_m \sum_n \alpha_{mn} Z_{im} Z_{jn} + \sum_m \sum_n \alpha_{gm} Z_{gm} Z_{in} \\
 & + \mathbf{Z}_g' \alpha_G + \sum_m \sum_k \alpha_{km} M_{ij,k} Z_{im} + \mathbf{M}_{ij}' \alpha_m + \alpha_{field(j)} + \alpha_{inst(j)},
 \end{aligned} \tag{16}$$

where D_{ij} is a dummy for the institution being in a different region (distance proxy), and A_{ij} is the student's relative ability in program j . $M_{ij} = (D_{ij}, A_{ij})$ is a vector of student-program matched variables.

C.2 Conditional Value Function

Conditional value function making use of a terminal action (dropping out) when log effort is the underlying index of an ordered logit model.

$$v_j(x_{it}, e_{it}) = u_j(x_{it}, e_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (\gamma + v_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g})))$$

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) - c(x_{it})e(x_{it}) + \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (\gamma + v_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g})))$$

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) - \beta \sum_{\bar{g} \in \mathcal{G}} \frac{\partial \phi^{\bar{g}_{ijt}}(e_{it})}{\partial e_{it}} (\gamma + v_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) e(x_{it})$$

$$+ \beta \sum_{\bar{g} \in \mathcal{G}} \phi^{\bar{g}}(e_{it}) (\gamma + v_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g})))$$

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) + \beta \gamma$$

$$+ \beta \sum_{\bar{g} \in \mathcal{G}} \left((\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \left(\phi^{\bar{g}}(e_j^*(x_{it})) - \frac{\partial \phi^{\bar{g}_{ijt}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right) \right)$$

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) + \beta \gamma$$

$$+ \beta \sum_{\bar{g} \in \mathcal{G}} ((\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g})))$$

$$\times (\Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})))$$

$$- (\Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it}))) - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it}))))$$

$$\times e_j^*(x_{it}))$$

where $\Lambda(\cdot)$ denotes the standard logistic CDF. The general expression for ordered logit

probabilities are, denoting by $\alpha_{\bar{g}}$ the estimated cutoff points,

$$\begin{aligned}\phi_{ijt}^{\bar{g}}(e_j^*(x_{it})) &= \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})) \\ &= \frac{1}{1 + \exp(e_j^*(x_{it}) - \alpha_{\bar{g}})} - \frac{1}{1 + \exp(e_j^*(x_{it}) - \alpha_{\bar{g}-1})}\end{aligned}$$

The derivative wrt $e_j^*(x_{it})$ writes

$$\begin{aligned}\left. \frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \right|_{e_{it}=e_j^*(x_{it})} &= -\Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it}))) \\ &\quad - (-\Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})))) \\ \left. \frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \right|_{e_{it}=e_j^*(x_{it})} &= \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}-1} - e_j^*(x_{it}))) \\ &\quad - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}} - e_j^*(x_{it})))\end{aligned}$$

Computing the marginal cost and EMAX expression

$$\sum_{\bar{g} \in \mathcal{G}} (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \left(\phi_{ijt}^{\bar{g}}(e_j^*(x_{it})) - \left. \frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \right|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right)$$

For $g = 1$

$$\begin{aligned}&(\Lambda(\alpha_1 - e_j^*(x_{it})) + (\Lambda(\alpha_1 - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_1 - e_j^*(x_{it})))) \times e_j^*(x_{it}) \\ &(\Lambda(\alpha_1 - e_j^*(x_{it})) + (\Lambda(\alpha_1 - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_1 - e_j^*(x_{it})))) \times e_j^*(x_{it})\end{aligned}$$

$$\begin{aligned}
& (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(1))) \times \\
& \left(\Lambda(\alpha_1 - index_j(x_{it})) - \frac{\partial \phi_{ijt}^{\bar{g}}(e_j(x_{it}))}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} \times e_j^*(x_{it}) \right)
\end{aligned}$$

We defined effort as

$$e_j(x_{it}) = \frac{1 - \Lambda(\alpha_1 - index_j(x_{it}))}{\Lambda(\alpha_1 - index_j(x_{it}))} \quad (17)$$

$$\Lambda(\alpha_1 - index_j(x_{it})) = \frac{1}{1 + e_j(x_{it})} \quad (18)$$

$$\alpha_1 - index_j(x_{it}) = \ln \left(\frac{\frac{1}{1+e_j(x_{it})}}{1 - \frac{1}{1+e_j(x_{it})}} \right) \quad (19)$$

$$\alpha_1 - index_j(x_{it}) = \ln \left(\frac{1}{e_j(x_{it})} \right) = -\ln(e_j(x_{it})) \quad (20)$$

$$\alpha_1 - \ln(e_j(x_{it})) = index_j(x_{it}) \quad (21)$$

then the derivative wrt effort looks like

$$\begin{aligned}
& \left(\Lambda(\alpha_1 - index_j(x_{it})) - \left(\lambda(\alpha_1 - index_j(x_{it})) \times \frac{-1}{e_j(x_{it})} \right) \times e_j(x_{it}) \right) \\
& = \frac{1}{1 + e_j(x_{it})} + \frac{e_j(x_{it})}{(1 + e_j(x_{it}))^2}
\end{aligned}$$

For $g \in (2, 4)$

$$\begin{aligned} & \left(\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g})) \right) \times \\ & (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) - \\ & (\Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it}))) - \Lambda(\alpha_{\bar{g}} - index_j(x_{it})) \times (1 - \Lambda(\alpha_{\bar{g}} - index_j(x_{it})))) : \end{aligned}$$

