

Equilibrium Effects of Housing Subsidies: Evidence from a Policy Notch in Colombia

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Cornerstone

ARE MARKET-ORIENTED HOUSING POLICIES EFFECTIVE?

- ▶ Governments implement various **market-oriented** policies to promote housing construction and home-ownership.
 - Subsidies or tax incentives

QUESTIONS:

1. Housing market effect?
 - Prices, quantities, **type of housing**
 2. Does incentivizing home-ownership work?
 - Are there any unintended consequences?
 - What happens if these policies are removed?
 - How big are the efficiency costs?
- ▶ I use a quasi-experiment to estimate a housing market model.
 - ▶ Counterfactual policy evaluation and welfare analysis.



COLOMBIAN HOUSING POLICY

► Policy tools:

1. Subsidies to low-income households low-cost housing.
2. Tax incentives to developers who build low-cost housing.
3. A price cap defining low-cost housing.

135 monthly minimum wages (*mMW*) \approx 40,000 \$USD

► Empirical advantages:

1. Price cap
 - Discontinuous incentives for developers and households to bunch at the cutoff.
2. Unique and novel data
 - Census data for all new construction projects.
 - Administrative records for the subsidies.
3. Subsidy expansion (2006-18)

THIS PAPER

I. Descriptive evidence

- Policy description and characterization of observed equilibrium
- Evidence of housing market responding to the subsidy scheme

II. Hedonic equilibrium of housing supply and demand

- Product differentiation and heterogeneous developers and households
- Identification:

Step 1: Equilibrium characterization \rightarrow *Detailed data and literature best practices*

Step 2: Preferences and cost function parameters $\xrightarrow{\text{new}}$ *bunching and policy tools*

III. Proposed policy counterfactual and welfare

- Colombian 2021 tax reform – remove tax incentives to developers
 - Policy change – phasing out price caps
- \rightarrow Effects on households and developers

RESULTS

I. Behavioural responses induced by the subsidy scheme

- Bunching at price cutoff
- Larger response as the subsidies increase → market share at cutoff went from 1% to 7%
- Households downsize → they buy units up to 30% percent smaller to benefit from the subsidy

II. Estimate a model that rationalizes the market observed equilibrium

- Elasticity of substitution between housing and consumption is 0.9

III. Effects of the proposed policy reforms

- Colombian 2021 tax reform proposal could create a housing shortage
- Removing the price cap increases welfare

LITERATURE AND CONTRIBUTION

Integrates the *bunching* and *hedonic* literatures to propose a method to think about welfare consequences of *housing policies*

Bunching	Hedonic	Housing Policy
<ul style="list-style-type: none">▶ Housing market▶ Link to model▶ Supply and demand	<ul style="list-style-type: none">▶ Policy notch▶ Supply side▶ Identification	<ul style="list-style-type: none">▶ Evidence▶ Welfare▶ Counterfactuals
<ul style="list-style-type: none">- Housing market applications Best et al. (2019), DeFusco and Paciorek (2017)- Methodology <i>Notches >> Kinks</i>: Kleven (2016), Bertanha et al. (2021), Blomquist et al. (2021)	<ul style="list-style-type: none">- Seminal paper S. Rosen (1974), Epple (1987)- Recent Contributions Bajari and Benkard (2005), Heckman et al. (2010), Epple et al. (2020), Chernozhukov et al. (2021)- Reviews Kuminoff et al. (2013), Greenstone (2017)	<ul style="list-style-type: none">- Developers subsidies Baum-Snow and Marion (2009), Soltas (2020), Sinai and Waldfoegel (2005)- Households Subsidies Carozzi et al. (2020)- Incidence and welfare Poterba (1992), Galiani et al. (2015)

I. DESCRIPTIVE ANALYSIS:
DATA, POLICY AND OBSERVED
EQUILIBRIUM

POLICY TOOLS

1. Supply Subsidies

- Value Added Tax (VAT) refund

2. Demand Subsidies

- Downpayment
- Interest rate

Income \leq 4 monthly minimum wages (mMW) classify

3. Targeting tool for the subsidy:

- Only new *low cost* units are eligible

$$\text{Low cost} = \begin{cases} 1 & \text{if } P_t \leq 135 \text{ mMW}_t \\ 0 & \text{if } P_t > 135 \text{ mMW}_t \end{cases}$$

Note: 135 mMW \approx 40,000 \$USD



DATA

1. Administrative Records from Minister of Housing

- Subsidy size
- Mortgage information

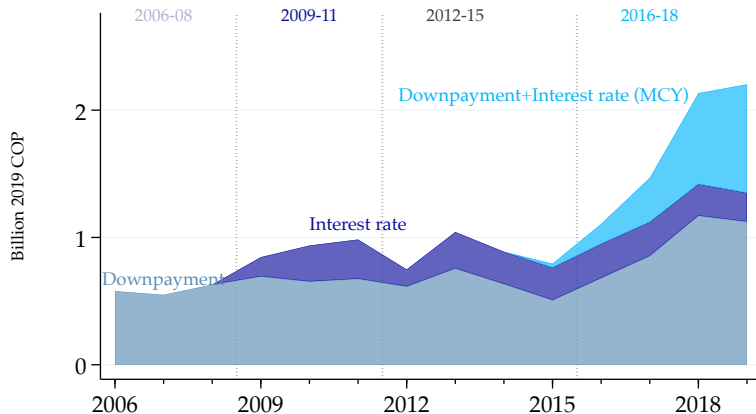
→ Government expenditure on each subsidy

2. New Construction Census (Camacol)

- 126 Municipalities
- Years: 2006-2018
- Sale prices and quantities
- Unit characteristics: **size**, location, # rooms, # bathrooms, etc.
- Development characteristics: lot size, # towers, # floors, developer id, etc.

Everything measured in monthly minimum wages (*mMW*) or 2019 *COP* Inflation and *mMW* change

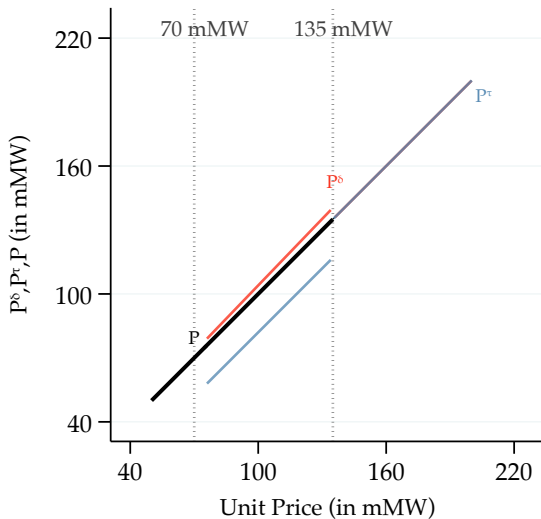
GOVERNMENT EXPENDITURE AND POLICY EXPANSION



► Four different periods of expansion.

- 2006-08: Downpayment (only for formal employees)
- 2009-11: + Interest rate subsidy
- 2012-15: Focus on extremely poor population (subsidies targeted at 70mMW)
- 2016-18: + *Mi Casa Ya*–MCY (downpayment **and** interest for informal and formal employees)

THE NOTCH



Transaction Price

P

Developers Price

$$P^\delta = P \cdot (1 + \delta):$$

δ = Tax refund

Households price

$$P^\tau = P - \tau$$

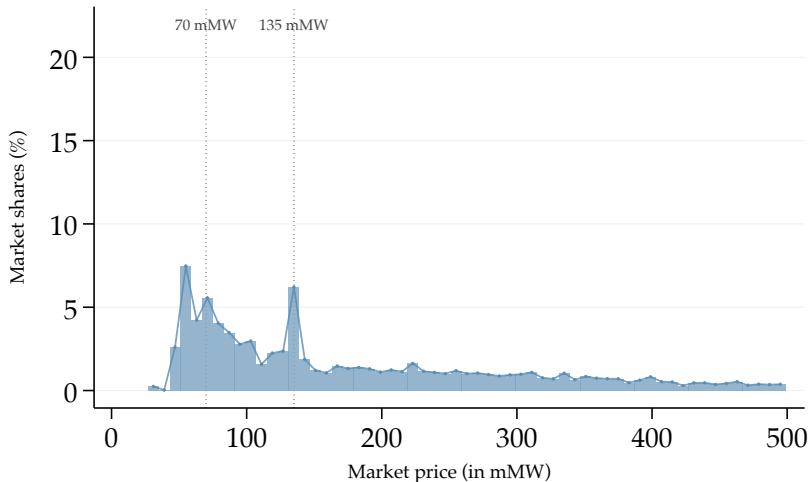
τ = Subsidy

Agents benefit from buying/selling *low cost housing* ($P \leq 135 \text{ mMW}$)

