Ghost Imaging at Uniandes

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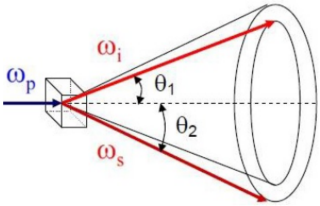
SPDC

The **SPDC** is a quantum process in which two photons are produced by pumping a non-linear media. This process satisfy:

$$\omega_p = \omega_i + \omega_s \quad \text{y} \quad k_p = k_i + k_s + \frac{2\pi}{\Lambda(T)}$$



SPDC

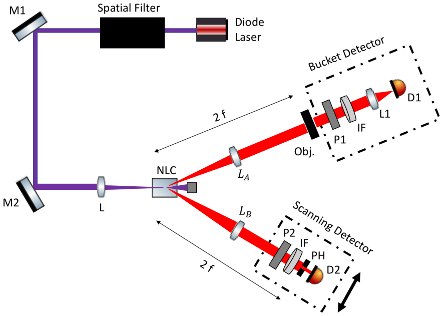


Figure: Approximatedn Experimental Setup

Biphoton

$$|\Psi\rangle = \int dq_s dq_i d\Omega_s d\Omega_i$$
$$\times \left[\Phi(q_s, \Omega_s; q_i, \Omega_i)\hat{a}^\dagger(\Omega_s, q_s)\hat{a}^\dagger(\Omega_i, q_i)\right.$$
$$\left. + \Phi(q_i, \Omega_i; q_s, \Omega_s)\hat{a}^\dagger(\Omega_s, q_s)\hat{a}^\dagger(\Omega_i, q_i)\right]|0\rangle$$

(1)

Where $\Phi(q_s, \Omega_s; q_i, \Omega_i)$ are the mode fuctions or Biphotons, a fuctions that contain all the information about the correlations. $\hat{a}^\dagger(\Omega_n, q_n)$ the creation of a photon with tranverse momentum q_n and frequency Ω_n

$$\Phi(q_s, \Omega_s; q_i, \Omega_i) \propto E_p(q_p, \Delta_0) B_p(\Omega_p) \mathcal{C}_{spatial}(q_s) \mathcal{C}_{spatial}(q_i)$$

$$\text{x } \mathrm{F}_{frequency}(\Omega_s) \mathcal{F}_{frequency}(\Omega_i) sinc\left(\frac{\Delta_k \mathcal{L}}{2}\right) (2)$$

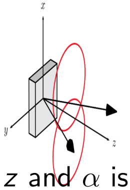
where $B_p(\omega_p^0 + \Omega_p)$ and $E_p(q_p)$ are the frequency and transverse momentum distribution of the pump. $\mathcal{C}_{spatial}(q_n)$ spatial filtering. $\mathcal{F}_{frequency}(\Omega_n)$ frequency filter function.

Phase matching conditions

$$\Delta_0 = q_s^y cos\varphi_s + q_i^y cos\varphi_i + k_s sin\varphi_s - k_i sin\varphi_i;$$

$$\Delta_k = k_p - k_s cos\varphi_s - k_i cos\varphi_i - q_s^y sin\varphi_s + q_i^y sin\varphi_i$$

$$+(q_s^x + q_i^x) tan\rho_0 cos\alpha + \Delta_0 tan\rho_0 sin\alpha$$



where $k_n = [(\omega_n^0 n_n/c)^2 - |q_n|^2]^{\frac{1}{2}}$ is the longitudinal wavevector inside the crystal. φ_s and φ_i are the propagation directions of the generated photons inside the crystal with respect to the pump direction z and α is the azimuthal angle.

Gaussian approximations

Taking into account the Gaussian nature of the pump, that's $E_p(q_p^x, q_p^y) \approx exp\left[-\frac{w_p^2}{4}({q_p^x}^2 + {q_p^y}^2)\right]$.

approximating the sinc function by a Gaussian function with the same width at $\frac{1}{e^2}$ of its maximum, i.e., $sinc(x) \approx exp(-\gamma x^2)$ with γ equal 0.193.

