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15 Ejercicios.

5 Ecuaciones diferenciales homogéneas.

1. $\frac{dy}{dx} = \frac{y}{x}$

$f(x,y) = \frac{y}{x}$

$f(tx, ty) = \frac{ty}{tx}$

$= \frac{y}{x}$

$= t^0 f(x,y)$

$\frac{y}{x} = u \quad y = ux \quad \frac{dy}{dx} = u + x\frac{du}{dx}$

$\frac{dy}{dx} = \frac{y}{x}$

$u + x\frac{du}{dx} = \frac{ux}{x}$

$u + x\frac{du}{dx} = u$

$x\frac{du}{dx} = u - u$

$x\frac{du}{dx} = 0$

$\frac{du}{dx} = \frac{0}{x}$

$\frac{du}{dx} = 0$

$\frac{u}{x} = 0$

$u = \frac{0}{x}$

$u = C$

$y = ux$

$y = Cx$

2. $\frac{dy}{dx} = \frac{x+y}{x}$

$dy = \frac{x+y}{x} dx$

$(x+y)dx - x\,dy = 0$

$M(x,y) = x+y \quad ; \quad N(x,y) = -x$

$M(tx,ty) = tx + ty \qquad N(tx,ty) = -tx$

$= t(x+y) \qquad t(-x)$

$= t^1 M(x,y) \qquad t^1 N(x,y)$

$y = ux \quad ; \quad \frac{dy}{dx} = u + x\frac{du}{dx}$

$\frac{dy}{dx} = \frac{x+y}{x}$

$u + x\frac{du}{dx} = \frac{x+ux}{x}$

$u + x\frac{du}{dx} = \frac{\cancel{x}(1+u)}{\cancel{x}}$

$u + x\frac{du}{dx} = 1+u$

$x\frac{du}{dx} = 1 + u - u$

$x\frac{du}{dx} = 1$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}dx$

$\int du = \int \frac{1}{x} dx$

$u = \ln|x| + c$

$\frac{y}{x} = \ln|x| + c \quad = \quad y = x(\ln|x| + c)$

3. $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$

$dy = \frac{x^2 + y^2}{xy} dx$

$(x^2 + y^2) dx - xy\, dx = 0$

$M(x,y) = x^2 + y^2 \quad ; \quad N(x,y) = -xy$

$M(tx, ty) = (tx)^2 + (ty)^2 \quad ; \quad N(tx, ty) = -(tx)(ty)$

$= t^2x^2 + t^2y^2 \quad ; \quad = -t^2xy$

$= t^2(x^2 + y^2) \quad ; \quad = t^2(-xy)$

$= t^2 M(x,y) \quad ; \quad = t^2 N(x,y)$

$y = ux \qquad \frac{dy}{dx} = u + x\frac{du}{dx}$

$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$

$u + x\frac{du}{dx} = \frac{x^2 + (ux)^2}{x(ux)}$

$u + x\frac{du}{dx} = \frac{x^2 + u^2x^2}{x^2 u}$

$u + x\frac{du}{dx} = \frac{\cancel{x^2}(1+u)}{u\cancel{x^2}}$

$u + x\frac{du}{dx} = \frac{1+u}{u}$

$x\frac{du}{dx} = \frac{1+u}{u} - u$

$x\frac{du}{dx} = \frac{1 + u - u^2}{u}$

$x\frac{du}{dx} = \frac{1}{u}$

$u\,du = \frac{dx}{x}$

$\int u\,du = \int \frac{dx}{x}$

$\frac{u^2}{2} = \ln|x| + c$

$u^2 = 2(\ln|x| + c)$

$u^2 = 2\ln|x| + 2c$

$\left(\frac{y}{x}\right)^2 = 2\ln|x| + 2c$

$\frac{y^2}{x^2} = 2\ln|x| + 2c$

$y^2 = x^2(2\ln|x| + 2c)$

$y^2 = x^2(2\ln|x| + C)$

