

EQUILIBRIUM EFFECTS OF HOUSING SUBSIDIES: EVIDENCE FROM A POLICY NOTCH IN COLOMBIA

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Abstract

This paper studies how the housing market in Colombia responds to policies that aim to increase homeownership among low-income households. Private sector developers received tax incentives to build houses priced below a cut-off, and households received subsidies to buy houses below the same cutoff. To benefit from the policy, households change their housing consumption and spend less on housing, bunching at the cutoff. To rationalize this response, the paper models an equilibrium between heterogeneous developers building differentiated housing and heterogeneous households buying them. The model is estimated with the moments recovered by comparing the distribution with bunching to a counterfactual distribution estimated using the techniques from the bunching literature. I use the model and estimated parameters to evaluate the policy. I calculate the efficiency cost induced by the notched subsidy scheme and I show that without supply-side incentives, developers may exit the market; their profits would be up to 14 percent lower. However, the existence of these tax incentives artificially increases the profits of developers who would build low-cost housing even in their absence.

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I. INTRODUCTION

Many governments invest significant resources through subsidies or tax incentives to provide housing solutions for low-income households and promote homeownership. Approaches such as the mortgage interest deduction (MID) aim to encourage homeownership through tax incentives. However, they raise concerns because they primarily benefit the rich and there is little evidence that they increase homeownership (E. Glaeser & Shapiro, 2003; Gruber et al., 2021; OECD, 2021a). Alternative strategies include downpayment assistance, subsidized interest rates, and subsidies to developers to build affordable housing. We know little about the market effects of these alternative approaches, which face the same concerns as MID tax incentives. To understand the effectiveness of these policies, we need to know the market responses, who benefits from them, and the potential inefficiencies of the subsidies.

Studying the market response and recovering the structural parameters of a model that allows a welfare evaluation of these policies is challenging. Valid counterfactuals to a housing market without subsidies are rare. Additionally, a reduced form analysis would be insufficient to understand the effectiveness of the policies. We need to disentangle demand and supply responses and have a model to interpret them. Identifying market effects in markets with heterogeneous agents and differentiated products, such as the housing market, is particularly challenging.¹ A model that does not allow product differentiation could not account for changes in the type of housing built and consumed, which could be a relevant response to the government incentives.

Hedonic equilibrium models are a common approach to model differentiated product markets and have been widely used to model housing markets. However, existing identification approaches for estimating structural parameters rely on strong assumptions.² Few papers actually attempt to estimate structural parameters. Most of those that do focus on estimating hedonic regressions that provide equilibrium

¹Zoutman, Gavrilova, and Hopland (2018) shows that a single tax or subsidy can help to identify supply and demand responses in a market with homogeneous goods and agents. Implementing this approach to a market with differentiated products will require multiple instruments

²There are three main identification approaches in these types of models: (i) Excluded instruments and variation across markets (Epple, 1987; Brown & Rosen, 1982; Wooldridge, 2010), (ii) Functional forms and inversion methods (Bajari & Benkard, 2005; Yinger, 2015; Bishop & Timmins, 2019), (iii) Non parametric identification and single index reduction (Ekeland, Heckman, & Nesheim, 2004; Heckman, Matzkin, & Nesheim, 2010; Chernozhukov, Galichon, Henry, & Pass, 2021; Epple, Quintero, & Sieg, 2020). For more details see Chernozhukov et al. (2021). An approach that integrates the hedonic insights into a discrete choice framework is Bayer, Ferreira, and McMillan (2007) or Anagol, Ferreira, and Rexer (2021).

marginal willingness to pay (MWTP) for different housing characteristics (Greenstone, 2017). Although MWTP estimates can be informative, they do not allow for non-marginal policy evaluations or counterfactual policy analysis.

This paper studies the social housing policy in Colombia, which combines subsidies and tax incentives for developers and households buying and building low-cost housing. The policy design allows me to overcome the empirical challenges associated with the evaluation of housing subsidies. Low-cost housing is defined using a market price cut-off of 135 times the monthly minimum wages (roughly USD 40,000). This cutoff introduces notches, or discontinuous incentives, on both the supply and demand sides. This triggers bunching at the cutoff. I use this notch and the variation of the notch size overtime to provide evidence of the market response to these subsidies. I propose a model that rationalizes the observed equilibrium, and I integrate the bunching and hedonic equilibrium literature to propose a method to identify and estimate the structural parameters of the model.³ The model and estimated parameters are used to evaluate the effectiveness of the Colombian policy design and to evaluate counterfactual policies.

Between 2006-18, the policy expanded, doubling the subsidy amount and the number of households receiving it. To show the market response, I combine data from a construction census containing the universe of new housing developments between 2006 and 2018 and administrative records for the subsidies awarded from the Ministry of Housing.

I show strong evidence of bunching at the cutoff. Following the bunching literature, I estimate a counterfactual distribution of market shares by price to recover the behavioral responses induced by the subsidy. The households that change their housing consumption to receive the subsidy spend up to 85 percent less in housing to take advantage of the subsidy. Given the equilibrium prices, this is translated to a housing unit up to 90 percent smaller. Using the variation in the subsidy over time, I show that increasing government expenditure on the policy increases the share of units sold at the cutoff defining low-cost housing. The fact that the bunching amount increases as the generosity of the subsidies increases demonstrates that Colombia's social housing policy matters a lot and may provide credible identification of market structure.

³This is not the first paper suggesting to use bunching to identify hedonic or sorting equilibrium models. Kuminoff, Smith, and Timmins (2013, p.1009) wrote: "Equilibrium sorting models provide the means to implement both the original Blinder and Rosen (1985) idea and the Saez (2010) test and extend them for policies that target public goods or other amenities that affect agents differently." However, I am not aware of any paper that actually implements this approach.

During my study period, an interest rate subsidy was introduced, the downpayment subsidy increased, and eligibility expanded. Households received around 13 percent of the price of the house at the cutoff in 2006 and around 24 percent in 2018. As a result, the excess mass, or bunching, sold at the price cutoff increases from around 3 percent of the market share around 2006 to about 16 percent by 2018. I provide suggestive evidence that housing characteristics and in particular, housing size drive the behavioral responses resulting in the bunching.

To rationalize the observed equilibrium responses, I introduce and estimate a competitive housing market equilibrium model. The model includes the policy-induced notch, to a hedonic – or sorting – equilibrium model.⁴ Households are heterogeneous in income, developers in productivity level, and housing in size. I use the model to show how the notch creates incentives for developers and households to bunch at the threshold. Like in the observed equilibrium, buyers and developers in the model change the type of units they buy and build to take advantage of subsidies, and consequently the equilibrium density has bunching at the cutoff point.

I propose an identification strategy based on a two-step procedure suggested by [S. Rosen \(1974\)](#). The first step follows the standard practices in the literature to estimate the implicit price function for housing size and to use the reduced-form estimates from the first part of the paper. The main innovation of my paper is in the second step. In the second step, I use the discontinuity and estimated behavioral responses and adapt the identification strategy proposed in the literature using notches to estimate the structural parameters. By comparing the counterfactual distribution with the observed distribution, I learn about the trade-offs between developers and households. Using the model, I show that there is a marginal buncher who is indifferent to receiving the subsidy but consuming less housing and not receiving the subsidy and consuming their optimal housing type. This insight of the model allows me to use the counterfactual distribution to recover two points on the same indifference curves for the marginal buncher.⁵ Using the parameters of the first step and the marginal buncher indifference condition, I can estimate the shape of the utility and cost functions. Because the policy scheme has one subsidy targeted to developers and one for households using the same cutoff, I have three different prices. I have

⁴For a review of the general approach, see [Kuminoff et al. \(2013\)](#) or [Greenstone \(2017\)](#). For recent applications, see [Epple et al. \(2020\)](#) and [Chernozhukov et al. \(2021\)](#).

⁵[Best, Cloyne, Ilzetzki, and Kleven \(2019\)](#) use the same identification idea to estimate the intertemporal elasticity of substitution from the behavioral responses induced by notches in the interest rates for loan refinancing. Other examples are [Einav, Finkelstein, and Schrimpf \(2015\)](#) and [Chen, Liu, Suárez Serrato, and Xu \(2021\)](#) or [Kleven and Waseem \(2013\)](#). [Bertanha, McCallum, and Seegert \(2021\)](#) and [Blomquist, Newey, Kumar, and Liang \(2017\)](#) discuss how in contrast with changes in the slope, or kinks, notches allow to recover structural parameters.

the market price, the price received by developers and the price paid by households. This allows me to use the same logic that I used to estimate the demand parameters to estimate the parameters describing developers' marginal costs.⁶

The model and estimated parameters are used to evaluate how marginally subsidized households and developers benefit from the subsidy scheme. On the demand side, I compare the utility levels of the marginally subsidized households in two counterfactual scenarios. In the first counterfactual scenario, households do not get subsidies. Marginally subsidized households, that reduce their housing consumption to benefit from the subsidies could be better off if they receive the money without a restriction on the cost of the house. I calculate this welfare loss associated with the policy design. Quantifying this is relevant to assess if a notched policy design is better than a linear subsidy. [H. S. Rosen \(1985\)](#) show that depending on the elasticity of substitution, notched policy designs may be more effective than linear incentives at targeting subsidies. My structural parameters suggest an elasticity of substitution between housing and consumption of other goods of around higher than one. Therefore, housing and consumption of other goods are gross substitutes.

I compare the observed equilibrium with a counterfactual scenario with subsidized households but without tax refunds for developers. Developers would be worse off. Between 2006 and 2009, the profits for marginally subsidized developers would be 5 percent lower, and by 2016, after the subsidy's expansion, their profits would be 14 percent lower. The marginally subsidized developers have higher marginal costs when producing at the price cutoff. They are competing with more productive developers, which have profits when building low-cost housing even without subsidies. A price increase, desirable for both developer types, will lead to non-eligibility for the subsidies. Because of the price cap, marginally subsidized developers need tax incentives to build low-cost housing, and without them, the market may face a rationing problem. The existence of these tax incentives can prevent the exit of the marginally subsidized developers, however they artificially increase the profits in more than 5 percent for developers that would produce low-cost housing even in the absence of these incentives.

⁶To estimate the shape of the indifference curve and offer curve, I impose functional forms for the utility function and a cost function. The utility function is a CES utility function depending on consumption on housing and consumption on other goods, and the cost function depends on housing size and the number of units built. I observed equilibrium relationships non parametrically.

Contributions and Related Literature

I make several methodological and empirical contributions. My first contribution is to provide additional evidence of bunching in the housing market, which is a relatively unexplored setting. [Carozzi, Hilber, and Yu \(2020\)](#) provide evidence of bunching in response to a similar housing policy in the United Kingdom. [McMillen and Singh \(2020\)](#) show that apartment rents cluster at values near the fair market rent in Los Angeles, California. There is also evidence of bunching in the density of mortgages with notches in the interest rate schedule.⁷ This paper complements the bunching evidence around a price cutoff in the housing market.

The main contribution of this paper is it provides a method to recover structural parameters using bunching responses in a market equilibrium of a vertically differentiated product. The paper offers a new framework to use the observed bunching responses to do a welfare analysis of housing policies. The approach proposed in this paper can be applied to other settings with policy interventions with discontinuous incentives that cause bunching (e.g., [Carozzi et al., 2020](#); [McMillen & Singh, 2020](#)). The proposed method complements the approaches that use notches and bunching moments for identification by providing the same identification principle to recover model primitives in the sorting or hedonic models. I use the bunching evidence and moments to estimate the structural parameters of economic models, as done recently in other settings by [Einav et al. \(2015\)](#) for the drug market, [Best et al. \(2019\)](#) for the mortgage market, and [Chen et al. \(2021\)](#) for incentives for research and development in China.⁸ This paper brings this relatively novel approach to the housing market literature.

This paper makes important methodological contributions to the bunching and hedonic equilibrium models literature, but it also provides new empirical and theoretical insights into a first-order question. The findings of this paper can inform the design of housing policies aimed at providing affordable housing. The model presented in this paper allows me to estimate the welfare effects on developers and

⁷For example, [DeFusco and Paciorek \(2017\)](#) use these bunching responses to estimate the interest rate elasticity of mortgage demand. [Best and Kleven \(2017\)](#); [Kopczuk and Munroe \(2015\)](#); [Slemrod, Weber, and Shan \(2017\)](#) report housing transaction bunching responses around notches in transaction costs.

⁸In contrast to this approach, alternative approaches implemented, for example, by [Saez \(2010\)](#), [Chetty et al. \(2011\)](#), or [Chetty, Friedman, and Saez \(2013\)](#) use the bunching moments to derive reduced form elasticities and use them as sufficient statistics for welfare analysis. See [Kleven \(2016\)](#) for a review of the literature using bunching. Some recent applications include studies on minimum wage ([Cengiz, Dube, Lindner, & Zipperer, 2019](#); [Harasztosi & Lindner, 2019](#); [Jales, 2018](#)), overpay hours ([Goff, 2021](#); [Bachas & Soto, 2018](#); [Abel, Dey, & Gabe, n.d.](#)), marriage market ([Persson, 2020](#)), Crime ([Goncalves & Mello, 2021](#)) among others.

households, which has direct implications for policy design. While other papers investigate the effects of housing programs on households, my study contributes to the literature by also investigating the effects of these housing programs on developers.⁹ My setting and approach also allow me to understand the effect of these programs on the housing market itself. My findings suggest that the policy affects behavior on both sides of the market. The policy incentives shape the type of housing that is built and sold, which has implications for how the city grows and develops. This is particularly relevant in a world of increasing urbanization. Furthermore, the affordability crisis in many developed and developing countries highlights the importance of effective housing policies. It also informs the debate about the effectiveness of housing programs compared to other social assistance programs such as unconditional cash transfers (Olsen, 2003; Olsen & Zabel, 2015).

The paper has three parts. The first part introduces the reduced-form analysis. In the next section, I present the Colombian housing policy, institutional context and the discontinuities created by the subsidy scheme. Section III presents the housing market data and provides reduced-form evidence of the housing market response. The second part of the paper contains the housing equilibrium model and identification strategy. Section IV, introduces the model, section V, presents the identification strategy. The third part, presented in section VII, shows the estimates for the structural parameters, the policy counterfactuals, and welfare analysis.

II. INSTITUTIONAL CONTEXT AND DATA

This section introduces the Colombian housing policy, describes the subsidy expansion and shows how the discontinuity creates incentives to bunch at the price cutoff.

⁹Many papers study housing market policies implemented in the United States. For example, Baum-Snow and Marion (2009), Soltas (2021) and Sinai and Waldfogel (2005) study the LIHTC, Collinson and Ganong (2018), McMillen and Singh (2020) study housing vouchers and Gruber et al. (2021), E. Glaeser and Shapiro (2003) study mortgage interest deductions (MID). Olsen (2003) and Olsen and Zabel (2015) compares different approaches. H. S. Rosen (1985); Poterba (1992); Galiani, Murphy, and Pantano (2015); Quigley (1982); Geyer (2017) carry out incidence and welfare analysis on housing policies. In addition to housing subsidies, there is literature on alternative approaches to affordable housing including public housing (Kumar, 2021; Franklin, 2019; van Dijk, 2019), rent control (E. L. Glaeser & Luttmer, 2003; Autor et al., 2014; Diamond et al., 2019), maximum permitted construction (Anagol et al., 2021). OECD (2021b) describes the different approaches implemented around the world to promote affordable housing.

A. COLOMBIAN HOUSING POLICY

Institutional context. Colombian housing policy aims to provide a decent home and suitable living, reduce housing deficits, and achieve the dream of being a country of homeowners.¹⁰ Since the 1990s, Colombia and other Latin American countries have changed their approach, moving from state-provided housing to a market-oriented solution based on subsidies.¹¹ This policy approach aims to incentivize the purchase and construction of low-cost housing through subsidies to households and developers. On the demand side, there are two main policy tools: 1) mortgage assistance through a downpayment subsidy and 2) a subsidized interest rate. On the supply side, the policy tool is a tax refund for developers who build low-cost housing.¹²

Low-cost housing definition. The policy design is heavily based on the definition of low-cost housing, which is a unit with a market price below an arbitrary threshold $\underline{P} = 135$ times the monthly minimum wage (mMW).¹³ This arbitrary threshold is the same for all cities, and changes over time are associated with changes in the minimum wage.¹⁴ The subsidies apply only to households and developers who buy and build low-cost housing. There is an additional definition creating a similar discontinuity at a lower price cutoff. Housing units below $70 \times mMW$ (around US\$20,000) classify as priority low-cost housing. This cutoff defines eligibility for some subsidies for the extreme poor and those affected by forced displacement or natural disasters. Between 2012 and 2015 these subsidies included 100 thousand free housing units.

¹⁰The first and second goals are based on Article 51 of the Colombian Constitution. The goal of being a country of homeowners appears in the country's last three National Development Plans (see, for example p104 of the [National Development Plan](#) for 2002–06).

¹¹For example, in a 1993 report the World Bank said that “housing policy making must thus move away from its previously narrow focus on a limited engagement of government in the direct production of low-cost housing.” [World Bank Group](#) (1993, p.1) Following these recommendations, many Latin American countries, including Chile and Colombia, abandoned the construction of public housing and implemented a market-oriented approach called ABC (from Spanish, *Ahorro-Savings, Bonos-Bonds, Creditos-Credit*) ([A. Gilbert, 2014](#); [Cohen, Carrizosa, & Gutman, 2019](#)).

¹²The demand subsidies are similar to many *first time buyers programs* in the United States and some housing policies in the United Kingdom, such as *help to buy* ([Carozzi et al., 2020](#)). The supply subsidy could be compared to the Low Income Housing Tax Credit (LIHTC) in the United States ([Baum-Snow & Marion, 2009](#)). However, in contrast to LIHTC, the units built are not rental units, they are units to be occupied by the owner.

¹³In Colombia the minimum wage is adjusted every year based on the inflation, productivity growth and an agreement between different representatives of the different economic sectors. Appendix Figure A.1 shows the evolution of the minimum wage and inflation during my study period.

¹⁴This price limit is set by the government's National Development Plan. It was the same from 1997 until 2019. With [law 1467 of 2019](#), it increased to 150 mMW for the five largest cities (including the metropolitan areas) and remained the same in the other cities.

This paper focuses mostly on the subsidies targeting the population buying low-cost housing units.

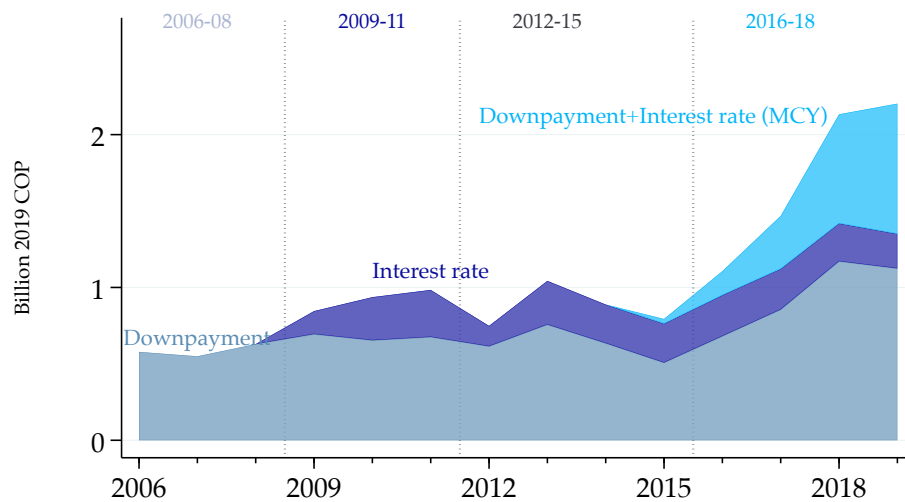
Subsidies over-time. During my study period, the demand side subsidies increased in generosity and were modified. The interest rate subsidy was introduced, the subsidy amount increased, and individuals in the informal sector became eligible. I use these changes to show how the housing market responds to changes in subsidies. I divide my study period into four sub-periods corresponding to the distinct set of policies available. The four periods are 1) **2006-08** downpayment subsidy available only to formal employees, 2) **2009-11** downpayment subsidy and interest rate subsidy available only to formal employees. 3) **2012-15** unstable period with rapid changes in interest rate subsidy and the existence of programs targeted at the extreme poor.¹⁵ 4) **2016-18** An increased downpayment subsidy for formal employees and interest rate subsidies. Additionally, the program *Mi Casa Ya* is available to all households with earnings of 4 mMW or less and automatically includes the downpayment and the interest rate subsidy.