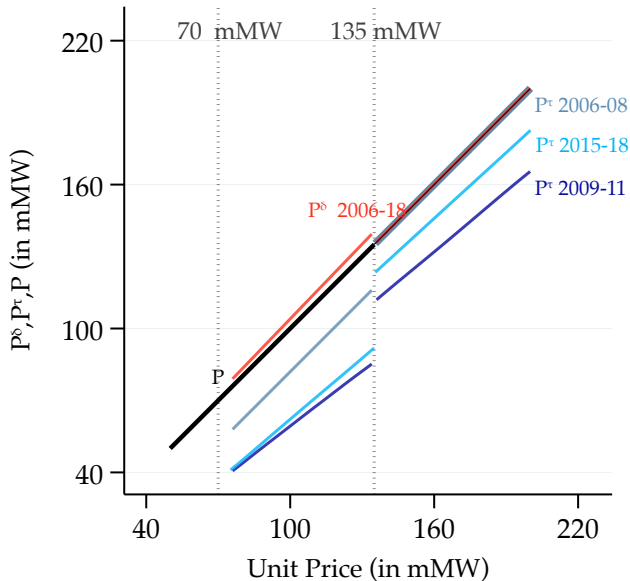
BUNCHING AT THE LOW-COST HOUSING PRICE LIMIT

Only downpayment subsidy

2006-08



THE DEMAND NOTCH INCREASES OVER TIME



Supply Notch δ

2006-18 4%

Demand Notch τ_t

2006-08: 18 mMW

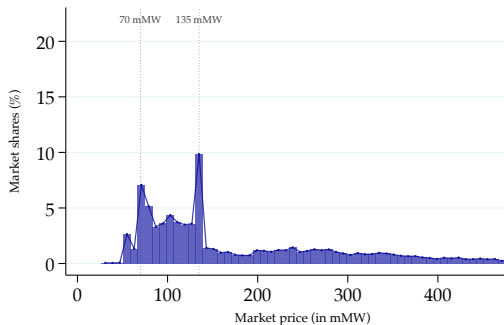
2009-11: 26 mMW

2016-18: 33 mMW

Note: 2012-15 Too many changes and free housing at 70 mMW

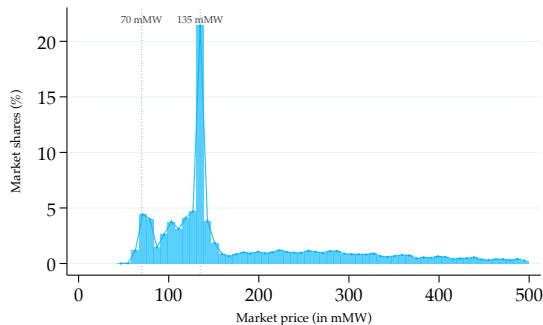
LARGER BUNCHING AS NOTCH INCREASES

Downpayment and interest rate subsidies
2009-11



Notch: 26 mMW

Subsidy expansion
2016-18

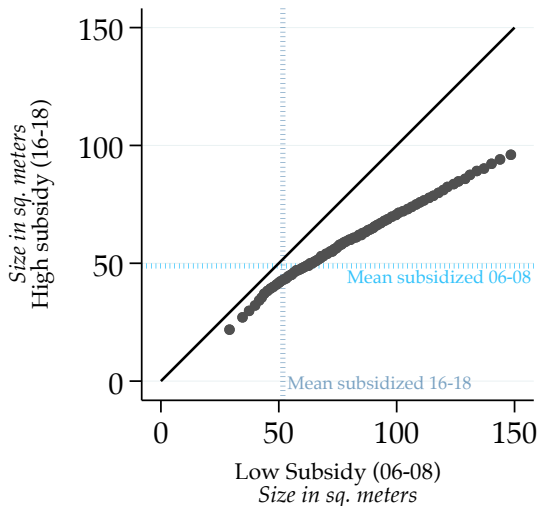


Notch: 33 mMW

CHANGES IN HOUSING STOCK CHARACTERISTICS

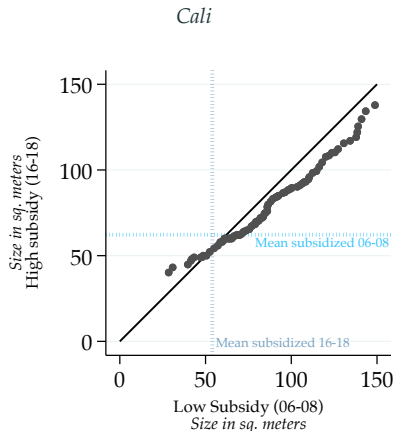
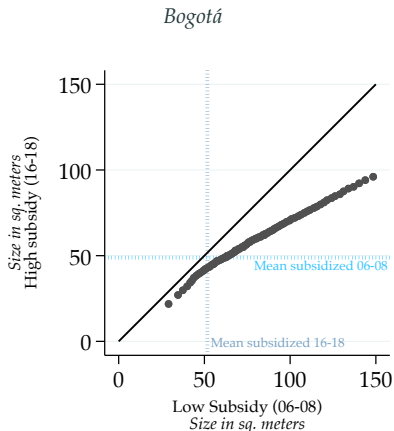
► Changes in unit size (quantile to quantile plot)

Bogotá



CHANGES IN HOUSING STOCK CHARACTERISTICS

► Changes in unit size



► Why size?

- Continuous, easy to measure, monotonic relationship with price and income.
- In contrast to most datasets, I observe it.

II. EQUILIBRIUM MODEL OF HOUSING SUPPLY AND DEMAND

HOUSING MARKET EQUILIBRIUM MODEL

1. Housing

- Differentiated product described by its size $h \in \mathcal{H}$
- Price depends on size $P(h)$

2. Households $i \in I$, Heterogeneous in Income $Y_i \sim F_Y$

- Choose h_i and consumption C_i to maximize Utility $U(C_i, h_i; \theta)$

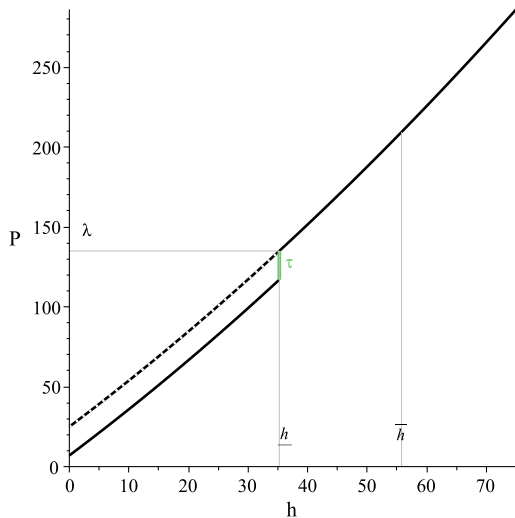
3. Developers $j \in J$, Heterogeneous in Productivity $A_j \sim G_A$

- Choose h_j to maximize profits
- Building costs $B(A_j, h_j, Q(h_j); \beta)$

4. Competitive Market Equilibrium

- Price function $P(h) \rightarrow$ clears the market $\forall h \in \mathcal{H}$

HOUSEHOLDS' OPTIMAL CHOICES



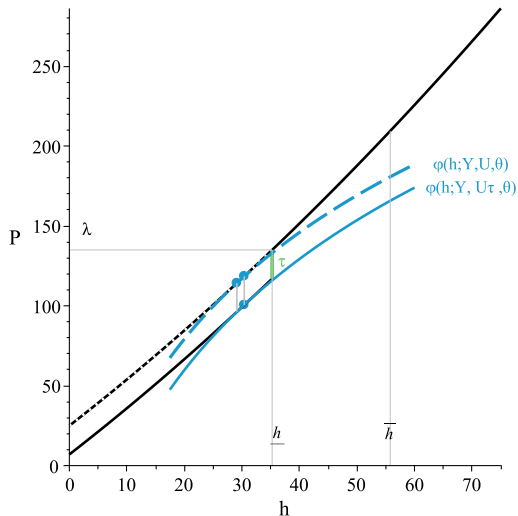
- **Implicit Price Function**

$$P(h)$$

- **Subsidy**

$$\tau$$

HOUSEHOLDS' OPTIMAL CHOICES



- **Implicit Price Function**

$$P(h)$$

- **Subsidy**

$$\tau$$

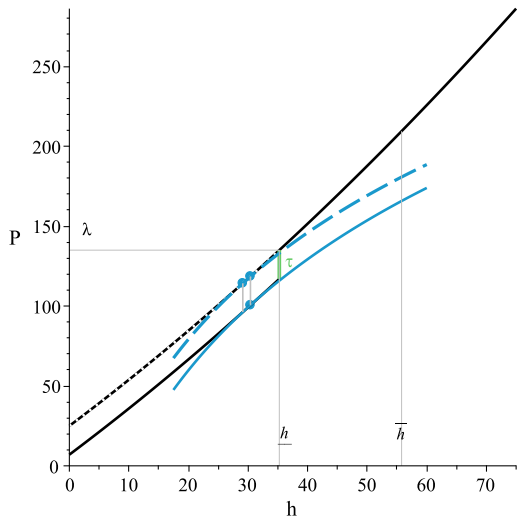
- **Bid functions**

$$\varphi_D(h, Y, \bar{U}; \theta)$$

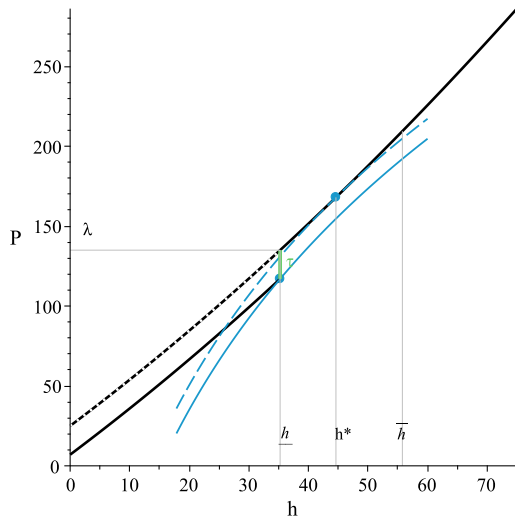
- $\bar{U} = U(h, Y_i - \varphi_D; \theta)$
- $\bar{U}_\tau = U(h, Y_i - \varphi_D + \tau; \theta)$

