Equilibrium Effects of Housing Subsidies: Evidence from a Policy Notch in Colombia

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ARE MARKET-ORIENTED HOUSING POLICIES EFFECTIVE?

- ► Governments implement various **market-oriented** policies to promote housing construction and home-ownership.
 - Subsidies or tax incentives

QUESTIONS:

- 1. Housing market effect?
 - Prices, quantities, type of housing
- 2. Does incentivizing home-ownership work?
 - Are there any unintended consequences?
 - What happens if these policies are removed?
 - How big are the efficiency costs?
- ▶ I use a quasi-experiment to estimate a housing market model.
- ► Counterfactual policy evaluation and welfare analysis.



COLOMBIAN HOUSING POLICY

▶ Policy tools:

- 1. Subsidies to low-income households low-cost housing.
- 2. Tax incentives to developers who build low-cost housing.
- 3. A price cap defining low-cost housing.

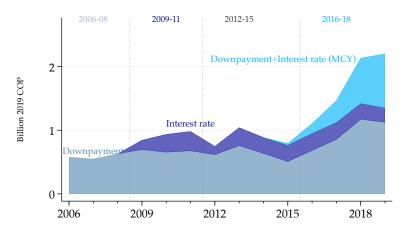
135 monthly minimum wages (mMW) $\approx 40,000 \$USD$

Empirical advantages:

- 1. Price cap
 - Discontinuous incentives for developers and households to bunch at the cutoff.
- 2. Unique and novel data
 - Census data for all new construction projects.
 - Administrative records for the subsidies.
- 3. Subsidy expansion (2006-18)

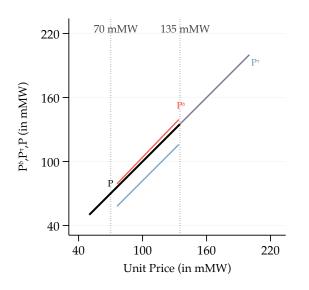


GOVERNMENT EXPENDITURE AND POLICY EXPANSION



- ► Four different periods of expansion.
 - 2006-08: Downpayment (only for formal employees)
 - 2009-11: + Interest rate subsidy
 - 2012-15: Focus on extremely poor population (subsidies targeted at 70mMW)
 - 2016-18: + Mi Casa Ya–MCY (downpayment and interest for informal and formal employees)

THE NOTCH



Transaction Price P Developers Price

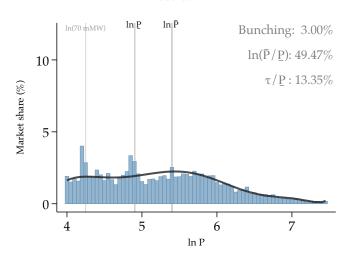
$$\mathbf{P}^{\delta} = P \cdot (1 + \delta)$$
:
 $\delta = \text{Tax refund}$
Households price

$$\mathbf{P}^{\tau} = P - \tau$$
$$\tau = \text{Subsidy}$$

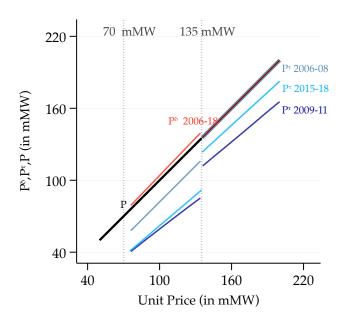
Agents benefit from buying/selling *low cost housing* ($P \le 135 \text{mMW}$)

BUNCHING AT THE LOW-COST HOUSING PRICE LIMIT

Only downpayment subsidy 2006-08



THE DEMAND NOTCH INCREASES OVER TIME



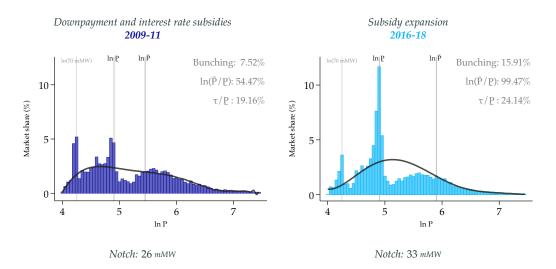
Supply Notch δ 2006-18 4%

Demand Notch τ_t

2006-08: 18 *mMW* 2009-11: 26 *mMW* 2016-18: 33 *mMW*

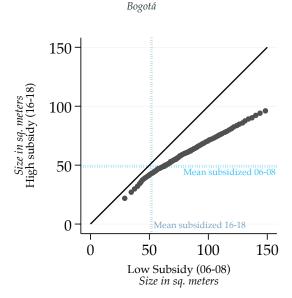
Note: 2012-15 Too many changes and free housing at 70 mMW

