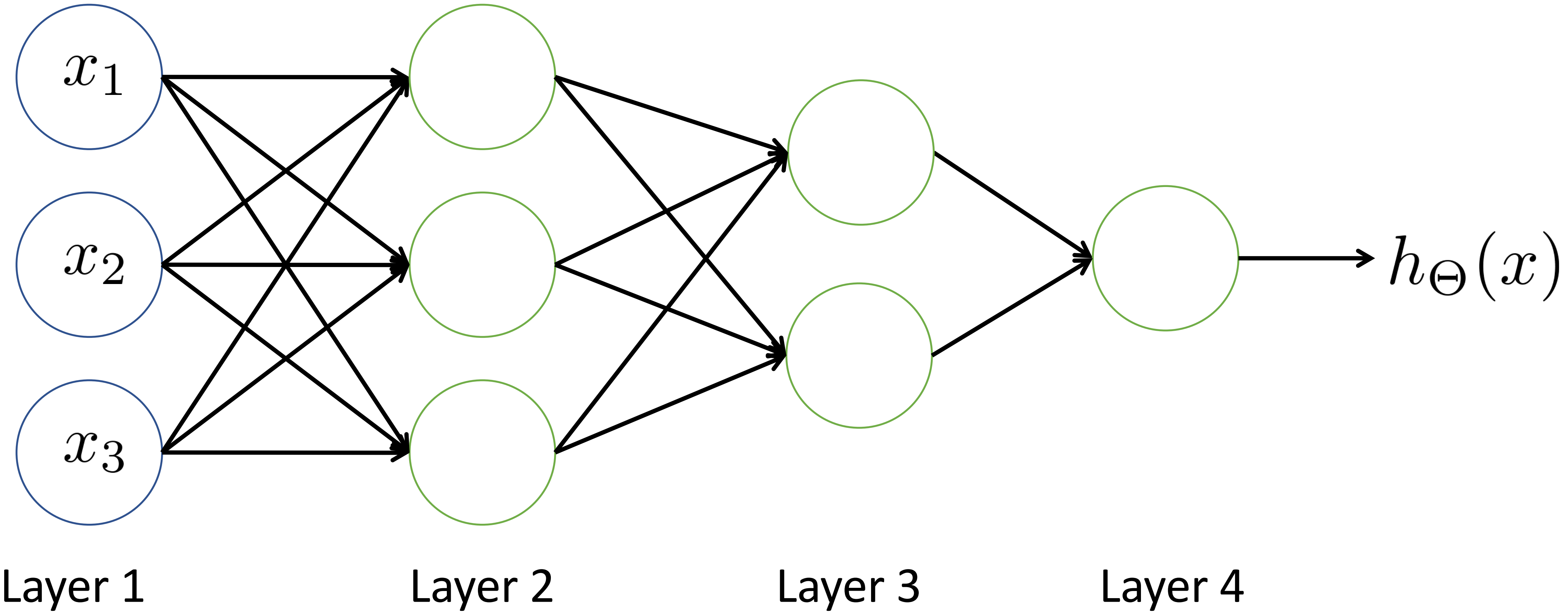
Lecture 5. Model training

MLP review



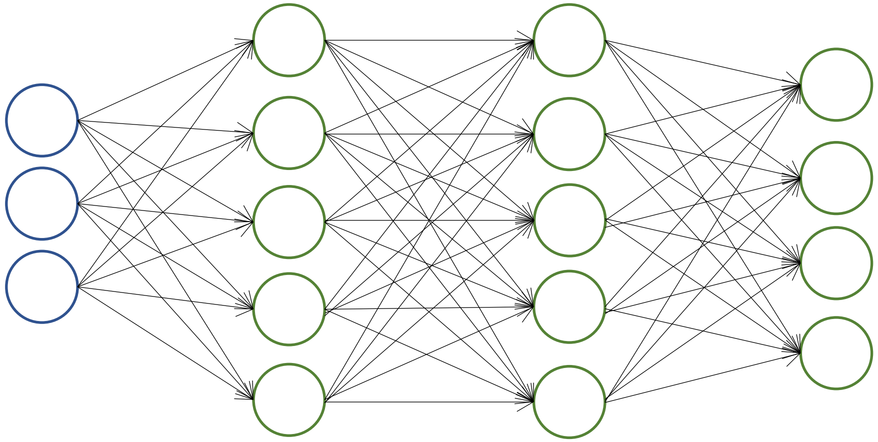
Multi-class classification

Pedestrian

Car

Motorcycle

Truck



$$h_\Theta(x) \in \mathbb{R}^4$$

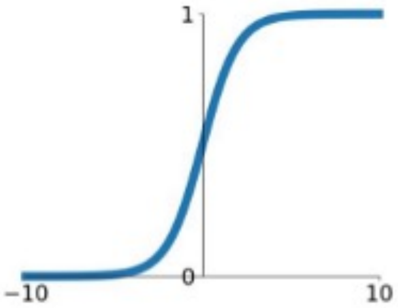
Want $h_\Theta(x) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $h_\Theta(x) \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $h_\Theta(x) \approx \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, etc.

when pedestrian when car when motorcycle

Activation Functions

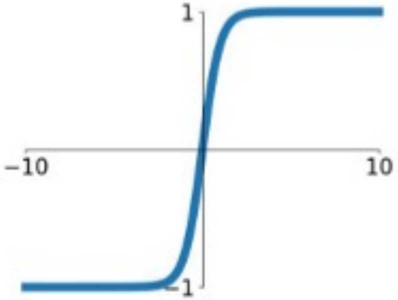
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



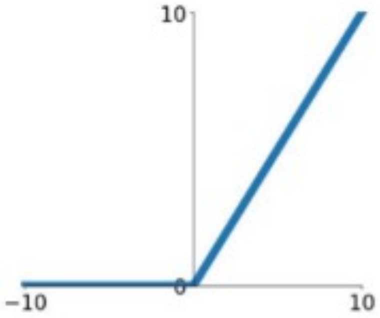
tanh

$\tanh(x)$



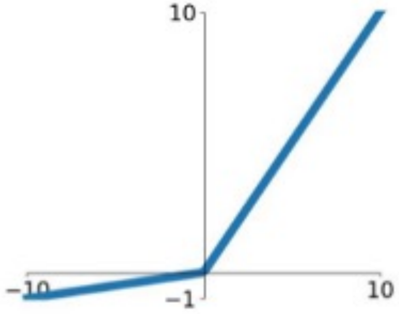
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