HOUSEHOLDS' OPTIMAL CHOICES

A. Subsidized

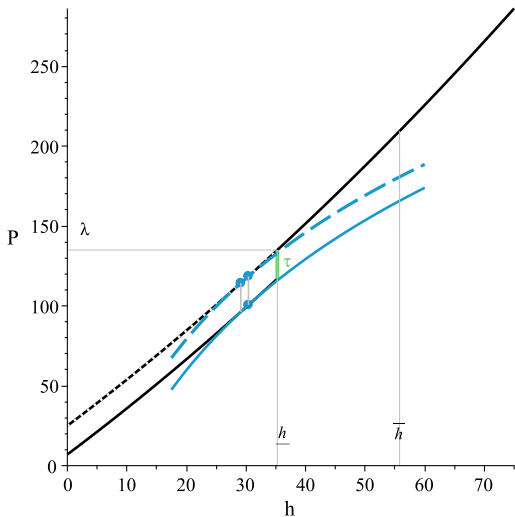


B. Marginally Subsidized and Bunchers

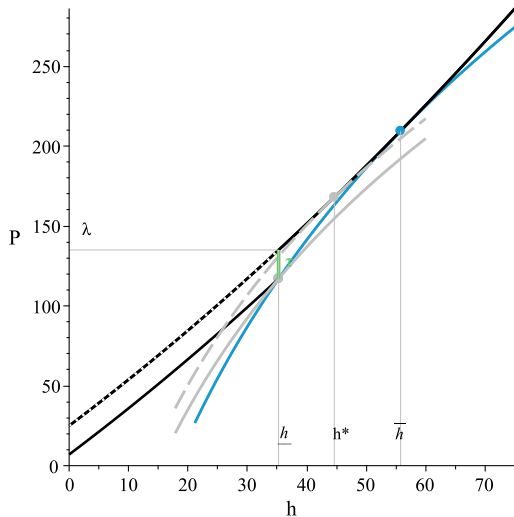


HOUSEHOLDS' OPTIMAL CHOICES

A. Subsidized

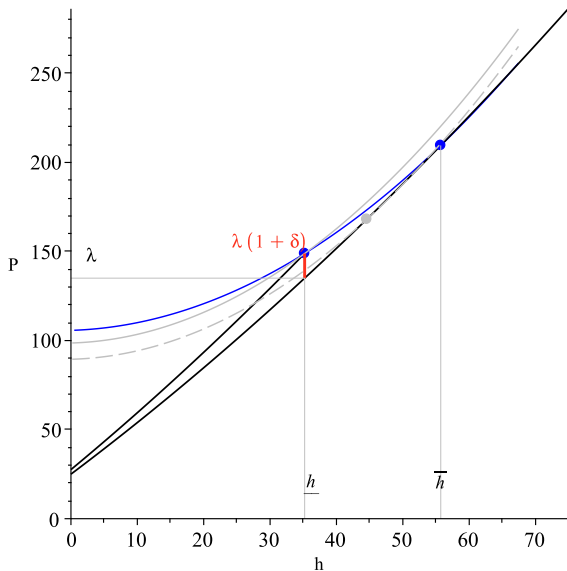


B. Marginally Subsidized and Bunchers



Housing demand function

DEVELOPERS' OPTIMAL CHOICES



- **Implicit Price Function**

$$P(h)$$

- **Tax incentives**

$$P(h) \cdot (1 + \delta)$$

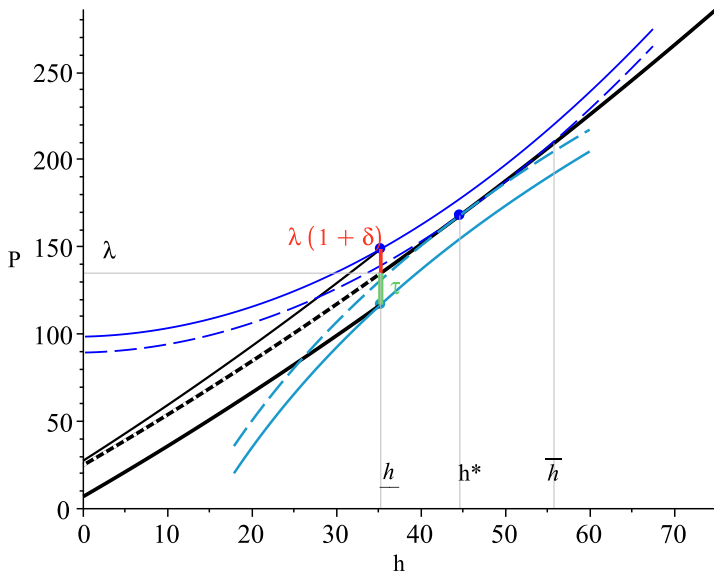
- **Offer Functions**

$$\varphi_S(h, A_j, \bar{\pi}; \beta)$$

$$\bar{\pi} = \pi(h, A_j, P(h); \beta)$$

$$\bar{\pi}_\delta = \pi(h, A_j, P(h) * (1 + \delta); \beta)$$

EQUILIBRIUM: DEVELOPERS AND HOUSEHOLDS MATCH

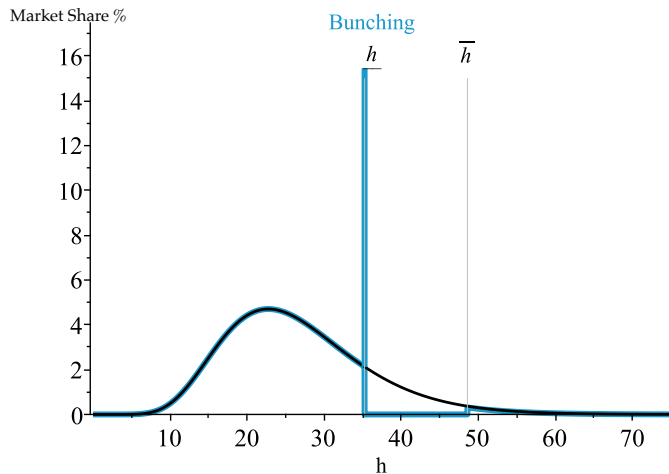


Implicit price: Envelop of offer and bid curves.

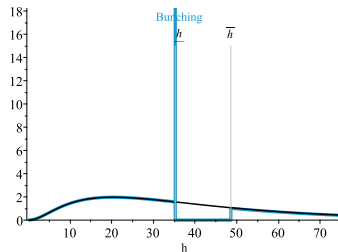
EQUILIBRIUM: AGGREGATE DEMAND AND SUPPLY DENSITY

How to aggregate? → Change of variable formula optimal choices (h^*) and the density of households (F_y) and developers (G_y) details

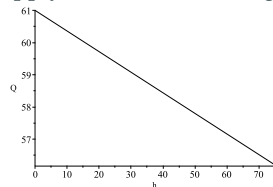
Demand Density Function



Housing types density function $g_h(h)$

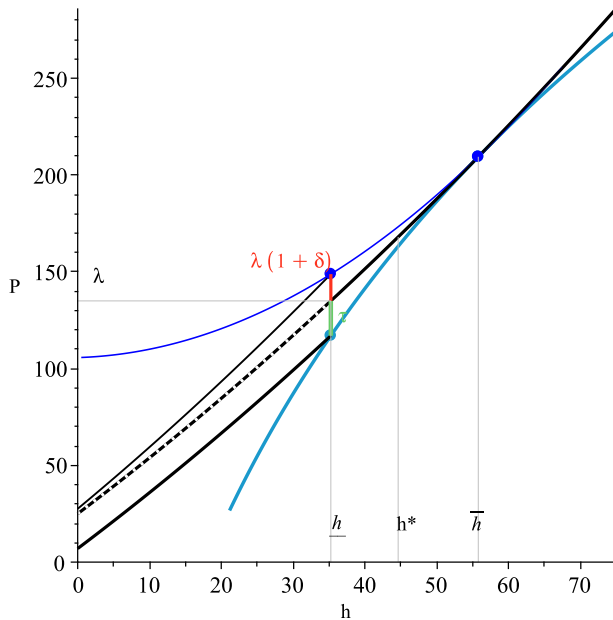


Unit supply function $Q(h)$ (exogenous)



