Can Two-Photon Correlation of Chaotic Light Be Considered as Correlation of Intensity Fluctuations?

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Two-photon correlation phenomena, including the historical experiment of Hanbury Brown and Twiss, may have to be described quantum mechanically, regardless of whether the source of radiation is classical or quantum. Supporting this point, we present a ghost imaging type of second-order spatial correlation experiment of chaotic light to show that the classical understanding based on the concept of statistical intensity fluctuations does not give a correct interpretation for the observation. From a practical point of view, this experiment demonstrates the possibility of having high contrast lensless two-photon imaging with chaotic light, suggesting imaging applications for radiations for which no effective lens is available.

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Unlike first-order correlation, which is considered as a coherent effect of the electromagnetic field, the second-order correlation of radiation is usually considered as the classical statistical correlation of intensity fluctuations. The first second-order correlation experiment was demonstrated in 1956 by Hanbury Brown and Twiss (HBT) with two different types of correlation: temporal and spatial [1]. The HBT experiment created quite a surprise in the physics community with an enduring debate about the classical or quantum nature of the phenomenon. It has been popular to consider that the HBT experiment measures the classical statistical correlation of the intensity fluctuations of the radiation:

$$\langle \Delta I_1 \Delta I_2 \rangle = \langle (I_1 - \bar{I}_1)(I_2 - \bar{I}_2) \rangle = \langle I_1 I_2 \rangle - \bar{I}_1 \bar{I}_2 \quad (1)$$

where $\bar{I}_1$ and $\bar{I}_2$ are the mean intensities of the radiation measured by photodetectors D_1 and D_2, respectively.

Figure 1(a) is a schematic of the historical HBT experiment which measures the second-order transverse spatial correlation of radiation of wavelength λ coming from a distant star with an angular size of $\Delta\theta$. The second-order transverse spatial correlation function $\Gamma^{(2)}(x_1, x_2)$ is expected to be

$$\Gamma^{(2)}(x_1, x_2) = \langle I_1 I_2 \rangle \sim I_0^2 \left\{ 1 + \text{sinc}^2\left[\frac{\pi \Delta\theta (x_1 + x_2)}{\lambda} \right] \right\} \quad (2)$$

where we have simplified the problem to 1D and assumed $\bar{I}_1 = \bar{I}_2 = I_0$. The second term in Eq. (2), $I_0^2\,\text{sinc}^2[\pi\Delta\theta(x_1 + x_2)/\lambda]$, is interpreted as the correlation of intensity fluctuations. This term is useful in astronomy for angular size measurement of stars. For short wavelengths, this function quickly drops from its maximum to minimum when $x_1 + x_2$ goes from zero to a value such that $\Delta\theta(x_1 + x_2)/\lambda = 1$. Thus, we effectively have a point-to-point relationship between the x_1 and x_2 plane. As a matter of fact, the planes of x_1 and x_2 are the far-field Fourier transform planes of the finite-size distant star. Therefore, the measured quantity is the correlation between the transverse $\mathbf{k}$ vectors of the radiation. The nonzero correlation corresponds to equal transverse wave vectors: $\mathbf{k_1} = \mathbf{k_2}$. This is consistent with the physics behind the model of classical correlation. It is natural to imagine that the radiation coming from the same mode of the electromagnetic field, passing through the same optical path, would have identical intensity fluctuations, while radiation coming from different modes, passing through different optical paths would not share the same intensity fluctuations.

The classical statistical interpretation has been widely accepted. Moreover, the concept of intensity fluctuation has been even extended to quantum models to take over the concept of two-photon coherence. The philosophy of "photon bunching" is essentially a phenomenological extension to quantum theory of the statistical correlation on photon number fluctuations.

In the past 20 years, the massive research on quantum entanglement [2,3] has brought new challenges to the classical statistical correlation interpretation. For example, replacing the chaotic light with an EPR-type two-photon

