Universidad Central del Ecuador
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Pedagogía de las Ciencias Experimentales Informática

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Resolver 10 Ecuaciones diferenciales homogéneas

1 $(x+y)\,dx - x\,dy = 0$

$M(x,y) = x + y \quad , N(x,y) = -x$

$M(tx,ty) = tx + ty \qquad N(tx,ty) = -(tx)$

$= t(x+y) \qquad = t(-x)$

$= t^1 M(x,y) \qquad = t^1 N(x,y)$

$u = \dfrac{y}{x}$

$y = ux$

$dy = u\,dx + x\,dx$

$(x+y)\,dx - x\,dy = 0$

$(x + ux)\,dx - x(u\,dx + x\,du) = 0$

$x(1+u)\,dx - ux\,dx + x^2\,du = 0$

$x\,dx + \cancel{ux\,dx} - \cancel{ux\,dx} + x^2\,du = 0$

$x\,dx - x^2\,du = 0$

$x\,(dx - x\,du) = 0$

$\dfrac{\cancel{x}(dx - x\,du)}{\cancel{x}} = \dfrac{0}{x}$

$dx - x\,du = 0$

$dx = x\,du$

$\int \dfrac{dx}{x} = \int du$

$\ln x = u + C$

$\ln x = \dfrac{y}{x} + C$

$\ln x - C = \dfrac{y}{x}$

$x(\ln x - C) = y$

$\boxed{y = x\ln(x) - Cx}$

2. $(x-y)\,dx + x\,dy = 0$

$M(x,y) = (x-y)$, $N(x,y) = x$

$M(tx,ty) = tx - ty$ $\quad N(tx,ty) = tx$

$= t(x-y)$ $\quad = t^1 N(x,y)$

$= t^1 M(x,y)$

$u = \frac{y}{x}$

$y = ux$

$dy = u\,dx + x\,du$

$(x-y)\,dx + x\,dy = 0$

$(x-ux)\,dx + x(u\,dx + x\,du) = 0$

$x\,dx - \cancel{ux\,dx} + \cancel{ux\,dx} + x^2\,du = 0$

$x\,dx + x^2\,du = 0$

$x(dx + x\,du) = 0$

$\frac{\cancel{x}(dx + x\,du)}{\cancel{x}} = \frac{0}{x}$

$dx + x\,du = 0$

$x\,du = -dx$

$\frac{\cancel{x}\,du}{\cancel{x}} = -\frac{dx}{x}$

$du = -\frac{dx}{x}$

$\int \frac{dx}{x} = \int -du$

$\ln|x| = -\int du$

$\ln|x| = -u + C$

$\ln|x| = -\frac{y}{x} + C$

$\frac{y}{x} = C - \ln|x|$

$y = x(C - \ln|x|)$

$\boxed{y = Cx - x\ln|x|}$

3. $y' = \frac{y}{x} + \frac{x}{y}$

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$$

$$\frac{dy}{dx} - \frac{x}{y} - \frac{y}{x} = 0$$

$$dx\left(\frac{dy}{dx} - \frac{x}{y} - \frac{y}{x} = 0\right) dx$$

$$\cancel{dx}\,\frac{dy}{\cancel{dx}} - \frac{x}{y}\,dx - \frac{y}{x}\,dx = 0$$

$$dy - \frac{x}{y}\,dx - \frac{y}{x}\,dx = 0$$

$$f(x,y) = \frac{x}{y} + \frac{y}{x}$$

$$f(tx,ty) = \frac{\cancel{t}x}{\cancel{t}y} + \frac{\cancel{t}y}{\cancel{t}x}$$

$$= \frac{x}{y} + \frac{y}{x}$$

$$= f(x,y)$$

$$\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}$$

$$u + x\frac{du}{dx} = \frac{\cancel{x}}{u\cancel{x}} + \frac{u\cancel{x}}{\cancel{x}}$$