LARGER BUNCHING AS NOTCH INCREASES



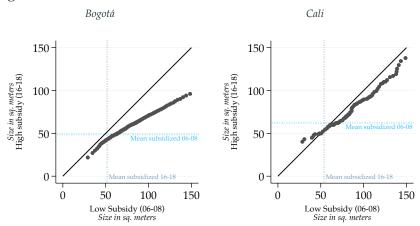
CHANGES IN HOUSING STOCK CHARACTERISTICS

► Changes in unit size (quantile to quantile plot)



CHANGES IN HOUSING STOCK CHARACTERISTICS

► Changes in unit size



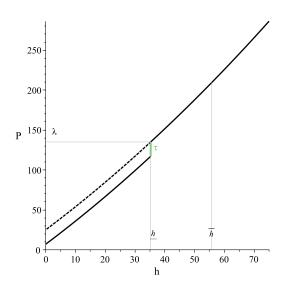
- ► Why size?
 - Continuous, easy to measure, monotonic relationship with price and income.
 - In contrast to most datasets, I observe it.

II. EQUILIBRIUM MODEL OF HOUSING SUPPLY AND DEMAND

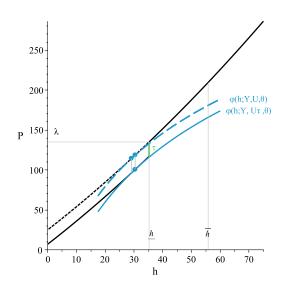
HOUSING MARKET EQUILIBRIUM MODEL

1. Housing

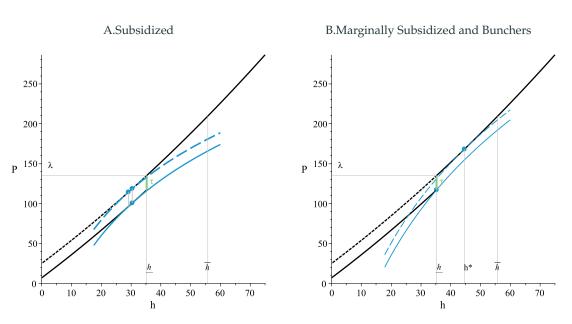
- Differentiated product described by its size $h \in \mathcal{H}$
- Price depends on size P(h)
- 2. **Households** $i \in I$, Heterogeneous in Income $Y_i \sim F_Y$
 - Choose h_i and consumption C_i to maximize Utility $U(C_i, h_i; \theta)$
- 3. **Developers** $j \in J$, Heterogeneous in Productivity $A_j \sim G_A$
 - Choose h_i to maximize profits
 - Building costs $B(A_j, h_j, Q(h_j); \beta)$
- 4. Competitive Market Equilibrium
 - Price function $P(h) \rightarrow$ clears the market $\forall h \in \mathcal{H}$

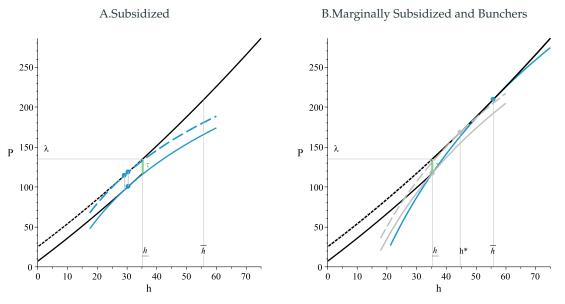


- **Implicit Price Function** *P*(*h*)
- Subsidy τ



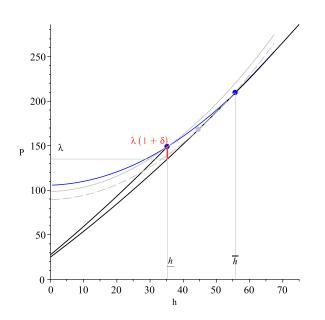
- **Implicit Price Function** *P*(*h*)
- Subsidy au
- Bid functions $\varphi_D(h, Y, \bar{U}; \theta)$
 - $\bar{U} = U(h, Y_i \varphi_D; \theta)$
 - $\bar{U}_{\tau} = U(h, Y_i \varphi_D + \tau; \theta)$