IDENTIFICATION

MARGINAL BUNCHER CONDITION



MARGINAL BUNCHER CONDITION

Marginal Buncher Condition

Household	$V_D = U\left(\bar{Y} - P(\bar{h}), \bar{h}; \theta\right) - U\left(\bar{Y} - P^r(h), \underline{h}; \theta\right) = 0$
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Developer	$V_S = \pi\left(Q(\bar{h}), \bar{A}, P(\bar{h}); \beta\right) - \pi\left(Q(h), \bar{A}; P^\delta(h); \beta\right) = 0$
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Optimality Conditions

Income	$\bar{Y} = \tilde{Y}\left(\bar{h}; \theta, P(h), \lambda\right)$
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Productivity	$\bar{A} = \tilde{A}\left(\bar{h}; \beta, P(h), \lambda\right)$
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Functional Forms

Implicit Price	$P = \rho_0 + \rho_1 \cdot h + \rho_2 \cdot h^2$
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Utility	$U = \left[\frac{1}{2} \cdot C^\theta + \frac{1}{2} \cdot h^\theta\right]^{\frac{1}{\theta}}$
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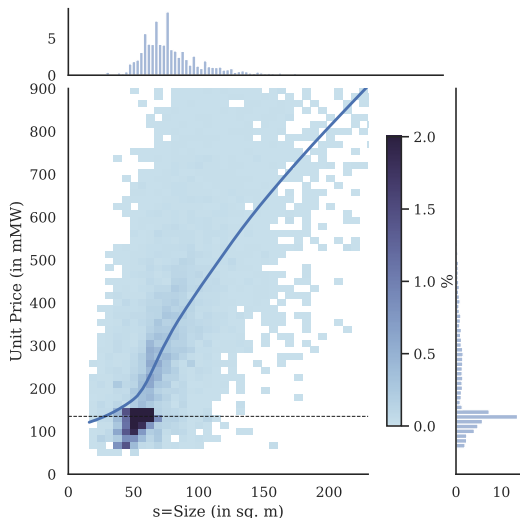
Unit Supply	$Q = \alpha_0 + \alpha_1 h$
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Cost	$B = A_j \cdot Q \cdot h^\beta$
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ESTIMATION

OBSERVED EQUILIBRIUM: PRICES, QUANTITIES, AND SIZE

2016-18



► Solid line: price vs size

→ hedonic price function

► Multiple characteristics

→ Reduce to a single characteristic

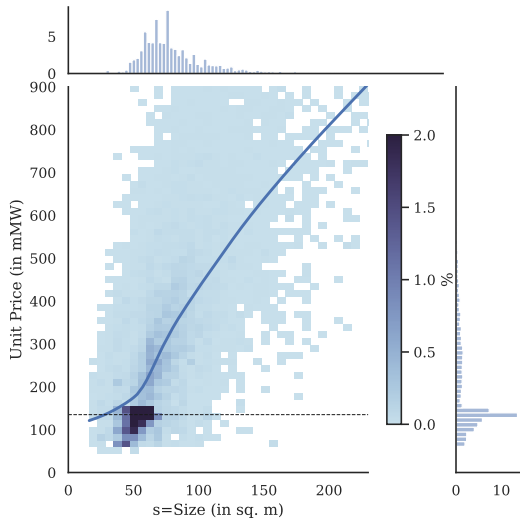
► Standard unit size (h):

→ Size of a unit with average characteristics that costs the same price

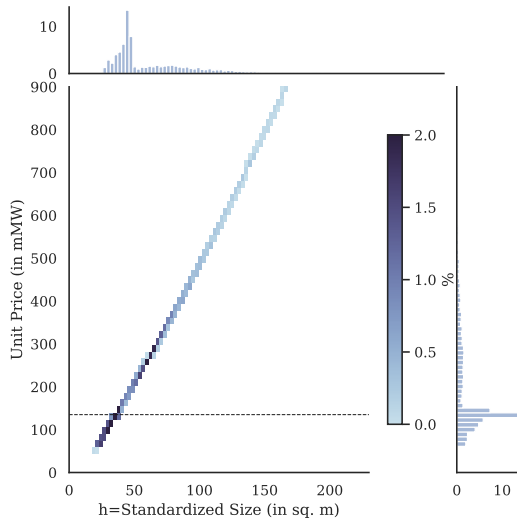
details

OBSERVED EQUILIBRIUM: PRICES, QUANTITIES, AND SIZE

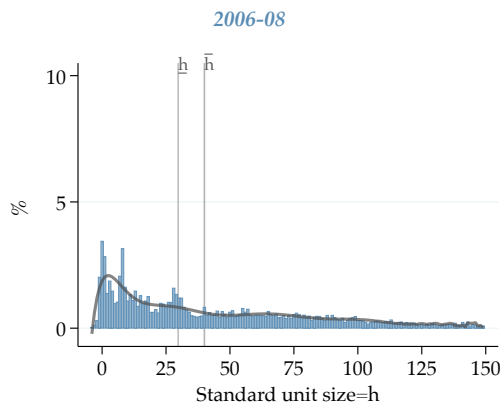
Observed size
2016-18



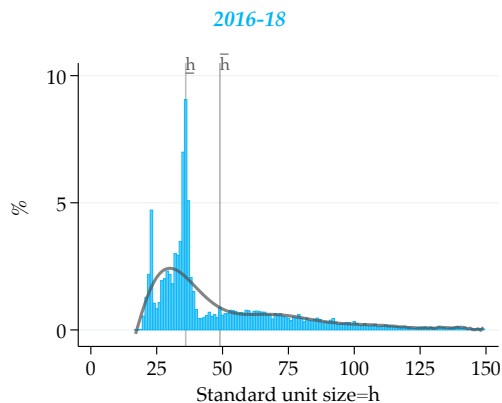
Standardized Unit
2016-18



BUNCHING IN HOUSING CHARACTERISTICS (SIZE OF STD. UNIT)



Notch: 19.7 *mMW*
Bunching: 1.53 % *market share*
 Δh 11.2 *m²*



Notch: 33.1 *mMW*
Bunching: 14.2 % *market share*
 Δh 13 *m²*

STEP I: EQUILIBRIUM CHARACTERIZATION

► Using the observed hedonic equilibrium

- Price function: $\rho_t = \rho_{0t}, \rho_{1t}, \rho_{2t}$
- Size threshold: $\underline{h} = P^{-1}(\lambda; \rho)$
- Standard Unit Size: h

► Behavioural Responses:

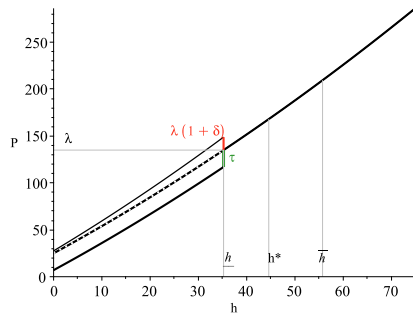
- Housing size for marginal buncher: \bar{h}

► Unit Supply Function:

- $Q = \alpha_0 + \alpha_1 \cdot h_{ltc}$ see

► Policy Parameters:

- Notches: τ_t, δ see



STEP II: STRUCTURAL PARAMETERS

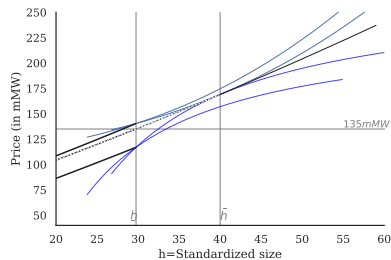
- $B = A_j \cdot Q \cdot h^\beta$
- $U = \left[\frac{1}{2} \cdot C^\theta + \frac{1}{2} \cdot h^\theta \right]^\frac{1}{\theta}$
- Elasticity of Substitution: $\sigma = \frac{1}{1 - \theta}$

Identification equations:

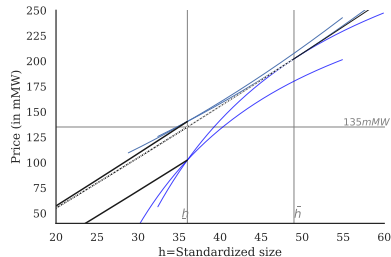
- $V_D \left(\theta | \underline{h}, \bar{h}, P(h), \tau, \lambda \right) = 0$
- $V_S \left(\beta | \underline{h}, \bar{h}, P(h), \alpha, \delta, \lambda \right) = 0$