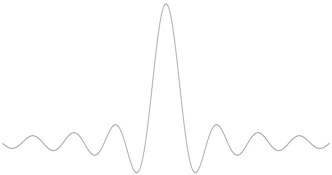


Figure: $f(x) = sinc(x)$

Gaussian approximations

To Observe the transverse correlations the frequency information has to be traced out.

$$\mathcal{F}_{frequency}(\Omega_n) \approx exp\left[-\frac{\Omega_n^2}{4\sigma_n^2}\right]$$

$$\tilde{\Phi}(q_s, q_i) = \int d\Omega_s d\Omega_i \mathcal{F}_s(\Omega_s)\mathcal{F}_i(\Omega_i)\Phi(q_s, \Omega_s; q_i, \Omega_i)$$

The Biphoton then takes a quadratic form:

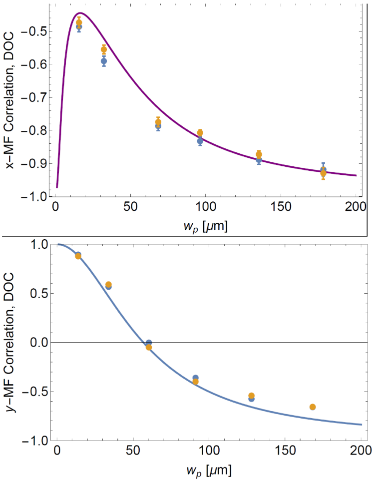
$$\tilde{\Phi}(q_s, q_i) = Nexp\left[-\frac{1}{2}x^T Ax + ib^T x\right] \tag{3}$$

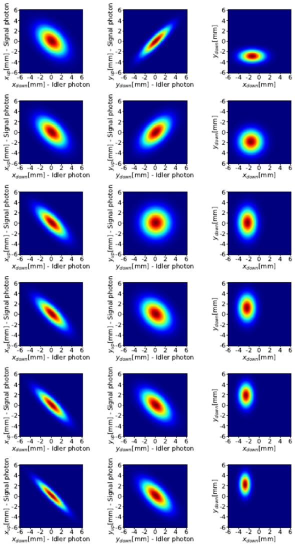
where N is a normalization constant, x is a 4-dimensional vector defined as $x = (q_s^x, q_s^y, q_i^x, q_i^y)$, A is a 4 real-valued, symmetric, positive definite matrix and b is a 4- dimensional vector. A and b are defined from the phase-matching conditions of the SPDC process. x^T and b^T denote the transpose of x and b. A and b are functions that depend of all the relevant parameters in the experiment such as the length of the crystal L, pump waist w_p, creation angles inside the crystal φ_n and the width of the spectral filter σ_n

A way to quantify the degree of spatial correlation we shall define 'correlation parameter':

$$K^{\lambda} = \frac{C_{si}^{\lambda}}{\sqrt{C_{ss}^{\lambda} C_{ii}^{\lambda}}}$$

calculated for each direction ($\lambda = x, y$) from the covariance matrix C^{λ} with elements $C_{kj}^{\lambda} = \langle q_k^{\lambda} q_j^{\lambda} \rangle - \langle q_k^{\lambda} \rangle \langle q_j^{\lambda} \rangle$.


x−MF Correlation, DOC
w_p [μm]
y−MF Correlation, DOC
w_p [μm]


x_{up}[mm] - Signal photon
x_{down}[mm] - Idler photon
y_{up}[mm] - Signal photon
y_{down}[mm] - Idler photon
y_{down}[mm]
x_{down}[mm]

Fresnel propagation

Fresnel Propagator: $h(r,z) = (-\frac{i}{\lambda z})e^{(i\frac{2\pi z}{\lambda})}\Psi(r,z)$ with $\Psi(r,z) = e^{(i\frac{\pi}{\lambda z})r^2}$. Thin-lens transfer function $L_f(r) = Psi(r,-f)$

$$G = \int d^2 r_1 \int d^2 r_0 h(r_f - r_1, f) L_f(r_1) h(r_1 - r_0, f)$$

Green Function ignoring temporal dependence
(4)

The propagation is done by determining the Green function of the optical path by which the beam will travel. The biphoton function in terms of transverse momenta $\Phi_1(q_s, q_i)$ after traveling through two arbitrary optical paths can be expressed in terms of the corresponding Green functions and the initial biphoton function $\Phi(q_s, q_i)$ as:

$$\Phi_1(q_s, q_i) = G_s(q_s, r_1) G_i(q_i, r_2) \Phi(q_s, q_i)$$
$$\Phi_1(r_1, r_2) = \int d^2 q_s d^2 q_i \Phi_1(q_s, q_i)$$