Housing demand function

DEVELOPERS' OPTIMAL CHOICES

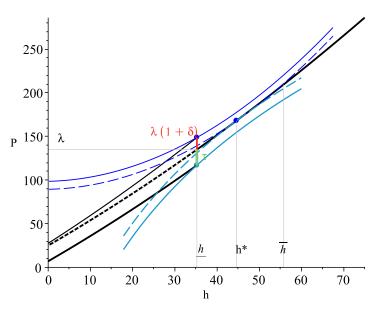


- **Implicit Price Function** *P*(*h*)
- Tax incentives $P(h) \cdot (1 + \delta)$
- Offer Functions $\varphi_S(h, A_i, \bar{\pi}; \beta)$

$$\bar{\pi} = \pi \left(h, A_j, P(h); \beta \right)$$

$$\bar{\pi}_{\delta} = \pi \left(h, A_j, P(h) * (1 + \delta) \right); \beta \right)$$

EQUILIBRIUM: DEVELOPERS AND HOUSEHOLDS MATCH

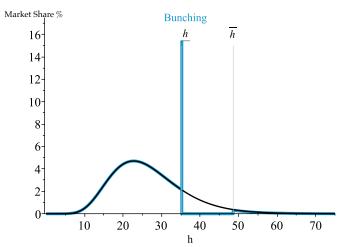


Implicit price: Envelop of offer and bid curves.

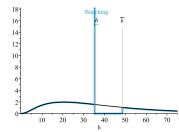
EQUILIBRIUM: AGGREGATE DEMAND AND SUPPLY DENSITY

How to aggregate? \rightarrow Change of variable formula optimal choices (h^*) and the density of households (F_v) and developers (G_v) details

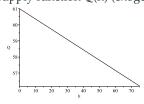
Demand Density Function



Housing types density function $g_h(h)$

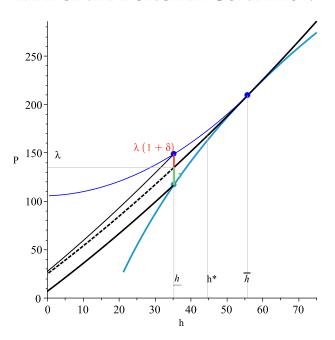


Unit supply function Q(h) (exogenous)



IDENTIFICATION

MARGINAL BUNCHER CONDITION

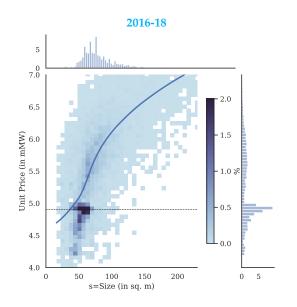


MARGINAL BUNCHER CONDITION

Marginal Buncher Condition					
Household	$V_D = U\left(\overline{Y} - P\left(\overline{h}\right), \overline{h}; \theta\right) - U\left(\overline{Y} - P^{\tau}\left(\underline{h}\right), \underline{h}; \theta\right) = 0$				
Developer	$V_{D} = U\left(\overline{Y} - P\left(\overline{h}\right), \overline{h}; \theta\right) - U\left(\overline{Y} - P^{\tau}(\underline{h}), \underline{h}; \theta\right) = 0$ $V_{S} = \pi\left(Q(\overline{h}), \overline{A}, P\left(\overline{h}\right); \beta\right) - \pi\left(Q(\underline{h}), \overline{A}; P^{\delta}(\underline{h}); \beta\right) = 0$				
Optimality Conditions					
Income	$\overline{Y} = \tilde{Y}\left(\overline{h}; \boldsymbol{\theta}, P(h), \lambda\right)$				
Productivity	$egin{aligned} \overline{Y} &= \widetilde{Y}\left(\overline{h}; oldsymbol{ heta}, P(h), \lambda ight) \ \overline{A} &= \widetilde{A}\left(\overline{h}; oldsymbol{eta}, P(h), \lambda ight) \end{aligned}$				
Functional Forn	ns				
Implicit Price	$P = \rho_0 + \rho_1 \cdot h + \rho_2 \cdot h^2$				
Utility	$U = \left[rac{1}{2}\cdot C^{ heta} + rac{1}{2}\cdot h^{ heta} ight]^{rac{1}{ heta}}$				
Unit Supply	$Q = \alpha_0 + \alpha_1 h$				
Cost	$B = A_j \cdot Q \cdot h^{\beta}$				

