EQUILIBRIUM EFFECTS OF HOUSING SUBSIDIES: EVIDENCE FROM A POLICY NOTCH IN COLOMBIA

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Abstract

This paper studies how the housing market in Colombia responds to policies aimed at increasing homeownership among low-income households. Private sector developers received tax incentives to build houses priced below a cutoff, and households received subsidies to buy houses below the same cutoff. The paper models an equilibrium between heterogeneous developers building differentiated housing and heterogeneous households purchasing homes. The model is estimated using bunching around the price cutoff induced by the policies. To benefit from the policy, households buy housing units up to 30 percent smaller and bunch at the policy cutoff. A counterfactual simulation shows that without supply-side incentives, developers may exit the market; their profits would be 14 percent lower.

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I. Introduction

In the past several decades, governments around the world have adopted market-oriented strategies to encourage homeownership.¹ Common strategies include down-payment assistance, low interest rates, and subsidies to developers to build affordable housing. The assumption is that a market-oriented strategy is more efficient than direct government intervention. While policy-makers adopt these approaches on that premise, little is known about how the market actually responds to these interventions. This paper investigates and estimates these responses.

It is difficult to isolate a policy's effect on the housing market and the human behavior that drives it, due to several empirical and theoretical challenges. The main empirical challenges of separating supply and demand responses and finding a valid counterfactual make estimating housing market elasticities difficult. It is hard to find a valid quasi-experimental variation in settings with detailed housing data.² Even with credible elasticities, incidence evaluations of such housing policies based on a standard economic framework with homogeneous goods and agents can be misleading. Housing is a differentiated product traded in a market with heterogeneous households and developers; including these features in a tractable economic model is challenging, but important (Greenstone, 2017). A potential market response to housing policies is that agents change the type of housing they buy or build. The economic model used to evaluate the welfare effects of the policy should be able to account for this type of response.

This paper overcomes these challenges by studying the unique features of a housing policy implemented in Colombia. Colombia has a price-capped policy designed to incentivize homeownership among low-income households. The policy design and data availability allow me to evaluate the housing market's response to this policy. The policy's price cutoff introduces discontinuous incentives – or a notch. I use this quasi-experimental variation to study the behavioral responses induced by the policy.

Because the policy offers subsidies and tax incentives to developers and households

¹For example, in a 1993 report the World Bank said that "housing policy making must thus move away from its previously narrow focus on a limited engagement of government in the direct production of low-cost housing." World Bank Group (1993, p. 1) Following these recommendations, many Latin American countries, including Chile and Colombia, abandoned the construction of public housing and implemented a market-oriented approach (A. Gilbert, 2014; Cohen, Carrizosa, & Gutman, 2019) similar to *help to buy* in the United Kingdom or some first time buyers programs in the United States.

²Some examples are Saiz (2010); Baum-Snow and Han (2021); Anagol, Ferreira, and Rexer (2021); Galiani, Murphy, and Pantano (2015).

using the same cutoff, I rely on economic theory, introducing a housing equilibrium model that allows me to untangle this policy's supply and demand responses. The model has heterogeneous buyers and sellers of vertically differentiated goods traded in a competitive market. I estimate the primitives of the model using the notch. I combine data from a construction census containing the universe of new housing developments between 2006 and 2018 and administrative records for the awarded subsidies from the Ministry of Housing.

In Colombia, low-income households can receive a cash transfer for the downpayment on a mortgage and a subsidized interest rate to buy low-cost housing. Developers who build low-cost housing receive a tax incentive from the government. A housing unit classifies as low-cost if its price is below 135 times the monthly minimum wage (roughly US \$40,000). This is the cutoff introducing discontinuous incentives for both developers and households to bunch at the price threshold. Over the last fifteen years, the policy expanded, doubling the subsidy amount and the number of households receiving it.

I show evidence of bunching at the price cutoff. Following the bunching literature, I estimate a counterfactual distribution of market shares by price to recover the behavioral responses induced by the subsidy. I provide suggestive evidence that housing characteristics and, in particular, housing size drive the behavioural responses reflected in the bunching. Households buy housing units up to 30 percent smaller to take advantage of the subsidy. Using the variation in the subsidy over time, I show that increasing government expenditure on the policy increases the share of units sold at the cutoff. During my study period, the interest rate subsidy was introduced, the downpayment subsidy increased, and eligibility expanded. As a result, the bunching of units sold at the price cutoff moves from less than 1 percent of the market share around 2006 to about 7 percent by 2018.

To rationalize the observed equilibrium responses, I introduce and estimate a housing market equilibrium model. The model includes the policy induced notch, creating discontinuous incentives, to an hedonic – or sorting – equilibrium model.³ Households are heterogeneous in income, developers in productivity level, and housing in size. I use the model to show how the notch creates incentives for developers and households to bunch at the threshold. In the model, buyers and developers change the type of units they buy and build to take advantage of the subsidies. The bunching moments provide information about developers' and house-

³For a review of the general approach, see Kuminoff, Smith, and Timmins (2013) or Greenstone (2017). For recent applications, see Epple, Quintero, and Sieg (2020) and Chernozhukov, Galichon, Henry, and Pass (2021).

holds' trade-offs between choosing their optimal housing size without subsidies and altering their choice to qualify for subsidies.

I propose an identification strategy based on a two-step procedure implemented in the hedonic equilibrium literature. The first step relies on the reduced-form analysis from the first part of the paper. I recover the implicit price function, the behavioral responses induced by the subsidy, the subsidy size, and the relationship between the number of units built and unit size. In the second step, I use the discontinuity and estimated behavioral responses and adapt the identification strategy proposed in the literature using notches to estimate the structural parameters of the model. The counterfactual distribution and the existence of behavioral responses allow me to observe two points on the same indifference curves for the marginal buncher who is indifferent between getting the subsidy or not. Using the parameters from the first step and the marginal buncher indifference condition, I can estimate the shape of the utility and cost functions.