Taking advantage of the 2-F system as a Fourier-Transform to reduce the amount of calculations. Solving 4 over r_0 and r_1 we have:

$$G(q, r_f) = Ce^{\frac{i\pi}{\lambda f} r_f^2} e^{\frac{i\lambda f}{4\pi} q^2} \delta(q - \frac{2\pi}{\lambda f} r_f) \tag{5}$$

where C is a complex constant that depends only on $\lambda = 2\pi c$ and f. Then we can define the Green Functions for each path:

$$G_1(q_s, r_1) = G(q_s, r_1) x T(r_1)$$

$$\mathrm{G}_2(q_i, r_2) = G(q_i, r_2)$$

Where $T(r_1)$ is the transfer function of the object.

Dectection

Gathering all the previous results we can obtain $\Phi_1(r_1, r_2) = C^2 T(r_1)\Phi(\frac{2\pi}{\lambda f} r_1, \frac{2\pi}{\lambda f} r_2)$, which describes the biphoton at the planes of the object and the scanning detector. It shows that the biphoton at the 2F plane in terms of r_1 and r_2 has the same form as the biphoton at the output face of the crystal with the relationship $q = \frac{2\pi}{\lambda f} r$. This allows to computationally simulate the biphoton at the 2-F plane by using Eq 3 without the need to computationally simulate its propagation through the 2-F system.

Detection

We are collecting all the light that interacts with the object by the means of a bucket detector, this from the mathematical point of view leave us with:

$\Phi_1(r_2) = C^2 \int d^2 r_1 T(r_1) \Phi(\frac{2\pi}{\lambda f} r_1, \frac{2\pi}{\lambda f} r_2)$

The coincidence counts that will be measured by the Detectors will be proportional to the magnitude square of the resulting biphoton function $\Phi_1(r_2)$.

$$S(r_2) \propto \left| \int d^2 r_1 T(r_1) \Phi(\frac{2\pi}{\lambda f} r_1, \frac{2\pi}{\lambda f} r_2) \right|^2 \qquad (6)$$

Detection

For non-ideal forms of $\Phi(q_s, q_i)$ we have the relation between $\Phi(q) \rightarrow \Phi(r)$ for a 2F system, Hence:

$$\Phi(r) = \frac{1}{\sqrt{det(\Sigma)(2\pi)^4}} e^{-\frac{1}{2} r^T \Sigma^{-1} r} e^{ibr}$$

$$\Sigma = \begin{bmatrix} \sigma_{sx}^2 & Cov(x_s, y_s) & Cov(x_s, x_i) & Cov(x_s, y_i) \\ Cov(y_s, x_s) & \sigma_{sy}^2 & Cov(y_s, x_i) & Cov(y_s, y_i) \\ Cov(y_i, x_s) & Cov(x_i, y_s) & \sigma_{iy}^2 & Cov(x_i, y_i) \\ Cov(y_i, x_s) & Cov(y_i, y_s) & Cov(y_i, x_i) & \sigma_{iy}^2 \end{bmatrix}$$

