UNIVERSIDAD DE LOS ANDES

THESIS

Two-Photon Imaging Using Tunable Spatial Correlations

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for the degree of Physicist

in the

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Physics Department

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Declaration of Authorship

I, Juan VARGAS, declare that this thesis titled, “Two-Photon Imaging Using Tunable Spatial Correlations” and the work presented in it are my own. I confirm that:

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- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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N.N

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Abstract

Science Faculty
Physics Department

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Two-Photon Imaging Using Tunable Spatial Correlations

by Juan VARGAS

Two-Photon Imaging is a well studied fenomena, where we take advantage of the diferent correlations in wich the light can be related to recontruct the image of certain objects. In this Thesis use diferent spatial correlations of a SPDC light source, where we change this correlations changing the pump waist.

Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor... s [1] ss [2] green [3] green here [3] Shih[4]

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List of Abbreviations

LAH	**L**ist **A**bbreviations **H**ere
WSF	**W**hat (it) **S**tands **F**or

Physical Constants

Speed of Light $c_0 = 2.997\,924\,58 \times 10^{8}\,\mathrm{m\,s^{-1}}$ (exact)

List of Symbols

a	distance	m
P	power	W ($\mathrm{J\,s^{-1}}$)
ω	angular frequency	rad

For/Dedicated to/To my…

Chapter 1

Introduction and Background

1.1 Nature of Source Light

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1.1.1 Entangled Light

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1.1.2 Thermal Light

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1.2 Two-photon Imaging with lens

1.3 Lens less Two-photon Imaging

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Chapter 2

Theoretical Discussion

2.1 Quantum Imaging

2.2 Light Source

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2.2.1 Biphoton

$$|\Psi\rangle = \int dq_s dq_i d\Omega_s d\Omega_i x[\Phi(q_s, \Omega_s; q_i, \Omega_i)\hat{a}^\dagger(\Omega_s, q_s)\hat{a}^\dagger(\Omega_i, q_i) + \Phi(q_i, \Omega_i; q_s, \Omega_s)\hat{a}^\dagger(\Omega_s, q_s)\hat{a}^\dagger(\Omega_i, q_i)] |0\rangle \tag{2.1}$$

taken like it appears on [1]

Where $\Phi(q_s, \Omega_s; q_i, \Omega_i)$ are the mode fuctions or Biphotons, a fuctions that contain all the information about the correlations. $\hat{a}^\dagger(\Omega_n, q_n)$ the creation of a photon with tranverse momentum q_n and frequency Ω_n

2.2.2 Mode Function

$$\Phi(q_s, \Omega_s; q_i, \Omega_i) \propto E_p(q_p, \Delta_0) B_p(\Omega_p) \mathcal{C}_{spatial}(q_s) \mathcal{C}_{spatial}(q_i) x \mathcal{F}_{frequency}(\Omega_s) \mathcal{F}_{frequency}(\Omega_i) sinc\left(\frac{\Delta_k \mathcal{L}}{2}\right) \tag{2.2}$$

where $B_p(\omega_p^0 + \Omega_p)$ and $E_p(q_p)$ are the frequency and transverse momentum distribution of the pump. $\mathcal{C}_{spatial}(q_n)$ spatial filtering. $\mathcal{F}_{frequency}(\Omega_n)$ frequency filter function.

2.2.3 Phase matching conditions

$$\Delta_0 = q_s^y cos\varphi_s + q_i^y cos\varphi_i + k_s sin\varphi_s - k_i sin\varphi_i; \tag{2.3}$$

$$\Delta_k = k_p - k_s cos\varphi_s - k_i cos\varphi_i - q_s^y sin\varphi_s + q_i^y sin\varphi_i + (q_s^x + q_i^x) tan\rho_0 cos\alpha + \Delta_0 tan\rho_0 sin\alpha \tag{2.4}$$

where $k_n = [(\omega_n^0 n_n/c)^2 - |q_n|^2]^{\frac{1}{2}}$ is the longitudinal wavevector inside the crystal. φ_s and φ_i are the propagation directions of the generated photons inside the crystal with respect to the pump direction z and α is the azimuthal angle.

2.2.4 Gaussian approximations

Taking into account the Gaussian nature of the pump, that's $E_p(q_p^x, q_p^y) \approx exp\left[-\frac{w_p^2}{4}(q_p^{x^2} + q_p^{y^2})\right]$.

approximating the sinc function by a Gaussian function with the same width at $\frac{1}{e^2}$ of its maximum, i.e., $sinc(x) \approx exp(-\gamma x^2)$ with γ equal 0.193.

$$\mathcal{F}_{frequency}(\Omega_n) \approx exp\left[-\frac{\Omega_n^2}{4\sigma_n^2}\right] \tag{2.5}$$

$$\tilde{\Phi}(q_s, q_i) = \int d\Omega_s d\Omega_i \mathcal{F}_s(\Omega_s)\mathcal{F}_i(\Omega_i)\Phi(q_s, \Omega_s; q_i, \Omega_i) \tag{2.6}$$

The Biphoton then takes a quadratic form:

$$\tilde{\Phi}(q_s, q_i) = Nexp\left[-\frac{1}{2}x^T Ax + ib^T x\right] \tag{2.7}$$

where N is a normalization constant, x is a 4-dimensional vector defined as $x = (q_s^x, q_s^y, q_i^x, q_i^y)$, A is a 4 x 4 real-valued, symmetric, positive definite matrix and b is a 4- dimensional vector. A and b are defined from the phase-matching conditions of the SPDC process. x^T and b^T denote the transpose of x and b. A and b are functions that depend of all the relevant parameters in the experiment such as the length of the crystal L, pump waist w_p, creation angles inside the crystal φ_n and the width of the spectral filter σ_n.