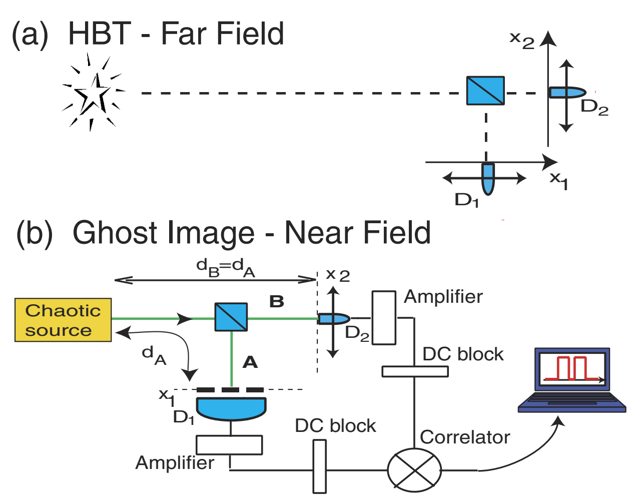

FIG. 1 (color online). (a) Hanbury-Brown–Twiss configuration. (b) Ghost imaging configuration.

entangled state in Fig. 1(a), the second-order transverse spatial correlation function turns out to be

$$\langle I_1 I_2 \rangle \sim I_0^2 \,\mathrm{sinc}^2\left[\frac{\pi \Delta\theta(x_1 - x_2)}{\lambda}\right]. \tag{3}$$

Based on the concept of classical statistical correlation of intensity fluctuation, the mean intensities $\bar{I}_1$ and $\bar{I}_2$ must be zero in this case, otherwise Eq. (1) leads to nonphysical conclusions. The measurements, however, never yield zero mean values of $\bar{I}_1$ and $\bar{I}_2$ in any circumstances.

Thus the concept of classical statistical correlation of intensity fluctuation may not work for entangled two-photon states. Two-photon correlation experiments with entangled photons have been explained in terms of the superposition of indistinguishable alternatives, two-photon probability amplitudes, that can lead to a joint-detection event [4]. Such alternatives, however, represent a troubling concept in classical theories, because they are nonlocal. If accepted, the nonlocal behavior of the radiation has been classified as a peculiar property of nonclassical sources.

More interestingly, we ask ourselves: does the statistical correlation of intensity fluctuation always work for chaotic light? We wish to report a two-photon imaging experiment aimed at answering this question [5]. We will conclude that two-photon correlation phenomena have to be described quantum mechanically, regardless if the source of radiation is "classical" or "quantum." As first-order correlation is a coherent effect of electric fields, the second-order correlation is a coherent effect of two-photon probability amplitudes.

Figure 1(b) illustrates the setup of the experiment. Radiation from a chaotic pseudothermal source [6,7] was divided in two optical paths by a nonpolarizing beam splitter. In arm A an object, a double slit ($d = 1.5$ mm and $a = 0.2$ mm) was placed at a distance $d_A = 139$ mm in front of a bucket detector (D_1). In arm B a point detector D_2 scanned the transverse plane at a distance $d_B = d_A$ from the source. The bucket detector D_1 was simulated by using a short focal length lens ($f = 25$ mm) to focus the light onto the active area of the detector while the point detector D_2 was obtained by a pinhole. The output current from the photodiodes was first amplified and dc blocked by a passive RC filter and then sent to a correlation circuit with an rf mixer and a low pass filter.

Figure 2 reports the measured two-photon image of the double-slit. The result shows a high visibility equal-size reproduction of the double slit when scanning photodetector D_2 along the x_2 axis which is located at distance $d_B = d_A$ from the source.

Let us first clarify the main differences of this measurement compared to Hanbury Brown and Twiss types of experiments [1]. The measurements of HBT are in the *far-field* zone, which measure the *momentum-momentum* correlation of the field. In the reported experiment, instead, we worked in the *near-field* zone ($\Delta\theta \sim 10\lambda/d$) and therefore we effectively measured the *position-position* correlation between the object plane and the image plane. The observation is a lensless two-photon "ghost" image.

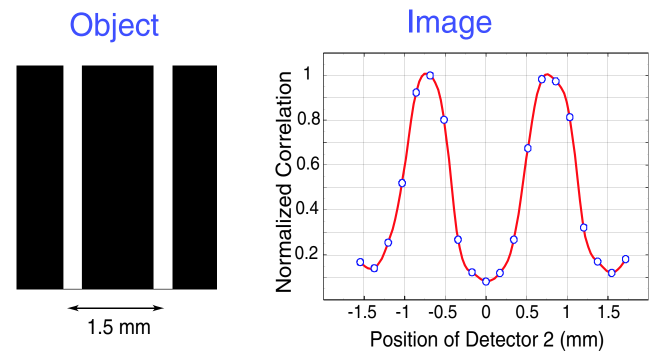

FIG. 2 (color online). Lensless imaging results. Normalized correlation of the photocurrents vs transverse position of D_2.