$$u + x\frac{du}{dx} = \frac{1}{u} + u$$

$$\cancel{u} + x\frac{du}{dx} \cancel{-u} = \frac{1}{u}$$

$$x\frac{du}{dx} = \frac{1}{u}$$

$$u\,du = \frac{dx}{x}$$

$$\int u\,du = \int \frac{dx}{x}$$

$$u = \frac{y}{x}$$

$$y = ux$$

$$\frac{dy}{dx} = \frac{d(ux)}{dx}$$

$$= \frac{du}{dx}x + u\frac{\cancel{dx}}{\cancel{dx}}$$

$$= x\frac{du}{dx} + u$$

$$= u + x\frac{du}{dx}$$

$$\frac{u^2}{2} = \ln|x| + C$$

$$u^2 = 2\ln|x| + 2C$$

$$\left(\frac{y}{x}\right)^2 = 2\ln|x| + 2C$$

$$\boxed{y^2 = x^2(2\ln|x| + 2C)}$$

4. $(x^2+y^2)dx - 2xydy = 0$

$M(x,y) = x^2+y^2 \quad ; N(x,y) = -2xy$

$M(tx,ty) = (tx)^2 + (ty)^2 \qquad N(tx,ty) = -2(tx)(ty)$

$= t^2x^2 + t^2y^2 \qquad = -2t^2xy$

$= t^2(x^2+y^2) \qquad = t^2(-2xy)$

$= t^2 M(x,y) \qquad = t^2 N(x,y)$

$\frac{y}{x} = u \; ; \; y = ux \; ; \; dy = udx + xdu$

$(x^2+y^2)dx - 2xydy = 0$

$(x^2+(ux)^2)dx - 2x(ux)(udx+xdu) = 0$

$(x^2+u^2x^2)dx - 2ux^2(udx+xdu) = 0$

$x^2(1+u^2)dx - 2ux^2(udx+xdu) = 0$

$\frac{\cancel{x^2}(1+u^2)dx - 2u\cancel{x^2}(udx+xdu)}{\cancel{x^2}} = \frac{0}{x^2}$

$(1+u^2)dx - 2u(udx+xdu) = 0$

$(1+u^2)dx - 2u^2dx - 2uxdu = 0$

$(1+u^2-2u^2)dx - 2uxdu = 0$

$(1-u^2)dx - 2uxdu = 0$

$(1-u^2)dx = 2uxdu$

$w = 1-u^2$
$dw = -2udu$
$2udu = -dw$

$\frac{dx}{x} = \frac{2u}{1-u^2}du$

$\int \frac{dx}{x} = \int \frac{2u}{1-u^2}du$

$\ln|x| = -\int \frac{dw}{w}$

$\ln|x| = -\ln|w| + C$

$\ln|x| = -\ln|1-u^2| + C$

$\ln|x| + \ln|1-u^2| + C$

$\ln|x(1-u^2)| = C$

$\ln\left|x\left(1-\frac{y^2}{x^2}\right)\right| = C$

$\ln\left|\cancel{x}\left(\frac{x^2-y^2}{\cancel{x^2}}\right)\right| = C$

$\ln\left|\frac{x^2-y^2}{x}\right| = C$

$\frac{x^2-y^2}{x} = e^C$

$x^2-y^2 = e^C \cdot x$

$\boxed{x^2-y^2 = Cx}$

5 $y' = \frac{xy}{x^2+y^2}$

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$$

$$(x^2+y^2)dy = xy\,dx$$

$$xy\,dx - (x^2+y^2)dy = 0$$

$M(x,y) = xy$; $N(x,y) = -(x^2+y^2)$

$M(tx,ty) = (tx)(ty)$; $N(tx,ty) = -((tx)^2+(ty)^2)$

$= t^2xy$ $\qquad = -(t^2x^2+t^2y^2)$

$= t^2M(x,y)$ $\qquad = -t^2(x^2+y^2)$

$\qquad = t^2N(x,y)$

$\frac{y}{x} = u \quad y = ux \quad dy = u\,dx + x\,du$

$$xy\,dx - (x^2+y^2)dy = 0$$

$$x(ux)dx - [x^2+(ux)^2](u\,dx+x\,du) = 0$$

$$ux^2dx - [x^2+u^2x^2](u\,dx+x\,du) = 0$$

$$ux^2dx - x^2(1+u^2)(u\,dx+x\,du) = 0$$

$$\frac{u\cancel{x^2}dx - \cancel{x^2}(1+u^2)(u\,dx+x\,du)}{\cancel{x^2}} = \frac{0}{x^2}$$

$$u\,dx - (1+u^2)(u\,dx+x\,du) = 0$$

$$u\,dx - u(1+u^2)dx - x(1+u^2)du = 0$$

$$[u - u(1+u^2)]dx - x(1+u^2)du = 0$$

$$[u-u-u^3]dx - x(1+u^2)du = 0$$

$$-u^3dx - x(1+u^2)du = 0$$

$$u^3dx + x(1+u^2)du = 0$$

$$u^3dx = -x(1+u^2)du$$

$$\frac{dx}{x} = -\frac{1+u^2}{u^3}du$$

$$\int\frac{dx}{x} = -\int\frac{1+u^2}{u^3}du$$

$$\ln|x| = -\int(u^{-3}+u^{-1})du$$

$$\ln|x| = -\left[\frac{u^{-2}}{-2} + \ln|u|\right] + c$$

$$\ln|x| = \frac{1}{2u^2} - \ln|u| + c$$

$$\ln|x| = \frac{1}{2\left(\frac{y}{x}\right)^2} - \ln\left|\frac{y}{x}\right| + c$$