A way to quantify the degree of spatial correlation we shall define 'correlation parameter':

$$K^\lambda = \frac{C_{si}^\lambda}{\sqrt{C_{ss}^\lambda C_{ii}^\lambda}} \tag{2.8}$$

calculated for each direction ($\lambda = x, y$) from the covariance matrix C^λ with elements $C_{kj}^\lambda = \langle q_k^\lambda q_j^\lambda \rangle - \langle q_k^\lambda \rangle \langle q_j^\lambda \rangle$.

2.2.5 Fresnel Propagator

Fresnel Propagator: $h(r,z) = (-\frac{i}{\lambda z})e^{(i\frac{2\pi z}{\lambda})}\Psi(r,z)$ with $\Psi(r,z) = e^{(i\frac{\pi}{\lambda z})r^2}$. Thin-lens transfer function $L_f(r) = \Psi(r, -f)$

$$G = \int d^2 r_1 \int d^2 r_0 h(r_f - r_1, f) L_f(r_1) h(r_1 - r_0, f) \tag{2.9}$$

The propagation is done by determining the Green function of the optical path by which the beam will travel. The biphoton function in terms of transverse momenta $\Phi_1(q_s, q_i)$ after traveling through two arbitrary optical paths can be expressed in terms of the corresponding Green functions and the initial biphoton function $\Phi(q_s, q_i)$ as:

$$\Phi_1(q_s, q_i) = G_s(q_s, r_1)G_i(q_i, r_2)\Phi(q_s, q_i) \tag{2.10}$$

$$\Phi_1(r_1, r_2) = \int d^2 q_s d^2 q_i \Phi_1(q_s, q_i) \tag{2.11}$$

Taking advantage of the 2-F system as a Fourier-Transform to reduce the amount of calculations. Solving 2.9 over r_0 and r_1 we have:

$$G(q, r_f) = Ce^{\frac{i\pi}{\lambda f}r_f^2}e^{\frac{i\lambda f}{4\pi}q^2}\delta(q - \frac{2\pi}{\lambda f}r_f) \tag{2.12}$$

where C is a complex constant that depends only on $\lambda = 2\pi c$ and f. Then we can define the Green Functions for each path:

$$G_1(q_s, r_1) = G(q_s, r_1)xT(r_1) \tag{2.13}$$

$$G_2(q_i, r_2) = G(q_i, r_2) \tag{2.14}$$

Where $T(r_1)$ is the transfer function of the object.

Gathering all the previous results we can obtain $\Phi_1(r_1, r_2) = C^2 T(r_1)\Phi(\frac{2\pi}{\lambda f}r_1, \frac{2\pi}{\lambda f}r_2)$, which describes the biphoton at the planes of the object and the scanning detector. It shows that the biphoton at the 2F plane in terms of r_1 and r_2 has the same form as the biphoton at the output face of the crystal with the relationship $q = \frac{2\pi}{\lambda f}r$. This allows to computationally simulate the biphoton at the 2-F plane by using Eq 2.7 without the need to computationally simulate its propagation through the 2-F system.

We are collecting all the light that interacts with the object by the means of a bucket detector, this from the mathematical point of view leave us with: $\Phi_1(r_2) = C^2 \int d^2 r_1 T(r_1)\Phi(\frac{2\pi}{\lambda f}r_1, \frac{2\pi}{\lambda f}r_2)$ The coincidence counts that will be measured by the Detectors will be proportional to the magnitude square of the resulting biphoton function $\Phi_1(r_2)$.

$$S(r_2) \propto |\int d^2 r_1 T(r_1)\Phi(\frac{2\pi}{\lambda f}r_1, \frac{2\pi}{\lambda f}r_2)|^2 \tag{2.15}$$

For non-ideal forms of $\Phi(q_s, q_i)$ we have the relation between $\Phi(q) \to \Phi(r)$ for a 2F system, Hence: $\Phi(r) = \frac{1}{\sqrt{det(\Sigma)(2\pi)^4}}e^{-\frac{1}{2}r^T\Sigma^{-1}r}e^{ibr}$

$$\Sigma = \begin{bmatrix} \sigma_{sx}^2 & Cov(x_s, y_s) & Cov(x_s, x_i) & Cov(x_s, y_i) \\ Cov(y_s, x_s) & \sigma_{sy}^2 & Cov(y_s, x_i) & Cov(y_s, y_i) \\ Cov(y_i, x_s) & Cov(x_i, y_s) & \sigma_{iy}^2 & Cov(x_i, y_i) \\ Cov(y_i, x_s) & Cov(y_i, y_s) & Cov(y_i, x_i) & \sigma_{iy}^2 \end{bmatrix}$$

2.3 Spatial Correlations

Chapter 3

Experimental Setup

3.1 SPDC Setup

3.2 Spatial Correlations Measurement Setup

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3.3 Two-Photon Imaging Setup

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Chapter 4

Results

4.1 Experimental Correlations

4.1.1 $w_p = ?$

4.2 Two-Photon Images

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4.2.1 mask1

4.2.2 mask2

4.2.3 mask3

Chapter 5

Discussions and Conclusion

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