ESTIMATION

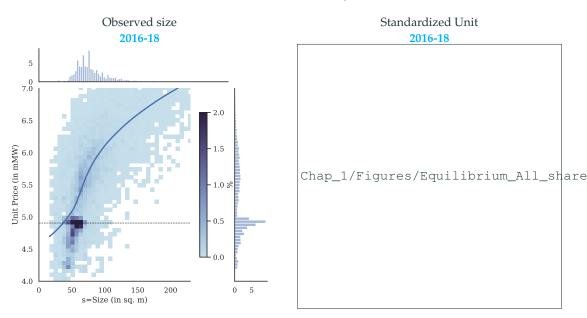
OBSERVED EQUILIBRIUM: PRICES, QUANTITIES, AND SIZE



- ► Solid line: price vs size
- \rightarrow hedonic price function
- ► Multiple characteristics
- \rightarrow Reduce to a single characteristic
- ► Standard unit size (*h*):
- → Size of a unit with average characteristics that costs the same price

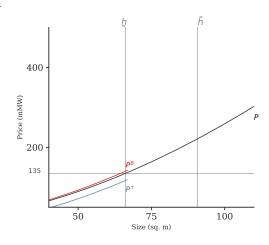
details

OBSERVED EQUILIBRIUM: PRICES, QUANTITIES, AND SIZE



STEP I: EQUILIBRIUM CHARACTERIZATION

- Using the observed hedonic equilibrium
 - Price function: P(h)
 - Size threshold: $\underline{h} = P^{-1} (\lambda = 135 \text{mMW})$
 - Standard Unit Size: h
- ► Behavioural Responses:
 - Housing size for marginal buncher: \bar{h}
- ► Unit Supply Function:
 - $Q = \alpha_0 + \alpha_1 \cdot h_{ltc}$ see
- ► Policy Parameters:
 - Notches: τ_t , δ see



STEP II: STRUCTURAL PARAMETERS

$$\triangleright B = A_i \cdot Q \cdot h^{\beta}$$

$$U = \left[\frac{1}{2} \cdot C^{\theta} + \frac{1}{2} \cdot h^{\theta}\right]^{\frac{1}{\theta}}$$

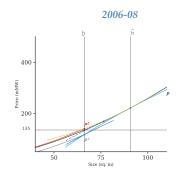
► Elasticity of Substitution: $\sigma = \frac{1}{1-\theta}$

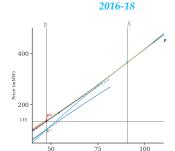
Identification equations:

$$V_D\left(\underline{\theta}|\underline{h},\overline{h},P(h),\tau,\lambda\right) = 0$$

$$V_S\left(\underline{\beta}|\underline{h},\overline{h},P(h),\alpha,\delta,\lambda\right) = 0$$

	Structural Parameters						
	2006-08	2009-11	2012-15	2016-18			
β	2.53	1.67	1.77	1.70			
σ	0.85	0.97	0.90	0.90			





III. POLICY EVALUATION:

COUNTERFACTUAL POLICY I: PROPOSED TAX REFORM

▶ Policy proposal: Remove the tax incentives to developers

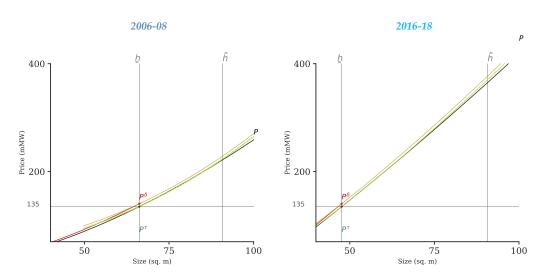
▶ Developers reaction:

"If these items are repealed, in Valle del Cauca we would go from having an offer of SH and sales of 23,000 homes, average year, to one of sales of 4,600 homes"

source: El Tiempo (2021)

▶ Question: What happens to the marginally subsidized developers?

EFFECT ON MARGINALLY SUBSIDIZED DEVELOPERS



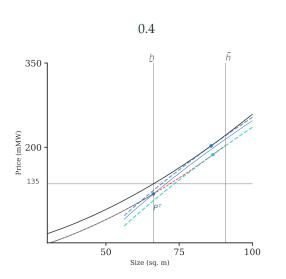
B. Excess Profit and Potential Losses

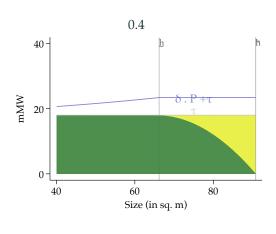
COUNTERFACTUAL POLICY II: REMOVE PRICE CUTOFF

► Same households get subsidy but they can buy any house.