Subsidy expansion. The government expenditure on these subsidies doubled during my study period. Figure 1 shows the total government expenditure from 2006 until 2018. The **gray blue** area shows the expenditure on downpayment subsidies. The expenditures were stable until 2015, when the subsidy's size increased. The **dark blue** area shows the total government expenditure on the subsidized interest rate. The number of households that received this subsidy was stable over time, but government expenditure decreased slightly due to the lower interest rate.¹⁶ The **light blue** area shows the expenditure related to the *Mi Casa Ya* program, which provides downpayment assistance and covers the interest rate discount.

¹⁵Including the provision of 100,000 free housing units and the country's primary mortgage downpayment subsidy program for the vulnerable population (VIPA). For more details, see Camacho, Caputo, and Sanchez (2020) and A. Gilbert (2014)

¹⁶To obtain the government expenditure, I calculate the total savings on mortgage payments induced by the discount at the interest rate. I calculate the monthly payments of each loan using the administrative records for the subsidy and the formula for monthly payments on a mortgage, $L_{monthly} = L \cdot \kappa(i, n)$ with $\kappa(i, n) = \frac{i}{12} \cdot \left(1 + \frac{i}{12}\right)^{12 \cdot n} / \left(\left(1 + \frac{i_h}{12}\right)^{12 \cdot n} - 1\right)$. Where i is the interest rate, $i_{subsidy}$ is the interest rate discount, n is the loan term in years, L is the loan amount. The government pays the difference in the amount paid by households ($L \cdot \kappa(i_\tau, n)$, with $i_\tau = i - i_{subsidy}$) and the amount received by the bank ($L \cdot \kappa(i_\tau, n)$). In particular, $\tau^i = \sum_{t=1}^{84} L_{month}(i, n)(i, n) - L \cdot \kappa(i_\tau, n)$, the sum of monthly payments for seven years, the period during which the subsidy applies. Figure A.6 shows the loan terms by unit price and Figure A.7 shows the market interest rate and the interest rate that households pay.

Figure 1: Total Government Expenditure on Demand Subsidies over Time



SOURCE: Administrative records from the Ministry of Housing. Appendix A provides more details about the data.

NOTE: This Figure shows the evolution of total government expenditure by type of subsidy. The **downpayments** are the subsidies awarded to employees affiliated to family funds. The **interest rate** represents the total amount paid by the government to the banks corresponding to the interest rates payments. I assigned the total amount to the year of the subsidy assignment. I calculated this amount using the administrative data containing detailed information on each loan. **Mi Casa YA** corresponds to the payments for the interest rate and the downpayment subsidy. Figure A.2 shows the number of assigned subsidies over time.

Supply subsidy–value-added tax (VAT) refund. To encourage developers to build low-cost housing, the government introduced a VAT refund. Developers get up to 4 percent of the sale price of each unit as a refund for taxes paid on construction materials. This subsidy, was introduced in 1995, a couple of years after the beginning of the downpayment subsidies.¹⁷

Comparison to other subsidies. As a reference point, the Colombian conditional cash transfer, *Familias en Acción*, benefited almost 4 million people with an expenditure of 3 billion COP in 2019 (DNP, 2018), while 100,000 of these housing subsidies cost 2 billion COP plus the tax benefits for developers. Becoming homeowners can confer substantial benefits to low income households and is therefore a legitimate goal. However, few households benefit from the subsidies to become homeowners, raising the question of whether it could be better to target a wider population with other subsidies like rent vouchers or unconditional cash transfers. For this reason,

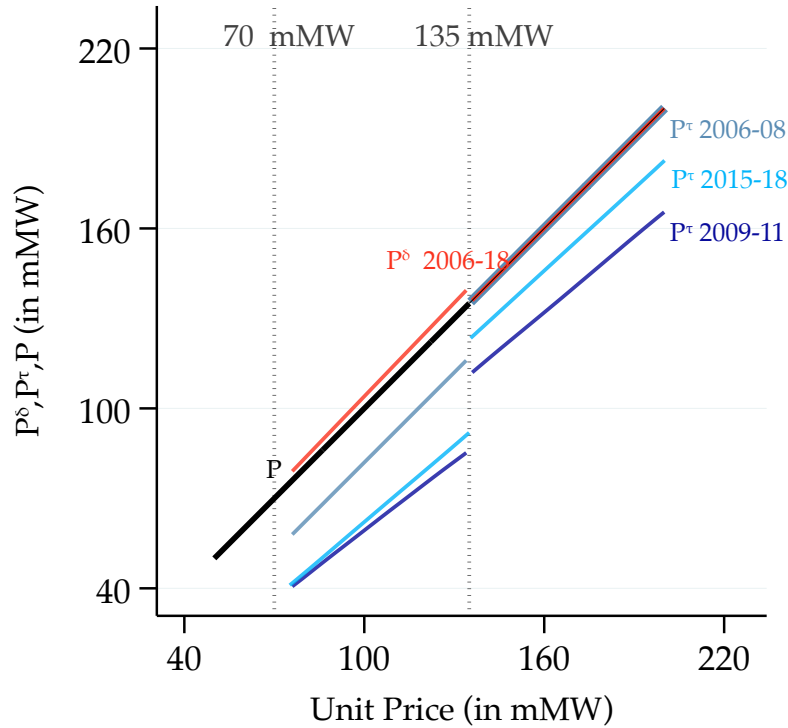
¹⁷This policy instrument was first introduced in the 1995. Even-thought it has been regulated by different laws and acts, for example, Law 1607 of 2012 or Act 2924 of 2013 (Camacol (2016) p.25.), it has always had the same incentive capped at 4 percent of the value of each unit.

it is crucial to understand the effects on the housing market and the welfare gains associated with these subsidies.¹⁸

B. THE NOTCH

The combination of the arbitrary definition of low-cost housing and supply and demand subsidies creates discontinuous incentives or notches around the low-cost housing cutoff point.

Figure 2: The Notch



NOTE: This figure compares the **market price** P , the price received by developers P^δ , and the price households pay net of subsidies (in blue). The x-axis represents the market price P , and the y-axis represents the price received by developers P^δ or paid by households P^τ . The 45-degree black line represents the market price. The three different blue lines correspond to the three subsidy schemes available during the study period, P^τ 2006-08, P^τ 2009-11 and P^τ 2016-18. The price paid by households is $P - \tau^m - \tau^i$, τ^m is a transfer from the government for the downpayment and does not depend on the price of the house. τ^i are the savings in interest rate payments, because this depends on the mortgage; it is calculated by taking a typical mortgage at each market price using administrative records from the Ministry of Housing.

¹⁸See Pattillo (2013) for discussion about housing as a commodity vs a right

Relevant prices. The subsidy scheme creates three different prices: transaction or market price, P ; developer price, P^δ , or the price per unit that developers receive after including tax refunds; and household price, P^τ , or the price households pay net of subsidies. Therefore, $P^\delta = P \cdot (1 + \delta)$ and $P^\tau = P - \tau$. Where, $\delta = 4$ percent is the tax refund rate, and $\tau = \tau^m + \tau^i$ is the total amount of money paid by the government for a housing unit. τ^m represents the downpayment subsidy and τ^i is the interest rate subsidy. The downpayment assistance is a fixed amount, independent of the housing price. The interest rate subsidy is related to the housing price to the extent that they depend on the size of the mortgage. There is an interest rate subsidy above the low-cost housing cutoff, but there is a jump in the subsidy at that cutoff. For example, in the 2016-18 period, the subsidy goes from 4pp for a house with a price below 135 *mMW* to 2.5 pp above that cutoff. I use administrative records to calculate the government expenditure on interest rate payments using a typical mortgage at each price level (see details in Appendix A).

The notch. The demand notch is the difference between the blue lines and black line in Figure 2. A household buying a housing unit below the cutoff qualifies for more subsidies; the blue lines are below the black lines because of the housing subsidies. The gap between the black line and the blue line is the money paid by the government τ . The supply notch is the difference between the red and black lines. Figure 2 illustrates how the subsidy scheme creates incentives for developers and households to build and buy housing units with a price at or below the cutoff. By reducing the housing consumption from above to below the cutoff, households and developers have a discontinuous jump in the price they pay or receive.

Notch over time. The notch on the demand side increases over time. The **gray blue** line shows the household price when only the downpayment subsidy was available between 2006-08. Before the government introduced the interest rate subsidy, households buying a unit priced above the cutoff paid the full price. In the figure, the black and blue lines coincide above the cutoff. The **dark blue** line shows the price paid by a household that gets the downpayment subsidy and the interest rate subsidy during 2009-11. The interest rate is also available if households get a unit above the price cutoff, but the discount is smaller. The two subsidies combined increase the discontinuity or notch at the cutoff. The **light blue** line shows the price paid by households who received the two subsidies after the *Mi Casa Ya* program was introduced and the increase in the downpayment subsidy. During this period, there was a drop in the interest rate and therefore the interest rate subsidy was lower. This explains why the price paid by households below the cutoff was similar during

2009-11 and 2016-18, even if the downpayment subsidy was higher. It also explains why the price in 2016-18 was lower above the cutoff. Despite these changes in the interest rate, the notch increased during this period. There were many changes during the period between 2012 and 2015. In addition to the 100,000 free housing units priced at $70 \times mMW$ or below the interest rate, the subsidy changed many times. For completeness, I include this period when presenting the data and results; however, I see it as a transition period and therefore pay little attention to it.

Notch size. Table 1 shows the size of the jump at the cutoff during the study period and the number of assigned subsidies for each program. Around 45,000 households received the downpayment subsidy each year, with slight variation across years, and around 22,000 households received the interest rate subsidy. Households can get both supports, but they have to apply separately to each program. Each year, around 17,000 households receive the subsidy from the *Mi Casa Ya* program, which grants both subsidies.

Table 1: Notch and number of subsidies by period

	Notch (in mMW)			Subsidies (in thousand)		
	τ^M	τ^i	τ	downpayment	interest rate	Mi Casa Ya
2006-2008	18.0	.	18.0	47.1	.	.
2009-2011	20.0	5.85	25.9	46.4	16.7	.
2012-2015	19.9	9.55	29.5	41.1	22.2	.
2016-2018	25.3	7.24	32.6	44.5	23.4	16.8

NOTE: This table shows the size of the notch in figure and by period and differentiating the discount coming from the interest rate subsidy and the discount from the downpayment assistance. It also shows the number of subsidies (in thousands) assigned to each type of program by year, downpayment, interest rate, and the two together with *Mi Casa Ya*. The value for each period is the average number. Figures A.7 and A.6 shows the loan terms and interest rate over time.

C. DATA

In addition to the administrative records for the subsidies that I presented above, the main analysis of the paper is based on a census of all new construction projects. This subsection introduces that census.

Data source. The data are from a monthly census, called *Coordenadas Urbanas*, collected by the Colombian Chamber of Construction-CAMACOL and containing all new construction units built in 126 Colombian municipalities between 2006 and

2018.¹⁹ The unit of observation is a housing unit type. For example, there may be three different apartment types in a housing development such as studios, one-bedrooms, and two-bedrooms. I observe the price and characteristics of each of them. I observe all housing development projects of at least 300 square meters of construction. The census excludes small, single-family homes and informal housing. It does not contain information on resales of existing housing units. Although this is a limitation of the data, the subsidies apply only to new housing, so the data covers the directly affected part of the market.

General characteristics. The data contain detailed information of the house such as the unit size; location, including the exact latitude and longitude coordinates; number of rooms; quality of appliances; estrato, which is an index summarizing neighborhood quality; and *developer and project characteristics*, like firm tax identifier and the number of units built in each project. The data also include detailed characteristics of the housing development, including the number of parking spots, the number of towers built, the lot size and an indicator function equal to 1 if the units are apartments and 0 if they are single family units, among other details. Finally, I observe the *sale price* at different stages of the construction process. To ease the comparison, I take the price at the beginning of the construction of the project. All prices are in 2019 COP or *mMW*. In Colombia, there is a national *mMW*, which is adjusted every year based on inflation (see Figure A.1). In most of the analysis, I express the price in *mMW* to make it comparable with the price cutoff defining low-cost housing units.

III. HOUSING MARKET RESPONSES

Section II shows how the Colombian social housing policy design creates incentives for households to bunch at the cutoff. In this section, I show the response of the housing market to those incentives. There is clear bunching at the price cutoff, and the bunching gets bigger as the generosity of the subsidies increases.

A. BUNCHING IN OBSERVED MARKET OUTCOMES.

Bunching around the price limit. Figure 2 shows how the subsidy scheme creates incentives for households and developers to buy and build housing units priced at or below the cutoff. The data allow me to differentiate the product choice from the

¹⁹Not all cities have information starting in 2006, the census expanded its coverage over time.

number of units that developers build. I leverage this advantage of the data in the model presented in section IV.

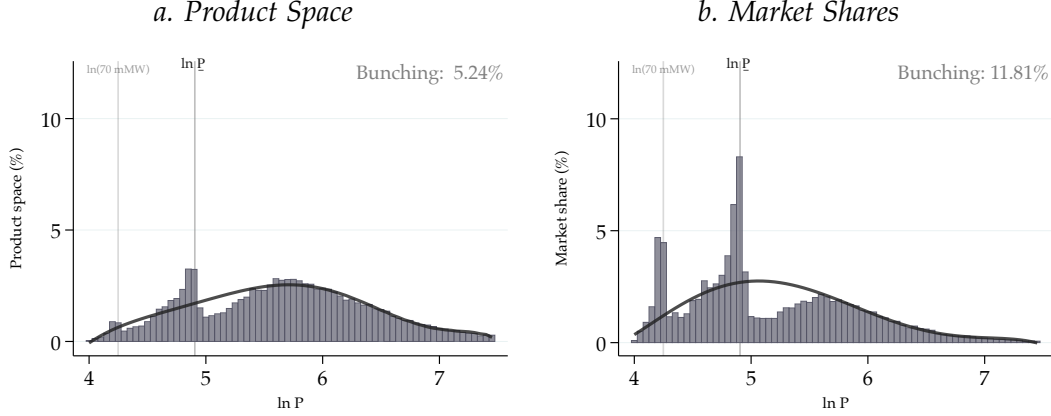
Figure 3 shows the market response to these subsidies. Figure 3a shows the distribution of the product space by unit price (i.e., type of units built). Figure 3b shows the distribution of market shares by unit price for all years and cities in the data. The difference between the two distributions is explained by the fact that developers who build cheaper housing units usually build more units. Apartment buildings often have cheaper housing than projects with single-family homes.

The figure shows a sharp and clear excess mass, or bunching, around the price cutoff defining low-cost housing. For the whole sample, about 12 percent of the market share moves from above the cutoff to below the cutoff. This response is the result of the notched policy design. From this figure it is not possible to determine if the policy induced new units to be built. However, there is a reallocation of the type of new construction. The subsidy moved people and developers from above to below the cutoff. In the absence of the policy, they would buy and produce more expensive housing, but they change their behavior to take advantage of the subsidy. From this, Figure 3 shows why a naive policy evaluation comparing the number of units to the left and to the right of the cutoff would be misleading. In this comparison, the treated group would be “inflated” and the control group “deflated” by households that modify their consumption, but in both scenarios buy a house. Therefore, accounting for this type of response becomes essential to understand the effect of the policy.

Counterfactual. The solid line in Figure 3 represents the counterfactual distribution. That is, the distribution of housing units in the absence of the subsidy. To construct the counterfactual distribution, I follow the standard techniques from the bunching literature (Kleven, 2016). The idea behind the estimation of the counterfactual distribution is to fit a flexible polynomial to fit the observed distributions and include dummies around the discontinuity. The distribution is the prediction of the flexible polynomial once we exclude the bins around the cutoff. To select the parameters related to the estimation, I follow a similar approach to Diamond and Persson (2016) and Chen et al. (2021). For a given bin size, I chose the degree of the polynomials and the number of bins to exclude to match the missing and excess mass. The details of the estimation approach are in Appendix B.

Figure 3: Bunching around the Low-Cost Housing Price Limit

All Data 2006-18



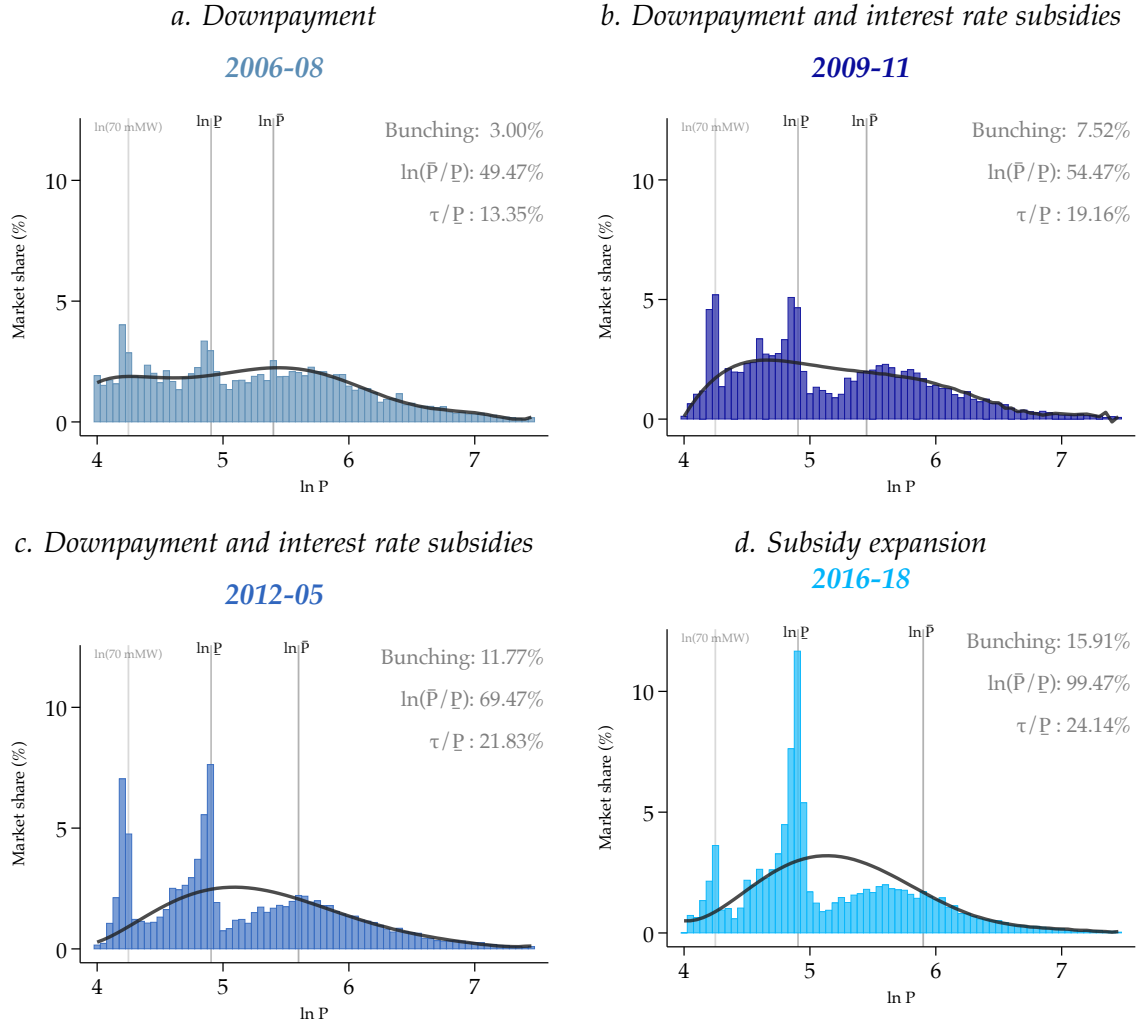
NOTE: This figure shows the distribution or the market share of housing units by sale price (expressed in logs (mMW)). The vertical lines are the cutoffs defining *low-cost housing* $\bar{P} = 135$ mMW and *priority low-cost housing* 70 mMW. The figure shows all the units from 2006 to 2018 in all the cities.

B. BUNCHING OVER TIME AND COUNTERFACTUAL DISTRIBUTION

An advantage of my setting is that I can see how the housing market responds to changes and increases in demand subsidies. The subsidy scheme evolved during my study period, and the demand notch almost doubled. Figure 4 shows the distributions of market share over time. This figure also shows the size of the notch, as the percentage of the price of a house at the cutoff (i.e., $\frac{\tau}{\bar{P}}$) and the maximum change in housing consumption (i.e., $\ln(\frac{\bar{P}}{\underline{P}})$). The relationship between these two magnitudes give me a reduced-form semi-elasticity, relating how much households are willing to change their housing consumption to get the subsidy.