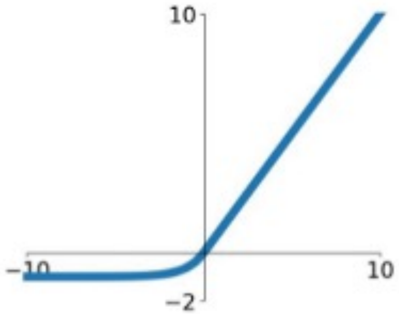


Maxout

$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Classification peformance metrics

$$\text{Accuracy} = \frac{\text{Number of Correct Predictions}}{\text{Total Number of Predictions}}$$

Cancer classification example

Train logistic regression model $h_\theta(x)$. ($y = 1$ if cancer, $y = 0$ otherwise)

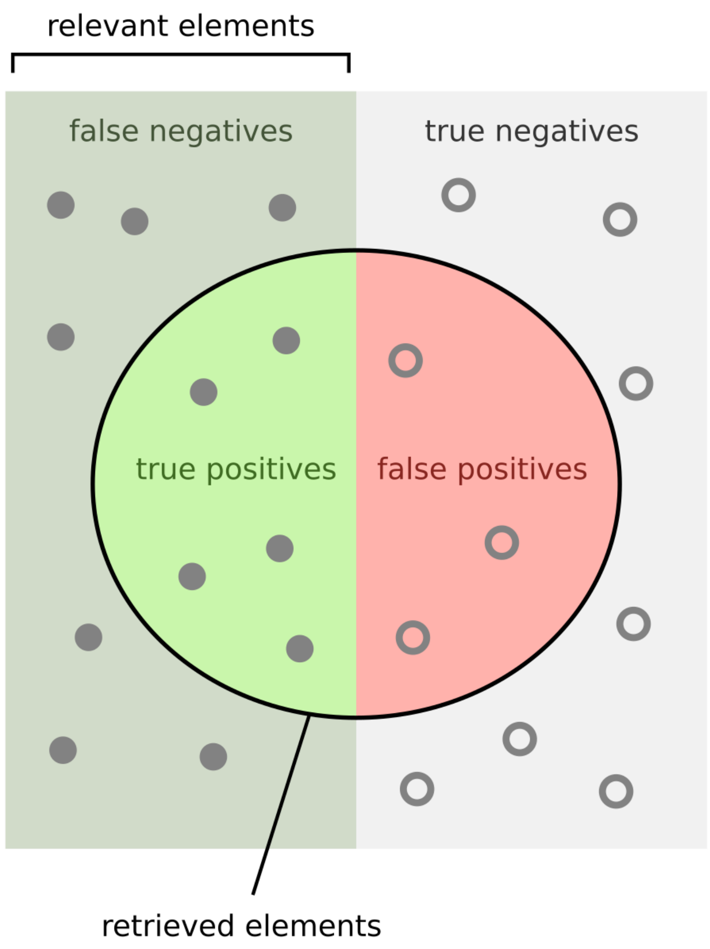
Find that you got 1% error on test set.

(99% correct diagnoses / **accuracy** = 0.99)

But only 0.50% of patients have cancer.

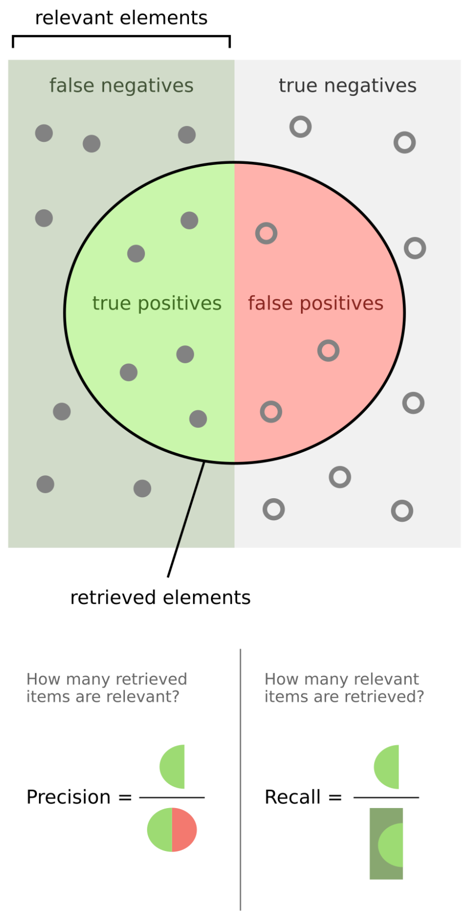
The confusion matrix

		Predicted condition	
	Total population $= P + N$	**Predicted Positive (PP)**	**Predicted Negative (PN)**
Actual condition	**Positive (P)** [a]	**True positive** (TP), hit[b]	**False negative** (FN), miss, underestimation
	Negative (N)[d]	**False positive** (FP), false alarm, overestimation	**True negative** (TN), correct rejection[e]



Precision/Recall

$y = 1$ in presence of rare class that we want to detect



Precision
(Of all patients where we predicted $y = 1$, what fraction actually has cancer?)

Recall
(Of all patients that actually have cancer, what fraction did we correctly detect as having cancer?)