I use the model and estimated parameters to evaluate how the marginally subsidized households and developers benefit from the subsidy scheme. On the demand side, I compare the utility levels of the marginally subsidized households in two counterfactual scenarios. In the first counterfactual scenario, households do not get subsidies. I compare the observed equilibrium with the counterfactual scenario. Throughout my study period, the utility level is 4.5 percent higher relative to the scenario with no subsidies. Despite the increase in the subsidy amount during my study period, the welfare of the responding households did not increase. In the second counterfactual scenario, I remove the price cutoff. Households receive a subsidy, but they do not need to reduce the housing size to access it. If the household did not reduce its

⁴There are three main identification approaches in these types of models: (i) Excluded instruments and variation across markets (Epple, 1987; Brown & Rosen, 1982; Wooldridge, 2010), (ii) Functional forms and inversion methods (Bajari & Benkard, 2005; Yinger, 2015; Bishop & Timmins, 2019), (iii) Non parametric identification and single index reduction (Ekeland, Heckman, & Nesheim, 2004; Heckman, Matzkin, & Nesheim, 2010; Chernozhukov et al., 2021; Epple et al., 2020). For more details see Chernozhukov et al. (2021). An approach that integrates the hedonic insights into a discrete choice framework is Bayer, Ferreira, and McMillan (2007) or Anagol et al. (2021).

⁵Best, Cloyne, Ilzetzki, and Kleven (2019) use the same identification idea to estimate the intertemporal elasticity of substitution from the behavioral responses induced by notches in the interest rates for loan refinancing. Other examples are Einav, Finkelstein, and Schrimpf (2015) and Chen, Liu, Suárez Serrato, and Xu (2021) or Kleven and Waseem (2013). Bertanha, McCallum, and Seegert (2021) and (Blomquist, Newey, Kumar, & Liang, 2017) discuss how in contrast with changes in the slope, or kinks, notches allow to recover structural parameters.

⁶To estimate the shape of the indifference curve and offer curve, I impose functional forms for the utility function and a cost function. The utility function is a CES utility function depending on consumption on housing and consumption on other goods, and the cost function depends on housing size and the number of units built.

consumption and still received the subsidy, its utility would be 2.8 percent higher.⁷ My structural parameters suggest an elasticity of substitution between housing and consumption of other goods of around 0.9. Therefore, housing and consumption of other goods are gross complements.

I compare the observed equilibrium with a counterfactual scenario with subsidized households but without tax refunds for developers. Developers would be worse off. Between 2006 and 2009, the profits for marginally subsidized developers would be 5 percent lower, and by 2016, after the subsidy's expansion, their profits would be 14 percent lower. The marginally subsidized developers have higher marginal costs when producing at the price cutoff. They are competing with more productive developers, which have profits when building low-cost housing even without subsidies. A price increase, desirable for both developer types, will lead to non-eligibility for the subsidies. Because of the price cap, marginally subsidized developers need tax incentives to build low-cost housing, and without them, the market may face a rationing problem.

Contributions and Related Literature

I make several methodological and empirical contributions. My first contribution is to provide additional evidence of bunching in the housing market, a relatively unexplored setting. Carozzi, Hilber, and Yu (2020) provide evidence of bunching in response to a similar housing policy in the United Kingdom. McMillen and Singh (2020) show that apartment rents cluster at values near the fair market rent in Los Angeles, California. There is also evidence of bunching in the density of mortgages with notches in the interest rate schedule.⁸ This paper complements the bunching evidence around a price cutoff in the housing market.

The main contribution is to provide a method to recover structural parameters using bunching responses in a market equilibrium of a vertically differentiated product. The paper offers a new framework to use the observed bunching responses to do a welfare analysis of housing policies. The approach proposed in this paper can be applied to other settings with policy interventions with discontinuous incentives that cause bunching (e.g., Carozzi et al., 2020; McMillen & Singh, 2020). The proposed method complements the approaches using notches and bunching moments

⁷This comparison could be more informative if it were expressed in dollars using compensated or equivalent variation. Including this is work in progress.

⁸For example, DeFusco and Paciorek (2017) use these bunching responses to estimate the interest rate elasticity of mortgage demand. Best and Kleven (2017); Kopczuk and Munroe (2015); Slemrod, Weber, and Shan (2017) report housing transaction bunching responses around notches in transaction costs.

for identification by providing the same identification principle to recover model primitives in the sorting or hedonic models. I use the bunching evidence and moments to estimate the structural parameters of economic models as done recently in other settings, by Einav et al. (2015) for the drug market, Best et al. (2019) for the mortgage market, and Chen et al. (2021) for incentives for research and development in China. This paper brings this relatively novel approach to the housing market literature.

This paper makes important methodological contributions to the bunching and hedonic equilibrium models literature, but it also provides new empirical and theoretical insights into a first-order question. The findings of this paper can inform the design of housing policies aimed at providing affordable housing. The model presented in this paper allows me to estimate the welfare effects on developers and households, which has direct implications for policy design. While other papers investigate the effects of housing programs on households, my study contributes to the literature by also investigating the effects of these housing programs on developers. My setting and approach also allow me to understand the effect of these programs on the housing market itself. My findings suggest that the policy affects behavior on both sides of the market. The policy incentives shape the type of housing that is built and sold, which has implications for how the city grows and develops. Understanding the housing market response can inform the design of better policies.