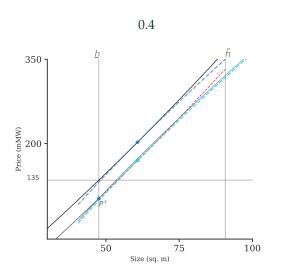
Question: How much better off households are?

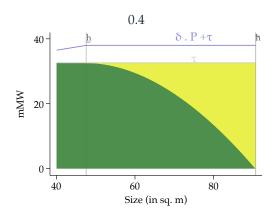
EFFECT ON MARGINALLY SUBSIDIZED HOUSEHOLDS 2006-08





EFFECT ON MARGINALLY SUBSIDIZED HOUSEHOLDS 2016-18





LITERATURE AND CONTRIBUTION

Integrates the *bunching* and *hedonic* literatures to propose a method to think about welfare consequences of *housing policies*

Bunching	Hedonic	Housing Policy
Housing marketLink to modelSupply and demand	Policy notchSupply sideIdentification	EvidenceWelfareCounterfactuals
 Housing market applications Best et al. (2019), DeFusco and Paciorek (2017) Methodology Notches >> Kinks: Kleven (2016), Bertanha et al. (2021), Blomquist et al. (2021) 	- Seminal paper Rosen (1974),Epple (1987) - Recent Contributions Bajari and Benkard (2005), Heckman et al. (2010), Epple et al. (2020), Chernozhukov et al. (2021) - Reviews Kuminoff et al. (2013), Greenstone (2017)	 Developers subsidies Baum-Snow and Marion (2009), Soltas (2021), Sinai and Waldfogel (2005) Households Subsidies Carozzi et al. (2020) Incidence and welfare Poterba (1992), Galiani et al. (2015)

CONCLUSION (I): THE PAPER

- ► Characterization of the equilibrium.
- ► Compelling evidence of the market responding to subsidies.
- An hedonic housing market equilibrium with heterogeneous agents can rationalize the response.
- ▶ Propose a identification strategy to recover the model parameters.
- ightharpoonup Model+estimates ightharpoonup Welfare.
- Policy design matters → need to be careful of how agents respond to incentives.

CONCLUSION (II): GENERALIZATION

- ► The method I propose could be used to evaluate housing policy more generally.
- ► Two facts suggest this could be potentially effective.
 - 1. There is increasing evidence to bunching responses to nonlinear incentives (e.g., help to buy, housing programs in the USA)
 - 2. Many other sources of non linear incentives in housing markets.
- ► Further, it can be applied to other markets (e.g., labor markets, drugs, etc.)

Appendix

Research Agenda

- ▶ In my research I exploit natural experiments using administrative and census data to study the impacts of large scales government investments.
 - What are effects on the population?
 - Are there any unintended consequences?
 - How do we evaluate costs?
 - What are the welfare effects?
 - How should we target subsidies?
 - Should governments invest directly or through subsidies?
 - Can we be more efficient in the way we spend the money?
- ► In my current projects I address these questions by studying subsidies to utilities, a push in internet expansion, the construction of the US interstate highway system and housing subsidies.

CURRENT PROJECTS

- ▶ Does the US have an Infrastructure Cost Problem? Evidence from the Interstate Highway System (2021) with *Neil Mehrotra* and *Matthew A. Turner*
- ► The Effect of Location-Based Subsidies on the Housing Market (2021)
- ► Internet Expansion and School Performance: Evidence from Colombia (2021) with *Aaron Weishrod*
- ► The Expansion of Higher Education in Colombia: Bad Students or Bad Programs? (2021) with *Adriana Camacho* and *Julian Messina*

WHAT'S NEXT?

I want to keep finding setting to explore these questions and propose methods to evaluate the effects of government expenditures.

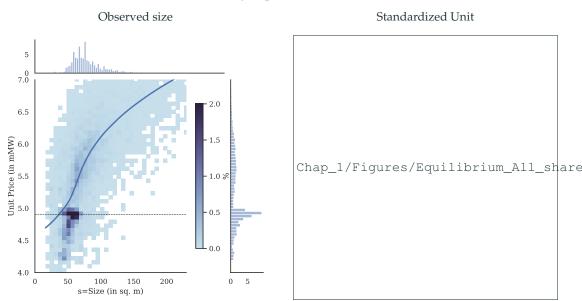
- ► Focus on housing subsidies
 - Study different policy approaches
 - Keep exploring the role of supply
 - Financial sector and inter-temporal decisions
 - Effects of housing policies on the labor market and other sectors
- ► Effects of highways or other policies like the *estratos* on urban shape and segregation patterns.