Trading off precision and recall

precision = $\dfrac{\text{true positives}}{\text{no. of predicted positive}}$

recall = $\dfrac{\text{true positives}}{\text{no. of actual positive}}$

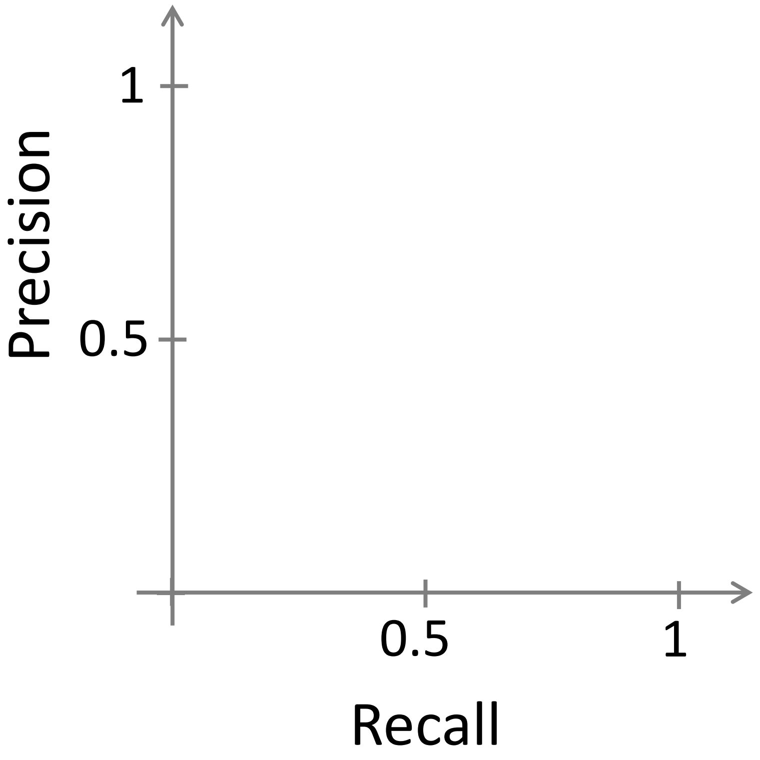
Logistic regression: $0 \leq h_\theta(x) \leq 1$

Predict 1 if $h_\theta(x) \geq 0.5$

Predict 0 if $h_\theta(x) < 0.5$

Suppose we want to predict $y = 1$ (cancer) only if very confident.

Suppose we want to avoid missing too many cases of cancer (avoid false negatives).



More generally: Predict 1 if $h_\theta(x) \geq$ threshold.

F_1 Score (F score)

How to compare precision/recall numbers?

	Precision(P)	Recall (R)
Algorithm 1	0.5	0.4
Algorithm 2	0.7	0.1
Algorithm 3	0.02	1.0

Average: $\frac{P+R}{2}$

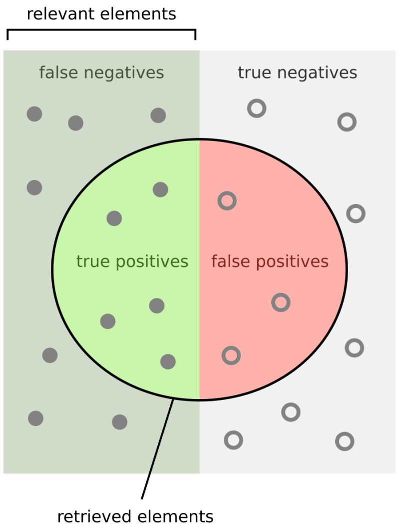
F_1 Score: $2\frac{PR}{P+R}$

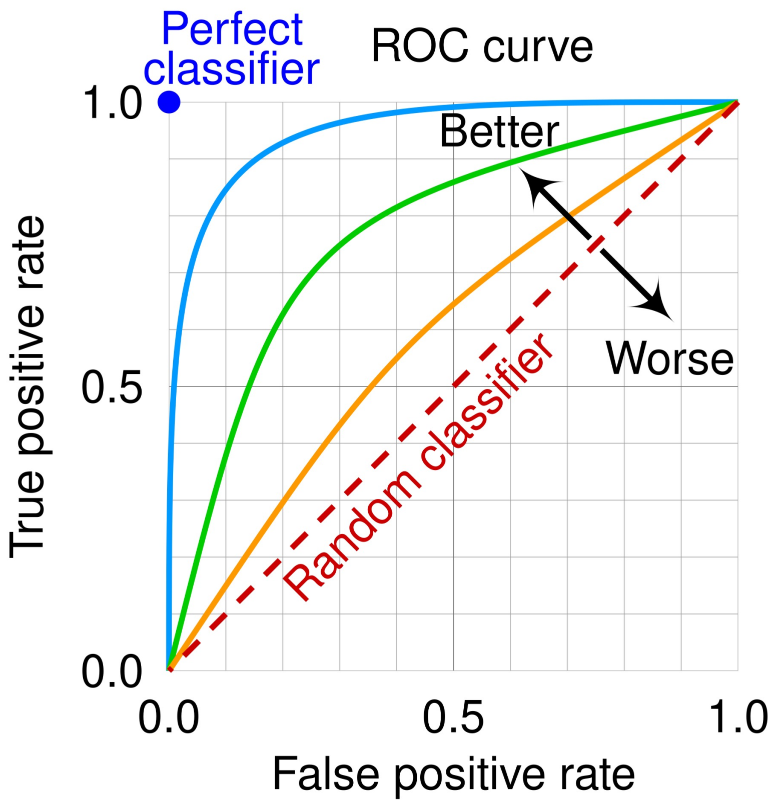
Receiver-operating characteristic curve (ROC)

		Predicted condition	
	Total population $= \mathrm{P} + \mathrm{N}$	Predicted Positive (PP)	Predicted Negative (PN)
Actual condition	Positive (P) [a]	**True positive** (TP), hit[b]	**False negative** (FN), miss, underestimation
	Negative (N)[d]	**False positive** (FP), false alarm, overestimation	**True negative** (TN), correct rejection[e]