Bunching over time results. Figure 4 shows these magnitudes based on comparisons between the observed distribution and the calculated counterfactual distributions. The figure provides compelling evidence that the housing market responds to the subsidy scheme. At the beginning of my period, when only the downpayment for formal employees was available, households reduced their housing consumption by up to 50 percent to receive a subsidy of 13 percent of the value of the house. The semi-elasticity is 3.85 . Bunching in that period is 3 percent. This number is equivalent to the market share that changes its behavior to take advantage of the subsidy. In 2009-11, when the interest rate subsidy appeared, the notch jumped to 19.2 percent of the house price at the cutoff. Consequently, the share of households responding in this dimension is . In 2012-15 there is a big bunching point at the cut-

Figure 4: Bunching over time



NOTE: This figure shows the distribution or the market share of housing units by sale price (expressed in log of mMW). The lines are the cutoffs defining *low-cost housing* $\bar{P} = 135$ mMW and *priority low-cost housing* 70 mMW. The additional lines shows the point, \bar{P} , where the counterfactual and observed distribution coincide again after the cutoff. The figure shows the different periods for all available cities.

off point of 70 mMW. This corresponds to the program of 100 thousand free housing units for the most vulnerable, which took place during that period (A. Gilbert, 2014; Camacho et al., 2020). The notch also increased, which triggered a larger share of the market to modify housing consumption to take advantage of the subsidy. In 2016, when the program *Mi Casa Ya* was introduced, households could receive almost a quarter of the value of the house if they reduced their consumption to qualify for the subsidy. Up to percent of households modify their behavior in this way. The semi-elasticity for this period is 3.85 .

C. BUNCHING AS AN EQUILIBRIUM RESPONSE

To be able to learn something about the market structure from these reduced-form evidence, it is crucial to know the specific responses of developers and households that lead to the observed market equilibrium.

Market adjustment on housing characteristics. The mechanism explored in this paper is that households and developers adjust the characteristics of the housing units they buy and produce to take advantage of subsidies. There are two main reasons for taking this approach. First, the construction sector is perceived as highly competitive and developers have no incentive to build larger units when, for the same price, households would buy smaller units.²⁰ Second, Figure 5 provides suggestive evidence that the subsidy scheme affects the characteristics of the housing stock.

Alternative explanations. There could be at least three explanations that explain this equilibrium. First, a change in housing characteristics is the main explanation explored in this paper. In this approach, households and developers change the type of housing they are consuming and building. This would include less quality, smaller, or fewer amenities. Second, a pure behavioral response. In the literature, the bunching is usually explained by reporting (Chetty et al., 2013) or relabeling (Chen et al., 2021). However, as I show in the next section, it can also be explained by real market response like changes in characteristics. In the institutional setting studied in this paper, there are many agents with competing interests; households, banks, developers and the government. This makes a simple reporting response costly and less likely.²¹ For example, banks do not have an incentive to under-report the price of the house for the mortgage. A third explanation could be a response only in prices. Households and developers buy and produce the same type of housing, but developers reduce the price for units above but close to the cutoff, allowing households to get the subsidy. This explanation would require an high degree of market power. Although the last two explanations are plausible, a detailed investigation of them is beyond the scope of this paper. The explanation explored in this paper rationalizes the observed equilibrium.

²⁰This argument will be clearer when I introduce the model in the next section. The argument applies for any characteristics that imply any cost for developers.

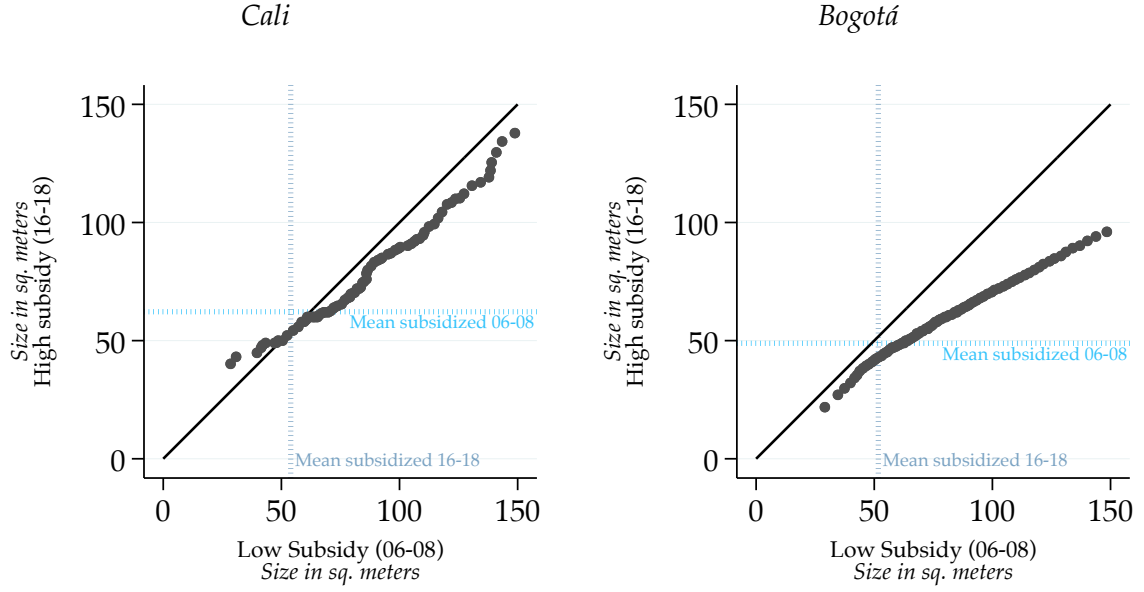
²¹Anecdotal evidence may suggest that some housing units sold in expensive neighborhoods as low-cost housing show miss-reporting (Metrocuadrado, 2022; radio, 2022). However, in many cases those houses do not have appliances when they are sold or are extremely small (20 square meters). In that sense, this type of response falls in the category of market response and not miss-reporting.

Housing characteristics. It is difficult to summarize housing units into a single variable, as they differ in many dimensions. However, focusing on a single characteristic makes the analysis more tractable. My data have exact size, which allows me to use this feature to analyze the structure of the housing market. Choosing size as the main variable in the analysis has several advantages. First, it is a concrete feature of the apartment itself, as opposed to the amenities of the neighborhood. Size is easy for builders to adjust in response to government policy, than neighborhood amenities. Second, the detail of the data allows to control for an unusually large set of variables that may be correlated with size, including apartment and building characteristics, neighborhood quality (estrato, exact location), and structural characteristics of the house such as number of rooms, if there is a porch, number of bathrooms, among others. Third, size has the strongest reduced-form association with price. Fifth, it allows estimation of a continuous implicit price function which is important for the modeling approach considered in this paper; 5) has a monotonic relationship with price, unlikely to be the case for other continuous variables (exact location). Finally, other characteristics such as quality are not detailed enough to allow for as plausible an analysis of overall market structure. Lastly, as Figure 5 shows, the market seems to respond in unit size.

Size response. Figure 5 shows that with the increase in subsidies, the housing size distribution is affected. Specifically, it is affected around the median size of the subsidized housing. Figure 5 has quantile-to-quantile plots for housing size at the beginning and end of the study period for two different cities (Figure C.10 in the appendix shows more cities). During this period, the notch induced by the policy increased from 18 *mMW* to 33 *mMW*. If the distribution of housing size did not change from the beginning to the end of the study period, the black dots would be on the 45-degree line. The blue dotted lines show the average size of a subsidized house. These figures suggest a change in the size distribution around the average subsidized unit. Changes in housing characteristics, particularly size, can explain the increase in bunching from 2006-08 to 2016-18.

Section takeaways. This section provides compelling evidence that the Colombian housing market responded to the discontinuous incentives generated by the subsidy scheme. The suggestive evidence supports the view taken in this paper that these responses come from changes in housing characteristics and, in particular, households and developers buying and building smaller housing units to take advantage of the subsidy. How does the supply side adjust to this change? How is the equilibrium price set? Is the subsidy to developers necessary to prevent housing rationing? Are

Figure 5: Quantile-to-Quantile Plots of Housing Size: Low versus High Subsidy Periods



NOTE: This figure shows the quantile-to-quantile plots for observed housing size in square meters for two representative cities, Cali and Bogotá. The y-axis shows the size at the end of the period, when subsidies are high, and the x-axis shows the size at the beginning of the period, when subsidies are low. The dotted vertical and horizontal lines show the average size of subsidized units. The dots represent the same quantiles in both years. If there are no changes in housing size, they would be on the 45-degree line. Instead, the figure shows how there are changes in size at the quantiles near the average subsidized house.

there any inefficiency gains or welfare losses associated with the subsidy scheme? The purpose of the remaining sections of this paper is to be able to address these questions.

IV. COMPETITIVE HOUSING MARKET EQUILIBRIUM MODEL

This section introduces a housing market equilibrium model. There are three main objectives for the model. First, provide a framework for rationalizing the observed equilibrium and understanding the economic behavior driving the market equilibrium. Second, it describes the equilibrium conditions for the model and the role of the hedonic price function. Third, it motivates a novel identification approach to recover the behavioral parameters of households and developers.

A. MODEL SETUP

The proposed model introduces the discontinuous incentives produced by the Colombian subsidy scheme into a standard hedonic equilibrium model or sorting model.²²

Housing. Housing is a vertically differentiated product characterized as a continuous variable h . In this case, all units are standard units that differ only in how large they are, therefore h represents the size of the house in square meters. The price of the housing unit P depends on the size h , and is described by the implicit price for the size $P(h)$, which can be nonlinear.

Households. Households looking to buy a *new* housing unit are indexed by i , are heterogeneous in their wealth level $Y_i \sim F_Y$.²³ Households decide how much housing to buy, h_i and how much to consume of other goods, C_i , to optimize utility $U(C_i, h_i; \theta)$, where θ is a preference parameter to be estimated.

Developers. Developers are indexed by j and heterogeneous in their productivity $A_j \sim G_A$. They decide what type of product they want to build. In particular, the size of housing units, h_j , to maximize profits. The number of units, Q_j , is determined exogenously by the function $Q(h_j)$. The difference between the choice of units they build and the type of units they build is behind the differences between the distribution of market shares in Figure 3. They face construction costs $B(h_j, Q(h_j); \beta)$ ²⁴ where β characterizes the cost function and is the supply parameter to be estimated.

Simplifying assumptions. I introduce three simplifying assumptions. First, I assume that the market is perfectly competitive, that is, developers cannot individually affect prices and $P(h)$ is independent of Q . Second, developers only choose the unit size they build. They follow a unit supply function that is exogenous and differentiable

²²This is a canonical version of a model with heterogeneous households and developers buying housing units of different sizes. For ease of exposition, I simplify it by assuming that a single variable describes the housing. For some examples of these types of models, without a notch in the budget set, see S. Rosen (1974), Epple (1987), Ekeland et al. (2004), Bajari and Benkard (2005), Heckman et al. (2010), Epple et al. (2020) or Chernozhukov et al. (2021). The literature based on this models is summarized by Kuminoff et al. (2013) and Greenstone (2017). For a survey of the empirical applications see Palmquist (2006).

²³I call Y_i wealth for simplicity. It is a measure containing wealth, assets and their returns, transfers, income, etc. F_Y is the *cdf* describing the wealth distribution

²⁴The cost function $B(Q, h, A_j; \beta)$ is derived from minimizing the production constraints related to producing Q units with characteristics h . A_j reflects underlying variables in the cost minimization, that is, factor prices and production function parameters. Different values of A express different factor prices or productivity among developers. For a discussion, see S. Rosen (1974, p.43)

$Q = Q^S(h)$. The number of units does not need to be predetermined since apartment size is an endogenous choice, but the allocation of property to developers is predetermined. This is, for a given lot, households need to decide the size of the units, and the regulation framework and construction and technological constraints will determine how many units they can build. Third, construction costs depend on $Q(h)$, h , and productivity levels, that is, $B = B(Q(h), h, A_j; \beta)$. The last two simplifying assumptions make it straightforward to specify functional forms for the profit function and to offer curves. Allowing for a completely endogenous choice of Q could be a better characterization, but obtaining a functional form for the offer curve, which is essential in the identification approach, is highly dependent on particular functional forms. Relaxing this assumption and allowing for imperfect competition is feasible, but beyond the scope of this paper.

Equilibrium. When households decide the type of units they buy, they choose the developer type from which to buy, and vice versa. Then, the equilibrium is an implicit price making the densities of the housing units demanded and produced match.

B. OPTIMAL CHOICES

Prices

Section II explained that given the subsidy scheme, there are three relevant prices. They are the market, household and developer price.

$$\text{Market:} \quad P(h) \quad (1)$$

$$\text{Household:} \quad P^\tau(h, \tau) = P(h) - \tau \cdot \mathbb{1}[P(h) \leq \underline{P}] \quad (2)$$

$$\text{Developer:} \quad P^\delta(h, \delta) = P(h) (1 + \delta \cdot \mathbb{1}[P(h) \leq \underline{P}]) \quad (3)$$

size threshold Note that given the price function $P(h)$, there is a maximum size that households can buy to qualify for the subsidy. This is the size threshold;

$$\underline{h} = P^{-1}(\underline{P}) \quad (4)$$

Differences in prices. A household buying a low-cost house pays a price $P^\tau(h, \tau)$ instead of $P(h)$, and developers who build low-cost houses can get back the VAT taxes paid for the construction materials. The reimbursement of VAT taxes cannot exceed a value $\delta = 4$ percent of the value of the house. In other settings where the price can increase and the limit is set in terms of size, market equilibrium could be

achieved by increasing the price and δ would represent a premium to build low-cost housing. The price function $P(h)$ can be a continuous and differentiable function for all $h \in \mathcal{H}$, but the developer and the household price functions, $P^\delta(h, \delta)$, and $P^\tau(h, \tau)$, are not differentiable at \underline{P} .

Decision Problem

Households. A household $i \in N$ maximizes its utility given its level of wealth Y_i . It solves the following optimization problem:

$$\begin{aligned} & \max_{h, C} \quad U(h, C; \theta) \\ \text{subject to:} \quad & Y_i = P^\tau(h, \tau) + C, \\ & h \geq 0. \end{aligned}$$

Bid functions (or indifference curves). $\varphi_D(h, Y, \bar{U}; \theta)$, represent all the combinations of prices P and unit size h that provide the same level of utility \bar{U} to a household with $Y = Y_i$. Therefore, φ_D is such that

$$\bar{U} = U(h, Y_i - \varphi_D; \theta) \quad (5)$$

Developers. Developer's profits $\pi(Q, h, A_j)$ are determined by the total revenue minus costs.

$$\begin{aligned} & \max_h \quad \pi(Q, h, A_j) \\ \text{subject to:} \quad & \pi = Q \cdot P^\delta(h, \delta) - B(Q, h, A_j; \beta) \\ & Q = Q(h) \end{aligned}$$

Offer function (or iso-profits) The offer function represents the indifference surface for all possible combinations of prices and size h providing the same profits. φ_j^s represents the price that developers are willing to accept at different unit sizes to obtain the same level of profits $\bar{\pi}_j$. To define the offer function, I replace the developers' price, $P^\delta(h, \delta)$, by φ_j^s , profits by $\bar{\pi}$, and solve for φ_j^s ,

$$\varphi_j^s = \frac{B(Q^s(h), A_j; \beta) + \bar{\pi}}{Q^s(h)} \quad (6)$$

Tangency Conditions

Households. On the demand side, households choose their housing size h to maximize their utility. Due to the notch in the budget set, the standard *tangency conditions* do not correspond to the optimal choice for all households. I define the *tangency conditions*,²⁵

$$\frac{\partial P(h)}{\partial h} = \frac{\frac{\partial U(h, C; \theta)}{\partial h}}{\frac{\partial U(h, C; \theta)}{\partial C}} \quad (7)$$

Assuming that equation 7 has a unique solution and using the budget constraint, $P^\tau(h, \tau) - Y_i = C_i$, we can solve for h^* , the choice of housing satisfying *tangency conditions*.²⁶

$$h^*(Y_i, \tau; \theta, \underline{P}) = \begin{cases} h(Y_i + \tau; \theta) & \text{if } P(h) \leq \underline{P} \\ h(Y_i; \theta) & \text{if } \underline{P} < P(h) \end{cases} \quad (8)$$

Developers. On the supply side, the design that satisfies the optimality conditions $h^*(A_j, \beta)$ for a given price function $P(h)$ is achieved when developers maximize profits subject to the developer's price being equal to the offer curve $P^\delta = \varphi^s$. The unit size that satisfies the tangency conditions $h^*(A_j, \beta)$ and the optimal profits $\bar{\pi}(A_j, \beta)$ are achieved when the price and offer curves are tangent.

$$\frac{\partial \varphi^s(h, A_j; \beta, \bar{\pi})}{\partial h} = \begin{cases} \frac{\partial P(h)}{\partial h} \cdot (1 + \delta) & \text{if } P(h) \leq \underline{P} \\ \frac{\partial P(h)}{\partial h} & \text{if } \underline{P} < P(h) \end{cases} \quad (9)$$

We can solve 9 for h , and obtain an expression for the tangency conditions,

$$h^*(A_j, \delta; \beta, \underline{P}) = \begin{cases} h(A_j, \delta; \beta) & \text{if } P(h) \leq \underline{P} \\ h(A_j; \beta) & \text{if } \underline{P} < P(h) \end{cases} \quad (10)$$

²⁵This follows by defining a Lagrangian and taking first-order conditions with respect to h and C and taking the ratio. I assume that the composite good has a price $p_c = 1$

²⁶It has been discussed in the literature that a sufficient condition for this to hold is to assume a Spence-Mirrlees type single crossing condition. See for example, Heckman et al. (2010, p.1573) or Kuminoff et al. (2013) for an overview.

C. MARGINAL BUNCHERS AND OPTIMIZER TYPES

The individual level demand and supply do not correspond to optimality conditions in this setting because there is a subset of households for which it is optimal to sacrifice housing consumption to obtain the subsidy. For developers, it is also beneficial to produce a smaller housing unit to benefit from the tax refund. There are three types of households and developers; *always-takers*, *marginally subsidized*, and *never-takers*. To define them, I use two key agent types; *marginal buncher* and *threshold optimizer* for both households and developers. The marginal buncher agents are a key component of the empirical approach of this paper; they define the identification approach presented in Section V. They are indifferent to changing their behavior and receiving the subsidies or not changing their behavior and buy and produce the housing unit satisfying the optimality conditions.

MARGINAL BUNCHER HOUSEHOLD: $Y_i = \bar{Y}$

$$h^*(\bar{Y}, \tau; \theta, \underline{P}) = \bar{h} \iff U(\bar{Y} - P^\tau(\bar{h}, \tau), \bar{h}; \theta) = U(\bar{Y} - P^\tau(\underline{h}, \tau), \underline{h}; \theta) \quad (11)$$

MARGINAL BUNCHER DEVELOPER: $A_j = \bar{A}$

$$h^*(A_j, \delta; \beta) = \bar{h} \iff \pi(Q(\underline{h}; \alpha), \bar{A}; \delta) = \pi(Q(\bar{h}; \alpha), \bar{A}; \delta) \quad (12)$$

Threshold optimizer are the households and developers optimizing at $P^{-1}(\underline{P}) = \underline{h}$. They have wealth and productivity $Y_i = \underline{Y}$ and $A_j = \underline{A}$ respectively.

Individual-Level Supply and Demand. The demand and supply function will be different for the three groups of agents. The *always-takers* with $Y_i \in (0, \underline{Y})$ and $A_j \in (0, \underline{A})$, receive subsidies and optimize at the tangency point. The *Marginally subsidized* with \underline{h} $Y_i \in (\underline{Y}, \bar{Y})$ and $A_j \in (\underline{A}, \bar{A})$, are the ones that bunch at the cutoff. The policy design induces a change in their behavior but they consume and produce less housing than the optimality conditions would suggest. An important component of the welfare analysis in Section VII is to calculate the size of this efficiency cost. The *never-takers* $Y_i > \bar{Y}$ and $A_j > \bar{A}$ do not find it beneficial to modify their behavior to take advantage of the subsidy.

Figure explanation. Figure 6 shows an example of the equilibrium choices of developers and households. The price function is the envelope of the offer curves when developers produce their optimal unit size and the assigned number of units.

The figure shows a representative marginal buncher household and developer. It also shows in gray marginally subsidized households and developers, which are the agents that change their behavior to take advantage of the subsidy. A developer type A_j matches with a household type Y_i in terms of their optimal choice of h when the dashed lines meet. However this is not an equilibrium choice because both developers and households can be better off if they reduce size h . Figure D.11a, shows the case of subsidized households and developers. Below \bar{h} , developers receive $P(1 + \delta)$ and households pay $P - \tau$. Developers and households increase their utility and profits as a result. The marginal bunching agents are indifferent between getting the subsidy or not. The identification approach in this paper relies on these agents and therefore the main identification strategy is conveyed in Figure 6. The idea is that the bunching in the observed equilibrium distribution allows me to recover \bar{h} . Therefore, I can observe two points, \underline{h} and \bar{h} , on the same indifference curves and offer functions and recover their shape. Figure D.11 shows the optimal choices for other types of developers and types of households.