4. $(x-y)\,dx + x\,dy = 0$

$M(x,y) = x - y \quad ; \quad N(x,y) = x$

$M(tx,ty) = tx - ty \quad ; \quad N(tx,ty) = tx$

$= t(x-y) \quad ; \quad = t(x)$

$= t\,M(x,y) \quad\quad = t\,N(x,y)$

$y = ux \quad\quad dy = d(ux) = u\,dx + x\,du$

$(x-y)\,dx + x\,dy = 0$

$(x-ux)\,dx + x\,(u\,dx + x\,du) = 0$

$x(1-u)\,dx + x\,(u\,dx + x\,du) = 0$

$x(1-u)\,dx + xu\,dx + x^2\,du = 0$

$x\,dx - \cancel{ux\,dx} + \cancel{ux\,dx} + x^2\,du = 0$

$x\,dx + x^2\,du = 0$

$x\,(dx + x\,du) = 0$

$dx + x\,du = \frac{0}{x}$

$dx + x\,du = 0$

$dx = -x\,du$

$\frac{dx}{x} = -du$

$\int \frac{dx}{x} = -\int du$

$\ln|x| = -u + C$

$\ln|x| = -\frac{y}{x} + C$

$\frac{y}{x} = C - \ln|x|$

$y = x\,(C - \ln|x|)$

5. $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

$2xdy = (y^2 - x^2)dx$

$(x^2 - y^2)dx + 2xy\,dy = 0$

$M(x,y) = x^2 - y^2$; $N(x,y) = 2xy$

$M(tx,ty) = (tx)^2 - (ty)^2$; $N(tx,ty) = 2(tx)(ty)$

$= t^2x^2 - t^2y^2$ $\qquad = 2t^2 xy$

$= t^2(x - y)$ $\qquad = t^2(2xy)$

$= t^2 M(x,y)$ $\qquad = t^2 N(x,y)$

$y = ux \qquad \frac{y}{x} = u \qquad dy = udx + xdu$

$(x^2 - (ux)^2)dx + 2x(ux)(udx + xdu) = 0$

$(x^2 - u^2x^2)dx + 2x^2u(udx + xdu) = 0$

$x^2dx - u^2x^2dx + 2x^2u^2dx + 2x^3udu$

$x^2dx + u^2x^2dx + 2x^3udu = 0$

$x^2(dx + u^2dx + 2xudu) = 0$

$dx + u^2dx + 2xudu = \frac{0}{x^2}$

$dx + u^2dx + 2xudu = 0$

$dx + u^2dx = -2xudu$

$(1 + u^2)dx = -2xudu$

$\frac{dx}{x} = -\frac{2u}{1+u^2}du$

$w = 1 + u^2$

$dw = 2u\,du$

$\int \frac{dx}{x} = -\int \frac{2u}{1+u^2}du$

$\ln|x| = -\int \frac{dw}{w}$

$\ln|x| = -\ln|w| + C$

$\ln|x| = -\ln|1 + u^2| + C$

$\ln|x| = -\ln|1 + (\frac{y}{x})^2| + C$

$\ln|x| = -\ln|\frac{x^2+y^2}{x^2}| + C$

$\ln|x| = -(\ln(x^2+y^2) - \ln|x^2|) + C$

$\ln|x| = -\ln(x^2+y^2) + 2\ln|x| + C$

$\ln|x| - 2\ln|x| = -\ln|x^2+y^2| + C$

$-\ln|x| = -\ln(x^2+y^2) + C$

$|x| = e^{\ln(x^2+y^2) - c}$

$|x| = \frac{x^2+y^2}{e^c} = |x| = \frac{x^2+y^2}{k}$

$k|x| = x^2 + y^2$

B Ecuaciones diferenciales exactas

1. $(2x+y)dx+(x+2y)dy=0$

$M(x,y)=2x+y$

$N(x,y)=x+2y$

$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(2y+y)$
$=0+1$
$=1$

$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(x+2y)$
$=1+0$
$=1$

$F(x,y)=\int(2x+y)dx$
$=x^2+xy+h(y)$

$\frac{\partial F}{\partial y}=\frac{\partial}{\partial y}(x^2+xy+h(y))$
$=x+h'(x)$