Structural Parameters				
	2006-08	2009-11	2012-15	2016-18
β	2.53	1.67	1.77	1.70
σ	0.85	0.97	0.90	0.90

2006-08



2016-18



III. POLICY EVALUATION:

COUNTERFACTUAL POLICY I: PROPOSED TAX REFORM

- ▶ Policy proposal: Remove the tax incentives to developers

- ▶ Developers reaction:

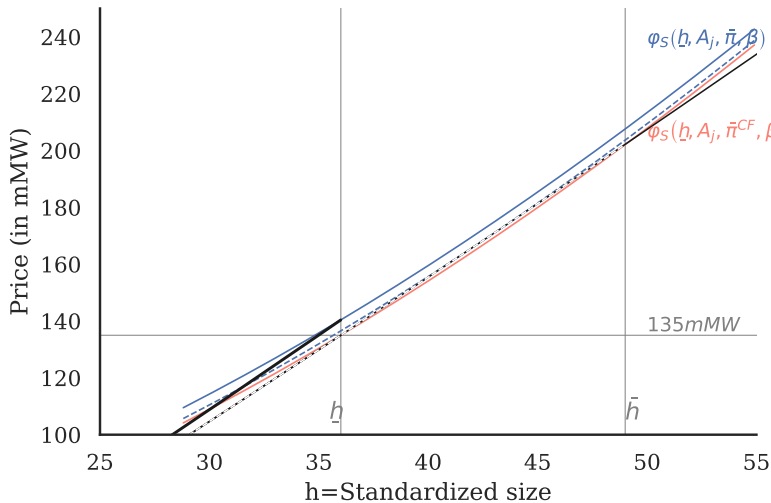
“If these items are repealed, in Valle del Cauca we would go from having an offer of SH and sales of 23,000 homes, average year, to one of sales of 4,600 homes”

source: El Tiempo (2021)

- ▶ Question: What happens to the marginally subsidized developers?

EFFECT ON marginALLY SUBSIDIZED DEVELOPERS

2016-18



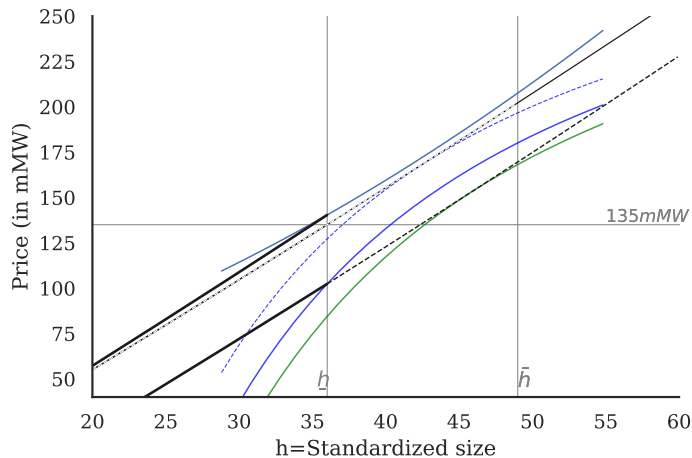
<i>Changes in profits (%)</i>	
period	$\frac{\pi - \pi^{CF}}{\pi}$
2006-08	-4.9
2009-11	-15.9
2012-15	-9.3
2016-18	-12.3

COUNTERFACTUAL POLICY II: REMOVE PRICE CUTOFF

- ▶ Same households get subsidy but they can buy any house.
- ▶ Question: How much better off households are?

EFFECT ON marginALLY SUBSIDIZED HOUSEHOLDS

2016-18



<i>Changes in welfare (mMW)</i>		
period	Welfare \uparrow	Efficiency \downarrow
2006-08	13.8	-10.2
2009-11	19.5	-12.1
2012-15	22.4	-16.2
2016-18	24.8	-17.9

CONCLUSION (I): THE PAPER

- ▶ Characterization of the equilibrium.
- ▶ Compelling evidence of the market responding to subsidies.
- ▶ An hedonic housing market equilibrium with heterogeneous agents can rationalize the response.
- ▶ Propose a identification strategy to recover the model parameters.
- ▶ Model+estimates \rightarrow Welfare.
- ▶ Policy design matters \rightarrow need to be careful of how agents respond to incentives.

CONCLUSION (II): GENERALIZATION

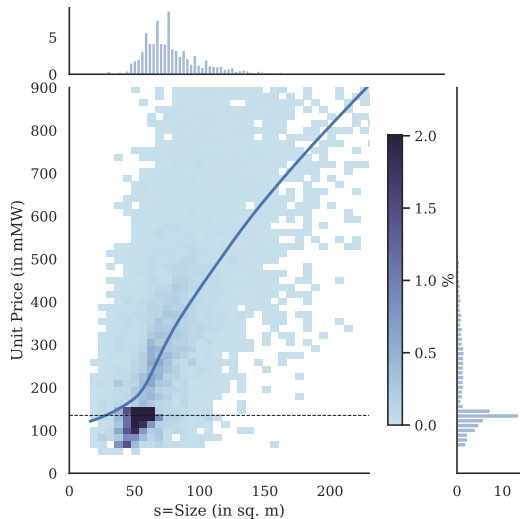
- ▶ The method I propose could be used to evaluate housing policy more generally.
- ▶ Two facts suggest this could be potentially effective.
 1. There is increasing evidence to bunching responses to nonlinear incentives (e.g., help to buy, housing programs in the USA)
 2. Many other sources of non linear incentives in housing markets.
- ▶ Further, it can be applied to other markets (e.g., labor markets, drugs, etc.)

Appendix

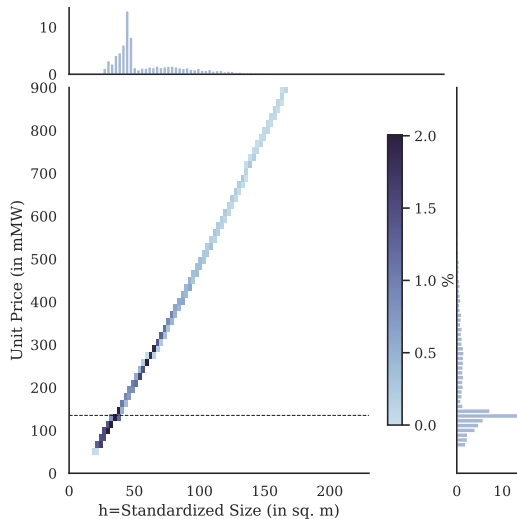
FROM SIZE s TO STANDARDIZED SIZE h

Subsidy expansion 2016-18

Observed size



Standardized Unit



HEDONIC PRICES AND STANDARDIZED HOUSING UNIT

- Hedonic price/Implicit price for housing size

$$P_{ltc} = \rho(s_{ltc}) + \Gamma'X_{ltc} + \omega_{ltc} \quad (1)$$

l , house type in a development, t year, c city

- Simplifying assumption: $\rho(s_{ltc}) = \rho_1 \cdot s_{ltc} + \rho_2 \cdot s_{ltc}^2$
- Identifying assumption: $E(s_{ltc}|X_{ltc}, \omega_{ltc}) = 0$

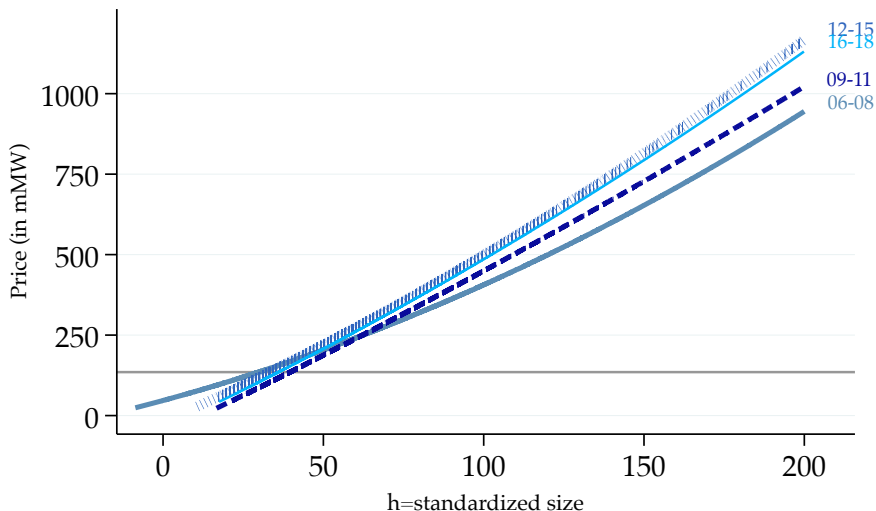
- Standard Unit Size h_{ltc}

$$\rho(h_{ltc}) + \Gamma'\bar{X} + \bar{\omega} = \rho(s_{ltc}) + \Gamma'X_{ltc} + \omega_{ltc} \quad (2)$$