D. MARKET-LEVEL SUPPLY AND DEMAND

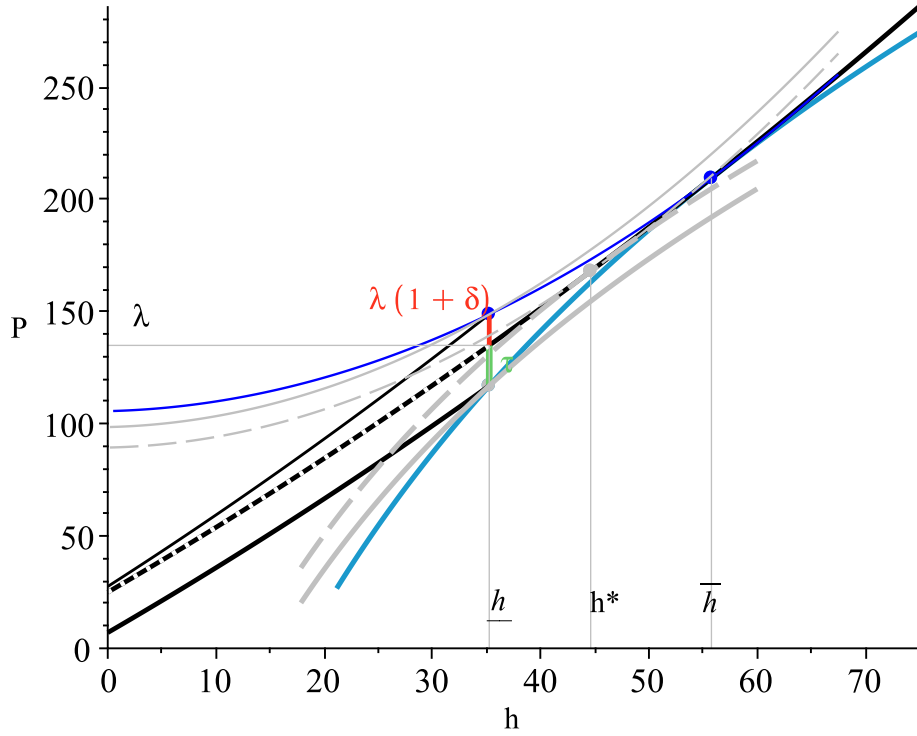
The market level demand and supply is defined by the individual demand and supply represented in Figure 6 and the distribution of wealth and productivity. The approach to derive the market-level supply and demand is to use the optimality conditions and the distributions F_Y and G_A and a change of variable formula.²⁷

Graphical Representation Figure 7c shows the product space or developer density, and the exogenous unit supply function. Figure 7a shows an example of the equilibrium density when f_Y and g_A follow a log-normal distribution. The equilibrium price makes the product of the functions in figures 7c and 7b to match the demand density in Figure 7c. The observed density function suggests that the market equilibrium has a discontinuous density and that this stylized model can explain the observed equilibrium represented in Figure 4.

Productivity and Income Mapping to Housing Size. Households and developers only differ in wealth Y_i , and productivity A_j . If $h^*(Y_i, \tau; \theta, \underline{P})$ is strictly monotone, there

²⁷Heckman et al. (2010, p.1571) derives the market demand and supply densities in this way for the case without a notch.

Figure 6: Marginally Subsidized and Marginal Buncher Agents' Choices



NOTE: This figure shows the optimal choices for the marginal buncher household and developer. The figure presents the intuition for the identification idea. The gray offer and bid functions represent the indifference curves for the marginally subsidized agents. These are the ones who can increase their profits or utility by increasing or reducing h to take advantage of the subsidy and tax incentives. The demand and supply functions are defined as follows:

$$h^D = \begin{cases} h^*(Y_i, \tau; \theta, \underline{P}) & \text{if } Y_i \leq \underline{Y} \\ \underline{h} & \text{if } Y_i \in (\underline{Y}, \bar{Y}) \\ h^*(Y_i, \tau; \theta, \underline{P}) & \text{if } \bar{Y} \leq Y_i \end{cases} \quad h^S = \begin{cases} h^*(A_j, \delta; \beta, \underline{P}) & \text{if } A_j \leq \underline{A} \\ \underline{h} & \text{if } A_j \in (\underline{A}, \bar{A}) \\ h^*(A_j, \delta; \beta, \underline{P}) & \text{if } \bar{A} \leq A_j \end{cases}$$

is a one to one mapping between Y_i and A_j and the optimality conditions.²⁸

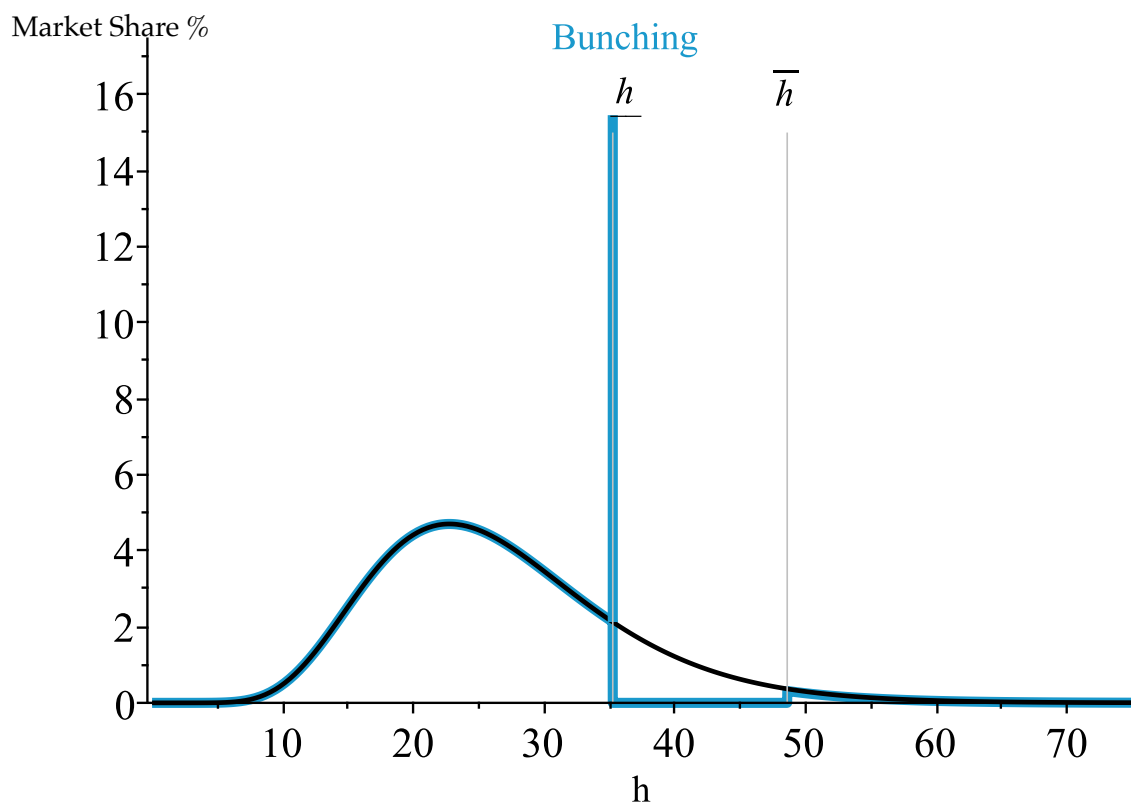
$$Y_i = \tilde{Y}(h, \tau; \theta, \underline{P}) = h^{*-1}(h_i, \tau; \theta, \underline{P}) \quad (13)$$

$$A_j = \tilde{A}(h; \beta, \delta) = h^{*-1}(A_j, Q(h); \beta, \delta, \underline{P}) \quad (14)$$

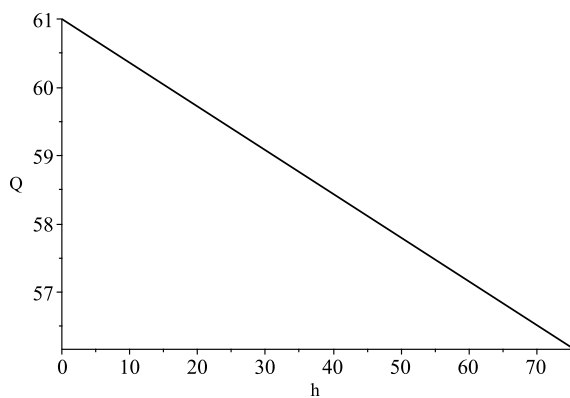
²⁸This mapping from housing consumption to income is a consequence of the assumption $\theta_i = \theta \forall i$. If I allow heterogeneity in θ , the same demand for housing h can come from different combinations of Y_i, θ_i .

Figure 7: Equilibrium Density, Developer's Choice Density and the Unit Supply Function

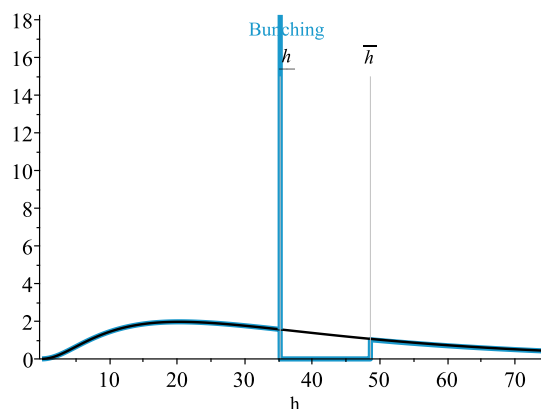
a. $Y \sim \log \text{ normal } Y$



b. Unit supply function $Q(h)$



c. Density function $g_h(h)$



NOTE: This figure shows the equilibrium market share or distribution of units by standard unit size for a given income density f_y following a log-normal distribution. The bottom 2 figures show the share of developers choosing to build at each unit size and the unit supply function.

From distribution of income and productivity to a size distribution. The share of households and developers choosing h determines the market-level demand and supply

densities. Using equations 13 and 14, we can get the distribution of market shares and the product space that satisfies the optimality conditions of the market.

$$f_{h^*} = \begin{cases} f_Y(\tilde{Y}(h, \tau \neq 0; \theta, \underline{P})) \frac{d}{dh} \tilde{Y}(h, \tau \neq 0; \theta, \underline{P}) & \text{if } h < \underline{h} \\ f_Y(\tilde{Y}(h, \tau = 0; \theta, \underline{P})) \frac{d}{dh} \tilde{Y}(h, \tau = 0; \theta, \underline{P}) & \text{if } \underline{h} < h \end{cases} \quad (15)$$

$$g_{h^*} = \begin{cases} g_A(\tilde{A}(h; \beta, \delta \neq 0)) \frac{d\tilde{A}(h; \beta, \delta \neq 0)}{dh} & \text{if } h < \underline{h} \\ g_A(\tilde{A}(h; \beta, \delta = 0)) \frac{d\tilde{A}(h; \beta, \delta = 0)}{dh} & \text{if } \underline{h} < h \end{cases} \quad (16)$$

Densities

The distributions f_{h^*} and g_{h^*} and the demand and supply functions, $h^D(Y_i; \tau, \theta, \underline{P})$ and $h^S(A_j, \delta; \beta, \underline{P})$, allow to derive a the market-level demand density function, $D_h(h; \tau, \theta, \underline{P})$, and the market-level supply function $S_h(h, \beta, \delta)$.

Aggregate Demand density. The demand for housing at the size limit \underline{h} contains the demand for the *threshold maximizing households*, $f_{h^*}(\underline{h}; \tau, \theta)$, and the *marginally subsidized households* $\int_{\underline{h}}^{\bar{h}} f_{h^*}(h; \tau, \theta, \underline{P}) dh$. Finally, there is no demand for housing units with $h \in (\underline{h}, \bar{h})$.

$$D_h = \begin{cases} f_{h^*}(h; \tau, \theta, \underline{P}) dh & \text{if } h < \underline{h} \\ f_{h^*}(\underline{h}; \tau, \theta, \underline{P}) dh + \int_{\underline{h}}^{\bar{h}} f_{h^*}(h; \tau, \theta, \underline{P}) dh & \text{if } \underline{h} = h \\ 0 & \text{if } h \in (\underline{h}, \bar{h}) \\ f_{h^*}(h; \tau, \theta, \underline{P}) dh & \text{if } \bar{h} \leq h \end{cases} \quad S_h = \begin{cases} g_{h^*}(h; \beta, \delta) \cdot Q(h) & \text{if } h < \underline{h} \\ g_{h^*}(\underline{h}; \beta, \delta) \cdot Q(h) + \int_{\underline{h}}^{\bar{h}} g_{h^*}(h; \beta, \delta) dh \cdot Q(h) & \text{if } \underline{h} = h \\ 0 & \text{if } h \in (\underline{h}, \bar{h}) \\ g_{h^*}(h; \beta, \delta) dh \cdot Q(h) & \text{if } \bar{h} \leq h \end{cases} \quad (17)$$

Given the hedonic price function $P(h) = P$ we can use a change of variable formula to get the market distribution in terms of price analogous to Figures 3 and 4.

E. MARKET EQUILIBRIUM

The housing market achieves an equilibrium E when, a given price scheme $P(h)$, market-level demand and supply are equal for all values of h :

$$E = \left\{ P(h) \in \mathcal{P} : D(h; \tau, \theta, \underline{P}) = S(h; A_j) \forall h \in \mathcal{H} \right\} \quad (18)$$

$D(h; \tau, \theta, \underline{P})$ to be equal. The equilibrium price function allows the match between types of households and developers that clears the market.

Existence of hedonic equilibrium. The existence of a hedonic equilibrium has received comparatively less attention than the identification of this type of model. [S. Rosen \(1974\)](#) and [Epple \(1987\)](#) show that under some specified utility functions, cost functions, and distributions for the unobserved heterogeneity, a closed-form solution for the equilibrium price function exists. [Heckman et al. \(2010\)](#) explicitly describe how the equilibrium price function depends on the distributions of observable characteristics of firms and workers. [Ekeland \(2010\)](#) shows an existence proof and provides a particular example of an equilibrium. Moreover, [Bajari and Benkard \(2005\)](#) prove that in equilibrium, the price of a differentiated product will be a function of its characteristics if the utility is continuously differentiable, monotonic in numeraire, and Lipschitz continuous. Using some particular functional forms, the model presented in this paper can have an analytical solution.²⁹

V. IDENTIFICATION AND ESTIMATION

The behavioral parameters to estimate are θ, β , which describe the curvature of the bid and offer curves. This section explains the identification and estimation of those parameters.

A. IDENTIFICATION OF THE STRUCTURAL PARAMETERS

Marginal Buncher indifference conditions. The identification argument in this paper follows the approach used by [Best et al. \(2019\)](#) and, more generally, the one suggested by [Bertanha et al. \(2021\)](#) and [Blomquist et al. \(2021\)](#). The idea is that the existence of the marginal buncher allows observing two points in the same bid and offer function.³⁰ Following the hedonic literature, the identification and estimation

²⁹A particular example is available upon request.

³⁰[Best et al. \(2019\)](#) proposition 1 and [Bertanha et al. \(2021\)](#) Theorem 1 both prove identification using the same identification idea as in this paper. The identification in my setting follows the same

approach follows a two-step procedure. In the first step, I use the analysis in Sections IV and III, to obtain the hedonic price function, the notch and the behavioral responses. In the second step, I use the marginal bunching condition to solve for the two parameters of interest from equations 11 and 12.

The estimation approach follows a two-step procedure. The first step estimates the hedonic price function at \bar{h} , $P(\bar{h})$, the three prices at \underline{h} , $P^\tau(\underline{h})$, $P(\underline{h})$, and $P^\delta(\underline{h})$, and the unit supply function $Q(h)$ at \underline{h} and \bar{h} , $Q(\underline{h})$, and $Q(\bar{h})$. The second step uses these estimates and the marginal buncher condition to recover structural parameters β and θ as the solutions of the marginal buncher equations. I do not observe \bar{Y} and \bar{A} , but I use the fact that, given the assumptions I impose in this paper, there is a one-to-one mapping between h and Y , and A , see equations 13, 14. This allows me to express \bar{Y} and \bar{A} in terms of observable characteristics. This requires estimates for $\frac{\partial P(h)}{\partial h}|_{h=\bar{h}} = p(\bar{h})$ and $\frac{\partial Q(h)}{\partial h}|_{h=\bar{h}} = q(\bar{h})$. Table 2 shows the functional forms that I use to recover θ and β and the elements that I need to estimate in the first step.

The two unobservable objects are the parameters that describe the utility and cost functions β and θ . All values summarized in Table 2 panel D, can be estimated. The parameter θ is not directly observed and it is assumed to be $\theta = \frac{1}{2}$. The identification of β , θ is achieved by solving two equations with two unknowns. The two equations are the ones in Table 2 panel A, after replacing panels B and C.

The existence of a marginal buncher allows me to address the main challenge of causal inference; observing the same agent at two different states of the world. Using the insights from the bunching literature, we estimate a counterfactual distribution that allows us to observe those to points in the data. Section V.B describes the estimation approach for the values described in Table 2 panel D, and explains how I apply this intuition to my data.

B. ESTIMATION

Observed Equilibrium. Figure 8 shows the joint densities of unit size and market price for all cities around 2006 when the subsidy notch on the demand side was

principal conditional on consistent estimates of the first step. Bertanha et al. (2021) describe the identification assumptions under which we can recover the structural parameters from the observed bunching. They argue that notches allow for the identification of elasticities, whereas kinks need additional assumptions about the unobserved heterogeneity. Blomquist et al. (2021) show the conditions under which elasticities can be identified under notches and kinks. They illustrate their approach using Saez (2010) setting. In contrast to Blomquist et al. (2021) who assume the *pdf* of heterogeneity is monotone, Bertanha et al. (2021) derive partial identification bounds by assuming the *pdf* has a bounded slope. Using censored regression models, covariates, and semi-parametric assumptions on the distribution of heterogeneity, they provide point estimation for kink points.

Table 2: Functional Form and Identification Equations

A. Marginal Buncher Condition	
Household	$V_D = U(\bar{Y} - \bar{P}, \bar{h}; \theta) - U(\bar{Y} - \underline{P}^\tau, \underline{h}; \theta) = 0$
Developer	$V_S = \pi(\bar{Q}, \bar{A}, \bar{P}; \beta) - \pi(\underline{Q}, \bar{A}, \underline{P}^\delta; \beta) = 0$
B. Functional Forms	
Utility	$U = [(1 - \theta) \cdot C^\theta + \theta \cdot h^\theta]^{\frac{1}{\theta}}$
Cost	$B = A_j \cdot Q \cdot h^\beta$
C. Optimality Conditions	
Income	$\bar{Y} = \bar{P} - \left(\frac{\theta h^{\theta-1}}{\bar{p}(\theta-1)} \right) \frac{1}{\theta-1}$
Productivity	$\bar{A} = \frac{(\bar{P} \cdot \bar{q} + \bar{p} \cdot \bar{Q}) \bar{h}^{(1-\beta)}}{\bar{q} \cdot \bar{h} + \bar{Q} \cdot \beta}$
D. First Step Estimates	
Marginal buncher thresholds	$\underline{h} = P^{-1}(\underline{P})$ and $\underline{h} = P^{-1}(\underline{P})$
Hedonic price	at \underline{h} : $\underline{P}^\tau = P(\underline{h}) - \tau, \underline{P} = P(\underline{h})$ $\underline{P}^\delta = P(\underline{h}) \cdot (1 + \delta)$ at \bar{h} : $\bar{P} = P(\bar{h}), \bar{p} = \frac{\partial P(h)}{\partial h} \big _{h=\bar{h}}$
Unit Supply Function	at \underline{h} : $\underline{Q} = Q(\underline{h})$ at \bar{h} : $\bar{Q} = Q(\bar{h}), \bar{q} = \frac{\partial Q(h)}{\partial h} \big _{h=\bar{h}}$

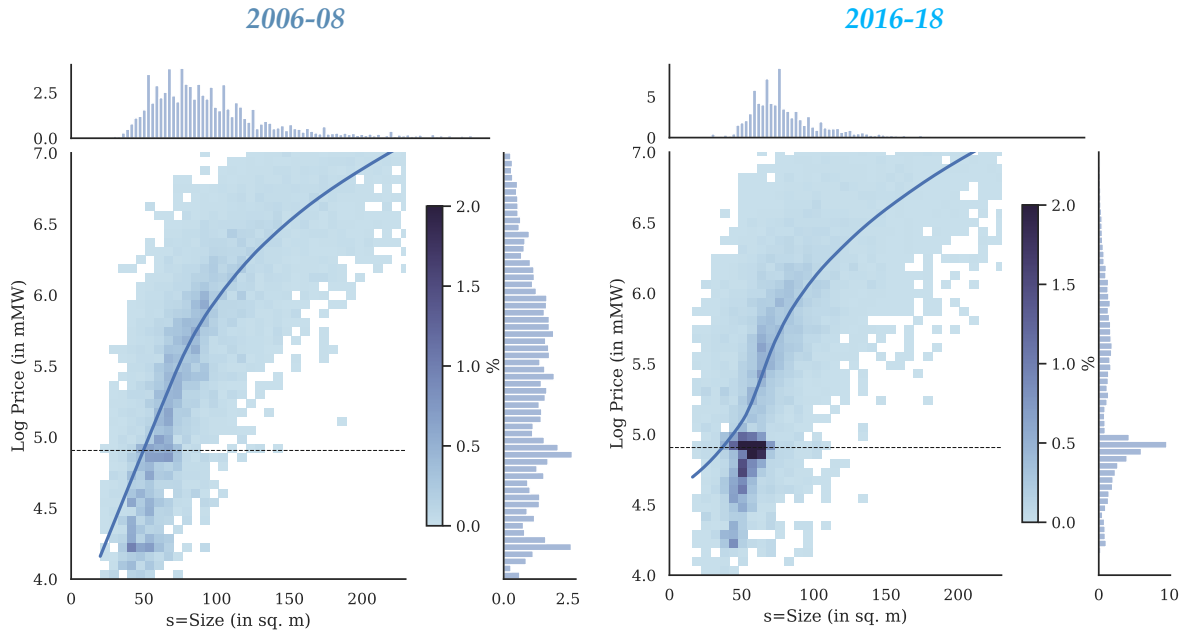
NOTE: This Table summarizes the functional forms used for the estimation of β and θ . θ is assumed to be $\frac{1}{2}$, but section VIII shows sensitivity to different numbers. This parameter corresponds to the share of expenditure on housing.

small and around 2016 when the subsidy was twice as big. In each market, heterogeneous agents buy and sell different housing units. The same money may buy larger housing units in separate submarkets; therefore, agents cluster at different housing sizes for which the sale price is at or below the cutoff point. This figure would be the analog of figures 6 7a, if the only characteristic of a house was size. However, there are other characteristics such as neighborhood quality or structural parameters such as the number of rooms or the availability of extra space such as a studio or a porch. Therefore, to apply the framework to the data, I need to reduce the characteristics of the house into a single characteristic and consistently estimate the implicit or hedonic price of size.