$x+h'(y)=x+2y$
$h'(y)=2y$
$h(y)=\int 2y\,dy$
$=y^2+C$

$F(x,y)=x^2+xy+y^2$
$F(x,y)=C$
$x^2+xy+y^2=C$

2. $(3x^2+y)dx+(x+4y^3)dy=0$

$M=(x,y)=3x^2+y$

$N=(x,y)=x+4y^3$

$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(3x^2+y)$
$=0+1$
$=1$

$\frac{\partial N}{\partial y}=\frac{\partial}{\partial}(x+4y^3)$
$=1+0$
$=1$

$\frac{\partial F}{\partial x}=M=3x^2+y$

$\frac{\partial F}{\partial y}=N=x+4y^3$

$F(x,y)=\int(3x^2+y)dx$
$=\int 3x^2dx+\int y\,dx$
$=x^3+xy+h(y)$

$\frac{\partial F}{\partial y}=\frac{\partial}{\partial y}(x^3+xy+h(y))$
$=0+x+h'(y)$
$=x+h'(y)$

$x+h'(y)=x+4y^3$
$h'(y)=4y^3$
$h(y)=\int 4y^3$
$=\frac{4y^4}{4}$
$=y^4+C$

$F(x,y)=x^3+xy+y^4$
$F(x,y)=C$
$x^3+xy+y^4=C$

3. $e^x \cos y\,dx - e^x \operatorname{sen} y\,dy = 0$

$M(x,y) = e^x \cos y$

$N(x,y) = -e^x \operatorname{sen} y$

$\frac{\partial F}{\partial x} = M = e^x \cos y$

$\frac{\partial F}{\partial y} = N = -e^x \operatorname{sen} y$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(e^x \cos y)$

$= e^x(-\operatorname{sen} y)$

$= -e^x \operatorname{sen} y$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(-e^x \operatorname{sen} y)$

$= -e^x \operatorname{sen} y$

$F(x,y) = \int e^x \cos y\,dx$

$= \cos y \int e^x\,dx$

$= \cos y \cdot e^x + h(y)$

$= e^x \cos y + h(y)$

$\frac{\partial F}{\partial y} = \frac{\partial}{\partial y}(e^x \cos y + h(y))$

$= e^x(-\operatorname{sen} y) + h'(y)$

$= -e^x \operatorname{sen} y + h'(y)$

$-e^x \operatorname{sen} y + h'(y) = -e^x \operatorname{sen} y$

$h'(y) = 0$

$h(y) = \int 0\,dy$

$h(y) = C$

$F(x,y) = e^x \cos(y)$

$e^x \cos y = C$

4. $\left(1 + \frac{y}{x^2}\right)dx - \frac{1}{x}\,dy = 0$

$M(x,y) = 1 + \frac{y}{x^2}$

$N(x,y) = -\frac{1}{x}$

$\frac{\partial F}{\partial x} = M = 1 + \frac{y}{x^2}$

$\frac{\partial F}{\partial y} = N = -\frac{1}{x}$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(1 + \frac{y}{x^2}\right)$

$= 0 + \frac{1}{x^2}$

$= \frac{1}{x^2}$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{1}{x}\right)$