$$\ln|x| = \frac{x^2}{2y^2} - \ln|y| + \ln|x| + c$$

$$0 = \frac{x^2}{2y^2} - \ln|y| + c$$

$$\boxed{\ln|y| = \frac{x^2}{2y^2} + c}$$

6. $(x+2y)\,dx - x\,dy = 0$

$M(x,y) = x+2y \quad ; \quad N(x,y) = -x$

$M(tx,ty) = tx + 2(ty) \quad ; \quad N(tx,ty) = -(tx)$

$= t(x+2y) \quad ; \quad = t(-x)$

$= t^1 M(x,y) \quad ; \quad = t^1 N(x,y)$

$\frac{y}{x} = u \quad y = ux \quad dy = u\,dx + x\,du$

$(x+2y)\,dx - x\,dy = 0$

$(x+2ux)\,dx - x(u\,dx + x\,du) = 0$

$x(1+2u)\,dx - ux\,dx - x^2\,du = 0$

$x(1+2u)\,dx - ux\,dx - x^2\,du = 0$

$[x(1+2u) - ux]\,dx - x^2\,du = 0$

$x(1+2u-u)\,dx - x^2\,du = 0$

$x(1+u)\,dx - x^2\,du = 0$

$\frac{\cancel{x(1+u)}\,dx}{x^{\cancel{2}}\cancel{(1+u)}} - \frac{\cancel{x^2}\,du}{\cancel{x^2}(1+u)} = \frac{0}{x^2(1+u)}$

$\frac{dx}{x} - \frac{du}{1+u} = 0$

$\frac{dx}{x} = \frac{du}{1+u}$

$\int \frac{dx}{x} = \int \frac{du}{1+u}$

$\ln|x| = \ln|1+u| + C_1$

$\ln|x| - \ln|1+u| = C_1$

$\ln\left|\frac{x}{1+u}\right| = C$

$\frac{x}{1+u} = e^C$

$\frac{x}{1+\frac{y}{x}} = C$

$\frac{x}{1+\frac{y}{x}} = C$

$\frac{x}{\frac{x+y}{x}} = C$

$\frac{x^2}{x+y} = C$

$x^2 = C(x+y)$

$x + y = \frac{x^2}{C}$

$\boxed{y = \frac{x^2}{C} - x}$

7 $y' = \frac{y}{x} + e^{y/x}$

$f(x,y) = \frac{y}{x} + e^{y/x}$

$f(tx, ty) = \frac{ty}{tx} + e^{\frac{ty}{tx}}$

$= \frac{y}{x} + e^{\frac{y}{x}}$

$f(tx,ty) = f(x,y)$

$\frac{y}{x} = u \quad y = ux \quad y' = u + x\frac{du}{dx}$

$u + x\frac{du}{dx} = \frac{ux}{x} + e^{ux/x}$

$u + x\frac{du}{dx} = \frac{ux}{x} + e^{ux/x}$

$u + x\frac{du}{dx} = u + e^{u}$

$x\frac{du}{dx} = e^{u}$

$\frac{du}{e^{u}} = \frac{dx}{x}$

$e^{-u}\,du = \frac{dx}{x}$

$\int e^{-u}\,du = \int \frac{dx}{x}$

$-e^{-u} = \ln|x|$

$-e^{-u} = \ln|x + C$

$-e^{-y/x} = \ln|x| + C$

$e^{-y/x} = -\ln|x| - C$

8. $xy' = y + \sqrt{x^2+y^2}$

$y' = f(x,y)$:

$$y' = \frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x}$$

$$f(x,y) = \frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x}$$

$$f(tx,ty) = \frac{ty}{tx} + \frac{\sqrt{(tx)^2+(ty)^2}}{tx}$$

$$= \frac{y}{x} + \frac{\sqrt{t^2x^2+t^2y^2}}{tx}$$

$$= \frac{y}{x} + \frac{t\sqrt{x^2+y^2}}{tx}$$

$$\frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x}$$

$$= f(x,y)$$

$y = ux \quad \frac{y}{x} = u \quad y' = u + x\frac{du}{dx}$

$$xy' = y + \sqrt{x^2+y^2}$$

$$x\left(u + x\frac{du}{dx}\right) = ux + \sqrt{x^2+(ux)^2}$$

$$xu + x^2\frac{du}{dx} = ux + \sqrt{x^2+u^2x^2}$$

$$xu + x^2\frac{du}{dx} = ux + \sqrt{x^2(1+u^2)}$$

$$x^2\frac{du}{dx} = \sqrt{x^2(1+u^2)}$$

$$x^2\frac{du}{dx} = x\sqrt{1+u^2}$$

$$x\frac{du}{dx} = \sqrt{1+u^2}$$

$$\frac{du}{\sqrt{1+u^2}} = \frac{dx}{x}$$

$$\int\frac{du}{\sqrt{1+u^2}} = \int\frac{dx}{x}$$

$$\ln|u+\sqrt{1+u^2}| = \ln|x|+C$$

$$\ln\left(\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right) = \ln|x|+C$$