To confirm that the observed results correspond to an image but not a projection shadow, we further constructed a secondary imaging system by using a $f = 85$ mm lens to image the ghost image onto a secondary image plane. The imaging lens was located in the transmitted arm B of Fig. 1(b) at a distance of $d_B \sim 253$ mm from the source. A magnified secondary image was observed at $d'_B \sim 330$ mm from the lens by scanning D_2 on the transverse plane. Figure 3 reports the measured magnified secondary image of the ghost image with the expected magnification $M = d'_B/(d_B - d_A) \sim 2.9$. For this examination we used two masks with more complicated structures: one, Fig. 3(a), with the starting letter of our cities (2.3×2.5 mm) and the other, Fig. 3(b), with the acronym of our institution (6×1.9 mm). Figure 3(c) shows the image obtained with the actual revealed correlation measurements to show the high contrast of the image, while Fig. 3(d) shows a "slice" of the image around the half maximum of the correlation.

The explanation in terms of statistical correlation of intensity fluctuations would not give an acceptable interpretation for this experiment. (a) Unlike the HBT experiment, the measurements are in the near-field zone, so the measured correlation can no longer be considered as the correlation of equal **k** vectors. As a consequence, the classical argument of trivial projection type "imaging" [8], according to which there is no correlation when the **k** vectors are blocked by the aperture, while the correlation is maximum when the **k** vectors are allowed to pass, does not work anymore. In our configuration, any point on the object plane is "hit" by many different **k** vectors; thus the projection-type image would be definitely blurred. (b) One might think that the two beams reaching the photodetectors are spatially correlated because they come from the splitting of the same source. However, in the two arms the split radiations are not experiencing the same spatial modulations because the object aperture is placed in only one arm. And it is exactly the spatial modulation introduced by the aperture that we retrieve in the measurement. This is the physics behind the term ghost image: although it is formed

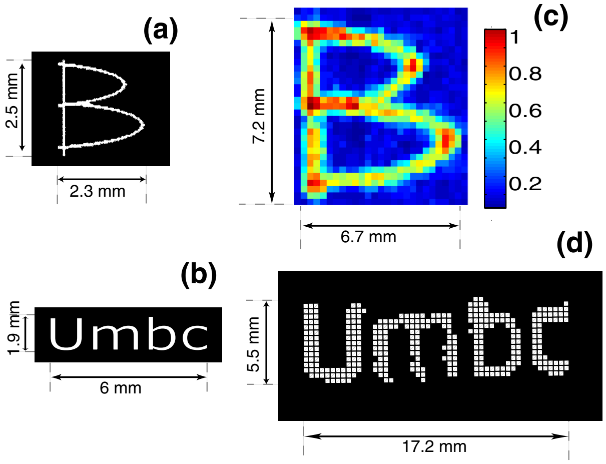

FIG. 3 (color online). Secondary imaging results. (a) and (b) are the two object used; (c) and (d) are the obtained images.

by the transmitted radiation, it reconstructs the spatial modulation experienced by the reflected radiation only. (c) The use of the bucket detector of D_1 further ensures the washing out of any classical spatial correlation. The bucket detector involves using a lens to focus all the light transmitted through the aperture onto the active area of the detector at once. Hence even if there exists a spot-by-spot classical correlation, such a correlation cannot be retrieved because in one arm all the ''spots'' are detected together.

On the other hand, the quantum model for this experiment is straightforward. In quantum theory of photodetection [9], the second-order correlation function is calculated as:

$$G^{(2)}(t_1, \vec{r}_1; t_2, \vec{r}_2) = \mathrm{Tr}[\rho E_1^{(-)}(t_1, \vec{r}_1) E_2^{(-)}(t_2, \vec{r}_2) \times E_2^{(+)}(t_2, \vec{r}_2) E_1^{(+)}(t_1, \vec{r}_1)]. \qquad (4)$$

where $E^{(-)}$ and $E^{(+)}$ are the negative-frequency and the positive-frequency field operators at space-time points $(\vec{r}_1, t_1)$ and $(\vec{r}_2, t_2)$ and ρ represents the density operator describing the radiation.