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From size s to Standardized Size h

Subsidy expansion 2016-18



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HEDONIC PRICES AND STANDARDIZED HOUSING UNIT

► Hedonic price/Implicit price for housing size

$$P_{ltc} = \rho \left(s_{ltc} \right) + \Gamma' X_{ltc} + \omega_{ltc} \tag{1}$$

l, house type in a development, *t* year, *c* city

- Simplifying assumption: $\rho(s_{ltc}) = \rho_1 \cdot s_{ltc} + \rho_2 \cdot s_{ltc}^2$
- Identifying assumption: $E(s_{ltc}|X_{ltc},\omega_{ltc})=0$
- ightharpoonup Standard Unit Size h_{ltc}

$$\rho\left(h_{ltc}\right) + \Gamma'\bar{X} + \bar{\omega} = \rho\left(s_{ltc}\right) + \Gamma'X_{ltc} + \omega_{ltc} \tag{2}$$

• Characteristics of the standard house: \bar{X} , $\bar{\omega}$

figures



BEHAVIOURAL RESPONSES INDUCED BY THE POLICY

▶ Recovered by comparing observed and counterfactual distribution

Observed
$$f_{h^*} \rightarrow \text{histogram}$$

Counterfactual $f_{h_0} \rightarrow$ predicted density excluding observations around the cutoff (Kleven, 2016)

$$h_b = \sum_{p=0}^{T} \hat{\iota}_p h_b^p + \sum_{k=L}^{H} \kappa_k \cdot \mathbb{1} \left[h_k = h_b \right] + v_b$$

$$\hat{f}_{h_0} = \sum_{p=0}^T \hat{\iota}_p h_b^p$$

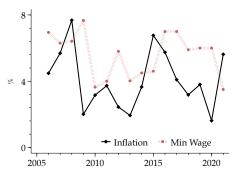
Choice parameters: bin size, bounds for excluded area (L,H) and polynomial degree p

Figures

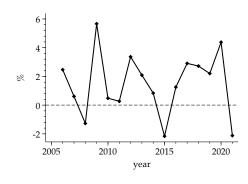
EQUILIBRIUM: DEVELOPERS AGGREGATE SUPPLY DENSITY

$$D\left(h\right) = \begin{cases} f_{h^*}(h) \, \mathrm{d}h & \text{if } h < \underline{h} \\ f_{h^*}(h) \, \mathrm{d}h & \text{if } \underline{h} = h \\ + \int\limits_{\underline{h}}^{\overline{h}} f_{h^*}(h) \, \mathrm{d}h & \text{if } \underline{h} = h \\ 0 & \text{if } h \in \left(\underline{h}, \overline{h}\right) \end{cases} \qquad S\left(h\right) = \begin{cases} g_{h^*}\left(h\right) \cdot Q\left(h\right) & \text{if } h < \underline{h} \\ \left(g_{\underline{h}^*}\left(\underline{h}\right) + \int\limits_{\underline{h}}^{\overline{h}} g_{h^*}\left(h\right) \, \mathrm{d}h\right) \cdot Q\left(\underline{h}\right) & \text{if } \underline{h} = h \\ 0 & \text{if } \underline{h} < h < \overline{h} \\ g_{h^*} \cdot Q\left(h\right) & \text{if } \overline{h} \leq h \end{cases}$$
 Equilibrium Figures

Inflation and minimum wages.



a. Min wage and Inflation

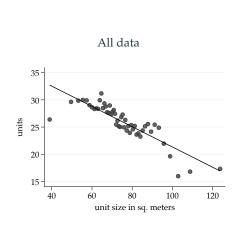


b. Min wage and Inflation

Data

DEVELOPERS CHOICES OF SIZE AND UNIT SUPPLY

Product type density (g_h)

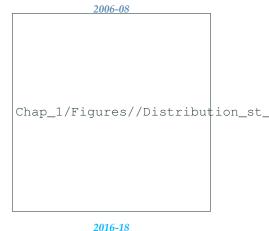


$$Q_{ltc} = \alpha_0 + \alpha_1 s_{ltc} + \alpha'_x X_{ltc} + \epsilon^{Q}_{ltc}$$

$$06-08 \quad 09-11 \quad 12-15 \quad 16-18$$

01 1

22 2





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NOTCHES

► Demand Notch Overtime

	Notch (in mMW)			# Subsidies (in thousand)		
	τ^{M}	$ au^i$	au	down payment	i rate	Mi Casa Ya
2006-08	18.0		18.0	47.1	•	•
2009-11	20.0	5.85	25.9	46.4	16.7	•
2012-15	19.9	9.55	29.5	41.1	22.2	
2016-18	25.3	7.24	32.6	44.5	23.4	16.8