C. HEDONIC PRICE FOR HOUSING SIZE

The estimation of the hedonic price function, subsidy notches, and the bunching analysis from section III are the two key components of the identification and estimation approach of this paper. The hedonic price function plays two roles. First, it allows to recover the function $P(h)$ and marginal willingness to pay for the size of the house $p(h)$ conditional on the other observed characteristics. Second, it allows me to use the bunching on the total price described in Section III to recover \underline{h} and \bar{h} .

Figure 8: Observed Market Equilibrium



NOTE: This figure shows the joint and marginal densities for housing size (x-axis) and price (y-axis). Darker dots inside the graph represent a higher market share. The figure contains all available cities in each period, and all the different unit types, that is, single-family homes, multifamily homes, condos, two bedrooms, one bedroom, and so forth. The solid line represents the non-linear relationship between housing size and price (using lowess).

The solid line in Figure 8 shows that the non-parametric bivariate relationship between price and size is positive. This pattern follows the expected positive relationship and suggests that it could be nonlinear. However, this unconditional relationship may not represent the marginal equilibrium willingness to pay for housing size. There could be observable and unobservable characteristics that affect size and price, creating bias. I follow common practice in the hedonic literature to estimate the equilibrium implicit—or hedonic—price of housing size.³¹ Equation 19 represents

³¹Bishop and Timmins (2019), Bajari and Benkard (2005), Epple et al. (2020) or Bajari, Fruehwirth,

a general specification for the price function. Where s_{ltc} is the size of the house, X_{ltc} is a vector containing all other characteristics of the house, and ω_{ltc} represents the residual containing unobserved characteristics.

$$P_{ltc} = P(h_{ltc}) + \Gamma'X_{ltc} + \varepsilon_{ltc} \quad (19)$$

I observe a type of unit l in the city c at time t . I assume that the housing price is additive and separable in the size of the house s_{ltc} , the observable characteristics are included in X_{ltc} , and ε_{ltc} represents the unobserved characteristics. X_{ltc} includes location, quality, number of rooms and neighborhood quality index (*estratos*),³² among others. $P(\cdot)$ is the implicit price function for the size of the housing. I follow Cattaneo, Crump, Farrell, and Feng (2019b) and Cattaneo, Crump, Farrell, and Feng (2019a) to estimate the function $P(h)$ non parametrically. Their approach also allows me to estimate $p(h)$ non parametrically.³³

To estimate the implicit or hedonic price, I rely on independence conditional on observable characteristics:³⁴

$$E(h_{ltc}|X_{ltc}, \varepsilon_{ltc}) = 0 \quad (20)$$

Independence conditional on observables. It is common to rely on conditional independence to recover the implicit price function of a certain characteristics. In my setting, I observe a rich and unique set of controls. This includes the exact location of the unit and general characteristics of the house, including the number of rooms and the neighborhood quality index. The assumption of conditional independence can be problematic in many settings. For example, Chay and Greenstone (2005) show that using a hedonic model to recover the marginal willingness to pay for air quality without using instruments generates biased results. Omitted variables could generate a bias in the current setting. However, I present two facts that are reassuring. First, in contrast to air quality, the hedonic regression does not show the opposite of the expected sign. Second, when I include characteristics, such as an indicator function equal to one, if the a house has an extra bathroom, or a studio or a porch,

Kim, and Timmins (2012)

³²The *estratos* are codes from 1 to 6. They summarize the quality of the block, for more details, see Uribe (2021)

³³An alternative estimation method Robinson (1988). We could also use a parametric approximation.

³⁴Bajari and Benkard (2005) propose three different identification assumptions; i) Independence conditional on observables, ii) Option packages and iii) instruments. My setting and data allows an implementation of each of the three identification approaches. However, the results presented in this paper rely on the first condition.

the magnitudes of the coefficients do not change. This type of characteristic is potentially unobserved by the econometrician in other settings, as it is related to size; so, it is reassuring that including it does not affect the size of the coefficients. However, this does not rule out the fact that other omitted variables could bias the results. For example, if changes in price generate the bunching with no change in size, the error term could be correlated with size, particularly for observations around the price cutoff.

D. UNIT SUPPLY FUNCTION NOTCH

How do developers respond? One of the principal objectives of the economic model is to address this question in more detail. Developers built more housing units when they built smaller housing units. One advantage of the data is that I observe the number of units built by unit type; therefore, I can get empirical estimates of the trade-off between unit size and the number of units and account for it in the model. I follow a similar strategy that I use to estimate the hedonic regression to estimate this relationship $Q(h)$ nonparametrically.

$$Q_{ltc} = Q(h_{ltc}) + \Omega' X_{ltc} + \epsilon_{ltc}^Q \quad (21)$$

Like in the case of the hedonic regression estimation, I rely on independence conditional on observables.

$$E(h_{ltc} | X_{ltc}, \epsilon_{ltc}) = 0 \quad (22)$$

I estimate $Q(h)$, nonparametrically using the approach proposed by [Cattaneo et al. \(2019b\)](#) and [Cattaneo et al. \(2019a\)](#). This approach also allows me to estimate the derivatives $q(h)$ that I required in the estimation of the structural parameters. In the set of controls, I include detail characteristics of each project such as number of towers, location of the lot, etc. The assumption of conditional independence is plausible in the sense that after controlling by all characteristics, the relationship between h and Q is given exogenously by regulatory constraints or the existent technology.

E. MARGINAL BUNCHING THRESHOLDS:

The relevant observable characteristics described in panel D of Table 2, need to be evaluated at \underline{h} and \bar{h} , which are not directly observable but can be recovered from the data using the estimates of the hedonic price function $\hat{P}(h)$ and the values of \underline{P}

and \bar{P} recovered in Section III.

$$\underline{h} = \hat{P}^{-1}(\underline{\hat{P}}) \text{ and } \bar{h} = \hat{P}^{-1}(\hat{\bar{P}}) \quad (23)$$

The implicit assumption is that I am comparing standard unit housings that only vary in size. The size distribution that would mirror the bunched price distribution is the distribution of the standardized size, not the observed size distribution. See Appendix IV.A for an example with a parametric specification for $P(h)$.

F. MISSING MASS: MODEL VS. DATA

The model predicts a missing demand and supply for housing units between \underline{h} and \bar{h} . However, in my setting, I only observe a partial missing mass in the distribution. This partial missing mass is common in bunching analysis using notches (Best et al., 2019; Kleven & Waseem, 2013). This is usually attributed to at least two potential factors, optimization frictions or heterogeneity in the behavioral parameters θ . Some households may not be aware of the subsidies, or the application costs may be too high. In my setting, there are a limited number of subsidies and not all eligible households receive it. It is also the case that some households receive the downpayment and the interest rate subsidy, but others get only one of the two. This means that the notch may vary between individuals due to different types of frictions. Moreover, households that are eligible may not see the benefits because living in a low-cost housing unit could create stigma and households may have a large dis-utility related to that.

There may be a preference heterogeneity across cities of family size. In this case, Best et al. (2019) suggests that the behavioral response can be interpreted as the average marginal response.

VI. RESULTS

This section presents the main estimation results. First, the equilibrium characterization, corresponding to the first step of the two-step estimation procedure is presented. Second, the structural estimates corresponding to the second step of the proposed estimation approach is presented.

A. FIRST STEP: EQUILIBRIUM CHARACTERIZATION

Describing observed equilibrium prices. Figure 9 illustrates the estimated implicit price function $\hat{P}(h)$ and the marginal willingness to pay $\hat{p}(h)$ by size. The figure shows a

change in the equilibrium price scheme. It is not possible to know if this change is only associated to the policy changes as other general demographic and economic factors changed during the same time period. Over my study period, housing became more expensive but particularly above the policy cutoff. The figure also shows that accounting for non-linearities in the estimation of $P(h)$. Note that in contrast to figure 8, I show the prices in levels and not logs. In terms of the marginal price for size, Figure 9b shows a difference, particularly around area of the marginally subsidized households.

Marginal bunching thresholds. The vertical lines show the value of \bar{P} for the different periods and \underline{P} . We can see in this figure how the estimation of the hedonic price function allows us to recover the marginal buncher thresholds in terms of size. This figure shows that the marginal buncher is willing to cut the size of the housing he buys almost in half to take advantage of the subsidy, which represents up to a 25 percent of the value of housing at the cutoff. This figure also shows that the equilibrium size that you can buy with the 135 *mMW* decreases overtime. In 2006, you could buy a house of around 66 square meters whereas in 2016 with the same money you can only buy a house of 47 square meters.

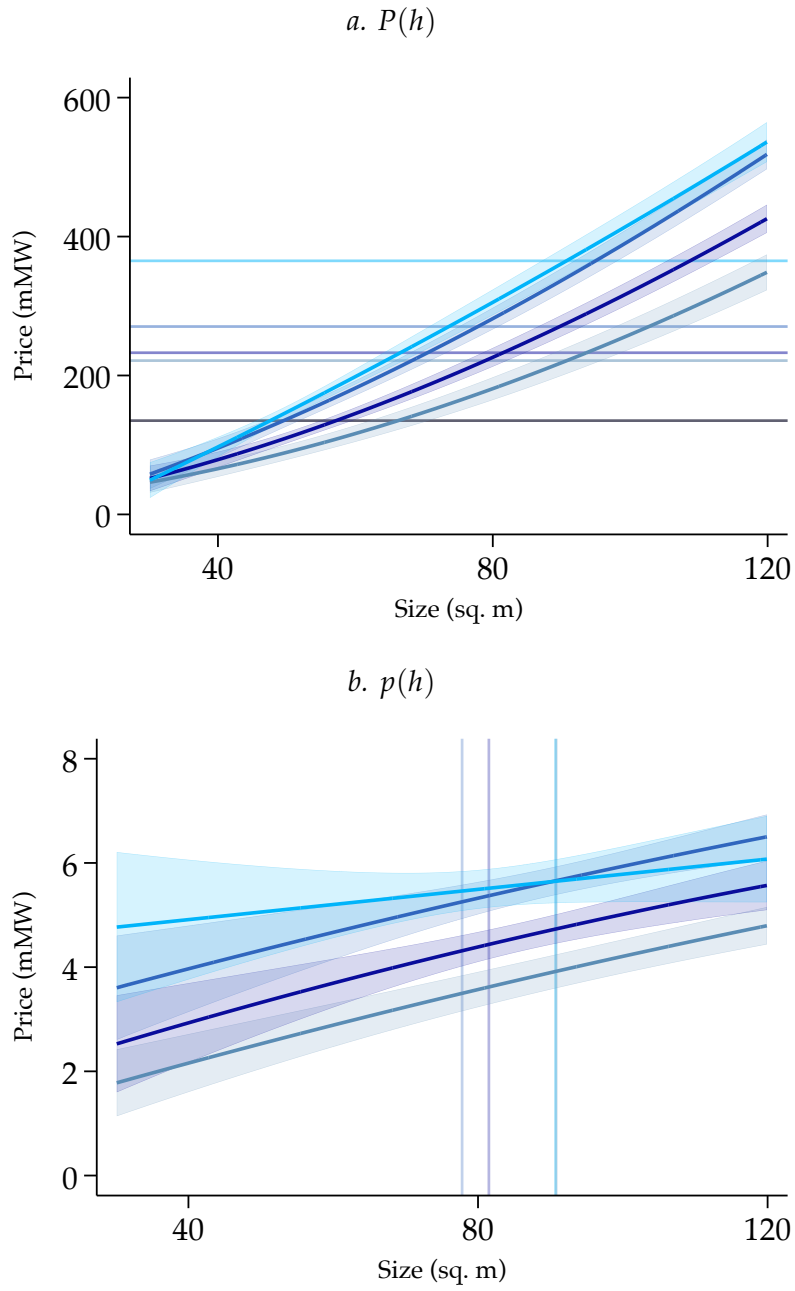
The three prices. Figures 2 and Table 1 together with the estimates in Figure 9 allow me to recover the developer's price $P^\delta(h)$ and the household's price P^τ .

Unit Supply Function. Figure 10 shows the unit supply function adjusted for the characteristics of the unit and the project using observations from all cities available in each period. The figure shows a negative relationship between unit size and the number of units, which is intuitive. Developers face a trade-off between building more but smaller units and fewer but larger units. There was a decrease in the number of units built at all levels. I do not have a clear explanation of this phenomenon, but it could be associated with lower availability of land, increases in the cost of building high or the fact that for the first years, the census covered mostly the main metropolitan areas whereas later years started to include smaller cities.

B. SECOND STEP: ESTIMATION OF θ , AND β

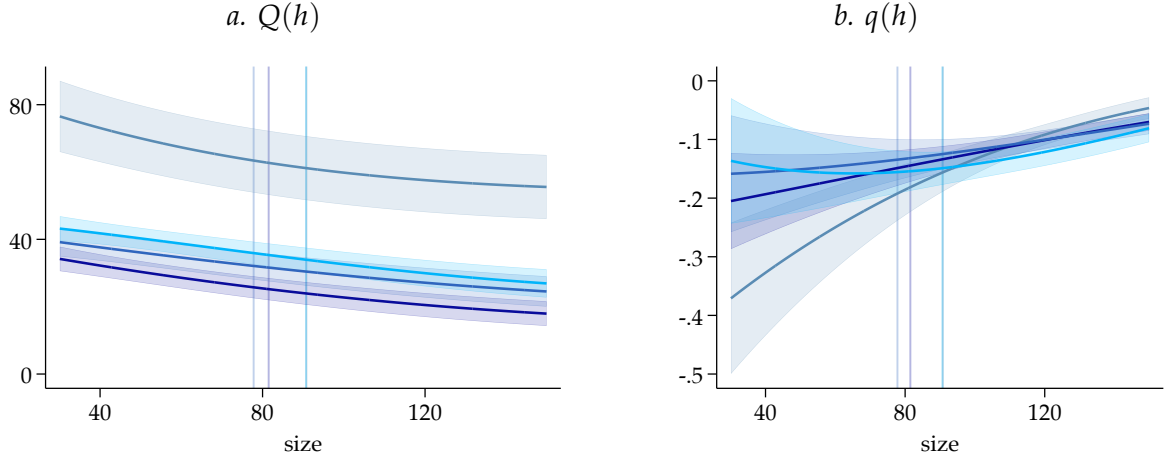
Using the functional forms and estimates for the values in panel D of Table 2 and presented in section VI.A, I can solve for θ and β . The marginal buncher functions do not have a closed-form solution; therefore, I use numerical methods to find the values of θ and β . I present the estimates separately for each subperiod with specific subsidy schemes.

Figure 9: Hedonic Price for Housing Size



NOTE: This figure shows $\hat{P}(h)|X$, and $\hat{p}(h)|X$ where X includes number of bathrooms, number of rooms, an indicator equal to 1 if the unit is a building, location; dummy variables equal to one if the unit has a porch, studio, storage unit, dressing room, service room, dining and living room, fireplace, kitchen, clothes areas, patio; location coordinates interacted with town fixed effects and metropolitan area fixed effects, lot size, number of building blocks, apartments per floor, number of floors, total parking spots, and number of building units. To estimate these figures I use the approach outlined in [Cattaneo et al. \(2019a\)](#).

Figure 10: Unit Supply Function



NOTE: This figure shows the bin scatter for the number of units and for unit size after controlling for observable characteristics. In this figure, I use the same controls as in Figure 9. This figure includes the observations for all years and all cities.

Structural Parameters. Figure 11 illustrates the equilibrium of the housing market and the preferences of households and the technology of developers using the estimated parameters presented in Table 3. The parameter $\sigma = 1/(1 - \theta)$ represents the constant elasticity of substitution for the specified utility function (CES). It represents how the relative consumption of housing varies when the relative price changes.

Households' parameters. The elasticity of substitution estimates was around 1.2 at the beginning of the period and increased substantially to 3.8 at the end of the period. This could be explained by the introduction of the Mi Casa Ya program later in the period. Under this program, subsidies became available to informal employees and applicants automatically received both the downpayment subsidy and interest rate subsidy. The estimated parameters are similar across years, which is reassuring considering that these are economic fundamentals and, therefore, very unlikely to drastically fluctuate over time. The increase in the estimated parameter at the end of the period is likely given the changes in the policy and the fact that informal employees were now eligible.

An elasticity of 1 corresponds to a Cobb-Douglas elasticity. Therefore, my estimates suggest that a Cobb-Douglas utility function would not be a bad representation, but would be imprecise, particularly at the end of my period. A negative value of θ corresponds to an elasticity of substitution σ less than one, which means that housing and other goods are gross complements. If θ is positive, the elasticity of

substitution is greater than one, and the housing and consumption of other goods would be gross substitutes.

Bayer et al. (2007) present an approach that integrates the hedonic insights into a discrete choice framework. As pointed out by Yinger (2015), their approach implicitly assumes a linear utility function, which violates the strict quasi-concavity postulate. In other approaches in the urban economics literature, the utility function is assumed to be Cobb-Douglas. In my setting, I allow for a less restrictive functional form, but my estimates suggest that the Cobb-Douglas utility function would be a close approximation in some cases but not always.