The paper has three parts. The first part introduces the reduced-form analysis. In the next section, I present the Colombian housing policy, institutional context and the discontinuities created by the subsidy scheme. Section III presents the housing

⁹In contrast to this approach, alternative approaches implemented, for example, by Saez (2010), Chetty et al. (2011), or (Chetty, Friedman, & Saez, 2013) use the bunching moments to derive reduced form elasticities and use them as sufficient statistics for welfare analysis. See Kleven (2016) for a review of the literature using bunching. Some recent applications include studies on minimum wage (Cengiz, Dube, Lindner, & Zipperer, 2019; Harasztosi & Lindner, 2019; Jales, 2018), overpay hours (Goff, 2021; Bachas & Soto, 2018; Abel, Dey, & Gabe, 2012), marriage market (Persson, 2020), Crime (Goncalves & Mello, 2021) among others.

¹⁰Many papers study housing market policies implemented in the United States. For example, Baum-Snow and Marion (2009), Soltas (2020) and Sinai and Waldfogel (2005) study the LIHTC, Collinson and Ganong (2018), McMillen and Singh (2020) study housing vouchers and Gruber, Jensen, and Kleven (2021), E. Glaeser and Shapiro (2003) study mortgage interest deductions (MID). H. S. Rosen (1985); Poterba (1992); Galiani et al. (2015); Quigley (1982); Geyer (2017) carry out incidence and welfare analysis on housing policies. In addition to housing subsidies, there is literature on alternative approaches to affordable housing including public housing (Kumar, 2021; Franklin, 2019; van Dijk, 2019), rent control (E. L. Glaeser & Luttmer, 2003; Autor et al., 2014; Diamond et al., 2019), maximum permitted construction (Anagol et al., 2021). OECD (2021) describes the different approaches implemented around the world to promote affordable housing.

market data and provides reduced-form evidence of the housing market response. The second part of the paper contains the housing equilibrium model and identification strategy. Section IV, introduces the model, section V, presents the identification strategy. The third part, presented in section VI, shows the estimates for the structural parameters, the policy counterfactuals, and welfare analysis.

II. Subsidies and Notch

This section introduces the subsidies and discontinuities that allow studying the effect of housing subsidies on the housing market. It describes the subsidy expansion and shows how the discontinuity creates incentives to bunch at the price cutoff.

A. Institutional Context and Discontinuity

Institutional context. Colombian housing policy aims to provide a decent home and suitable living, reduce housing deficits, and achieve the dream of being a country of homeowners. Since the 1990s, Colombia and other Latin American countries have changed their approach, moving from state-provided housing to a market-oriented solution based on subsidies. A market-oriented solution was promoted by multilateral organizations like the World Bank and the Inter-American Development Bank (IDB), and the particular approach implemented in Latin America was called ABC (from Spanish, Ahorro-Savings, Bonos-Bonds, Creditos-Credit). This policy approach incentivizes the purchase and construction of low-cost housing through subsidies to households and developers. The principal policy tools are mortgage assistance through a downpayment subsidy and a subsidized interest rate on the demand side. These types of policies are similar to many first-time home buyers programs in the United States and some housing policies in the United Kingdom, such as help to buy (Carozzi et al., 2020). On the supply side, the policy tool is a tax refund for developers who build low-cost housing. Such as help to developers who build low-cost housing.

The discontinuity or notch. The policy design relies heavily on the definition of low-cost housing. Most of the subsidies apply only to households and developers buying and building low-cost housing, which is a unit with a market price below an

¹¹The first and second goals are based on Article 51 of the Colombian Constitution. The goal of being a country of homeowners appears in the country's last three National Development Plans (see, for example p104 of the National Development Plan for 2002–06).

¹²This type of tax incentive could be compared to the Low Income Housing Tax Credit (LIHTC) in the United States (Baum-Snow & Marion, 2009). However, in contrast to LIHTC, the units built are not rental units but are made to be sold.

arbitrary threshold equal to 135 times the monthly minimum wage (mMW).¹³ This arbitrary threshold is the same for all cities, and the change over time is only because of changes in the minimum wage.¹⁴

B. Subsidies Over-Time

Subperiods. There were changes to the household subsidies during the study period. The interest rate subsidy was introduced, the subsidy amount increased, and individuals in the informal sector became eligible. As a result, the government expenditure targeting the purchase of low-cost housing more than doubled during this period. These changes allow me to study how the housing market responds to subsidy changes. I divide my study period into four subperiods corresponding to the distinct set of policies available. The details of the subsidies are in Appendix C.

The four periods are the following:

- 2006-08 Formal employee downpayment subsidy.
- **2009-11** Formal employee downpayment subsidy, and interest rate subsidy.
- 2012-15 Unstable period with rapid changes in the interest rate subsidy and the existence of programs targeted at the extreme poor.¹⁵
- 2016-18 Mi Casa Ya (MCY), an increase in downpayment subsidy for formal employees and interest rate subsidies. The program Mi Casa Ya is available to all households with earnings of 4 mMW or less and automatically includes the downpayment and interest rate subsidy.

¹³In Colombia the minimum wage is adjusted every year based on the inflation, productivity growth and an agreement between different representatives of the different economic sectors. Appendix Figure C.6 shows the evolution of the minimum wage an inflation during my study period.

¹⁴This price limit is set by the government's National Development Plan. The price limit was the same from 1997 until 2019. With law 1467 of 2019, the price limit increased to 150 mMW for the five largest cities (including the metropolitan areas) and remained the same in the other cities. A low-cost housing unit is a house whose total price is below the threshold of 135 times the monthly minimum wage (around US\$40,000). There is an additional definition creating a similar discontinuity at a lower price cutoff. Housing units below $70 \times mMW$ (around US\$20,000) classify as priority low-cost housing. This cutoff defines eligibility for some subsidies for the extreme poor and those affected by forced displacement or natural disasters. This paper focuses mostly on the subsidies targeting the population buying low-cost housing units.