True positive rate $= \frac{\mathrm{TP}}{\mathrm{P}}$

False positive rate $= \frac{\mathrm{FP}}{\mathrm{N}}$

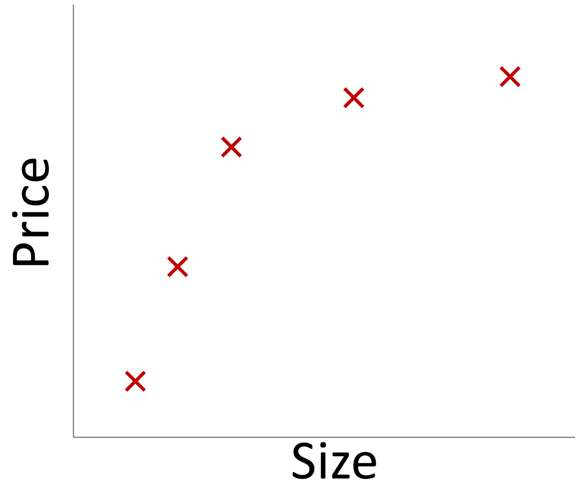




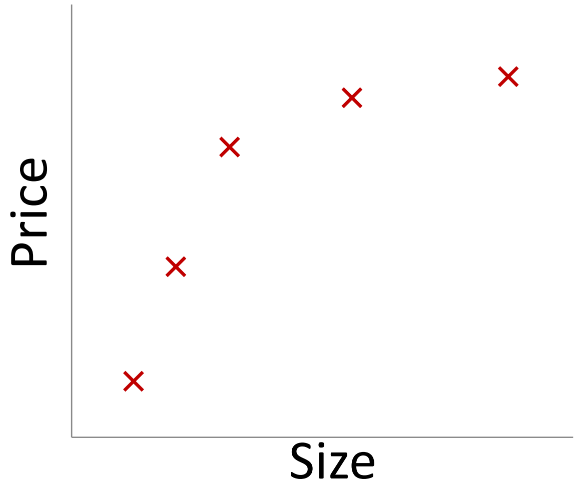
Regularization

The problem of overfitting

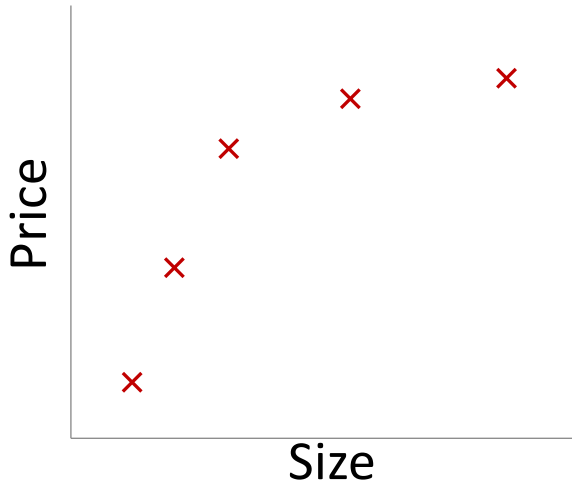
Example: Linear regression (housing prices)



$$\theta_0 + \theta_1 x$$



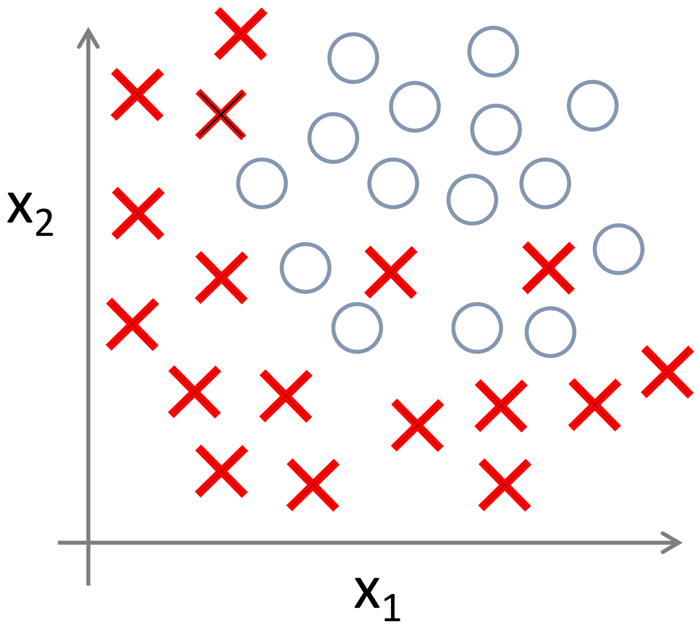
$$\theta_0 + \theta_1 x + \theta_2 x^2$$



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

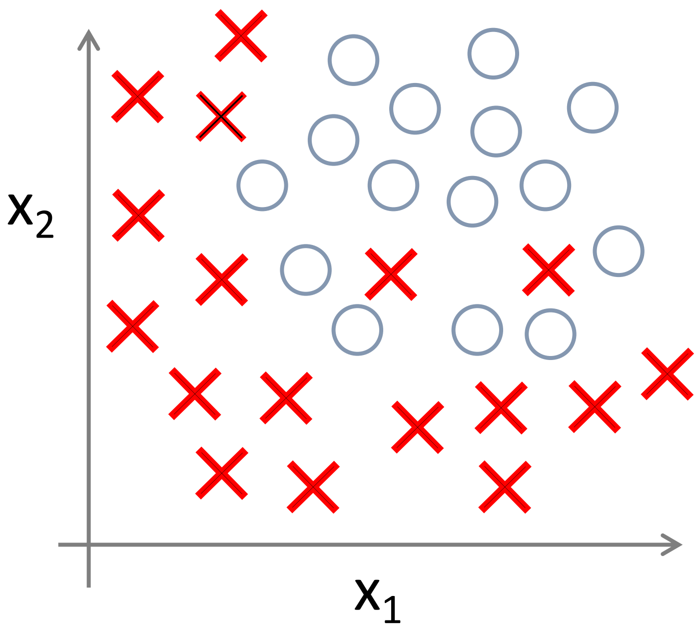
Overfitting: If we have too many features, the learned hypothesis may fit the training set very well ($J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 \approx 0$), but fail to generalize to new examples (predict prices on new examples).

Example: Logistic regression

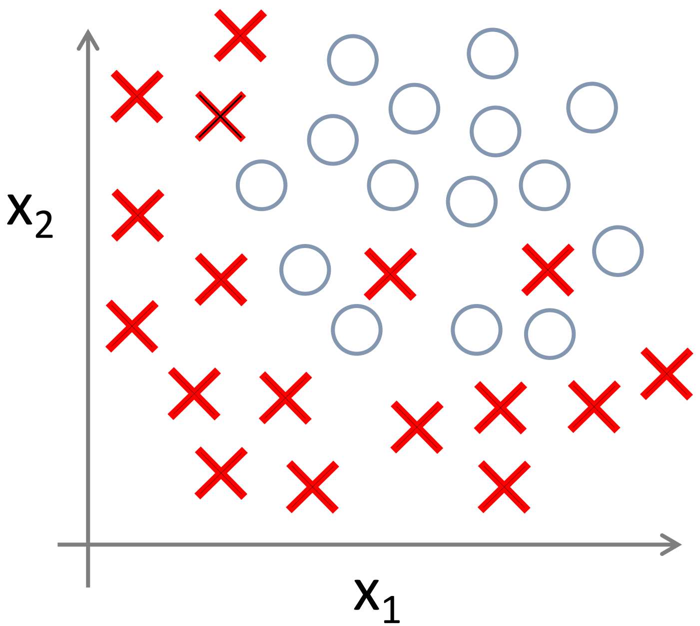


$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

(g = sigmoid function)



$$g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2 + \theta_5 x_1 x_2)$$



$$g(\theta_0 + \theta_1 x_1 + \theta_2 x_1^2 + \theta_3 x_1^2 x_2 + \theta_4 x_1^2 x_2^2 + \theta_5 x_1^2 x_2^3 + \theta_6 x_1^3 x_2 + \dots)$$