► Supply Notch: 4 percent

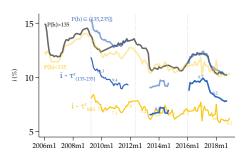
Step I

DATA: MORTGAGES AND INTEREST RATES

rent equivalent ((Poterba, 1984), (Bishop & Timmins, 2019) assume it is 0.05)

- ► Size of the mortgages and interest rate.
- ▶ Identifier for SIH.

Market interest rate i and subsidy τ^r



Monthly payments and monthly equivalent for relevant values. P(h) < 135



To convert the magnitudes into monthly payments I use:

$$X_{montly} = X \cdot \kappa(i, n); \kappa(i, n) = \frac{\frac{i}{12} \cdot \left(1 + \frac{i}{12}\right)^{12 \cdot n}}{\left(1 + \frac{i}{12}\right)^{n \cdot 12} - 1}$$

HOUSEHOLDS' DEMAND FUNCTION

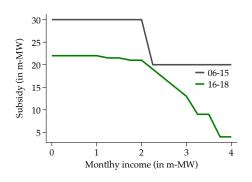
$$h^{\mathrm{D}}(Y_{i}) = \begin{cases} h^{*}(Y_{i}, \tau; \theta, \boldsymbol{\rho}, \lambda) & \text{if } Y_{i} \leq \underline{Y} \\ \\ \underline{h} & \text{if } \underline{Y} < Y_{i} < \overline{Y} \end{cases}$$
$$h^{*}(Y_{i}, \tau; \theta, \boldsymbol{\rho}, \lambda) & \text{if } \overline{Y} \leq Y_{i} \end{cases}$$

- Tangency conditions: $h^*(Y_i, \tau; \theta, \rho, \lambda)$
- Income and unit size: $Y_i = \tilde{Y}(h, \tau; \theta, \rho, \lambda) = h^{*-1}(h_i, \tau; \theta, \rho, \lambda)$

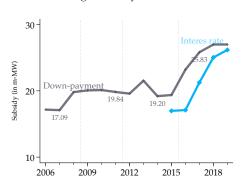
Graphs

THE NOTCH: DOWN PAYMENT SUBSIDY

Subsidy by household income



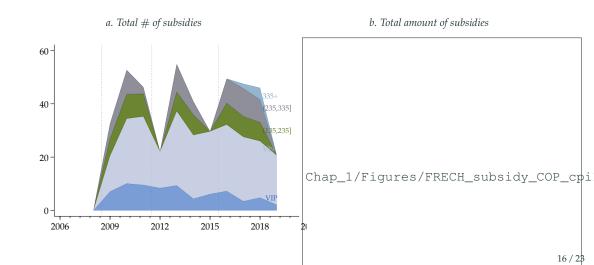
Average subsidy over time



- ► Varies by income.
- ► Increase in 2016.
- Expanded trough *mi casa YA*

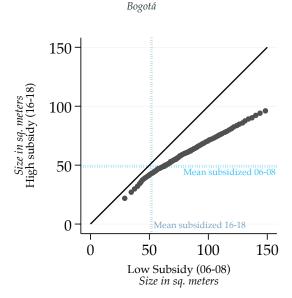
Subsidies and Government Expenditure (VIP-P(h) < 70)

This figure shows the interest rate subsidies for all different price levels



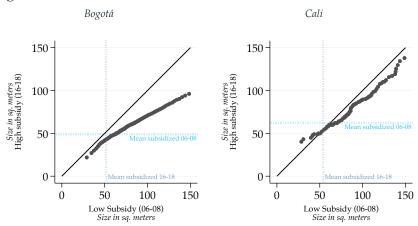
CHANGES IN HOUSING STOCK CHARACTERISTICS

► Changes in unit size (quantile to quantile plot)



CHANGES IN HOUSING STOCK CHARACTERISTICS

Changes in unit size



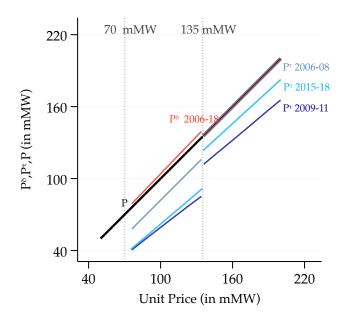
- ► Why size?
 - Continuous, easy to measure, monotonic relationship with price and income.
 - In contrast to most datasets, I observe it.