¹⁵Including the provision of 100,000 free housing units and the country's primary mortgage down-payment subsidy program for the vulnerable population (VIPA). More details are provided in Appendix C.

2 Down-payment + Interest rate

1 Down-payment - Interest rate

2 2006 2009 2012 2015 2018

Figure 1: Total Government Expenditure on Demand Subsidies over Time

Source: Administrative records from the Ministry of Housing. Appendix C provides more details about the data.

Note: This Figure shows the evolution of total government expenditure by type of subsidy. The downpayments are the subsidies awarded to employees affiliated to family funds. The interest rate represents the total amount paid by the government to the banks corresponding to the interest rates payments. I assigned the total amount to the year of the subsidy assignment. I calculated this amount using the administrative data containing detailed information on each loan. Mi Casa YA corresponds to the payments for the interest rate and the downpayment subsidy. Figure C.7 shows the number of assigned subsidies over time.

Subsidy expansion. Figure 1 shows the total government expenditure from 2006 until 2018. The gray blue area shows the expenditure on downpayment subsidies. The expenditures was stable until 2015, when the subsidy's size increased. The dark blue area shows the total government expenditure on the subsidized interest rate. The number of households receiving this subsidy was stable over time, but government expenditure slightly decreased due to the lower interest rate. The light blue area shows the expenditure related to the Mi Casa Ya program, which provides

¹⁶To obtain the government expenditure, I calculate the total savings on mortgage payments induced by the discount at the interest rate. I calculate the monthly payments of each loan using the administrative records for the subsidy and the formula for monthly payments on a mortgage, $L_{monthly} = L \cdot \kappa(i,n)$ with $\kappa(i,n) = \frac{\frac{i}{12} \cdot \left(1 + \frac{i}{12}\right)^{12 \cdot n}}{\left(1 + \frac{i_h}{12}\right)^{12 \cdot n}}$. Where i is the interest rate, $i_{subsidy}$ is the interest rate discount, n is the loan term in years, L is the loan amount. The government pays the difference in the amount paid by the households ($L \cdot \kappa(i_\tau, n)$, with $i_\tau = i - i_{subsidy}$) and the amount received by the bank ($L \cdot \kappa(i_\tau, n)$). In particular, $\tau^i = \sum_{t=1}^{84} L_{month}(i,n)(i,n) - L \cdot \kappa(i_\tau, n)$, the sum of monthly payments for seven years, the period when the subsidy applies. Figure C.11 shows the loan terms by unit price and Figure C.12 shows the market interest rate and the interest rate households pay.

downpayment assistance and covers a reduction in the interest rate.

Supply subsidy–value-added tax (VAT) refund. In 1995, a couple of years after the beginning of the downpayment subsidies, and to encourage developers to build low-cost housing, the government introduced a VAT refund. Developers get up to 4 percent of the sale price of each unit as a refund for taxes paid on construction materials.¹⁷

C. THE NOTCH

The combination of the arbitrary definition of low-cost housing and the different policy tools explained in the previous section creates discontinuous incentives or notches around the cutoff defining low-cost housing. Further, the size of the notch increases over time.

Relevant prices. The subsidy scheme has three different prices: transaction or market price, \mathbf{P} ; developers price, \mathbf{P}^{δ} , or the per unit price developers receive after including the tax refunds; and household price, \mathbf{P}^{τ} , or the price households pay net of subsidies. Formally,

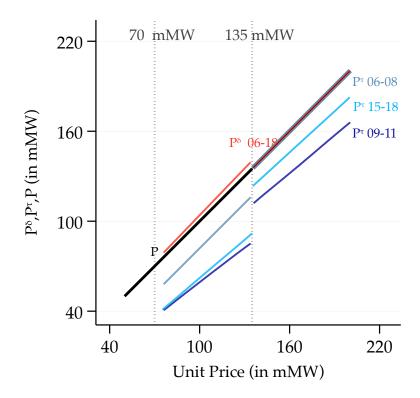
$$P^{\delta} = P \cdot (1 + \delta)$$
$$P^{\tau} = P - \tau$$

where, $\delta = 4$ percent is the tax refund rate, and $\tau = \tau^m + \tau^i$ is the total amount of money paid by the government for a housing unit. τ^m represents the downpayment subsidy, and τ^i is the interest rate subsidy. The demand subsidy depends on the downpayment assistance, which is a fixed amount that is independent of the housing price, and the savings from the interest rate discount, which depends on the size of the mortgage and therefore is related to the housing price.

Figure 2 shows the notch generated by the subsidy scheme. The x-axis represents the market price P, and the y-axis represents the price received by developers \mathbf{P}^{δ} or paid by households \mathbf{P}^{τ} . The 45-degree black line represents the market price. The red line is the price received by developers \mathbf{P}^{δ} , the blue lines represent the households' price \mathbf{P}^{τ} . There are three lines depending on the interest rate subsidy scheme available in each period.

¹⁷This policy instrument was first introduced in the 1995. Even-thought it has been regulated by different laws and acts, for example, Law 1607 of 2012 or Act 2924 of 2013 (Camacol (2016) p.25.), it has always had the same incentive capped at 4 percent of the value of each unit.

Figure 2: The Notch



Note: This figure compares the **market price P**, the price received by developers P^{δ} , and the price households pay net of subsidies (in blue). The three different blue lines correspond to the three subsidy schemes available during the study period, P^{τ} 2006-08, P^{τ} 2009-11 and P^{τ} 2016-18. The price paid by households is $P - \tau^m - \tau^i$, τ^m is a transfer from the government for the downpayment and does not depend on the price of the house. τ^i are the savings in interest rate payments, because this depends on the mortgage; it is calculated by taking a typical mortgage at each market price using administrative records from the Ministry of Housing.