- Characteristics of the standard house: $\bar{X}, \bar{\omega}$

figures

IMPLICIT PRICES FOR HOUSING SIZE OVER TIME



Plotted lines: $P_{ltc} = \hat{\rho}_1 \cdot h_{ltc} + \hat{\rho}_2 \cdot h_{ltc}^2 + \Gamma' \bar{X} + \bar{\omega}$

BEHAVIOURAL RESPONSES INDUCED BY THE POLICY

- Recovered by comparing observed and counterfactual distribution

Observed $f_{h^*} \rightarrow$ histogram

Counterfactual $f_{h_0} \rightarrow$ predicted density excluding observations around the cutoff
(Kleven, 2016)

$$h_b = \sum_{p=0}^T \hat{\iota}_p h_b^p + \sum_{k=L}^H \kappa_k \cdot \mathbb{1} [h_k = h_b] + v_b$$

$$\hat{f}_{h_0} = \sum_{p=0}^T \hat{\iota}_p h_b^p$$

Choice parameters: bin size, bounds for excluded area (L,H) and polynomial degree p

Figures

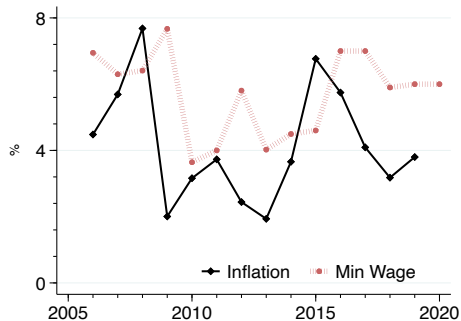
EQUILIBRIUM: DEVELOPERS AGGREGATE SUPPLY DENSITY

$$D(h) = \begin{cases} f_{h^*}(h) \, dh & \text{if } h < \underline{h} \\ f_{h^*}(h) \, dh + \int_{\underline{h}}^{\bar{h}} f_{h^*}(h) \, dh & \text{if } \underline{h} = h \\ 0 & \text{if } h \in (\underline{h}, \bar{h}) \\ f_h^*(h) \, dh & \text{if } \bar{h} \leq h \end{cases}$$

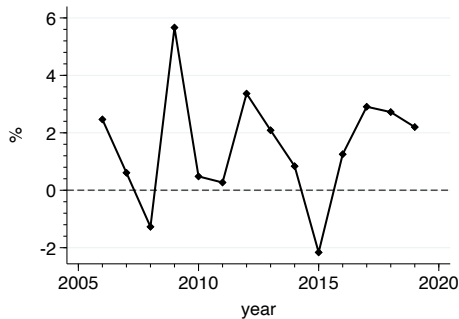
$$S(h) = \begin{cases} g_{h^*}(h) \cdot Q(h) & \text{if } h < \underline{h} \\ \left(g_{\underline{h}^*}(\underline{h}) + \int_{\underline{h}}^{\bar{h}} g_{h^*}(h) \, dh \right) \cdot Q(\underline{h}) & \text{if } \underline{h} = h \\ 0 & \text{if } \underline{h} < h < \bar{h} \\ g_{h^*} \cdot Q(h) & \text{if } \bar{h} \leq h \end{cases}$$

Equilibrium Figures

Inflation and minimum wages.



a. Min wage and Inflation

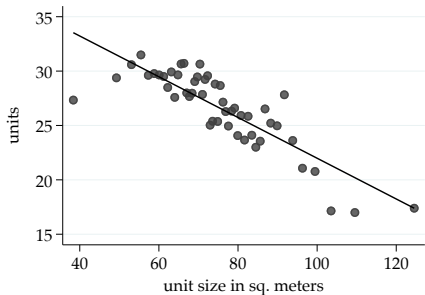


b. Min wage and Inflation

Data

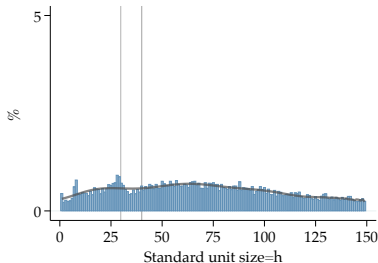
DEVELOPERS CHOICES OF SIZE AND UNIT SUPPLY

All data

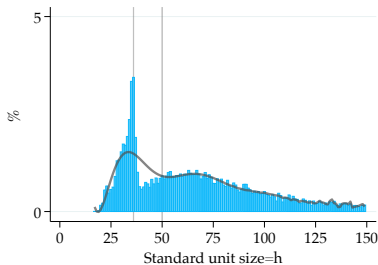


Product type density (g_h)

2006-08



2016-18



$$Q_{ltc} = \alpha_0 + \alpha_1 s_{ltc} + \alpha'_x X_{ltc} + \epsilon_{ltc}^Q$$

	06-08	09-11	12-15	16-18
α_0	70.5	12.7	81.1	33.3
α_1	-0.068	-0.020	-0.020	-0.042

NOTCHES

► Demand Notch Overtime

	Notch (<i>in mMW</i>)			# Subsidies (<i>in thousand</i>)		
	τ^M	τ^i	τ	<i>down payment</i>	<i>i rate</i>	<i>Mi Casa Ya</i>
2006-08	18.0	.	18.0	47.1	.	.
2009-11	20.0	5.85	25.9	46.4	16.7	.
2012-15	19.9	9.55	29.5	41.1	22.2	.
2016-18	25.3	7.24	32.6	44.5	23.4	16.8

► Supply Notch: 4 percent

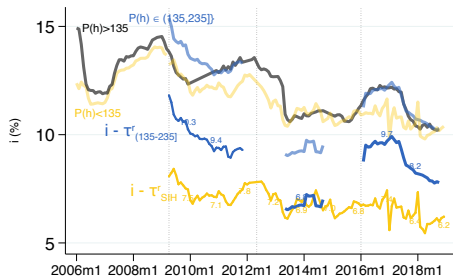
Step I

DATA: MORTGAGES AND INTEREST RATES

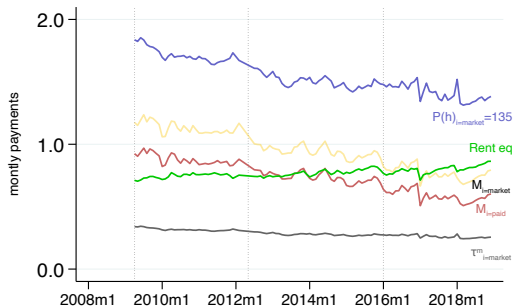
rent equivalent ((?, ?), (Bishop & Timmins, 2019) assume it is 0.05)

- Size of the mortgages and interest rate.
- Identifier for SIH.

Market interest rate i and subsidy τ^r



Monthly payments and monthly equivalent for relevant values. $P(h) < 135$



- To convert the magnitudes into monthly payments I use:

$$X_{\text{monthly}} = X \cdot \kappa(i, n); \kappa(i, n) = \frac{\frac{i}{12} \cdot \left(1 + \frac{i}{12}\right)^{12 \cdot n}}{\left(1 + \frac{i}{12}\right)^{n \cdot 12} - 1}$$

HOUSEHOLDS' DEMAND FUNCTION

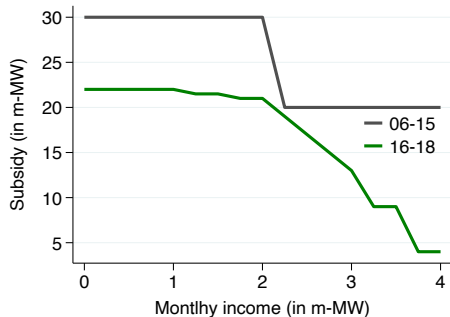
$$h^D(Y_i) = \begin{cases} h^*(Y_i, \tau; \theta, \boldsymbol{\rho}, \lambda) & \text{if } Y_i \leq \underline{Y} \\ \underline{h} & \text{if } \underline{Y} < Y_i < \bar{Y} \\ h^*(Y_i, \tau; \theta, \boldsymbol{\rho}, \lambda) & \text{if } \bar{Y} \leq Y_i \end{cases}$$

- Tangency conditions: $h^*(Y_i, \tau; \theta, \boldsymbol{\rho}, \lambda)$
- Income and unit size: $Y_i = \tilde{Y}(h, \tau; \theta, \boldsymbol{\rho}, \lambda) = h^{*-1}(h_i, \tau; \theta, \boldsymbol{\rho}, \lambda)$