$= -\frac{d}{dx}(x^{-1})$

$= -(-1)x^{-2}$

$= \frac{1}{x^2}$

$F(x,y) = \int\left(1 + \frac{y}{x^2}\right)dx$

$\int 1\,dx + y\int x^{-2}\,dx$

$x + y\left(-\frac{1}{x}\right) + h(y)$

$x - \frac{y}{x} + h(y)$

$$\frac{\partial F}{\partial y} = \frac{\partial}{\partial y}\left(x - \frac{y}{x} + h(y)\right)$$

$$= 0 - \frac{1}{x} + h(y)$$

$$= -\frac{1}{x} + h'(y)$$

$$-\frac{1}{x} + h'(y) = -\frac{1}{x}$$

$$h'(y) = 0$$

$$h(y) = \int 0 \, dy$$

$$h(y) = C$$

$$F(x,y) = x - \frac{y}{x}$$

$$x - \frac{y}{x} = C$$

$$x^2 - y = Cx$$

$$x^2 - y = Cx$$

5. $2xy\,dx + (x^2 + 3y^2)\,dy = 0$

$M(x,y) = 2xy$

$N(x,y) = x^2 + 3y^2$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy) = 2x(1) = 2x$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^2 + 3y^2) = 2x$$

$$\frac{\partial F}{\partial x} = M = 2xy$$

$$\frac{\partial F}{\partial y} = N = 2x^2 + 3y^2$$

$$F(x,y) = \int 2xy\,dx$$

$$= 2y \int x\,dx$$

$$= 2y\,\frac{x^2}{2} + h(y)$$

$$= x^2 y + h(y)$$

$$x^2 + h'(y) = x^2 + 3y^2$$

$$h'(y) = 3y^2$$

$$h(y) = \int 3y^2\,dy$$

$$= 3 \cdot \frac{y^3}{3}$$

$$= y^3 + C$$

$$F(x,y) = x^2 y + y^3$$

$$x^2 y + y^3 = C$$

5 Ejercicios Ecuaciones diferenciales lineales.

1. $\frac{dy}{dx} + 2y = 4$

$P(x) = 2$
$Q(x) = 4$

$\mu(x) = e^{\int P(x)dx}$

$\int P(x)dx = \int 2dx$

$\int P(x)dx = 2x + C$

$\mu(x) = e^{2x}$

$\mu'(x) = 2e^{2x}$

$e^{2x}\left(\frac{dy}{dx}\right) + e^{2x}(2y) = e^{2x}(4)$

$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 4e^{2x}$

$\frac{d}{dx}(e^{2x}y) = e^{2x}\frac{dy}{dx} + 2e^{2x}y$

$\frac{d}{dx}(e^{2x}y) = 4e^{2x}$

$\int \frac{d}{dx}(e^{2x}y)dx = \int 4e^{2x}dx$

$e^{2x}y = \cancel{4}^{2} \cdot \frac{1}{\cancel{2}_{1}} e^{2x} + C$

$e^{2x}y = 2e^{2x} + C$

$y = \frac{2e^{2x} + C}{e^{2x}}$

$y = 2 + Ce^{-2x}$

2. $\frac{dy}{dx} + \frac{1}{x}y = x$

$P(x) = \frac{1}{x}$

$Q(x) = x$

$\mu(x) = e^{\int P(x)dx}$

$\mu(x) = e^{\int \frac{1}{x}dx}$

$\int P(x)dx = \int \frac{1}{x}dx$

$\int P(x)dx = \ln|x| + C$

$\mu(x) = e^{\ln x}$

$= x$

$x\left(\frac{dy}{dx}\right) + \cancel{x}\left(\frac{1}{\cancel{x}}y\right) = x(x)$