Notch. If the price is below the cutoff and developers get the tax incentives, then the red line is above the black line. A household buying a housing unit below the cutoff qualifies for the subsidy and pays less than the market price. The gap between the black line and the blue lines is the money paid by the government τ . Because housing units with a price above the price cutoff can get an interest rate subsidy, the blue lines on the right of the cutoff are below the black line. However, there is always a notch for the household's price at the cutoff. The periods with the interest rate subsidies have a different slope in the household's price because the subsidy is higher as the mortgage increases. I calculate the government expenditure on interest rate payments using a typical mortgage at each price level. ¹⁸ This figure illustrates

¹⁸For more details on the data sources and the mortgages, see Appendix C.

how the subsidy scheme creates incentives for developers and households to build and buy housing units with a price at or below the cutoff.

Notch over time. The notch on the demand side increases over time. The gray blue line shows the household price when only the downpayment subsidy was available. Before the government introduced the interest rate subsidy, households buying a unit priced above the cutoff paid the full price. In the figure, the black and blue lines coincide above the cutoff. The dark blue line shows the price paid by a household that gets the downpayment subsidy and the interest rate subsidy during 2009-11. The interest rate is also available if households get a unit above the price cutoff, but the discount is smaller. The two subsidies combined increase the discontinuity or notch at the cutoff. The light blue line shows the price paid by households who received the two subsidies after the Mi Casa Ya program was introduced and the increase in the downpayment subsidy. During this period, there was a drop in the interest rate and therefore the interest rate subsidy was lower. This explains why the price paid by households below the cutoff was similar during 2009-11 and 2015-18 even if the downpayment subsidy was higher. It also explains why the price in 2015-18 was lower above the cutoff. Despite these changes in the interest rate, the notch increased during this period.

There were many changes during the period between 2012 and 2015. In addition to the 100,000 free housing units priced at $70 \times mMW$ or below the interest rate, the subsidy changed many times. For completeness, I include this period when presenting the data and results; however, I see it as a transition period and therefore pay little attention to it.

Table 1: Notch and number of subsidies by period

	Notch (in mMW)			Subsidies (in thousand)			
	$ au^M$	$ au^i$	τ	downpayment	i rate	Mi Casa Ya	
2006-2008	18.0		18.0	47.1	•	•	
2009-2011	20.0	5.85	25.9	46.4	16.7	•	
2012-2015	19.9	9.55	29.5	41.1	22.2	•	
2016-2018	25.3	7.24	32.6	44.5	23.4	16.8	

Note: This table shows the size of the notch in figure and by period and differentiating the discount coming from the interest rate subsidy and the discount from the downpayment assistance. It also shows the number of subsidies (in thousands) assigned to each type of program by year, downpayment, interest rate, and the two together with *Mi Casa Ya*. The value for each period is the average number. Figures C.12 and C.11 shows the loan terms and interest rate over time.

Notch size. Table 1 shows the size of the jump at the cutoff during the study period and the number of assigned subsidies for each program. Around 45,000 households received the downpayment subsidy each year, with slight variation across years, and around 22,000 households received the interest rate subsidy. Households can get both supports, but they have to apply separately to each program. Each year, around 17,000 households receive the subsidy from the *Mi Casa Ya* program, which grants both subsidies.

III. Housing Market Responses

This section introduces the housing market data and the main descriptive facts. I present evidence of bunching in housing price and introduce the hedonic price for housing characteristics and the definition of a standardized housing unit. I use this transformation to show the bunching over time, construct a counterfactual distribution, and explore the behavioral responses induced by the subsidy.

A. Data

Data source. The data are from a monthly census, called *Coordenadas Urbanas*, collected by the Colombian Chamber of Construction-CAMACOL and containing all new construction units built in 126 Colombian municipalities from 2006 to 2018.¹⁹ The unit of observation is a housing unit type, for example, if there are three different apartments in a housing development, a studio, one-bedroom, and two bedrooms, I observe the price and characteristics of each of these. I observe all housing developments of at least 300 square meters of construction. The census excludes small single-family homes and informal housing. It does not contain information on resales of existing housing units. Although this is a limitation of the data, the subsidies apply only to new housing, so the data covers the directly affected part of the market.

I observe *general characteristics* of the house such as the unit size; location, including the exact latitude and longitude coordinates; number of rooms; quality of appliances; estrato, which is an index summarizing neighborhood quality; and *developer and project characteristics*, like firm tax identifier and the number of units built in each project. The data also include detailed characteristics of the housing development, including the number of parking spots, the number of towers built, the lot size, and an indicator function equal to 1 if the units are apartments and 0 if they are single family units, among other details of each project. Finally, I observe the *sale price* at

¹⁹Not all cities have information starting in 2006, the census expanded its coverage over time.

different stages of the construction process. To ease the comparison, I take the price at the beginning of the construction of the project. All prices are in 2019 COP or mMW. In Colombia, there is a national mMW, which is adjusted every year based on inflation (see Figure C.6). In most of the analysis, I express the price in mMW to make it comparable with the price cutoff defining low-cost housing units.

B. Bunching in Observed Market Outcomes.

Bunching around the price limit. Figure 2 shows how the subsidy scheme creates incentives for households and developers to buy and build housing units priced at or below the cutoff. To explore the market response to these subsidies, Figure 3 shows the distribution of units by market price for all years and cities in the data. The figure shows a sharp and clear excess mass, or bunching, around the price cutoff defining low-cost housing.

