Rotating and scaling up image

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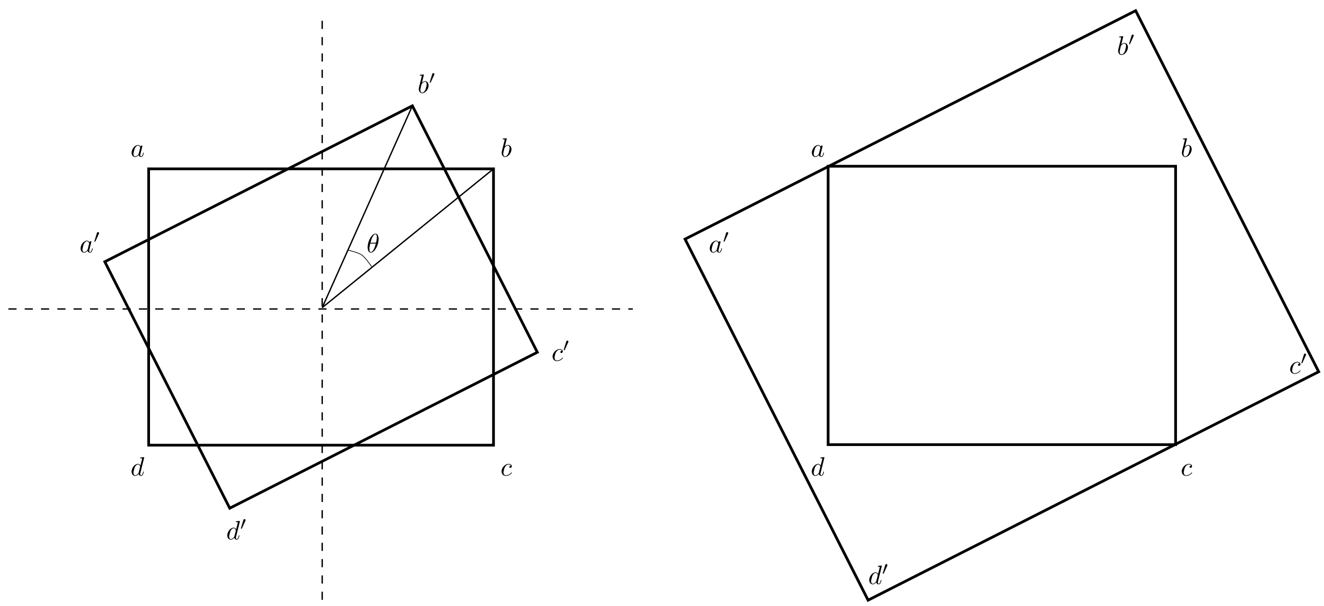

Figure 1

Image $abcd$ is rotated θ to make image $a'b'c'd'$ as shown in Figure 1 left. The objective is to scale up $a'b'c'd'$ such that the whole area of image $abcd$ is covered as shown in Figure 1 right. To do that the strategy is the following.

Let's imagine that θ is positive and we will focus our attention on the line $a'b'$. The objective is that we move the line $a'b'$ so that point a is in the line as in in Figure 1 right. The line will be moved according to the vector of growth of the image. For this exercise we will assume that the image will be scaled up proportionally to its width and height.

The growth vector (ϑ) will be defined as $[w, h]^T$ where w is the width of the image and h is the height. The idea is to find the intersection point between the lines l_1 and l_2 defined as

$$l_1 : a' + s\,u \qquad l_2 : a + t\,\bar{\vartheta}$$

where $\bar{\vartheta} = \vartheta/|\vartheta|$ and $u = \frac{b'-a'}{|b'-a'|}$. To find the value for s and t that identifies the intersection point we solve the linear system

$$\begin{bmatrix} a_1 - a_1' \\ a_2 - a_2' \end{bmatrix} = \begin{bmatrix} u_1' & -\bar{\vartheta}_1 \\ u_2' & -\bar{\vartheta}_2 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

Let $\hat{s}$ and $\hat{t}$ the solution of such linear system. Then, the intersection point is $I = a' + \hat{s}\,u$. Finally, a_1/I_1 is the scaling up factor necessary to make line l_1 reach point a