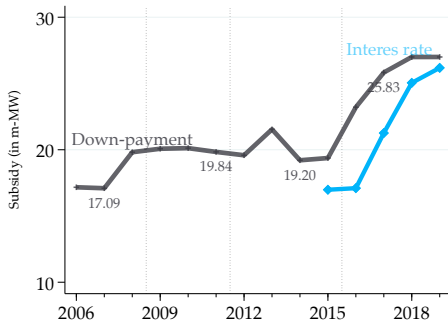
Graphs

THE NOTCH: DOWN PAYMENT SUBSIDY

Subsidy by household income

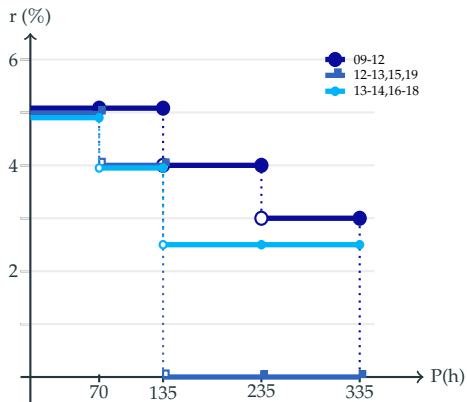


Average subsidy over time

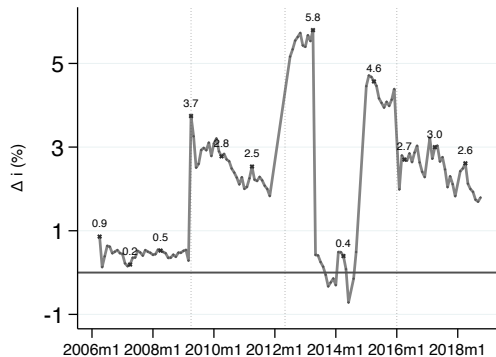


- Varies by income.
- Increase in 2016.
- Expanded trough *mi casa* YA

THE NOTCH: INTEREST RATE SUBSIDY



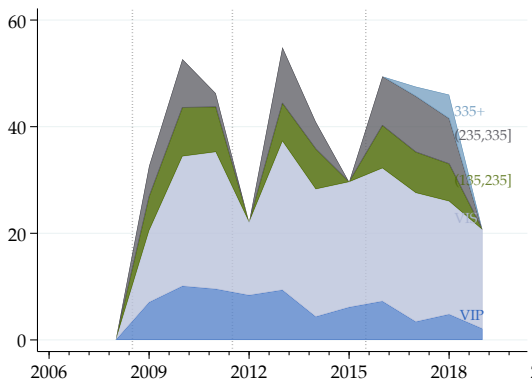
Comparing monthly payments around $P(h)=135$ m-MW



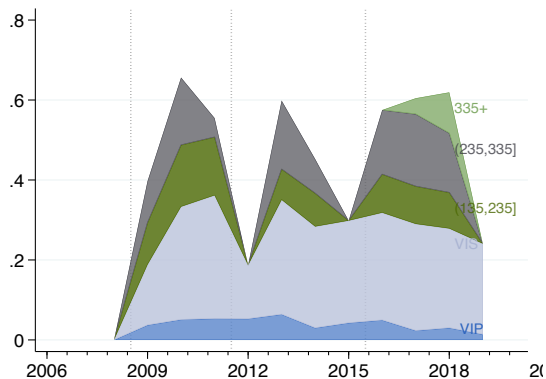
SUBSIDIES AND GOVERNMENT EXPENDITURE (VIP- $P(h) < 70$)

This figure shows the interest rate subsidies for all different price levels

a. Total # of subsidies



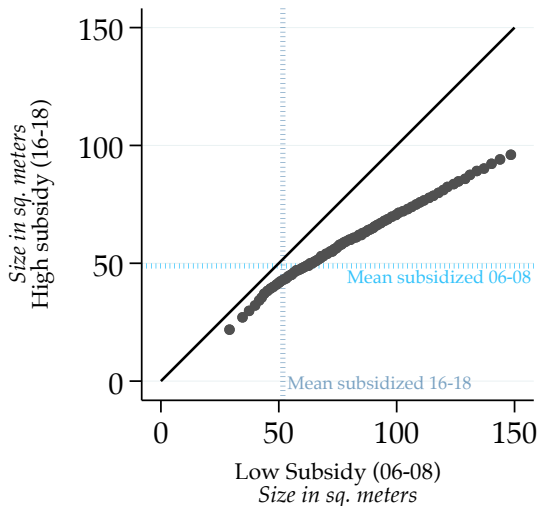
b. Total amount of subsidies



CHANGES IN HOUSING STOCK CHARACTERISTICS

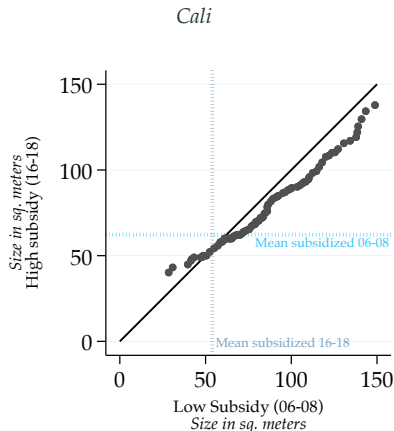
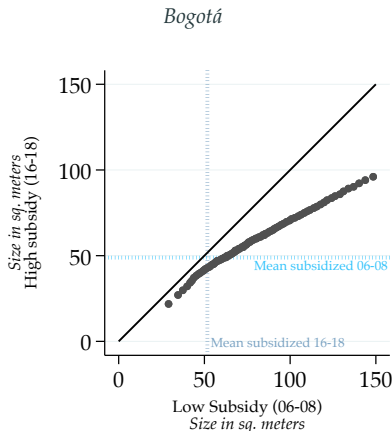
► Changes in unit size (quantile to quantile plot)

Bogotá



CHANGES IN HOUSING STOCK CHARACTERISTICS

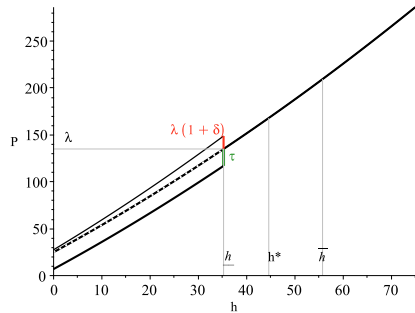
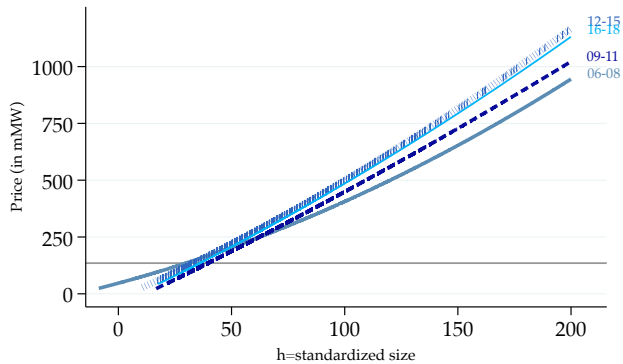
► Changes in unit size



► Why size?

- Continuous, easy to measure, monotonic relationship with price and income.
- In contrast to most datasets, I observe it.

PRICES



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THE POLICY EFFECT ON OBSERVED OUTCOMES

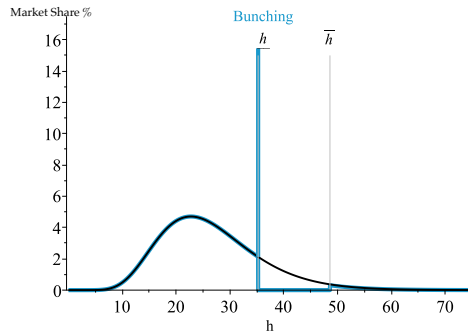
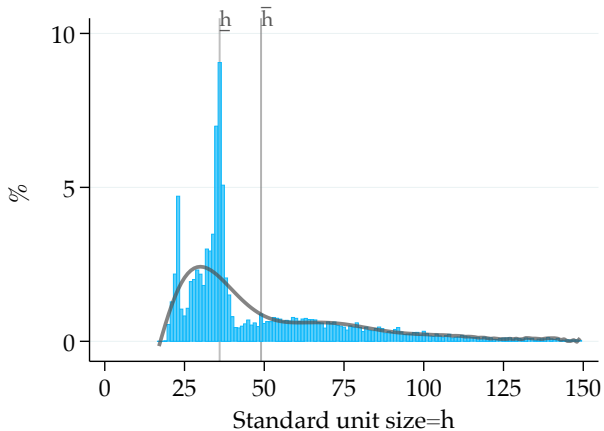
Table 1: Behavioral Responses Estimates'