Let us calculate the second-order correlation for a simple quantum mechanical model of chaotic light:

$$\hat{\rho}_{\text{chaotic}} \propto \sum_{\vec{q}} \sum_{\vec{q}'} |1_{\vec{q}} 1_{\vec{q}'}\rangle\langle 1_{\vec{q}} 1_{\vec{q}'}|. \qquad (5)$$

Basically we are modeling the light source as an incoherent statistical mixture of two photons with equal probability of having any transverse momentum $\vec{q}$ and $\vec{q}'$.

The transverse spatial part of the second-order correlation function can be rewritten as:

$$\begin{aligned} G^{(2)}(\vec{x}_1; \vec{x}_2) &= \sum_{\vec{q}, \vec{q}'} \langle 1_{\vec{q}} 1_{\vec{q}'} | E_1^{(-)}(\vec{x}_1) E_2^{(-)}(\vec{x}_2) E_2^{(+)}(\vec{x}_2) \\ &\quad \times E_1^{(+)}(\vec{x}_1) | 1_{\vec{q}} 1_{\vec{q}'} \rangle \\ &= \sum_{\vec{q}, \vec{q}'} |\langle 0 | E_2^{(+)}(\vec{x}_2) E_1^{(+)}(\vec{x}_1) | 1_{\vec{q}} 1_{\vec{q}'} \rangle|^2 \qquad (6) \end{aligned}$$

where $\vec{x}_j$ is the transverse coordinate of the jth detector. The transverse part of the electric field operator can be written as follows:

$$\vec{E}_j^{(+)}(\vec{x}_j) \propto \sum_{\vec{q}} g_j(\vec{x}_j; \vec{q}) \hat{a}(\vec{q}) \qquad (7)$$

where $\hat{a}(\vec{q})$ is the annihilation operator for the mode corresponding to $\vec{q}$ and $g_j(\vec{x}_j; \vec{q})$ is the Green's function associated to the propagation of the field from the source to the jth detector [10].

Substituting the field operators into Eq. (6) we obtain:

$$\begin{aligned} G^{(2)}(\vec{x}_1; \vec{x}_2) = \sum_{\vec{q}, \vec{q}'} &| g_2(\vec{x}_2, \vec{q}) g_1(\vec{x}_1, \vec{q}') \\ &+ g_2(\vec{x}_2, \vec{q}') g_1(\vec{x}_1, \vec{q})|^2. \qquad (8) \end{aligned}$$

This expression represents a key result toward understanding the phenomenon. In fact, it expresses the interference between two alternatives, different yet equivalent, which lead to a joint photodetection: (1) $\vec{q}$ and $\vec{q}'$ are annihilated at $\vec{x}_2$ and $\vec{x}_1$, respectively, and (2) $\vec{q}'$ and $\vec{q}$ are annihilated at $\vec{x}_2$ and $\vec{x}_1$, respectively. The interference phenomenon is not, as in classical optics, due to the superposition of electromagnetic fields at a local point of space-time. It is due to the superposition of $g_2(\vec{x}_2, \vec{q}) g_1(\vec{x}_1, \vec{q}')$ and $g_2(\vec{x}_2, \vec{q}') g_1(\vec{x}_1, \vec{q})$, the so-called two-photon amplitudes, nonclassical entities that involve both arms of the optical setup.

Equation (8), can be further simplified in the form of

$$\begin{aligned} G^{(2)}(\vec{x}_1; \vec{x}_2) &\propto \sum_{\vec{q}} |g_1(\vec{x}_1, \vec{q})|^2 \sum_{\vec{q}'} |g_2(\vec{x}_2, \vec{q}')|^2 \\ &\quad + |\sum_{\vec{q}} g_1^*(\vec{x}_1, \vec{q}) g_2(\vec{x}_2, \vec{q})|^2 \\ &= G_{11}^{(1)}(\vec{x}_1) G_{22}^{(1)}(\vec{x}_2) + |G_{12}^{(1)}(\vec{x}_1; \vec{x}_2)|^2. \qquad (9) \end{aligned}$$

The second expression of Eq. (9) highlights the link with the standard form of second-order correlation function of chaotic light in terms of the first-order correlation functions $G_{ij}^{(1)}$.

Although the model is at two-photon level, it is explicative of the physics of the phenomenon also at higher light intensities. In fact, the result in Eq. (9) is the same result obtained in Ref. [7] where we used the standard model of chaotic sources with many photons.