Recall that rearranging the effort measure in [Equation 17](#), we can express $index_j(x_{it}) = \alpha_1 - \ln(e_j(x_{it}))$

$$\begin{aligned}
& (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \\
& (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) - \\
& \left. \frac{\partial \phi_{ijt}^{\bar{g}}(e_j(x_{it}))}{\partial e_{it}} \right|_{e_{it}=e_j^*(x_{it})} (\Lambda(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it}))) - \Lambda(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it}))) \times e_j^*(x_{it}))) \\
& = (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \\
& (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) - \\
& \left(\lambda(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it}))) \times \left(\frac{-1}{e_j^*(x_{it})} \right) - \lambda(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it}))) \times \left(\frac{-1}{e_j^*(x_{it})} \right) \right) \times e_j^*(x_{it})) \\
& = (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \\
& (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) - \\
& \left(\frac{-e_j^*(x_{it})}{e_j^*(x_{it})} \right) \times \left(\frac{\exp(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it})))}{(1 + \exp(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it}))))^2} - \frac{\exp(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it})))}{(1 + \exp(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it}))))^2} \right)) \\
& = (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \\
& (\Lambda(\alpha_{\bar{g}} - index_j(x_{it})) - \Lambda(\alpha_{\bar{g}-1} - index_j(x_{it})) + \\
& \left(\frac{\exp(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it})))}{(1 + \exp(\alpha_{\bar{g}} - \alpha_1 - \ln(e_j^*(x_{it}))))^2} - \frac{\exp(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it})))}{(1 + \exp(\alpha_{\bar{g}-1} - \alpha_1 - \ln(e_j^*(x_{it}))))^2} \right))
\end{aligned}$$

For $g = 5$

$$\begin{aligned}
& (\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(5))) \times \\
& (1 - \Lambda(\alpha_4 - e_j^*(x_{it})) - (\Lambda(\alpha_4 - e_j^*(x_{it})) \times (1 - \Lambda(\alpha_4 - e_j^*(x_{it})))) \times e_j^*(x_{it}))
\end{aligned}$$

$$\sum_{\bar{g} \in \mathcal{G}} \left(\phi_{ijt}^{\bar{g}}(e_j^*(x_{it})) - \frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right)$$

Notice from the above expression that

$$\sum_{\bar{g} \in \mathcal{G}} (\phi_{ijt}^{\bar{g}}(e_j^*(x_{it}))) = 1$$

Since we are integrating over all possible states and that

$$\begin{aligned} & \sum_{\bar{g} \in \mathcal{G}} \left(\frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right) \\ &= \lambda(\alpha_1 - index(x_{it})) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) + \\ & (\lambda(\alpha_2 - index(x_{it})) - \lambda(\alpha_1 - index(x_{it}))) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) + \\ & (\lambda(\alpha_3 - index(x_{it})) - \lambda(\alpha_2 - index(x_{it}))) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) + \\ & (\lambda(\alpha_4 - index(x_{it})) - \lambda(\alpha_3 - index(x_{it}))) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) + \\ & (-\lambda(\alpha_4 - index(x_{it}))) \times (-1/e_j^*(x_{it})) \times e_j^*(x_{it}) \end{aligned}$$

since $index = \ln(e_j^*(x_{it})) + \alpha_1$ and that $\frac{\partial \lambda(\alpha_g - index(x_{it}))}{\partial e_{it}} = \lambda(\alpha_g - index(x_{it})) \times (-1/e_j^*(x_{it}))$.

This can be rewritten as

$$\begin{aligned}
& \sum_{\bar{g} \in \mathcal{G}} \left(\frac{\partial \phi_{ijt}^{\bar{g}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right) \\
&= -\lambda(\alpha_1 - index(x_{it})) + \\
&\quad - (\lambda(\alpha_2 - index(x_{it})) - \lambda(\alpha_1 - index(x_{it}))) + \\
&\quad - (\lambda(\alpha_3 - index(x_{it})) - \lambda(\alpha_2 - index(x_{it}))) + \\
&\quad - (\lambda(\alpha_4 - index(x_{it})) - \lambda(\alpha_3 - index(x_{it}))) + \\
&\quad - (-\lambda(\alpha_1 - index(x_{it}))) \\
&= 0
\end{aligned}$$

Conditional value function depending on t and j

$$v_j(x_{it}, e_{it}) = -FC(x_{it}, OP) + \beta\gamma + \beta \sum_{\bar{g} \in \mathcal{G}} \left((\alpha_w wage_0(x_{it+1}) - \ln \Pr(d_{it+1}^0 | x_{it+1}(\bar{g}))) \times \left(\phi^{\bar{g}}(e_j^*(x_{it})) - \frac{\partial \phi^{\bar{g}_{ijt}}(e_{it})}{\partial e_{it}} \Big|_{e_{it}=e_j^*(x_{it})} e_j^*(x_{it}) \right) \right)$$

- if $t = 1$ & $j = 0$: $v_0(x_{it}, e_{it}) = \beta \alpha_w wage_0(x_{it})$
- if $t > 1$ & $j = 0$: $v_0(x_{it}, e_{it}) = \beta \alpha_w wage_0(x_{it})$
- if $t = last$ & $t \neq T$ & $d_{it} = d_{it-1}$: $v_j(x_{it}, e_{it}) = \beta \alpha_w wage_1(x_{it})$
- if $t = T$ & $d_{it} = d_{it-1}$: $v_j(x_{it}, e_{it}) = \beta \alpha_w (wage_0(x_{it}) \text{ or } wage_1(x_{it}))$ depending on graduation (um accumulation variable).

In the last period ($T = 7$)

$$v_0(x_{it}, e_{it}) = \alpha_w wage_0$$

$$v_1(x_{it}, e_{it}) = \alpha_w wage_1$$