Addressing overfitting:

$x_1 =$ size of house

$x_2 =$ no. of bedrooms

$x_3 =$ no. of floors

$x_4 =$ age of house

$x_5 =$ average income in neighborhood

$x_6 =$ kitchen size

$\vdots$

x_{100}

Addressing overfitting:

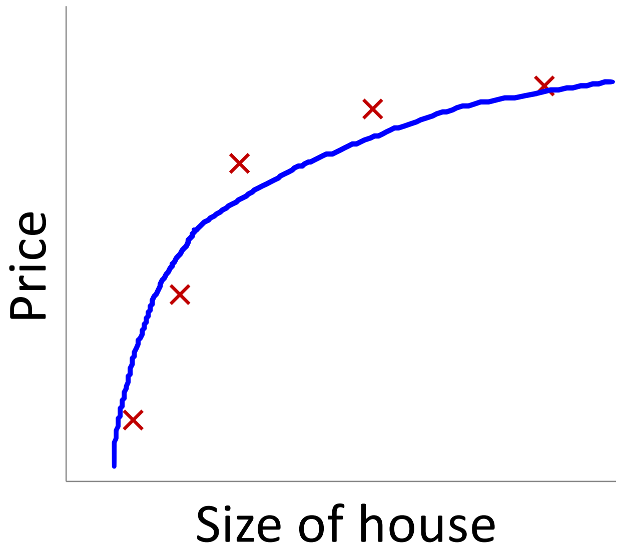
Options:

1. Reduce number of features.
   - Manually select which features to keep.
   - Model selection algorithm (later in course).
2. Regularization.
   - Keep all the features, but reduce magnitude/values of parameters θ_j.
   - Works well when we have a lot of features, each of which contributes a bit to predicting y.

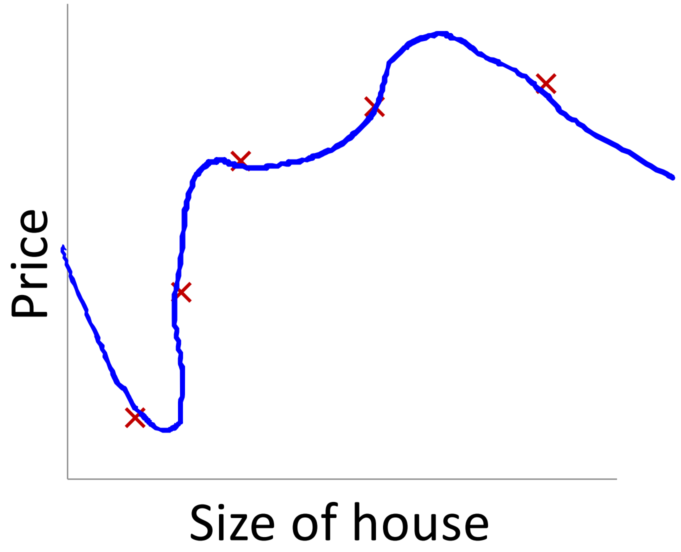
Regularization

Cost function

Intuition



$$\theta_0 + \theta_1 x + \theta_2 x^2$$



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

Suppose we penalize and make θ_3, θ_4 really small.

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

Regularization.

Small values for parameters $\theta_0, \theta_1, \ldots, \theta_n$

- "Simpler" hypothesis
- Less prone to overfitting

Housing:

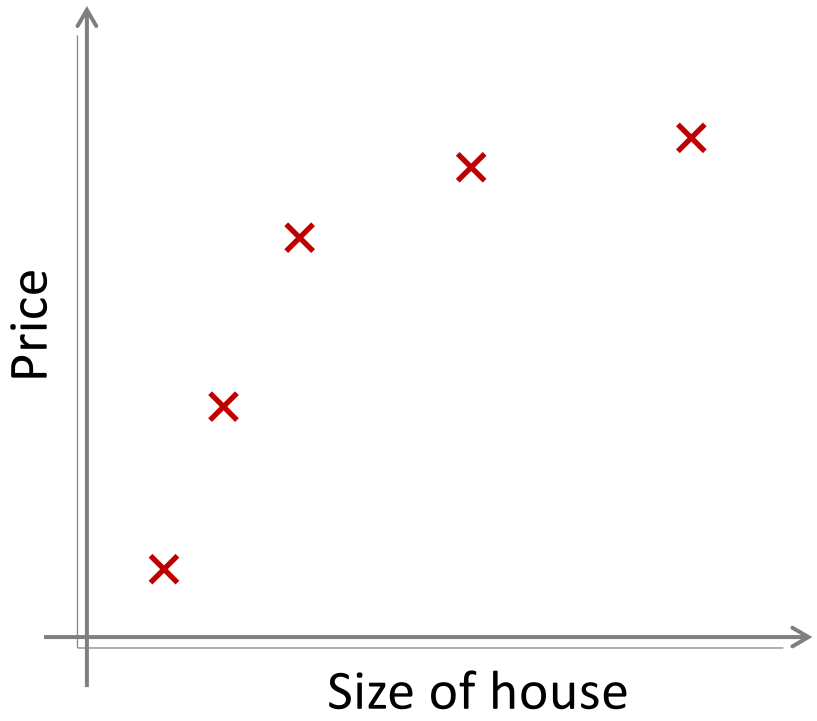
- Features: $x_1, x_2, \ldots, x_{100}$
- Parameters: $\theta_0, \theta_1, \theta_2, \ldots, \theta_{100}$

$$J(\theta) = \frac{1}{2m} \quad \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

Regularization.