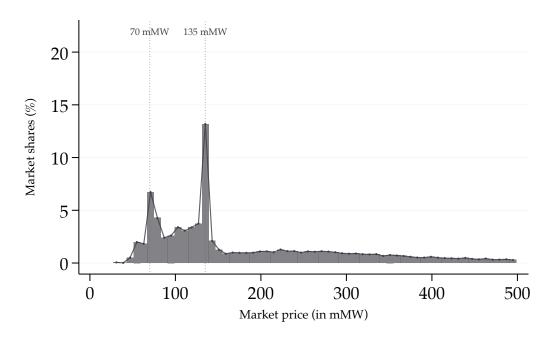
Market adjustment. How does the market adjust to reach this equilibrium? The mechanism explored in this paper is that households and developers adjust the characteristics of the housing units they buy and produce to take advantage of the subsidies, Figure 4 provides suggestive evidence that the subsidy scheme affects the characteristics of the housing stock. Moreover, the construction sector is perceived as highly competitive and developers have no incentive to build bigger units when, for the same price, households would buy smaller ones. An alternative explanation is that households and developers buy and produce the same type of units they would buy and produce in the subsidy's absence, but developers reduce the price for units above but close to the cutoff, allowing households to get the subsidy. Although this is a possibility, the arguments in this paragraph suggest that this is an unlikely scenario and therefore I focus on changes in housing characteristics.

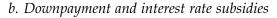
²⁰This argument will be clearer when I introduce the economic model in the next section. The argument applies for any characteristics that imply any cost for developers.

Figure 3: Bunching around the Low-Cost Housing Price Limit

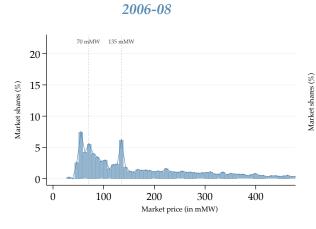
a. All Data

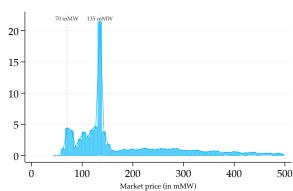
2006-18





c. Subsidy expansion 2016-18



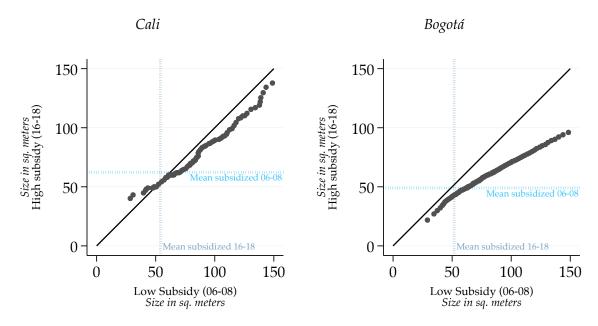


Notch: 18 mMW

Notch: 33 mMW

Note: This figure shows the distribution or the market share of housing units by sale price (expressed in (mMW). The dashed lines are the cutoffs defining *low-cost housing* 135 mMW and *priority low-cost housing* 70 mMW. The figure shows all the units from 2006 to 2018 in all the cities. The spike at *lowest cost housing* is mostly explained by the 100 thousand free housing units granted by the government between 2012 and 2015.

Figure 4: Quantile-to-Quantile Plots of Housing Size: Low versus High Subsidy Periods



Note: This figure shows the quantile-to-quantile plots for observed housing size in square meters for two representative cities, Cali and Bogotá. The y-axis shows the size at the end of the period, when subsidies are higher, and the x-axis shows the size at the beginning of the period, when subsidies are low. The dotted vertical and horizontal lines show the average size of subsidized units. The dots represent the same quantiles in both years. If there are no changes in housing size, they would be on the 45-degree line. Instead, the figure shows how there are changes in size at the quantiles near the average subsidized house.

Size response. Figure 4 shows the quantile-to-quantile plots for housing size at the beginning and end of the study period for two different cities (Figure A.2 shows more cities). During this period, the notch induced by the policy increased from $18 \, mMW$ to $33 \, mMW$. If the distribution of housing size did not change from the beginning to the end of the study period, the black dots should be on the 45-degree line. The blue dotted lines show the average size of a subsidized house. These figures suggest a change in the size distribution around the average subsidized unit. Changes in housing characteristics, particularly size, can explain the increase in bunching from 2006-08 to 2016-18.

Figure 5 shows the joint densities of unit size and total price for all years, cities, and unit types. The marginal densities are next to the axis. The price density in the x-axis is equivalent to Figure 3. Color intensity represents the joint density. In each submarket, heterogeneous agents buy and sell different housing units. The same money may buy bigger housing units in separate submarkets; therefore, agents cluster at different housing sizes for which the sale price is at or below the cutoff.

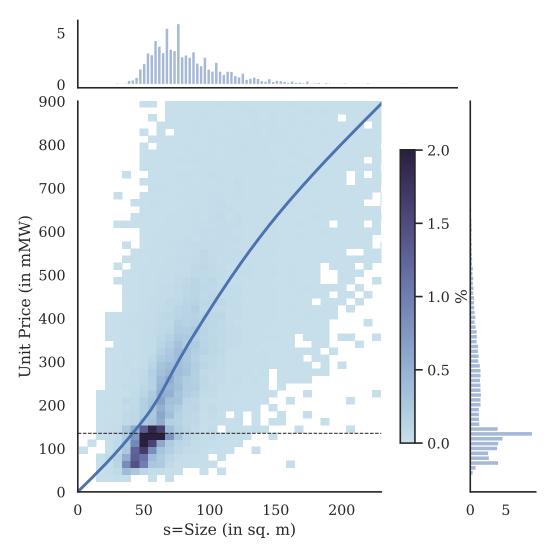


Figure 5: Observed Market Equilibrium

Note: This figure shows the joint and marginal densities for housing size (x-axis) and price (y-axis). Darker dots inside the graph represent a higher market share. The figure contains all cities, in all years, and all the different unit types, that is, single-family homes, multifamily homes, condos, two bedrooms, one bedroom, and so forth. The solid line represents the non-linear relationship between housing size and price (using lowess).