The first term in Eq. (9) is the product of the average intensities measured by the two detectors (blocked in the detection circuit). The second term, which corresponds to the ''intensity fluctuation'' in Eq. (1), is nothing but the two-photon interference term. Although this term can be expressed with first-order correlation functions, $G_{12}^{(1)}$ and $G_{21}^{(1)}$ in Eq. (9) do not correspond to the superposition of two electromagnetic fields at one local space-time point. On the contrary, they are measured by two independent photodetectors. The second-order interference, or superposition, happens between different yet equivalent two-photon amplitudes which lead to a joint-detection event of two photodetectors at distant space-time points ($\mathbf{r_1}$, $\mathbf{t_1}$) and ($\mathbf{r_2}$, $\mathbf{t_2}$). There is no counterpart for such a concept in classical electromagnetic theory.

Equation (9) also indicates another difficulty for classical interpretation. The radiation is completely chaotic, spatially incoherent, yet the second term in Eq. (9), the one retrieved in the measurement for two-photon imaging, represents a coherent superposition. This is quite a surprise by any classical means. In quantum theory, we have shown that this is simply a two-photon interference phenomenon. The superposition takes place between quantities $g_2(\vec{x}_2, \vec{q})g_1(\vec{x}_1, \vec{q}')$ and $g_2(\vec{x}_2, \vec{q}')g_1(\vec{x}_1, \vec{q})$ in Eq. (8), namely, the two-photon amplitudes. The two-photon amplitudes can be considered as effective two-photon fields; and, in terms of these nonclassical two-photon fields, indeed we have a coherent effect.

Equation (9) is a general expression for chaotic radiation [11]. Regarding the historical debate about quantum vs classical interpretation for second-order correlation measurement, we have shown that the second-order correlation of chaotic light is a two-photon interference phenomenon. In this respect, the argument that led to Eq. (9) extends from two-photon imaging to the historical HBT and to all second-order coherence phenomena.

In the specific case of two-photon ghost imaging, the experimental results can be accurately calculated from Eq. (9). In fact, in the experimental setup of Fig. 1(b), the Green's functions can be written as:

$$
\begin{aligned}
g_1(\vec{x}_1; \vec{q}) &\propto \Psi\left(\vec{q}, -\frac{c}{\omega} d_A\right) T(\vec{x}_1) e^{i\vec{q}.\vec{x}_1} \\
g_2(\vec{x}_2; \vec{q}) &\propto \Psi\left(\vec{q}, -\frac{c}{\omega} d_B\right) e^{i\vec{q}.\vec{x}_2}
\end{aligned}
\tag{10}
$$

where $T(\vec{x}_1)$ is a function describing the aperture in the plane of x_1; $\Psi(\vec{q}, \frac{\omega}{c} p)$ is proportional to the transfer function of the linear system and the paraxial approximation has been used; ω is the light frequency, and c is the speed of light.

If the distances from the source to the two detectors are equal ($d_A = d_B$), the second term of Eq. (9), after the integration of the bucket detector, reduces to

$$G_M^{(2)}(\vec{x}_2) = \int d\vec{x}_1 |T(\vec{x}_1)\delta(\vec{x}_1 - \vec{x}_2)|^2 = |T(\vec{x}_2)|^2. \tag{11}$$

Thus we have successfully explained the experimental observation in terms of quantum two-photon interference.

From a practical point of view this result may be useful because the chaotic source produces an equal-size reproduction of the object in the plane at equal distance from the source without the necessity of using a lens and we have shown that such an image can be obtained with high contrast. The situation seems quite promising for imaging applications in which no effective lens is available. In addition, the existence of the imaging condition $d_A = d_B$ would allow the possibility of reconstructing a 3D structure of an object by scanning D_2 on various transverse planes, layer by layer.

The result of this Letter is generally interesting and important if read in the context of the historical debate about quantum vs classical interpretation for second-order correlation measurements triggered by the observation of Hanbury Brown and Twiss. We have presented experimental and theoretical evidence to show that the second-order correlation measurement of chaotic radiation is, in general, a quantum phenomenon involving superposition between indistinguishable two-photon alternatives, rather than a classical effect due to the statistical correlation between intensity fluctuations. The two-photon coherent effects are *observed in* the intensity fluctuations, however, they are *not caused by* the statistical correlation of the intensity fluctuations.

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