	06-08	09-11	12-15	16-18
$\int_{h_{min}}^{\underline{h}} T(h)dh$	1.03	0.86	3.80	7.28
$\hat{T}(\underline{h})$	0.50	2.02	4.01	6.97
$\int_{\underline{h}}^{\underline{h}} T(h)dh$	1.54	2.88	7.81	14.2
$\int_{\underline{h}}^{\underline{h}} T(h)dh$	-0.12	-6.23	-4.27	-3.38
$h_{h^0}(\underline{h})$	0.73	1.28	1.07	1.43
h_{min}	26	37	29	32
\underline{h}	29.8	39.4	33.0	36.0
\overline{h}	40	53	45	49

back

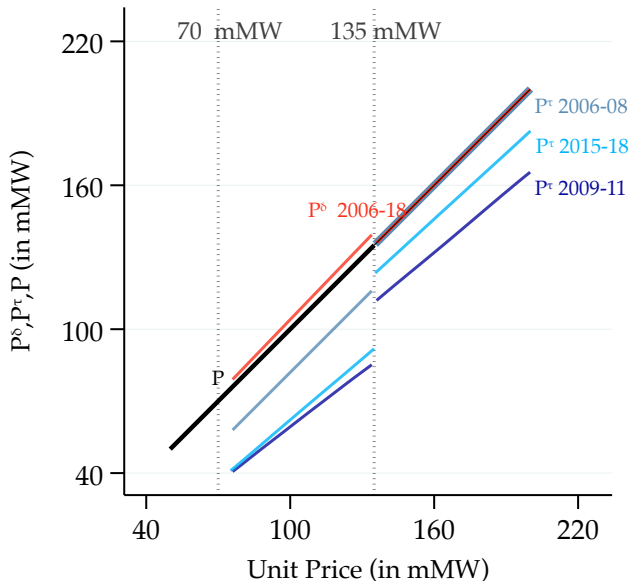
BEHAVIORAL RESPONSES

2016-18



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THE DEMAND NOTCH INCREASES OVER TIME



Supply Notch δ

2006-18 4%

Demand Notch τ_t

2006-08: 19.7 mMW

2009-11: 26.4 mMW

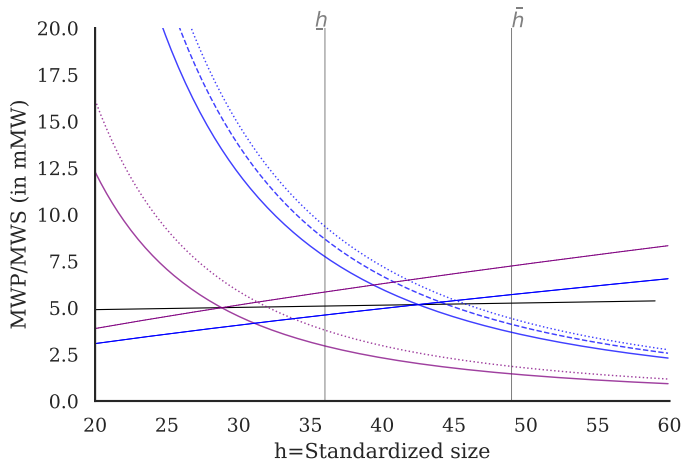
2016-18: 33.1 mMW

2012-15 Too many changes and
free housing at 70mMW

ALTERNATIVE REPRESENTATION OF THE EQUILIBRIUM.

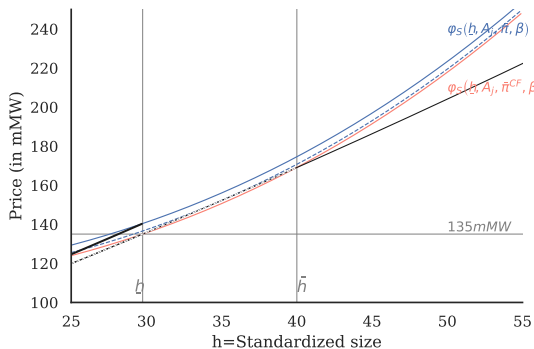
DEMAND AND SUPPLY FOR SIZE

Expansion period Mi Casa Ya 2016-18

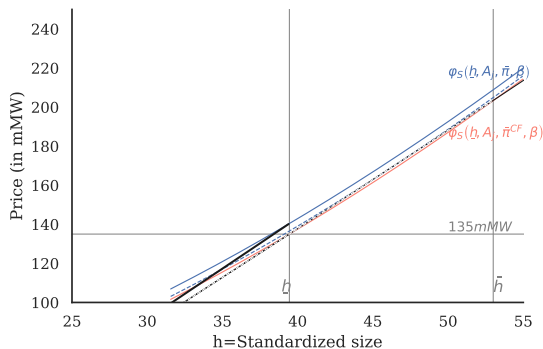


EFFECT ON marginALLY SUBSIDIZED DEVELOPERS

2006-08



2016-18

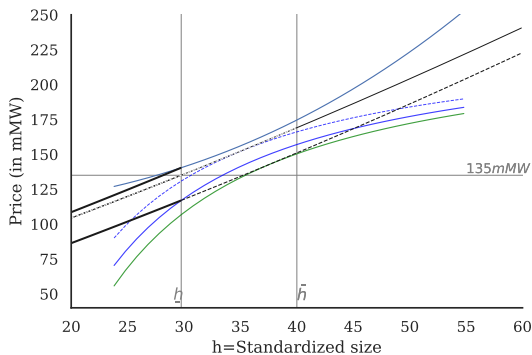


Changes in profits (%)

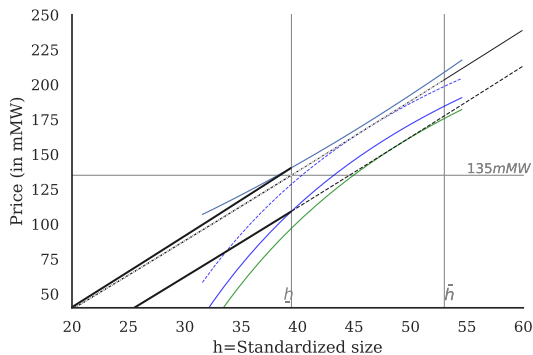
	2006-08	2009-11	2012-15	2016-18
$\frac{\pi - \pi^{PC}}{\pi}$	4.9	15.9	9.3	12.3

EFFECT ON marginALLY SUBSIDIZED HOUSEHOLDS

2006-08



2009-11



EQUATIONS

Optimality Conditions

$$\begin{aligned} \text{Income} \quad \bar{Y} &= (2 \cdot h \cdot \rho_2 + \rho_1)^{-\frac{1}{\theta-1}} \cdot h + h^2 \cdot \rho_2 + h \cdot \rho_1 + \rho_0 \\ \text{Productivity} \quad \bar{A} &= \frac{3 \cdot (\delta + 1) \cdot (h \cdot \alpha_1 + \alpha_0) \cdot \left(\left(h^2 \cdot \rho_2 + \frac{2}{3} \cdot h \cdot \rho_1 + \frac{1}{3} \cdot \rho_0 \right) \cdot \alpha_1 + \frac{2 \cdot (h \cdot \rho_2 + \frac{\rho_1}{2}) \cdot \alpha_0}{3} \right)}{h^{\beta_1+1} \cdot \alpha_1^2 + h^{\beta_1} \cdot \alpha_0 \cdot \alpha_1 + 2 \cdot h \cdot (h \cdot \alpha_1 + \alpha_0)^2} \end{aligned}$$

Marginal Buncher Condition

$$\begin{aligned} \text{Household} \quad 0 &= \left(\frac{\underline{h}^\theta + \left(\bar{h}^2 \rho_2 - \rho_2 \underline{h}^2 + \bar{h} \rho_1 + \bar{h} (2\bar{h} \rho_2 + \rho_1)^{\frac{1}{1-\theta}} - \rho_1 \underline{h} + \tau \right)^\theta}{2} \right)^{\frac{1}{\theta}} - \left(\frac{\left((2\bar{h} \rho_2 + \rho_1)^{\frac{\theta}{1-\theta}} + 1 \right) \bar{h}^\theta}{2} \right)^{\frac{1}{\theta}} \\ \text{Developer} \quad 0 &= \frac{\bar{h}^{\beta_1+1} \alpha_1 - \underline{h}^{\beta_1+1} \alpha_1 + \alpha_0 (\bar{h}^{\beta_1} - \underline{h}^{\beta_1})}{\bar{h}^{\beta_1+1} \alpha_1^2 + \bar{h}^{\beta_1} \alpha_0 \alpha_1 + 2\bar{h} (\bar{h} \alpha_1 + \alpha_0)^2} - \\ &\quad \frac{\left(-\rho_2 (1+\delta) \underline{h}^3 - \rho_1 (1+\delta) \underline{h}^2 - \rho_0 (1+\delta) \underline{h} + \bar{h} \left(\bar{h}^2 \rho_2 + \bar{h} \rho_1 + \rho_0 \right) \right) \alpha_1 + \alpha_0 \left(-\rho_2 (1+\delta) \underline{h}^2 - \rho_1 (1+\delta) \underline{h} + \bar{h}^2 \rho_2 + \bar{h} \rho_1 - \delta \rho_0 \right)}{3 \left(\left(\bar{h}^2 \rho_2 + \frac{2}{3} \bar{h} \rho_1 + \frac{1}{3} \rho_0 \right) \alpha_1 + \frac{2(\bar{h} \rho_2 + \frac{\rho_1}{2}) \alpha_0}{3} \right) (\bar{h} \alpha_1 + \alpha_0)} \end{aligned}$$

Main table