Main housing characteristic. It is difficult to summarize housing units into a single variable as they differ in many dimensions. However, focusing on a single characteristic makes the empirical analysis and modelling tractable. In this paper, I focus on housing size for several reasons. First, my data includes it, which is an advantage. In contrast with most datasets, my data has exact size. Second, it is a continuous variable whose measurement is incontestable, square meters. Finally, conditional on other characteristics, people prefer larger houses which guarantees a monotonic

relationship between size and prices.²¹ Additionally, for developers, it is costly to produce additional square meters.

C. Hedonic Price and Standardized Unit

This subsection introduces the hedonic price function, which plays two roles. First, it allows me to recover the marginal willingness to pay for housing size and the other observed characteristics. Second, I use this price function to convert all housing units into a standard unit that only changes in size.

Implicit Price for Housing Size

The solid line in Figure 5 shows that the nonparametric bivariate relationship between price and size is positive. This pattern follows the expected positive relationship and suggests a nonlinear relationship. However, this unconditional relationship may not represent the equilibrium marginal willingness to pay for housing size, as there may be observable and unobservable characteristics affecting size and price, creating bias. I follow common practice in the hedonic literature to estimate the equilibrium implicit price of size. Equation 1 represents a general specification for the price function. Where s_{ltc} is housing size, X_{ltc} is a vector containing all other housing characteristics, and ω_{ltc} represents the residual containing unobserved characteristics.

$$P_{ltc} = \rho \left(s_{ltc} \right) + \Gamma' X_{ltc} + \omega_{ltc} \tag{1}$$

I observe a unit type l in city c at time t. I assume that the housing price is additive and separable in housing size s_{ltc} , the observable characteristics are included in X_{ltc} , and ω_{ltc} represents the unobserved characteristics. X_{ltc} , includes location, quality, number of rooms, and the neighborhood quality index (*estratos*), ²³ among others. $\rho(.)$ is the implicit price function for housing size. I assume a parametric function

²¹Housing quality or the distance to the central business district is a good alternative variable. However, quality is hard to measure, and distance to the central business district may have a non-monotonic relationship with income.

²²Bishop and Timmins (2019), Bajari and Benkard (2005), Epple et al. (2020) or Bajari, Fruehwirth, Kim, and Timmins (2012)

²³The *estratos* are codes from 1 to 6 called that summarize the quality of the block, they also have different property taxes (in some cities) and prices for utilities. For more details, see Uribe (2021)

specification for ρ .²⁴ In particular,

$$\rho\left(s_{ltc}\right) = \rho_1 \cdot s_{ltc} + \rho_2 \cdot s_{ltc}^2 \tag{2}$$

To estimate the implicit or hedonic price, I rely on independence conditional on observable characteristics:²⁵

$$E\left(s_{ltc}|X_{ltc},\omega_{ltc}\right) = 0 \tag{3}$$

Independence conditional on observables. It is common to rely on conditional independence to recover the implicit price function of a certain characteristics. In my setting, I observe a rich and unique set of controls. This includes the exact location of the unit and general characteristics of the house, including the number of rooms and the neighborhood quality index.

The assumption of conditional independence can be problematic in many settings. For example, Chay and Greenstone (2005) show that using a hedonic model to recover the marginal willingness to pay for air quality without using instruments generates biased results. Omitted variables could generate a bias in the current setting; however, I present two facts that are reassuring. First, in contrast to air quality, the hedonic regression does not show the opposite of the expected sign. Second, when I include characteristics, such as an indicator function equal to one, if a house extension is possible (for example, an extra bathroom or bedroom), the magnitudes of the coefficients do not change. This type of characteristic is potentially unobserved by the econometrician as it is related to size; so, it is reassuring that including it does not affect the size of the coefficients. However, this does not rule out the fact that other omitted variables could bias the results. For example, if changes in price generate the bunching with no change in size, the error term could be correlated with size, particularly for observations around the price cutoff.

²⁴The unconditional relationship between size and price in Figure 5 suggest that the implicit price function is not quadratic. However, once I include controls in a partially linear model, a quadratic function seems accurate. This parametric approximation makes the inverse function straightforward to calculate.

²⁵Bajari and Benkard (2005) propose three different identification assumptions; i) Independence conditional on observables, ii) Option packages and iii) instruments. My setting and data allows an implementation of each of the three identification approaches. However, the results presented in this paper rely on the first condition.

Standardized Housing and Unit Size

To make all the housing units comparable, I use the hedonic price function to standardize all housing units. In particular, I use the estimates of equation 1 to convert all housing units into a standard unit.

This hedonic price estimation decomposes the unit price into observed and unobserved characteristics. The standardized housing size, which I call *h*, is the size of a housing unit with average characteristics that will cost the same as the observed price.

$$\rho (h_{ltc}) + \Gamma' \bar{X} + \bar{\omega} = \rho (s_{ltc}) + \Gamma' X_{ltc} + \omega_{ltc}$$
(4)

 \bar{X} , are the means of the observable characteristics and $\bar{\omega}$ equals the average residual. Solving for h in the equation 4, I get the following measure of the standardized size measure:

$$h_{ltc} = \rho^{-1} \left(\rho \left(s_{ltc} \right) + \Gamma' \left(X_{ltc} - \bar{X}_{ltc} \right) + \left(\omega_{ltc} - \bar{\omega}_{ltc} \right) \right) \tag{5}$$

Intuitively, this means that if a house is more expensive because it has certain amenities or more bathrooms, I convert this characteristic into the equivalent square meters that the household could get if they had a standard house.

In my application, I standardize the units in a way that $\bar{P} = \rho (\bar{s}_{ltc}) + \Gamma' \bar{X}_{ltc} + \bar{\omega}_{ltc}$ is the observed average price for the average house. For the implicit price function, I use a parametric approximation $\rho(s) = \rho_0 + \rho_1 \cdot s + \rho_2 \cdot s^2$.