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda\sum_{j=1}^{n}\theta_j^2\right]$$

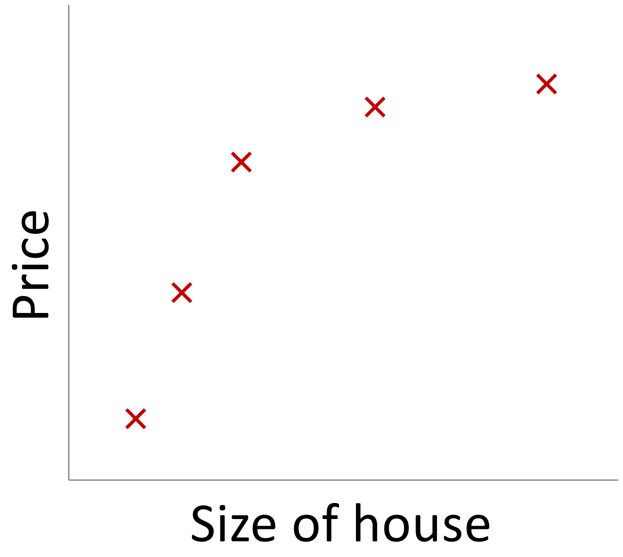
$$\min_\theta J(\theta)$$



In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda\sum_{j=1}^{n}\theta_j^2\right]$$

What if λ is set to an extremely large value (perhaps for too large for our problem, say $\lambda = 10^{10}$)?

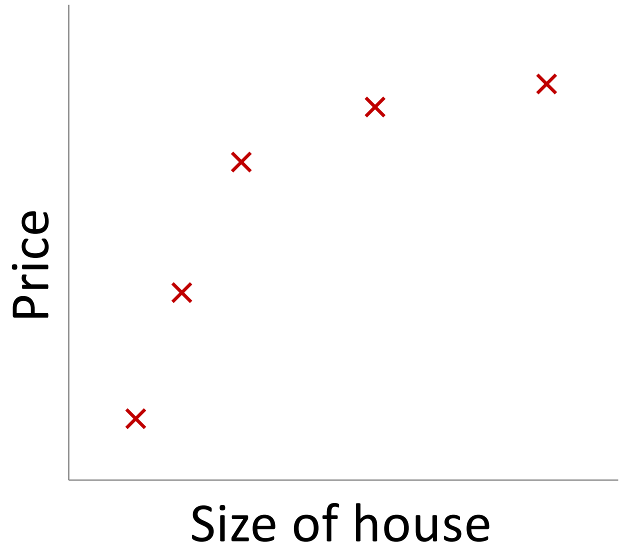


$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^{n}\theta_j^2\right]$$

What if λ is set to a very small value?



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

L2 Regularization (Ridge)

$$\text{Loss} = \text{Original Loss} + \lambda \sum_{i=1}^{n} w_i^2$$

L1 Regularization (Lasso)

$$\text{Loss} = \text{Original Loss} + \lambda \sum_{i=1}^{n} |w_i|$$

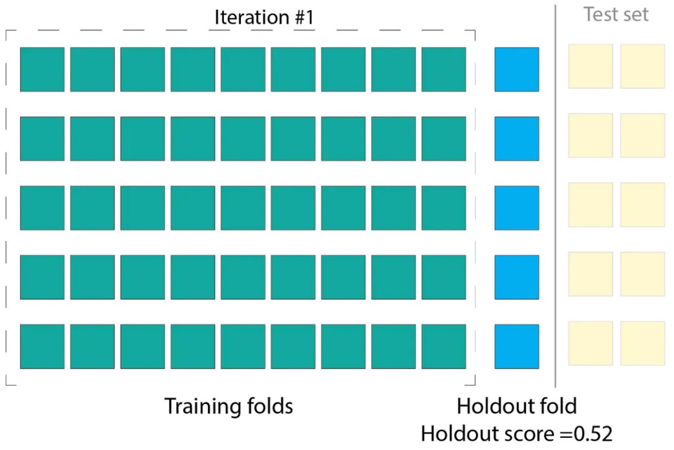
Elastic Net

$$\text{Loss} = \text{Original Loss} + \lambda_1 \sum_{i=1}^{n} |w_i| + \lambda_2 \sum_{i=1}^{n} w_i^2$$

L0 Regularization (L0 Norm)

$$\text{Loss} = \text{Original Loss} + \lambda \sum_{i=1}^{n} I(w_i \neq 0)$$

Pick the best regularization parameter with cross validation



Total Cross Validation Score = 0.497

Penalty Strength	CV Score
0.2	0.49
0.3	0.56
0.4	0.59
0.5	0.56
0.6	0.54
0.7	0.50

Neural Networks: Learning

Backpropagation algorithm

Gradient computation

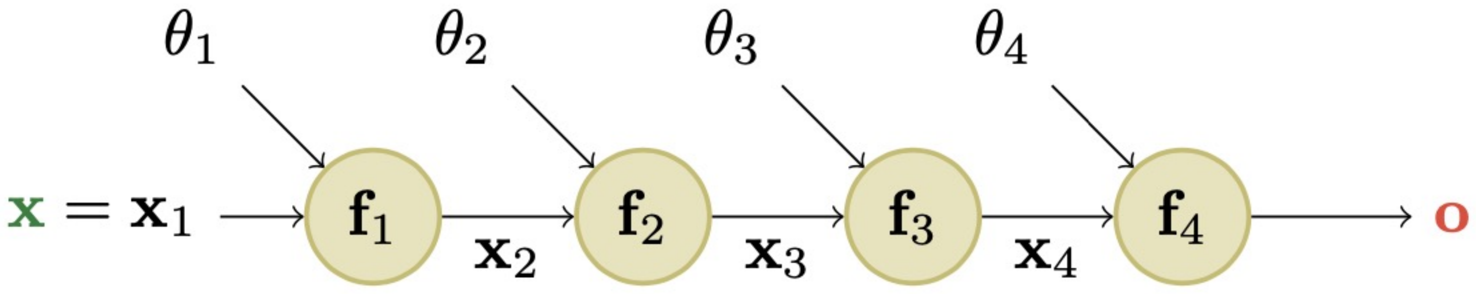
$$J(\Theta) = -\frac{1}{m}\left[\sum_{i=1}^{m}\sum_{k=1}^{K} y_k^{(i)} \log h_\theta(x^{(i)})_k + (1 - y_k^{(i)}) \log(1 - h_\theta(x^{(i)})_k)\right]$$
$$+\frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(\Theta_j^{(l)})^2$$