Experiment at Uniandes: Results

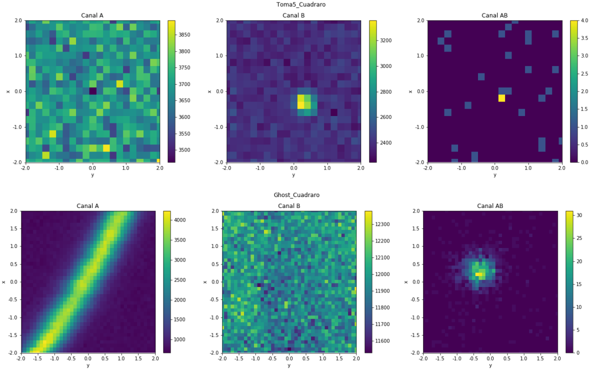


Figure: Alignment and Ghost Image square 4x4 μm

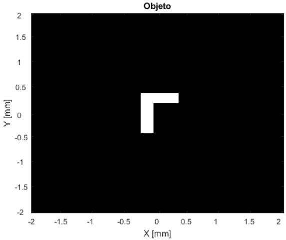


Figure: Mask Used so far

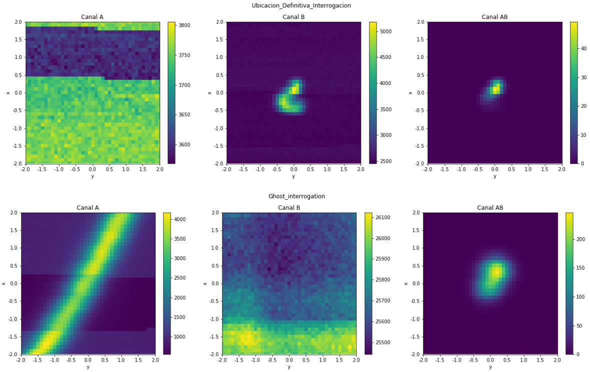


Figure: Alignment and Ghost Image interrogation

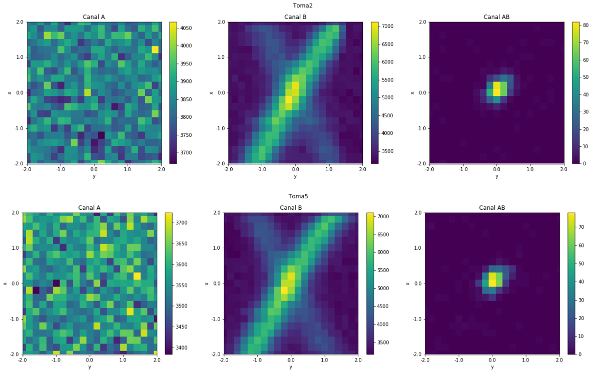


Figure: Alignment for the L

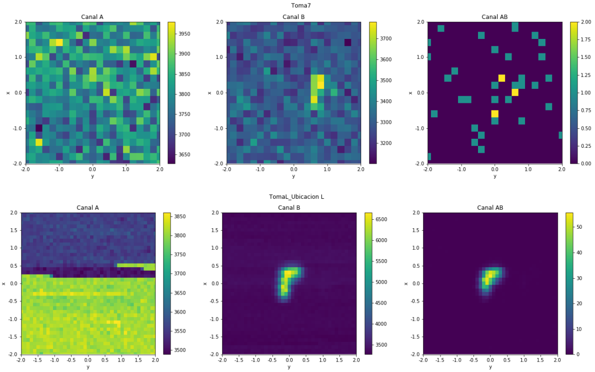


Figure: Alignment for the L

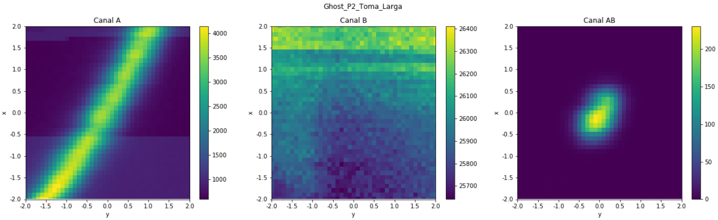


Figure: Ghost Long Measurement

Experimental vs. Simulation Results

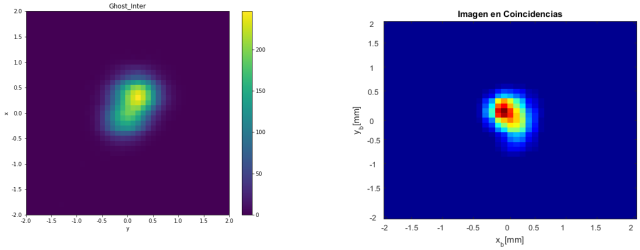
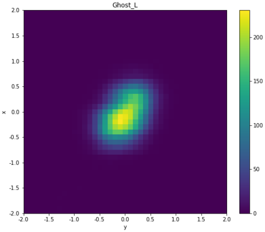


Figure: Comparison for the Interrrogation



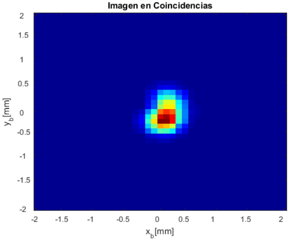


Figure: Comparison for the L

Bibliography