$$\ln\left(\frac{y+\sqrt{x^2+y^2}}{x}\right) = \ln|x|+C$$

$$\ln(y+\sqrt{x^2+y^2}) - \ln x = \ln|x|+C$$

$$\ln(y+\sqrt{x^2+y^2}) = 2\ln|x|+C$$

$$\ln(y+\sqrt{x^2+y^2}) = \ln|x^2|+C$$

$$\ln(y+\sqrt{x^2+y^2}) = \ln(kx^2)$$

$$\boxed{y+\sqrt{x^2+y^2} = kx^2}$$

9. $(x^2 - y^2)dx + 2xy\,dy = 0$

$M(x,y) = x^2 - y$; $N(x,y) = 2xy$

$M(tx,ty) = (tx)^2 - (ty)^2$; $N(tx,ty) = 2(tx)(ty)$

$= t^2x^2 - t^2y^2$; $= 2t^2xy$

$= t^2(x^2 - y^2)$; $= t^2 N(x,y)$

$= t^2 M(x,y)$;

$\frac{y}{x} = u \quad y = ux \quad dy = u\,dx + x\,du$

$(x^2 - (ux)^2)dx + 2x(ux)(u\,dx + x\,du) = 0$

$(x^2 - u^2x^2)dx + 2ux^2(u\,dx + x\,du) = 0$

$x^2(1-u^2)dx + 2ux^2(u\,dx + x\,du) = 0$

$x^2(1-u^2)dx + 2u^2x^2dx + 2ux^3du = 0$

$x^2[1 - u^2 + 2u^2]dx + 2ux^3du = 0$

$x^2(1+u^2)dx + 2ux^3du = 0$

$dx + \frac{2ux^3}{x^2(1+u^2)}du = 0$

$dx \quad \frac{2ux}{1+u^2}du = 0$

$\frac{dx}{x} + \frac{2u}{1+u^2}du = 0$

$\int \frac{dx}{x} + \int \frac{2u}{1+u^2}du = C$

$\ln|x| + \ln(1+u^2) = C$

$\ln(|x|(1+u^2)) = C$

$\ln\left(|x| \cdot \frac{x^2+y^2}{x^2}\right) = C$

$\frac{x^2+y^2}{x} = e^C$

$x^2 + y^2 = k|x| = \boxed{x^2 + y^2 = kx}$

10. $$y' = \frac{x^2 + xy + y^2}{x^2}$$

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

$$dy = \frac{x^2 + xy + y^2}{x^2}\, dx$$

$$(x^2 + xy + y^2)\, dx - x^2 dy = 0$$

$$M(x,y) = x^2 + xy + y^2 \quad ; \quad N(x,y) = -x^2$$

$$M(tx, ty) = (tx)^2 + (tx)(ty) + (ty)^2 \quad ; \quad N(tx, ty) = -(tx)^2$$

$$= t^2x^2 + t^2xy + t^2y^2 \quad ; \quad = -t^2x^2$$

$$t^2(x^2 + xy + y^2) \quad ; \quad = t^2 N(x,y)$$

$$t^2 M(x,y) \quad ;$$

$$y = ux \quad \frac{y}{x} = u \quad dy = u\,dx + x\,du$$

$$(x^2 + x(ux) + (ux)^2)\, dx - x^2(u\,dx + x\,du) = 0$$

$$(x^2 + ux^2 + u^2x^2)\, dx - ux^2 dx - x^3 du = 0$$

$$x^2(1 + u + u^2)\, dx - ux^2 dx - x^3 du = 0$$

$$x^2(1 + u + u^2 - u)\, dx - x^3 du = 0$$

$$x^2(1 + u^2)\, dx - x^3 du = 0$$

$$(1 + u^2)\, dx - x\, du = 0$$

$$(1 + u^2)\, dx = x\, du$$

$$\frac{dx}{x} = \frac{du}{1 + u^2}$$

$$\int \frac{dx}{x} = \int \frac{du}{1 + u^2}$$

$$\ln|x| = \arctan u + C$$

$$\arctan\left(\frac{y}{x}\right) = \ln|x| - C$$

$$\frac{y}{x} = \tan(\ln|x| - C) = \boxed{y = x\tan(\ln|x| + k)}$$