Figure 11: Equilibrium Choices using the estimated parameters

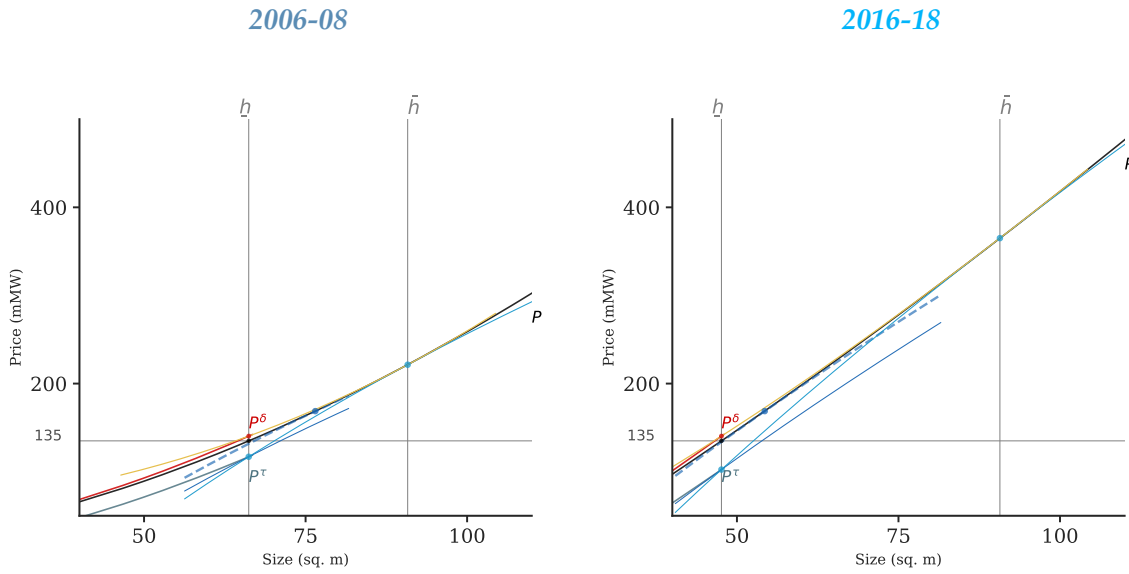


Table 3: Structural parameters

	06-08	09-11	12-15	16-18
β	2.34	2.03	1.65	1.29
θ	0.55	0.40	0.55	0.74
σ	2.23	1.68	2.22	3.88

NOTE: This figure uses the estimated parameters presented in Table 3 and creates the empirical analog of Figure 6 for the marginally subsidized households and developers. The figure represents the equilibrium choices and bid and offer functions estimated at the beginning and end of the study period. The elasticity of substitution implied by the CES utility function is $\sigma = 1/(1 - \theta)$. θ is assumed to be 0.5.

Developers' parameters. On the developer side, the estimated parameter β , does not change much overtime. In the first period, β is 2.34 , this decreases to around 1.26 in

the following periods. This change means that the costs of building bigger houses decreased over time. It is hard to compare these estimates to the literature, as the paper that use an hedonic approach to estimate the housing market usually takes the supply function as given and does not allow for heterogeneity (Bishop & Timmins, 2019) or do not allow for product differentiation (Saiz, 2010). There are not many papers that consider the developer's decisions regarding how many units to build and which unit to build separately.

VII. WELFARE AND POLICY EVALUATION

A goal of this paper is to evaluate the effectiveness of the policy scheme implemented in Colombia. The framework and estimated parameters presented in this paper allow for a different type of counterfactual policy evaluation and allows for an assessment of how much households and developers benefit from these policies. In this section, I illustrate the potential of this framework as a policy evaluation tool. I use it towards two aims. First, I compare the how much the government spends on these subsidies to how much the beneficiary households are willing to pay to increase their utility in an equivalent magnitude. The focus on the developer side also allows me to evaluate the efficiency loss induced by the notched subsidy scheme (Blinder & Rosen, 1985). Second, I explore the role of the subsidy on the supply side. I show what happens if the supply side subsidy is removed, and demonstrate that the notch incentive designed in Colombia requires this subsidy to prevent a shortage problem. However, once we account for this additional government expenditure, it is not clear that households value the subsidy enough to justify the government expenditure in these type of policies. I close the section with a discussion of other type of policies that could be evaluated, like the effect of a quality or size limit on the subsidized units.

A. BENEFITS AND EFFICIENCY LOSSES: THE HOUSEHOLDS' PERSPECTIVE

To evaluate the benefits of the subsidy, I focus on the effect of marginally subsidized households. Because the market response is a change in housing characteristics, the price limit creates efficiency losses. If the response was a pure price reduction without changes in characteristics, the policy would induce a transfer of welfare from developers to households. Figure 11 illustrates how a representative marginally subsidized household benefits from the subsidy. Households reduce their expenditure on housing to obtain the subsidy. By doing this, the household reaches a higher utility level, the indifference curve moves to the right from the dashed line to the solid blue line. Without the existence of the price limit and if the marginally subsidized

household gets the subsidy without being forced to reduce its consumption, a household could increase its utility even more, as illustrated in the graph. This means the notched scheme introduces an inefficiency. However, from a targeting perspective, this type of inefficiency could be justified. Without the price cutoff, richer households could receive the subsidy to buy expensive units which undermines the objective of the program. That is to provide opportunities for low income households to become homeowners.

Blinder and Rosen (1985) shows examples where notches can be preferred to alternatives targeting approaches such as slope changes. In that paper we explore under what circumstances a notch scheme is preferable to a conventional linear subsidy. They show that the elasticity of substitution matters when comparing different targeting approaches. They show that a lower elasticity of substitution increases the efficiency of notches. Based on the elasticities estimated in this paper, it seems that the effectiveness of these subsidy scheme is fading over time as the elasticity of substitution is getting bigger. Moreover, I show that the efficiency losses (yellow area in 12) is larger at the end of the period. The dollar amount households would pay for the increase in utility is around the same value of the notch τ , however, once we include the extra cost induced by the tax refunds to developers we can see that the households do not value the changes in housing units as much as the government expenditure in a per unit basis. To calculate the total losses and gains we could multiply the share of buncher households by size and calculate their welfare gains and benefit losses.

What happens in equilibrium.

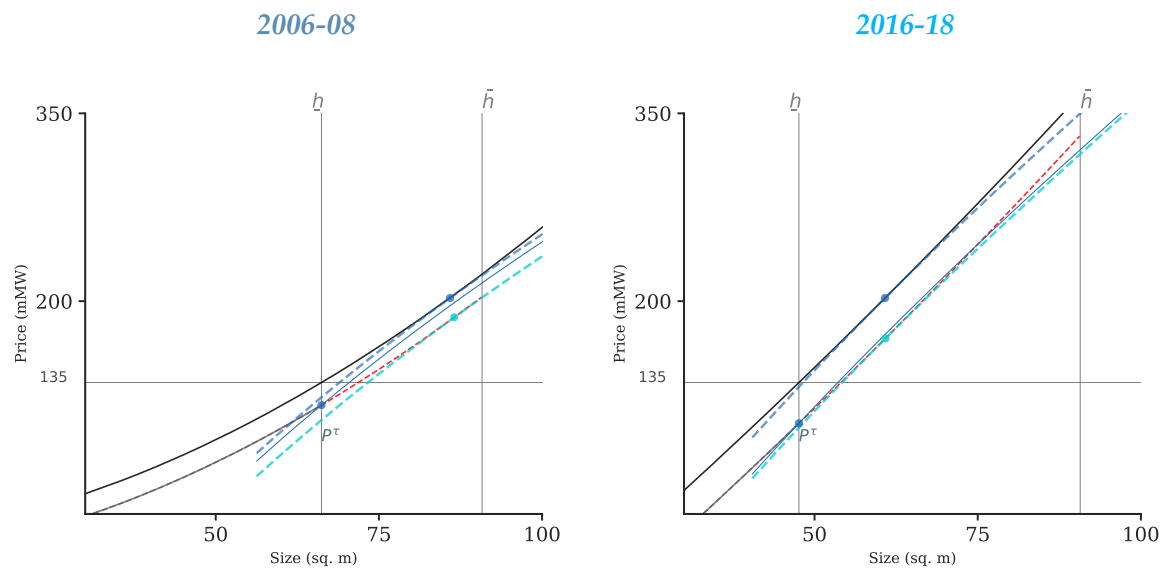
An advantage of this paper is that it allows me to think about market equilibrium. For example, in Section VII.A I showed the efficiency losses that would arise if there is a notched scheme that could be reduced with a linear subsidy. What happens in that case with the developers that bunched? Under that scenario, there would not be any need to have tax incentives for developers. To show how these are very relevant in the type of subsidy scheme in Colombia, the next section explores what happens if the tax incentives are removed.

B. THE EFFECT OF REMOVING DEVELOPERS' TAX INCENTIVES

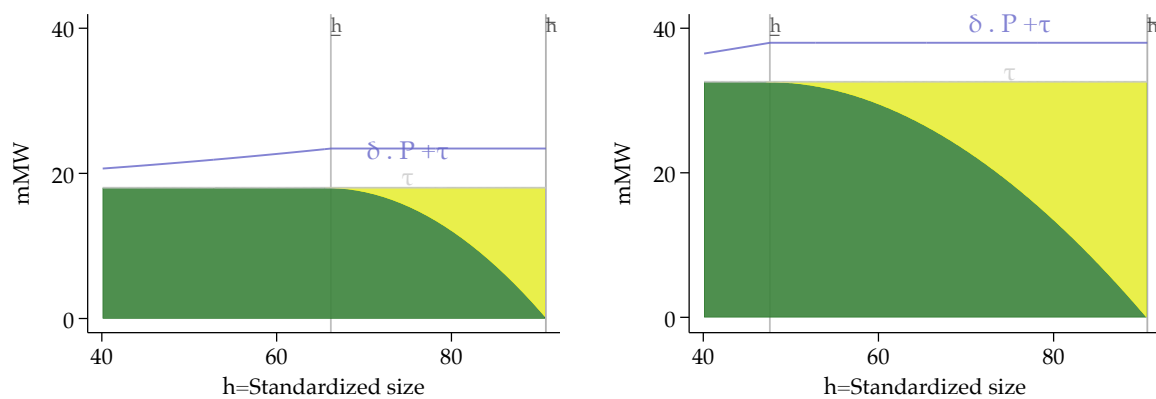
An important policy debate related to housing policy is whether the use of tax incentives for developers is an effective redistributive tool. In the USA there is the Low Income Housing Tax Credit (LIHTC) which is intended to produce. These types of subsidies could be ineffective or very expensive. They could also benefit

Figure 12: Welfare Gains and Efficiency Losses for the Marginally Subsidized Households

A. Welfare gains and losses for the marginally subsidized households



B. Equivalent Variation by Unit Size



NOTE: Panel A illustrates the changes in utility for a representative marginally subsidized household. Panel B shows how much households are willing to pay for their increase in utility (evaluated at \bar{h} for households at different levels. The green are the welfare gains and the yellow area represents the efficiency losses induced by the notch scheme.)

developers more than households which would make them hard to justify. [Soltas \(2021\)](#) shows that these types of subsidies could be very expensive as they force to build low-income housing in expensive areas. [Sinai and Waldfogel \(2005\)](#) shows that Tenant-based housing programs, such as Section 8 Certificates and Vouchers, are more effective than project-based programs such as developers' subsidies. The type

of developers subsidy implemented by the Colombian government is a little different, however, as it coexists with a demand-side subsidy targetted at households. The existence of these subsidies is an active policy debate in Colombia. I use the framework developed in this paper to show that these subsidies are required to avoid a shortage problem in a priced-capped policy scheme like the one implemented in Colombia.

In 2021, under the need for tax reforms, there was a policy proposal to remove these subsidies. However, developers actively opposed them, claiming that this would create a shortage problem.

“If these items are repealed, in Valle del Cauca we would go from having an offer of low-cost housing and sales of 23,000 homes, average year, to one of sales of 4,600 homes” ³⁵

The framework developed in this paper shows that this could be the case. Figure 13 illustrates the role of these tax incentives. Without the tax incentive, and under the existence of the price cap, developers that would produce housing units of size \underline{h} and \bar{h} would face no demand or a reduced demand. They build cheaper housing units to keep supply the households that changed the type of housing they buy. However, because they would be building units usually build by more productive developers they would have to reduce there profits or leave the market. In this sense the tax incentives guarantees that developers do not leave the market. The dashed yellow line, represents the equilibrium in the absence of the subsidies, the solid yellow line is the observed response and the green line represents the iso-profits if developers stay in the market but they cannot increase the price.

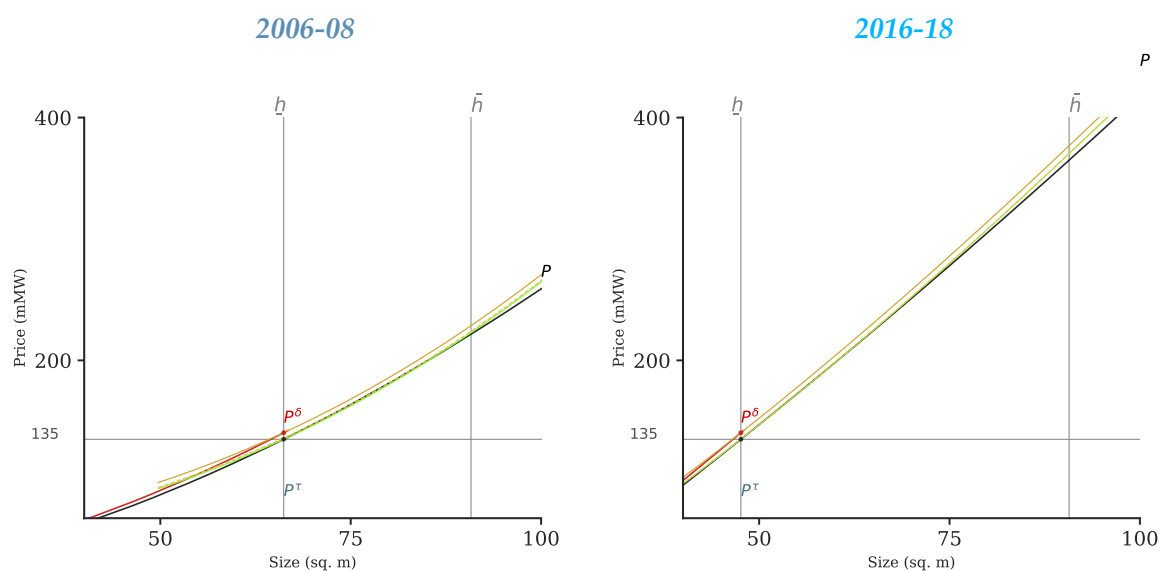
Figure 13 shows these responses at the beginning of my study period and at the end. The top panel illustrates the decision choice of a representative marginally subsidized developer under the observed scenario and two counterfactual scenarios, changing the type of housing to satisfied the subsidy induced demand at h and the scenario under no subsidies. The figure in panel B, shows what would be the changes in profits for households producing different housing units if they do not get the subsidy and reduce their consumption. The figure show that at the beginning of the period the losses would have been around 5 percent. However, at the end the losses could be up to 15 percent of the profits they would have in the absence of the subsidy scheme. The gray-red area is the increase in profits that they receive instead. The gray would produce low cost housing even in the absence of the subsidy. This analysis shows that the tax incentives may in fact prevent the exit of some developers and avoid a shortage problem. However, by doing this they artificially increase the

³⁵source: El Tiempo (2021)

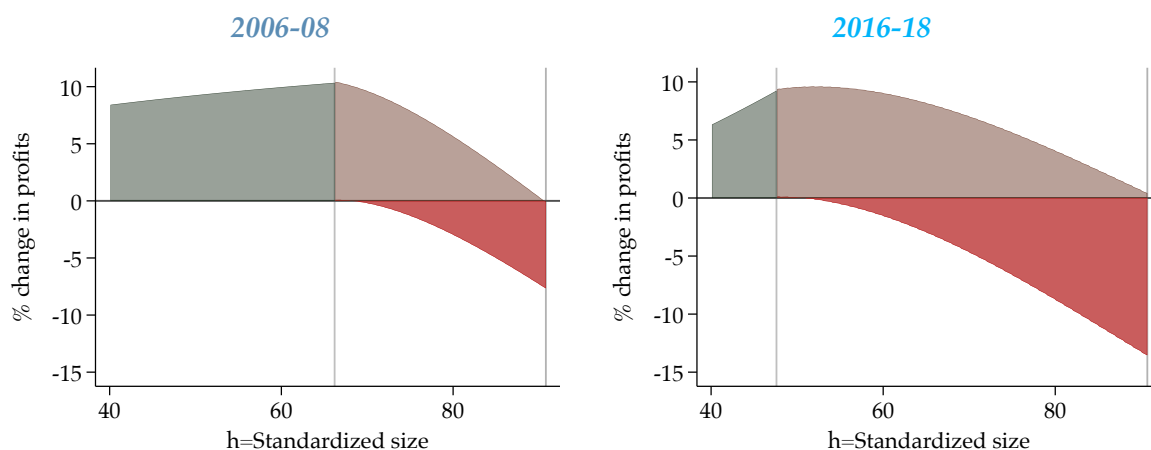
profits of developers that would build low-cost housing even in the absence of the subsidy and they make the potential exiters better off than in the absence of the subsidy.

Figure 13: Developer Response to Tax Incentives

A. Developers Incentives



B. Excess Profit and Potential Losses



NOTE: Panel A shows the incentives of a marginally subsidized developer if the tax incentives are removed. Panel B) shows the developers if there is not tax incentives as a percentage of their current profits (in red) and the induced excess profits to prevent the exit of those developers (in Green)

VIII. ROBUSTNESS AND SENSITIVITY ANALYSIS

This section presents sensitivity analysis for the bunching and structural estimates.

A. BUNCHING ESTIMATES AND STRUCTURAL PARAMETERS

The estimation of the structural parameters in this paper rely heavily on the estimation of the counterfactual distribution of market shares. These estimates depends on the selection of different parameters. The bandwidth for the bins, the number of omitted bins to the right and the left of the cutoff and the degree of the polynomial. To select these parameters, I fix a bandwidth and select the the excluded number of bins to minimize the difference between the excess mass and missing mass. Figures B.8 and B.9 in the appendix show the bunching analysis forthree different bin size and 2 different criteria to select the excluded bins and polynomial degree. Table ?? shows the sensitivity of the structural estimates to different approaches to estimate the bunching and the upper limit for the marginal buncher \bar{P} . It also shows sensitivity to different values of ϑ which represents the share of income devoted to housing. Overall the parameters are relatively stable to different approaches to estimate bunching. Regarding the share of income devoted to housing. The table shows that different values of ϑ does not affect the value of the elasticity of substitution. As expected, the elasticity drops as the share of consumption devoted to housing falls.

IX. CONCLUSIONS

This paper presents compelling evidence of the market responding to subsidies. I rely on detailed data on the universe of new housing, data on subsidies to both households and developers, the policy cutoff inducing discontinuous incentives, and the variation of the subsidy over time. I use the behavioral responses induced by the subsidy and introduce a novel identification approach to estimate a hedonic housing market equilibrium with heterogeneous agents and housing that rationalizes the observed responses. The model-guided estimation approach translates the bunching reflecting the behavioral responses and the reduced form estimates into parameters of both household utility and developers' production function. I use those estimates to illustrate the type of welfare analysis that the estimation approach allows.

I find that households and developers changed their housing consumption to take advantage of the policy. The price cap, which could be important if the response does not induce a change in housing consumption, induced welfare losses. House-

Table 4: Structural Parameters Using Different Estimation Approaches for P and Different Values for Consumption Shares ϑ

	$\vartheta = 60$	$\vartheta = 50$	$\vartheta = 40$	$\vartheta = 30$	
	σ				β
06-08					
$bw = 0.05$ (at \underline{P})	1.53	1.32	1.19	1.09	2.70
$bw = 0.05$ (all P)	0.80	0.72	0.66	0.61	5.00
$bw = 0.03$ (around \underline{P})	2.52	2.23	2.08	1.99	2.34
$bw = 0.07$ (around \underline{P})	2.34	2.07	1.91	1.81	2.35
09-11					
$bw = 0.05$ (at \underline{P})	1.13	1.00	0.90	0.82	2.51
$bw = 0.05$ (all P)	1.32	1.18	1.08	0.99	1.13
$bw = 0.03$ (around \underline{P})	1.78	1.57	1.43	1.33	2.08
$bw = 0.07$ (around \underline{P})	1.96	1.73	1.59	1.49	2.00
12-15					
$bw = 0.05$ (at \underline{P})	2.06	1.85	1.72	1.63	1.69
$bw = 0.05$ (all P)	2.64	2.46	2.37	2.32	1.25
$bw = 0.03$ (around \underline{P})	2.27	2.06	1.94	1.86	1.67
$bw = 0.07$ (around \underline{P})	2.43	2.22	2.11	2.04	1.65
16-18					
$bw = 0.05$ (at \underline{P})	2.47	2.28	2.17	2.11	1.32
$bw = 0.05$ (all P)	3.64	3.54	3.51	3.50	1.29
$bw = 0.03$ (around \underline{P})	1.69	1.50	1.38	1.29	1.40
$bw = 0.07$ (around \underline{P})	3.64	3.54	3.51	3.50	1.29

holds would have been better off if they received the subsidy without reducing their housing consumption. The welfare analysis also suggested that in a world with developer heterogeneity, subsidizing the demand side of the market may be insufficient. Developers need to be compensated to produce low-cost housing, which they can produce but at a higher marginal cost. The type of welfare analysis allowed by this approach goes beyond the examples presented here. Because I recovered the income and productivity levels of households and developers together with parameters describing their preferences and costs, the approach allow for the evaluation of different housing policies. The method could apply to other markets with vertical differentiation and price caps, such as labor markets. In this case, the policy induced a change in the type of housing bought and built. The housing stock accumulated smaller housing units purchased by households who would prefer bigger houses. Considering that housing is a durable asset that affects urban structure and city planning, this could translate into significant consequences for cities reaching a suboptimal equilibrium. The findings of this paper suggest that a careful evaluation

of the market structure matters for effective policy design. Understanding how the policy affects the housing market's incentives is crucial to understanding how the observed equilibrium outcomes inform us about the effects of the policy.