$$\min_{\Theta} J(\Theta)$$

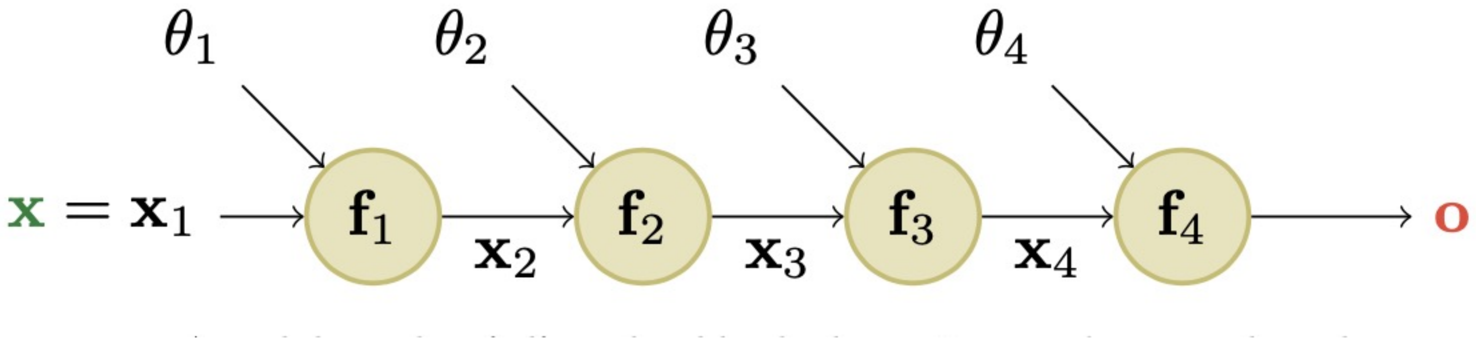
Need code to compute:

- $J(\Theta)$
- $\frac{\partial}{\partial\Theta_{ij}^{(l)}} J(\Theta)$

Forward Propagation



Back Propagation (backprop)



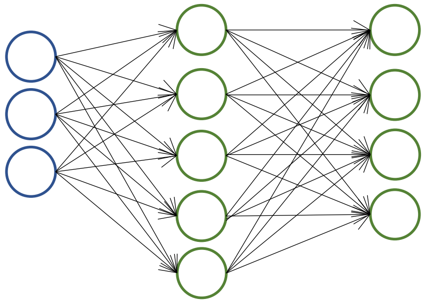
Additional reading: Murphy 13.3

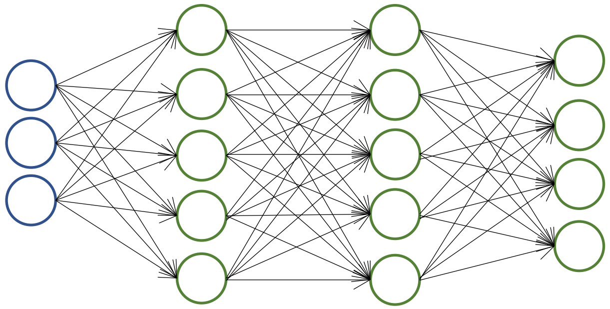
Neural Networks: Learning

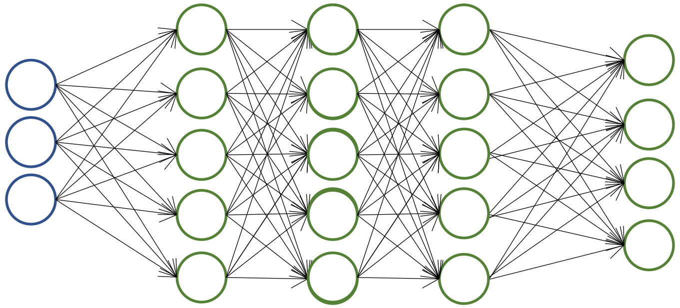
Putting it together

Training a neural network

Pick a network architecture (connectivity pattern between neurons)







No. of input units: Dimension of features $x^{(i)}$

No. output units: Number of classes

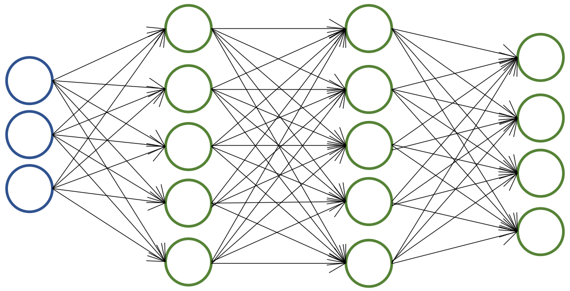
Reasonable default: 1 hidden layer, or if >1 hidden layer, have same no. of hidden units in every layer (usually the more the better)

Training a neural network

1. Randomly initialize weights
2. Implement forward propagation to get $h_\Theta(x^{(i)})$ for any $x^{(i)}$
3. Implement code to compute cost function $J(\Theta)$
4. Implement backprop to compute partial derivatives $\frac{\partial}{\partial\Theta_{jk}^{(l)}} J(\Theta)$