$x\frac{dy}{dx} + y = x^2$

$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$

$\frac{d}{dx}(x,y) = x^2$

$\int \frac{d}{dx}(xy)dx = \int x^2 dx$

$xy = \frac{x^3}{3} + C$

$$y = \frac{x^3}{3x} + \frac{C}{x}$$

$$y = \frac{x^2}{3} + \frac{C}{x}$$

3. $$\frac{dy}{dx} - y = e^x$$

$P(x) = -1$

$Q(x) = e^x$

$$\mu(x) = e^{\int(-1)dx} = e^{-x}$$

$$\int P(x) = \int -1\,dx = -x$$

$$e^{-x} \cdot \frac{dy}{dx} - e^{-x} \cdot y = e^{-x} \cdot e^x$$

$$e^{-x}\frac{dy}{dx} - e^{-x}y = e^0$$

$$e^{-x}\frac{dy}{dx} - e^{-x}y = 1$$

$$\frac{d}{dx}(\mu(x) \cdot y) = \mu(x)\frac{dy}{dx} + \mu'(x)y$$

$\mu(x) = e^{-x}$

$\mu'(x) = e^{-x}$

$$\frac{d}{dx}(e^{-x}y) = e^{-x}\frac{dy}{dx} + (e^{-x})'y$$

$$\frac{d}{dx}(e^{-x}y) = e^{-x}\frac{dy}{dx} - e^{-x}y$$

$$\frac{d}{dx}(e^{-x}y) = 1$$

$$\int \frac{d}{dx}(e^{-x}y)dx = \int 1\,dx$$

$$e^{-x}y = x + C$$

$$y = (x + C)e^x$$

4 $\frac{dy}{dx} + 3x^2y = x^2$

$P(x) = 3x^2$ $\quad$ $\mu(x) = e^{\int P(x)dx}$ $\quad$ $\int P(x)dx = \int 3x^2 dx$

$Q(x) = x^2$ $\quad$ $= e^{\int 3x^2 dx}$ $\quad$ $= 3\frac{x^3}{3}$

$= e^{x^3}$ $\quad$ $= x^3$

$e^{x^3}\left(\frac{dy}{dx}\right) + e(3x^2y) = e^{x^3}(x^2)$ $\quad$ $\mu(x)y = e^{x^3}y$

$$\frac{d}{dx}(e^{x^3}y) = e^{x^3} \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(e^{x^3})$$

$$\frac{d}{dx}(e^{x^3}y) = e^{x^3}\frac{dy}{dx} + 3x^2e^{x^3}y \; , \; \frac{d}{dx}(e^{x^3}) = e^{x^3} \cdot 3x^2$$

$$\frac{d}{dx}(e^{x^3}y) = x^2e^{x^3}$$

$$\int \frac{d}{dx}(e^{x^3}y)dx = \int x^2e^{x^3}dx$$ $\quad$ $u = x^3$

$$e^{x^3}y = \int e^u \cdot \frac{1}{3}du$$ $\quad$ $du = 3x^2dx$

$x\,dx = \frac{1}{3}du$

$$e^{x^3}y = \frac{1}{3}e^u + C$$

$$e^{x^3}y = \frac{1}{3}e^x$$

$$y = \frac{1}{3} + Ce^{-x^3}$$

6. $x\dfrac{dy}{dx} + 2y = 5x$ $\qquad P(x) = \dfrac{2}{x} \qquad Q(x) = 5.$

$$\frac{1}{\cancel{x}}\left(\cancel{x}\frac{dy}{dx} + 2y\right) = 5\cancel{x}\left(\frac{1}{\cancel{x}}\right)$$

$$\mu(x) = e^{\int P(x)dx} = e^{\int \frac{2}{x}dx} = e^{\ln|x^2|} = x^2$$

$$\int P(x)dx = \int \frac{2}{x}dx = 2\ln|x| = \ln(x^2)$$

$$\frac{dy}{dx} + \frac{2y}{x} = 5$$

$$x^2 \cdot \frac{dy}{dx} + x^2 \cdot \frac{2}{\cancel{x}}y = x^2 \cdot 5$$

$$x^2\frac{dy}{dx} + 2xy = 5x^2$$

$$\frac{d}{dx}(x^2y) = 2xy + x^2\frac{dy}{dx}$$

$$\frac{d}{dx}(x^2y) = 5x^2$$

$$\int \frac{d}{dx}(x^2y)dx = \int 5x^2 dx$$

$$x^2y = 5 \cdot \frac{x^3}{3} + C$$

$$x^2y = \frac{5}{3}x^3 + C$$

$$y = \frac{5}{3}x + \frac{C}{x^2}$$