Appendix

I. COLOMBIAN HOUSING POLICY: ADDITIONAL DETAILS

A. MINIMUM WAGE AND INFLATION

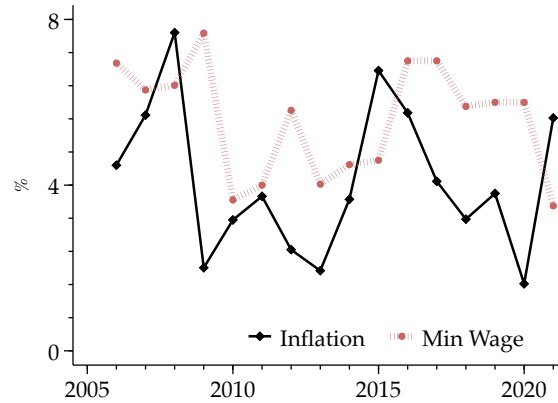
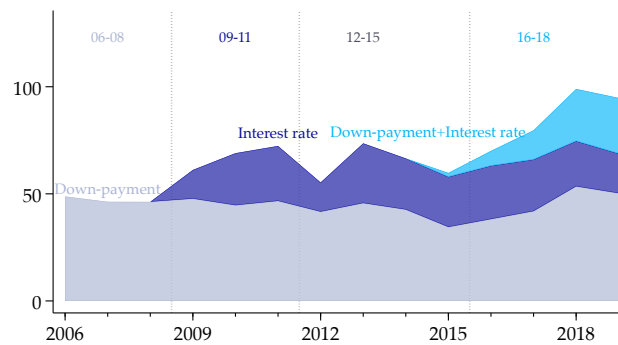


Figure A.1: Inflation and Minimum Wage Over-Time

B. NUMBER OF SUBSIDIES OVER TIME

Figure A.2: Total number of Subsidies Over Time



SOURCE: Minvivienda, FRECH

C. DETAILS ON SUBSIDIES

downpayment subsidy. The down payment subsidy was introduced at the beginning of the nineties and is available to formal employees who contribute to the family compensation funds.³⁶ The gray blue area in Figure 1 shows the number of sub-

³⁶

sidies and total government expenditure from 2006 to 2019. The number of downpayment subsidies to formal employees was more or less stable during the study period, but the government spending increased in 2015 due to an increase in the size of the subsidy. Only formal households earning less than four times the minimum wage (mMW) are eligible for the subsidy, and the subsidy can only be used to buy a low-cost housing units.

Interest rate subsidy. In 2009, the government introduced a program to subsidize mortgage' interest rates. This program, called *FRECH*, started as a program to incentivize economic growth after the crisis, but it became a permanent policy. In contrast to the downpayment subsidies, interest rate subsidies were also available to households buying housing units above the $135 \times mMW$ threshold. However, the subsidy is larger if the households buy a low-cost housing unit, that is, the price is less than $135 \times mMW$. If a household receives the subsidy, the government pays the bank the corresponding amount during the first seven years of the loan. Three different schemes existed during the study period, but in all the schemes, there was a discontinuity in the subsidy at the cutoff defining low-cost housing. The **dark blue** area in Figure 1 shows the number of subsidies and total government expenditure from 2011 to 2019. The subsidies were more or less stable over time; around 20,000 households received this subsidy. This subsidy represents lower government expenditure and expenditure has slightly decreased overtime partly due to lower interest rates.

In contrast to the downpayment subsidy, interest rate subsidies were also available to households buying housing units above the $135 \times m-MW$ threshold. However, there is a notch at $135 \times m-MW$. Figure A.3 shows the interest rate subsidies for the all the house price ranges.

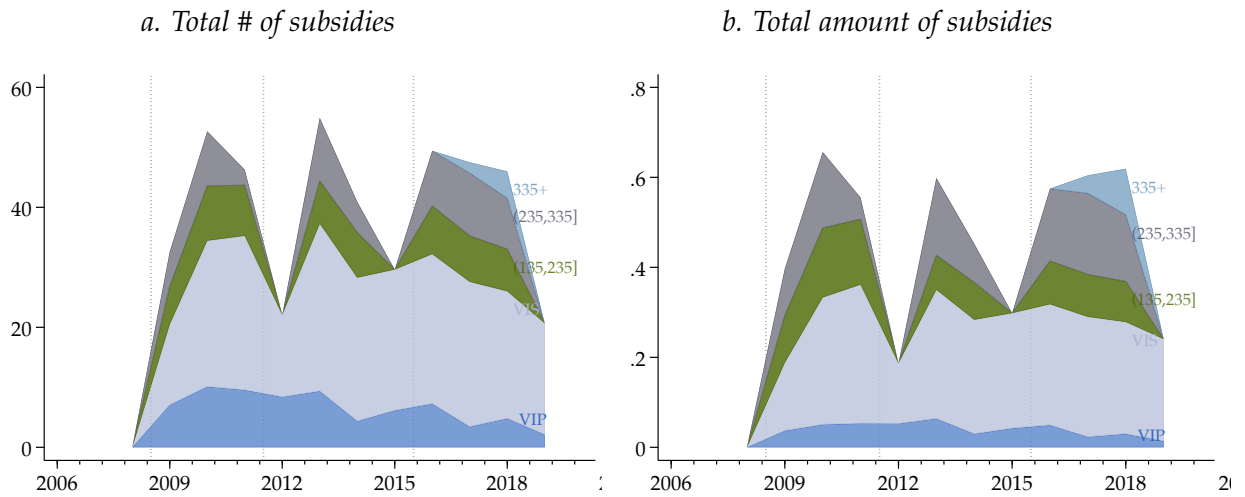


Figure A.3: FRECH subsidies for different price ranges

The subsidy expansion—Mi Casa Ya. In 2015, the government doubled the effort and introduced a new program *Mi Casa Ya*, (My House Now). Before this program was introduced, the down payment subsidy was only available to formal employees-contributing to the family compensation funds.³⁷ This program extended the coverage of the downpayment to noncontributing households. The households participating in this program get the downpayment subsidy and interest rate subsidy automatically with a single application. The **light blue** area in Figure 1 shows the number of subsidies and government expenditure, which is the sum of the downpayment and the total expenditures with the interest rate discount. The figure shows that the increase in the number of subsidies and government expenditure that started in 2015 was mainly driven by the introduction of this program and the increase in the down payment subsidy to formal employees.

Supply subsidy—value added tax (VAT) tax refund. A couple of years after the demand subsidies were introduced, to encourage developers to build low-cost housing, the government introduced a VAT tax refund. Developers get up to 4 percent of the sale price of each unit in the refund of taxes paid on construction materials. I include this subsidy in the analysis. Accounting for this subsidy introduces discontinuous incentives on the supply side.

³⁷In theory informal household could get access to housing subsidies. However, *fonvivienda*, the institution in charge of these subsidies, assigned mostly to vulnerable populations. The vulnerable populations are displaced by armed conflict and affected by natural disasters. I include the equivalent plots for those subsidies in the Appendix A.

Other subsidies. The Colombian housing policy includes other subsidies excluded from the main analysis of this paper.³⁸ These are mainly subsidies to disadvantaged populations. These subsidies exist to follow a constitutional mandate to provide housing to people affected by forced displacement and environmental disasters. They are for cheaper housing units and households in extreme poverty. These subsidies can be used to buy *priority low-cost housing*, which is housing units with a market price of $70 \times mMW$ or less. The approach of using subsidies as an incentive to promote construction and purchase of housing units was mostly ineffective to provide this type of housing. As a result, in 2014, a program to build 100'000 free housing units was launched. The goal was to satisfy the constitutional mandate and provide housing to the disadvantaged population that was neglected by the previous policy approaches. A. G. Gilbert (2014) describes this program, *100 mil viviendas gratis*, and evaluates its potential effectiveness. Camacho et al. (2020) study the effect of this conditional transfer on the economic outcomes of the receiving households. The appendix Figure ?? shows the evolution of those subsidies.³⁹ The program of 100 thousand free housing units occurred between 2012-2015. There is a program for rural housing and subsidies for the military that I ignore in this paper.

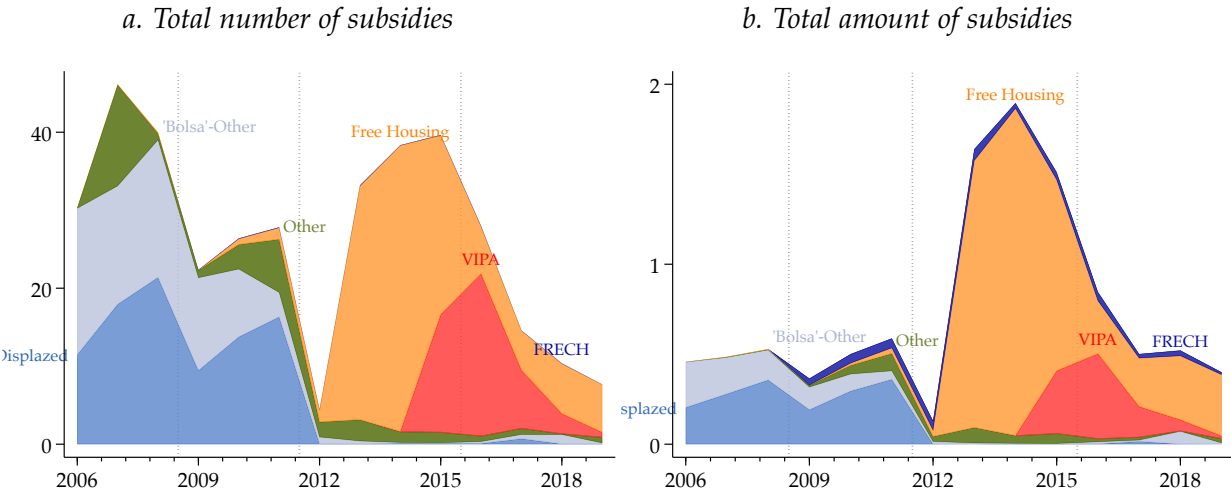


Figure A.4: Subsidies for the Vulnerable Population. Housing Units Priced Below 70 mMW.

Subsidy scheme. Figure A.5c shows the subsidy scheme for the interest rate. Three different schemes existed during my study period. Each scheme is represented in the figure by a different line. The x-axis is the monthly minimum wage and the y axis is

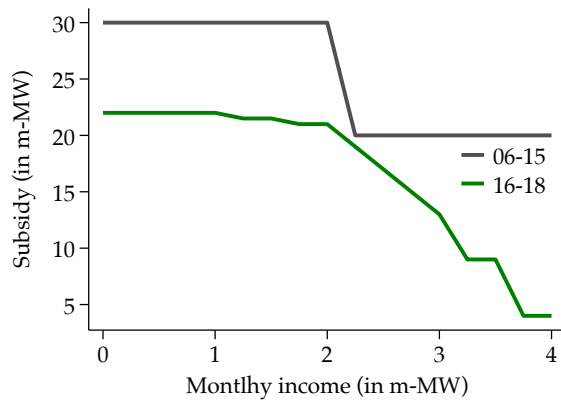
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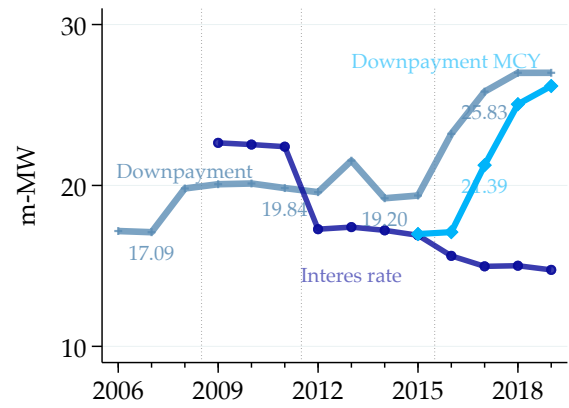
the discount in the interest rate. If a household gets the subsidy, the government pays the bank the corresponding amount during the first seven years of the loan.

Targeting instruments. The authorities use two different tools to determine eligibility; the households' income and the total price of the housing unit. A unit can be subsidized only if the market price is below the low-cost housing threshold, 135 times the monthly minimum wage (m-MW). This arbitrary threshold is the same for all cities.⁴⁰ Regarding income, only households earning below four times the monthly minimum wage can get the subsidy. Figure A.5a shows the subsidy scheme. Before 2015, the subsidy was decreasing on income, and the maximum possible subsidy was $22 \times \text{m-MW}$. In 2016 the generosity increased, the limit increase to $30 \times \text{m-MW}$ for individuals with income below $2 \times \text{m-MW}$ and $20 \times \text{m-MW}$ for individuals with income between $2 - 4 \times \text{m-MW}$. As the Figure 1 shows, the increase in the limit is reflected in higher government expenditure. Figure A.5b the average subsidy during my study period. We can see that the average subsidies were about 20 percent before 2015 where the mean subsidy is about 26 percent.

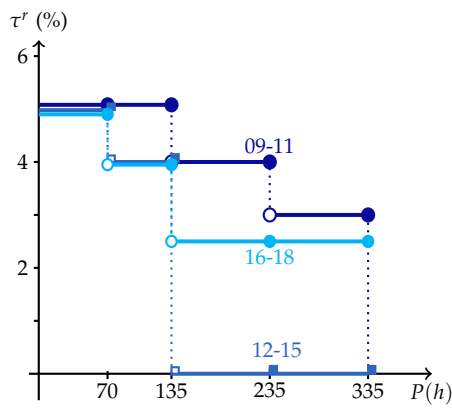
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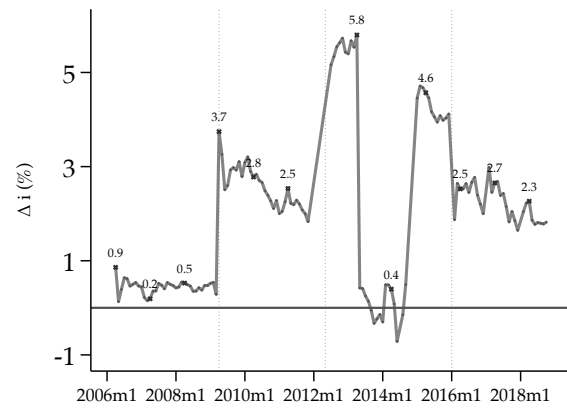
a. downpayment Subsidy Scheme Subsidy



b. Mean Subsidy Over Time



c. Interest rate subsidy scheme



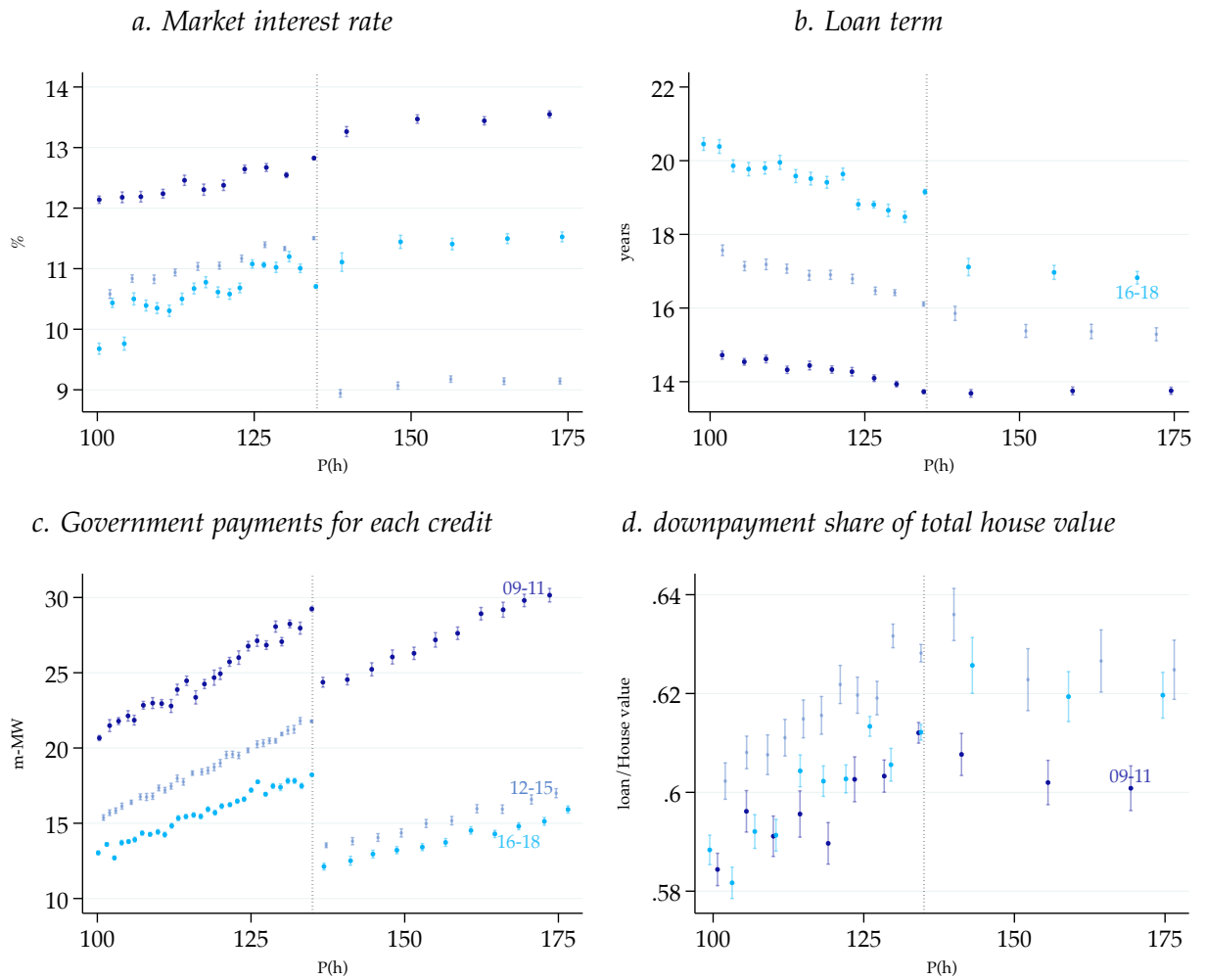
d. Differences in Market interest rate i tau^r

Figure A.5: Subsidy Scheme and Observed Mean Differences

NOTE: This figure shows the subsidy scheme and the evolution overtime of the subsidies for the interest rate and downpayment subsidy.

D. MORTGAGE TERMS:

Figure A.6: Loan Terms by House Prices



NOTE: This figure shows the subsidy scheme and the evolution overtime of the subsidies for the interest rate and downpayment subsidy.

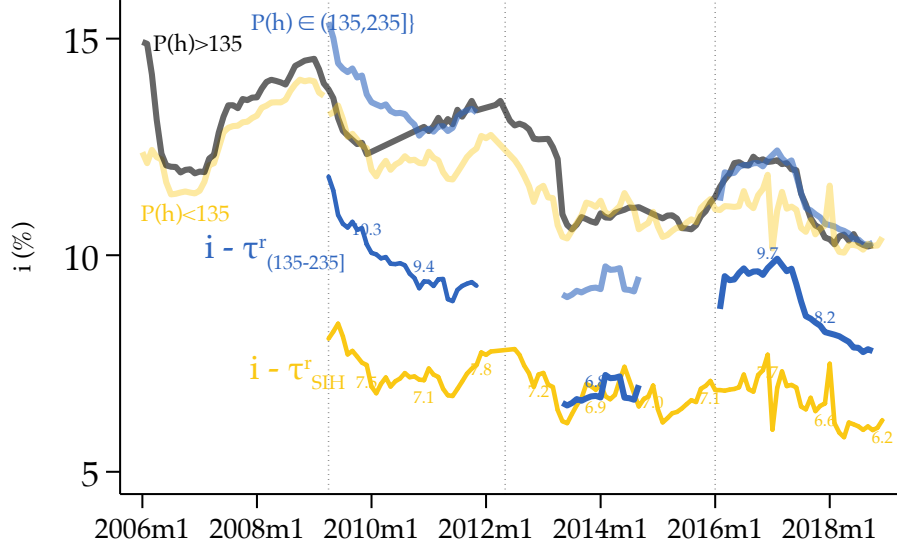


Figure A.7: Market interest rate i and subsidy τ^r

NOTE: This figure shows the interest rates with and without the subsidy over time.