PRICES

Chap_1/Figures/Price_function_All_main.pdf

Chap_1/Figures/M/1st_step.pdf

THE DEMAND NOTCH INCREASES OVER TIME



Supply Notch δ 2006-18 4%

Demand Notch τ_t

2006-08: 19.7 mMW 2009-11: 26.4 mMW 2016-18: 33.1 mMW

2012-15 Too many changes and free housing at 70mMW

ALTERNATIVE REPRESENTATION OF THE EQUILIBRIUM. DEMAND AND SUPPLY FOR SIZE

Expansion period Mi Casa Ya 2016-18

Chap_1/Figures/MgFigure_period_16-18.pdf

EFFECT ON MARGINALLY SUBSIDIZED DEVELOPERS



Changes in profits (%)

EFFECT ON MARGINALLY SUBSIDIZED HOUSEHOLDS

2006-08	2009-11
	<u>OMapO3.√Fdig</u> ures/welfare_demand_period

EQUATIONS

Optimality Conditions

Income
$$\overline{Y} = (2 \cdot h \cdot \rho_2 + \rho_1)^{-\frac{1}{\theta - 1}} \cdot h + h^2 \cdot \rho_2 + h \cdot \rho_1 + \rho_0$$
Productivity
$$\overline{A} = \frac{3 \cdot (\delta + 1) \cdot (h \cdot \alpha_1 + \alpha_0) \cdot \left(\left(h^2 \cdot \rho_2 + \frac{2}{3} \cdot h \cdot \rho_1 + \frac{1}{3} \cdot \rho_0 \right) \cdot \alpha_1 + \frac{2 \cdot \left(h \cdot \rho_2 + \frac{\rho_1}{2} \right) \cdot \alpha_0}{3} \right)}{h^{\beta + 1} \cdot \alpha_1^2 + h^{\beta} \cdot \alpha_0 \cdot \alpha_1 + 2 \cdot h \cdot (h \cdot \alpha_1 + \alpha_0)^2}$$

Marginal Buncher Condition

$$\text{Household} \qquad 0 = \left(\frac{\underline{h}^{\theta} + \left(\overline{h}^{2}\rho_{2} - \rho_{2}\underline{h}^{2} + \overline{h}\rho_{1} + \overline{h}\left(2\overline{h}\rho_{2} + \rho_{1}\right)^{\frac{1}{1-\theta}} - \rho_{1}\underline{h} + \tau\right)^{\theta}}{2}\right)^{\frac{1}{\theta}} - \left(\frac{\left(\left(2\overline{h}\rho_{2} + \rho_{1}\right)^{\frac{\theta}{1-\theta}} + 1\right)\overline{h}^{\theta}}{2}\right)^{\frac{1}{\theta}} \\ \text{Developer} \qquad 0 = \frac{\overline{h}^{\theta+1}\alpha_{1} - \underline{h}^{\theta+1}\alpha_{1} + \alpha_{0}\left(\overline{h}^{\theta} - \underline{h}^{\theta}\right)}{\overline{h}^{\theta+1}\alpha_{1}^{2} + \overline{h}^{\theta}\alpha_{0}\alpha_{1} + 2\overline{h}\left(\overline{h}\alpha_{1} + \alpha_{0}\right)^{2}} - \\ \frac{\left(-\rho_{2}(1+\delta)\underline{h}^{3} - \rho_{1}(1+\delta)\underline{h}^{2} - \rho_{0}(1+\delta)\underline{h} + \overline{h}\left(\overline{h}^{2}\rho_{2} + \overline{h}\rho_{1} + \rho_{0}\right)\right)\alpha_{1} + \alpha_{0}\left(-\rho_{2}(1+\delta)\underline{h}^{2} - \rho_{1}(1+\delta)\underline{h} + \overline{h}^{2}\rho_{2} + \overline{h}\rho_{1} - \delta\rho_{0}\right)}{3\left(\left(\overline{h}^{2}\rho_{2} + \frac{2}{3}\overline{h}\rho_{1} + \frac{1}{3}\rho_{0}\right)\alpha_{1} + \frac{2\left(\overline{h}\rho_{2} + \frac{\rho_{1}}{2}\right)\alpha_{0}}{3}\right)\left(\overline{h}\alpha_{1} + \alpha_{0}\right)}$$

Main table