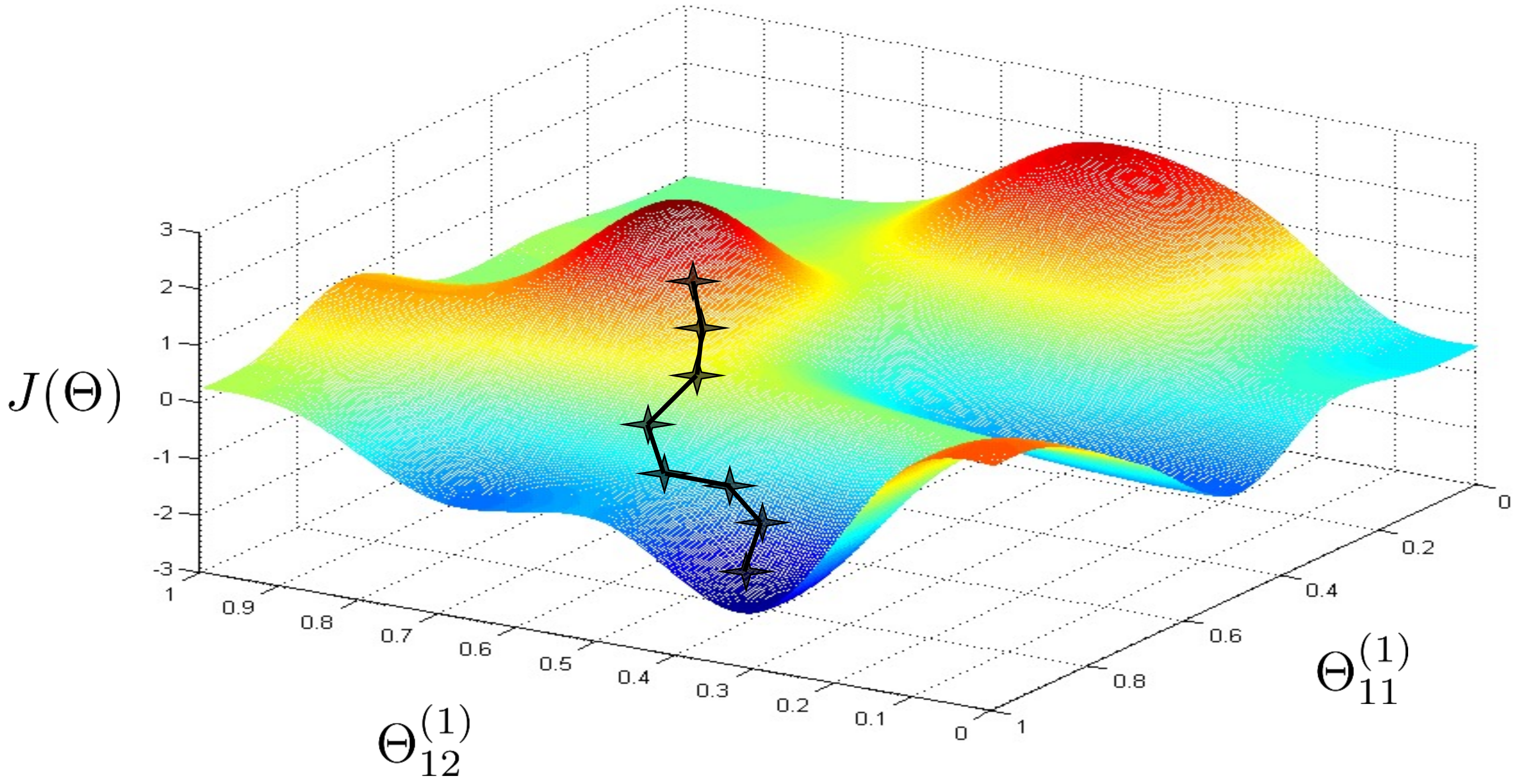
```
for i = 1:m
```

Perform forward propagation and backpropagation using example

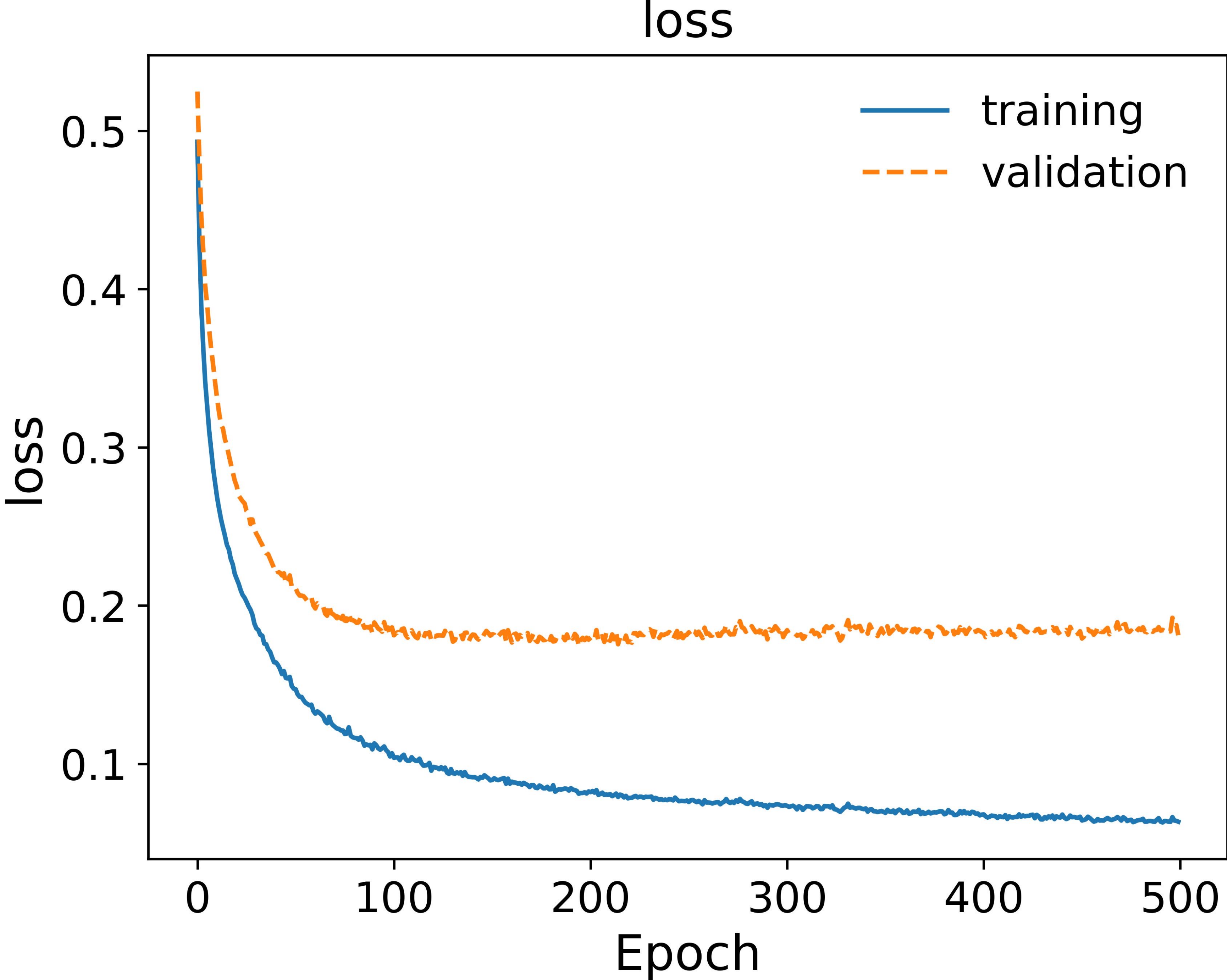


Training a neural network

5. Use gradient descent or advanced optimization method with backpropagation to try to minimize $J(\Theta)$ as a function of parameters Θ



$J(\Theta)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\Theta_{12}^{(1)}$
0
0.2
0.4
0.6
0.8
1
$\Theta_{11}^{(1)}$


loss
training
validation
0.5
0.4
0.3
0.2
0.1
loss
0
100
200
300
400
500
Epoch