Observed differences in monthly payments. I use the administrative records of these subsidies and administrative records on all loans to check that subsidies are reflected in the lower interest rates paid by households. The administrative records for the subsidies contain relevant information about the mortgages. It has the market interest rate i , the loan L , the term n , the discount in the interest rate τ^r , and the house price P . The administrative records for all loans contain less detailed information, but I observe the interest rate of each loan and the average loan amount. I use the loans for housing, which have an indicator variable equal to 1 if the house is low-cost housing and 0 otherwise.

II. BUNCHING ESTIMATION AND MORE BUNCHING RESULTS

Counterfactual Distribution Estimation

In contrast with Figure 3, in this section, I present the market shares by standardized unit size, and not unit price, and by period. By doing that, I can interpret the changes induced by the subsidies as changes in the size of a standard unit. Developers and households build and purchase smaller houses to take advantage of the subsidy.⁴¹

⁴¹I assume all changes are in terms of the housing size of a standard unit. Although I acknowledge this is a strong simplifying assumption, it makes the analysis tractable. The setup and economic

Define $T(h)$ to be the difference between the observed (f_{h^*}) and counterfactual densities (f_{h_0}),

$$T(h) = f_{h^*} - f_{h_0} \quad (24)$$

and Δh as the maximum change agents made to take advantage of the subsidy.

$$\Delta h = \bar{h} - \underline{h} \quad (25)$$

Intuition. The counterfactual is the distribution that would exist in the subsidy's absence. I calculate it fitting a flexible polynomial to the observed density and excluding the observations close to the cutoff. The differences between the counterfactual distribution and the observed distribution reflect the behavioral responses to the subsidy scheme's discontinuous incentives.

Estimation. To estimate f_{h^*} and f_{h_0} , I rely on standard techniques from the bunching literature. To estimate the empirical distribution \hat{f}_{h^*} , I calculate the share of units in each bin h_b of size $2 \cdot \epsilon$,

$$h_b = \frac{1}{N} \sum_{l=1}^N \mathbb{1} [h_l \in (b - \epsilon, b + \epsilon)] \quad (26)$$

The estimated observed equilibrium distribution is

$$\hat{f}_{h^*}(h) = h_b$$

To estimate the counterfactual distribution, \hat{f}_{h_0} , I predict the observed values for h_b using a flexible polynomial, $l(h_b) = \sum_{p=0}^T \iota_p h_b^p$ and excluding a region around the cutoff. The function $o(h_b; L, H)$ includes all the indicator variables for the bins between L and H , the lower and the upper bound, respectively, of the excluded area. $o(h_b; L, H) = \sum_{k=L}^H \mathbb{1} [h_k = h_b] h_b$

$$h_b = l(h_b) + o(h_b; L, H) + v_b \quad (27)$$

framework introduced in section IV can be the basis to extend the analysis to separate changes in multiple characteristics, such as location or quality, and to include the possibility of changes prices in response to a setting with developer frictions or imperfect competition.

Counterfactual distribution. The counterfactual distribution is the predicted density using only the flexible polynomial.

$$\hat{f}_{h_0} = \hat{l}(h_b) = \sum_{p=0}^T \hat{l}_p h_b^p \quad (28)$$

Bunching. Using the estimated distributions, I can get an expression for bunching or excess mass at \underline{h} , and calculate the maximum behavioral change induced by the subsidy Δh :

$$\hat{T}(\underline{h}) = \hat{f}_{h^*}(\underline{h}) - \hat{f}_{h_0}(\underline{h}) \quad (29)$$

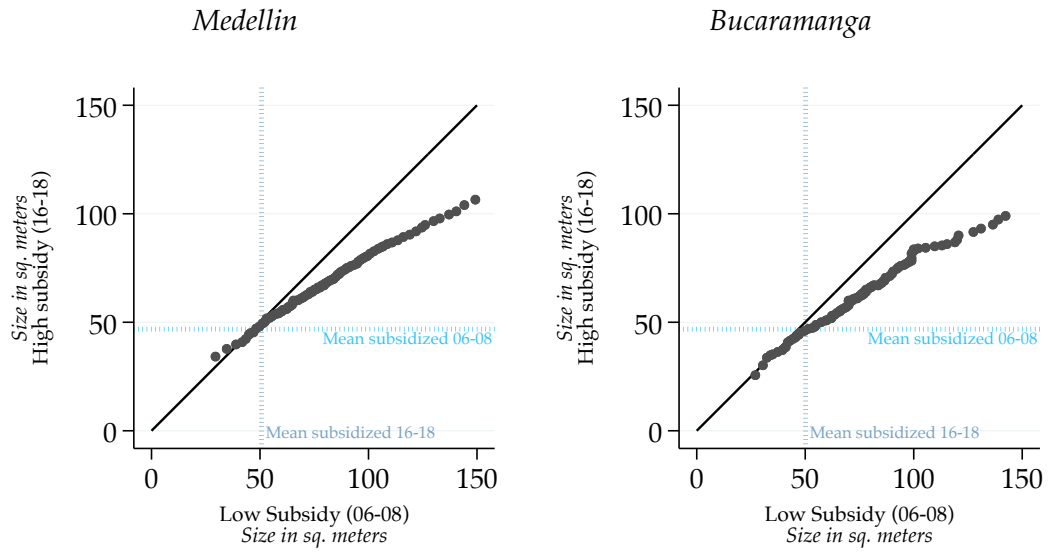
Equation 29 is the difference between the observed distribution and the counterfactual distribution at the discontinuity point, \underline{h} , and it represents the share of individuals who would consume $h \in (\underline{h}, \bar{h})$ in the absence of the subsidy, but consume h in a subsidy scenario.

Maximum behavioral response. The maximum behavioral response, \bar{h} , is obtained when the counterfactual and observed distributions coincide:

$$\bar{h} = \min[h : h > \underline{h} \text{ and } \hat{f}_{h_0}(\underline{h}) - \hat{f}(h_b) = 0]$$

III. RESPONSE ON SIZE: ADDITIONAL CITIES.

Figure C.10: Quantile-to-Quantile Plots of Housing Size: Low versus High Subsidy Periods



IV. MODEL APPENDIX:

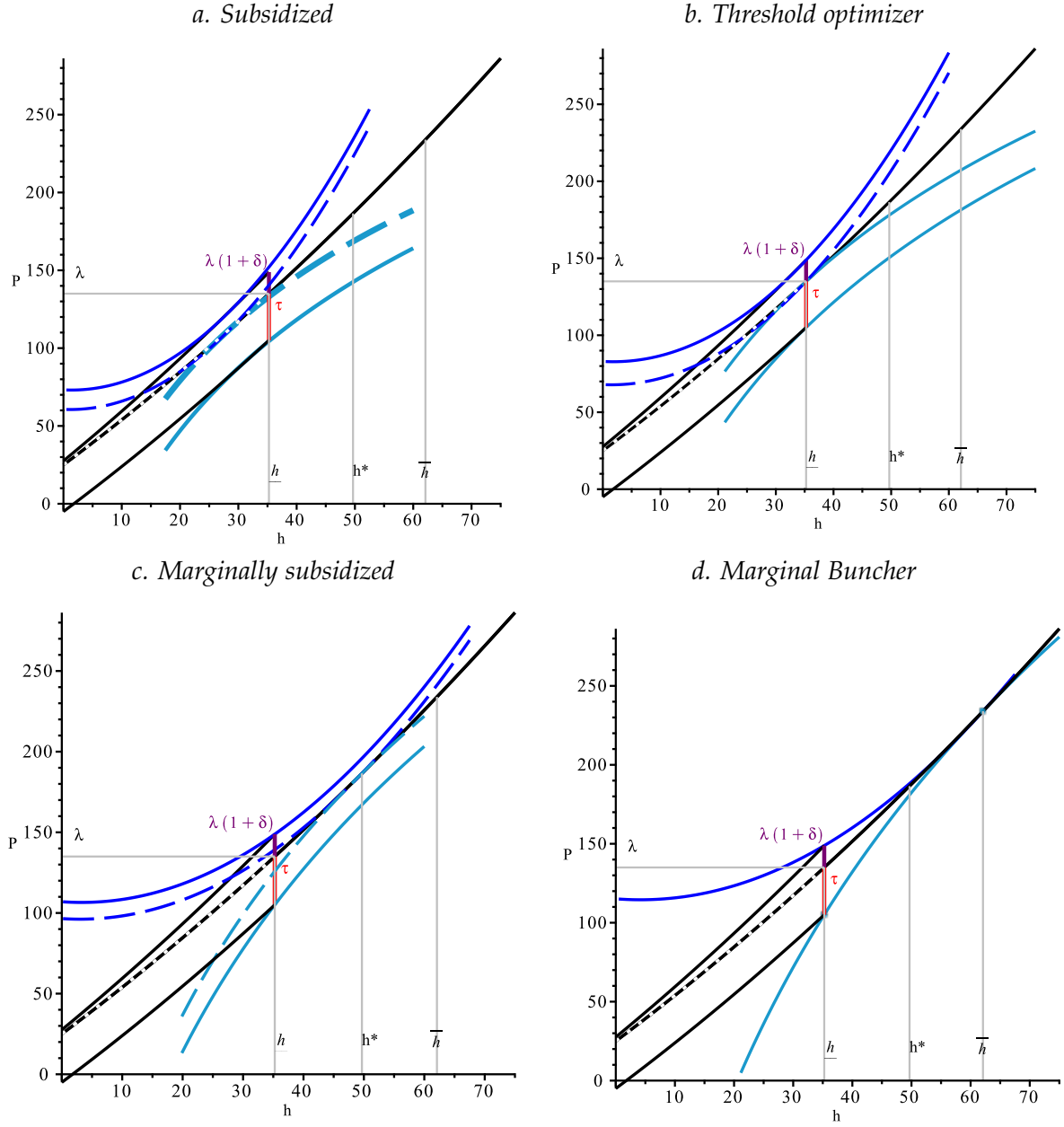


Figure D.11: Graphical representation of equilibrium choices

The offer function functional form is: $\varphi_j^S = \frac{\bar{\pi}_j \cdot h^2}{\bar{\beta}_1} + A_j$. This results from a cost function $C(Q(h), A_j) = A_j * Q(h)$, and $Q(h) = \frac{\alpha}{h^2}$

A. STANDARDIZED HOUSING AND UNIT SIZE

To make all the housing units comparable, I use the hedonic price function to standardize all housing units. In particular, I use the estimates of equation 19 to convert all housing units into a standard unit.

This hedonic price estimation decomposes the unit price into observed and unobserved characteristics. The standardized housing size, which I call h , is the size of a housing unit with average characteristics that will cost the same as the observed price.

$$\rho(h_{l_{tc}}) + \Gamma' \bar{X} + \bar{\omega} = \rho(s_{l_{tc}}) + \Gamma' X_{l_{tc}} + \omega_{l_{tc}} \quad (30)$$

\bar{X} , are the means of the observable characteristics and $\bar{\omega}$ equals the average residual. Solving for h in the equation 30, I get the following measure of the standardized size measure:

$$h_{l_{tc}} = \rho^{-1}(\rho(s_{l_{tc}}) + \Gamma'(X_{l_{tc}} - \bar{X}_{l_{tc}}) + (\omega_{l_{tc}} - \bar{\omega}_{l_{tc}})) \quad (31)$$

Intuitively, this means that if a house is more expensive because it has certain amenities or more bathrooms, I convert this characteristic into the equivalent square meters that the household could get if they had a standard house.

In my application, I standardize the units in a way that $\bar{P} = \rho(\bar{s}_{l_{tc}}) + \Gamma' \bar{X}_{l_{tc}} + \bar{\omega}_{l_{tc}}$ is the observed average price for the average house. For the implicit price function, I use a parametric approximation $\rho(s) = \rho_0 + \rho_1 \cdot s + \rho_2 \cdot s^2$.

Figure 9 shows the functional form of the estimated price function for the four different periods. The Figure 9 shows that the implicit price function has become steeper overtime.

Implied Maximum Size of a Standard Subsidized Unit

0.32

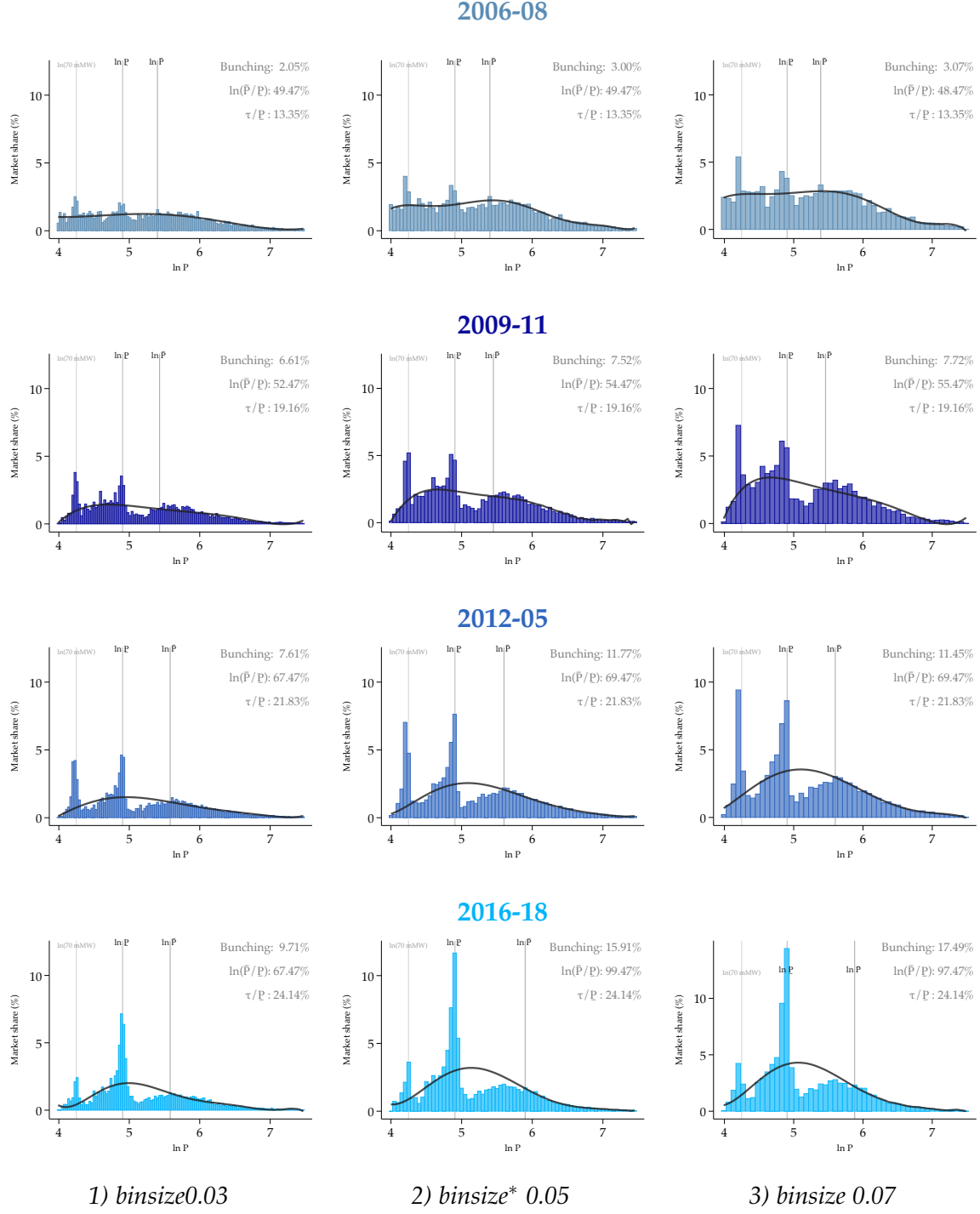
$$135 \times mMW = \lambda = \rho(\underline{h})$$

Given a particular assumed functional form,

$$\underline{h} = \frac{-\rho_1 + \sqrt{\rho_1^2 - 4 \cdot \rho_2 \cdot (\bar{P} - \lambda)}}{2 \cdot \rho_2}, \quad (32)$$

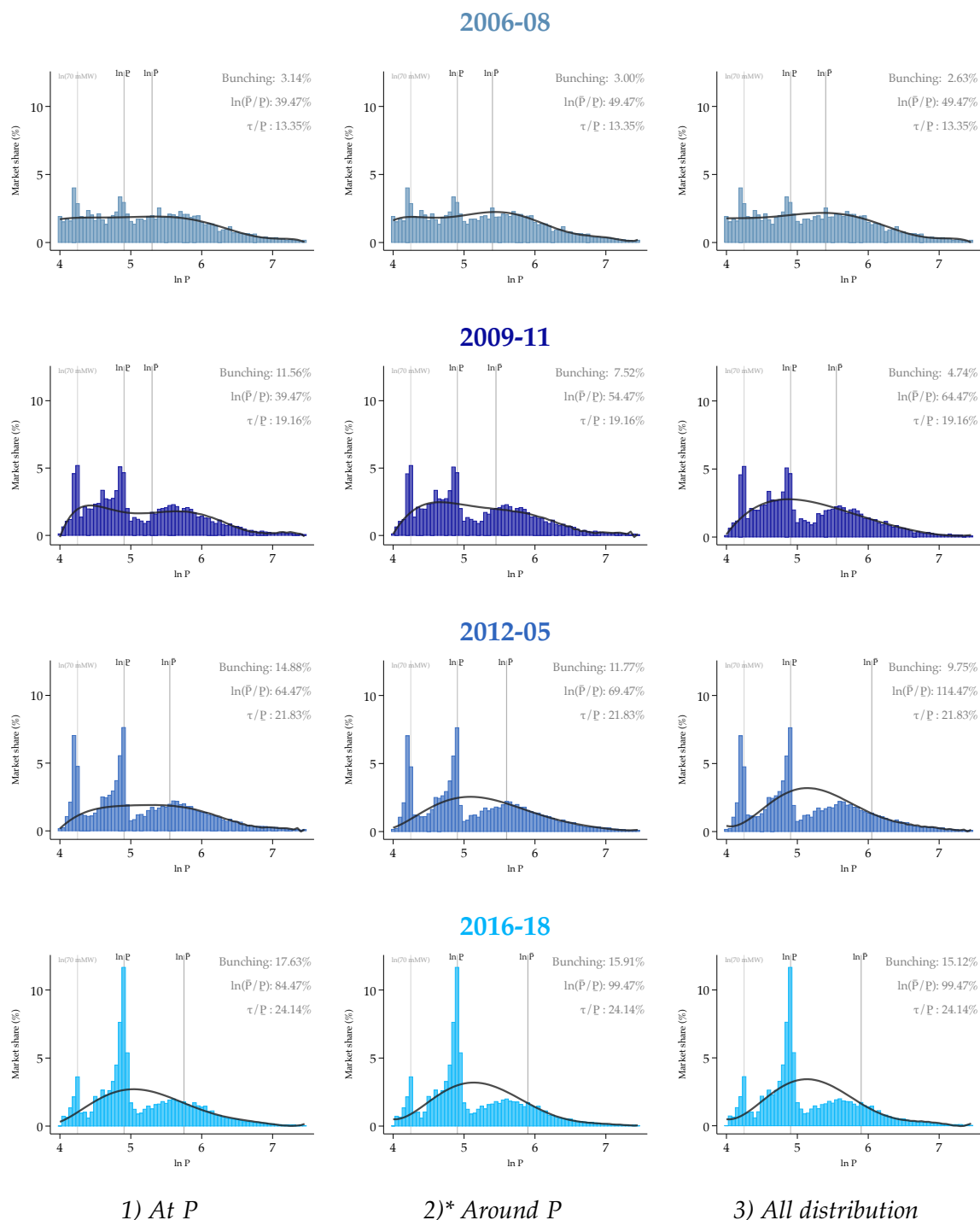
In Figure 9, \underline{h} corresponds to the value of h at which the implicit price intersects the price cutoff (gray horizontal line).

Figure B.8: Bunching Over Time Using Different Binsize



NOTE: This figure shows the distribution or the market share of housing units by sale price (expressed in log of mMW). The lines are the cutoffs defining *low-cost housing* $\underline{P} = 135$ mMW and *priority low-cost housing* $\underline{P} = 70$ mMW. The additional lines shows the point, \bar{P} , where the counterfactual and observed distribution coincide again after the cutoff. The figure shows for the different period for all available cities.

Figure B.9: Bunching Over Time Using Different Criteria of Missing=excess mass to Select Estimation Parameters



NOTE: This figure shows the distribution or the market share of housing units by sale price (expressed in log of mMW). The lines are the cutoffs defining *low-cost housing* $\underline{P} = 135$ mMW and *priority low-cost housing* 70 mMW. The additional lines shows the point, \bar{P} , where the counterfactual and observed distribution coincide again after the cutoff. The figure shows for the different period for all available cities.

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