Figure 6 shows the functional form of the estimated price function for the four different periods. The Figure 6 shows that the implicit price function has become steeper overtime.

Implied Maximum Size of a Standard Subsidized Unit

Given the standardized unit definition, the maximum size for a unit to qualify for the subsidy at the observed equilibrium's marginal willingness to pay and produce is *h*:

$$135 \times mMW = \lambda = \rho \left(\underline{h} \right)$$

Given a particular assumed functional form,

$$\underline{h} = \frac{-\rho_1 + \sqrt{\rho_1^2 - 4 \cdot \rho_2 \cdot (\bar{P} - \lambda)}}{2 \cdot \rho_2},\tag{6}$$

In Figure 6, \underline{h} corresponds to the value of h at which the implicit price intersects the price cutoff (gray horizontal line).

1000 - 750 - 500 - 250 - 0 50 100 150 200 h=standardized size

Figure 6: Implicit Price for Housing Size

Note: This figure shows the estimated implicit prices for a unit of standardized size h in each period based on equation 2. The standardized size comes from equation 4, where X includes number of bathrooms, number of rooms, an indicator equal to 1 if the unit is a building, location; dummy variables equal to one if the unit has a porch, studio, storage unit, dressing room, service room, dining and living room, fireplace, kitchen, clothes areas, patio, and potential to extend to an extra bathroom or bedroom; location coordinates, lot size, number of building blocks, apartments per floor, number of floors, total parking spots, and number of building units.

D. Unit Supply Function

How do developers respond? One of the principal objectives of the economic model is to address this question in more detail. However, before introducing the model and estimating the equilibrium responses in terms of changes in housing characteristics, I explore the relationship between the unit size and the number of units built. Developers built more housing units when they decided to produce smaller housing

sizes. One advantage of the data is that I observe the number of units built by unit type; therefore, I can get empirical estimates of the trade-off between unit size and the number of units and account for it in the model.

Figure 7 shows a bin scatter plot of the unit supply function adjusted for unit and project characteristics using observations from all years and cities. The figure shows a negative relationship between unit size and the number of units, which is intuitive and makes sense. Developers face a trade-off between building more but smaller units and fewer but bigger units.

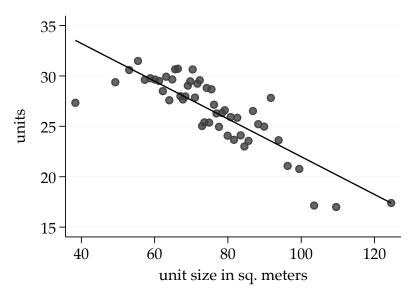


Figure 7: Unit Supply Function

Note: This figure shows the bin scatter for the number of units and for unit size after controlling for observable characteristics. In this figure, I use the same controls as in Figure 6. This figure includes the observations for all years and all cities. Table 2 shows the estimated parameters for this relationship by period with and without controls.

Table 2 show estimates of the model in equation 7 by period and with and without the set of controls X_{ltc} .

$$Q_{ltc} = \alpha_0 + \alpha_1 s_{ltc} + \alpha_x' X_{ltc} + \epsilon_{ltc}^Q$$
 (7)

The coefficients in Table 2 shows the same negative relationship between the number of units built by developers. Including the controls, X_{ltc} , changes the coefficients substantially, in particular, the controls related to the housing development, such as number of buildings and lot size, among others. The slope does not change much during this period, but the constant does change.

Table 2: Unit Supply Function

	06-08 09-12				13-15		16-18	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α_1	-0.17***	-0.05***	-0.13***	-0.04***	-0.16***	-0.05***	-0.18***	-0.07***
	(0.007)	(0.006)	(0.005)	(0.005)	(0.006)	(0.005)	(0.008)	(0.010)
α_0	43.66***	63.32**	33.74***	15.01	38.22***	83.92***	42.24***	35.85**
	(1.169)	(20.350)	(0.734)	(9.459)	(0.835)	(18.227)	(1.003)	(11.375)
X_{ltc}	No	Yes	No	Yes	No	Yes	No	Yes
N	11,906	11,889	15,056	15,029	20,950	20,881	15,756	15,418

Note: This table shows the coefficients from equation 7 by period. The set of controls X_{ltc} is the same as the one detailed in Figure 6.

E. Changes in Housing Characteristics to Benefit from the Subsidy

The estimated hedonic price, standardized size, and the implied size cutoff, \underline{h} , allows me to rationalize the bunching responses in terms of changes in the housing size of a standard unit. This subsection shows the evolution of the behavioral response in terms of reduction in housing size as the subsidy increases and eligibility grows. To quantify the changes induced by the subsidy, I follow the bunching literature to estimate the excess mass and behavioral responses, taking the differences between the observed distribution of market shares by size and an estimated counterfactual distribution.

Counterfactual Distribution Estimation

In contrast with Figure 3, in this section, I present the market shares by standardized unit size, and not unit price, and by period. By doing that, I can interpret the changes induced by the subsidies as changes in the size of a standard unit. Developers and households build and purchase smaller houses to take advantage of the subsidy.²⁶

Define T(h) to be the difference between the observed (f_{h^*}) and counterfactual den-

²⁶I assume all changes are in terms of the housing size of a standard unit. Although I acknowledge this is a strong simplifying assumption, it makes the analysis tractable. The setup and economic framework introduced in section IV can be the basis to extend the analysis to separate changes in multiple characteristics, such as location or quality, and to include the possibility of changes prices in response to a setting with developer frictions or imperfect competition.

sities (f_{h_0}) ,

$$T(h) = f_{h^*} - f_{h_0} \tag{8}$$

and Δh as the maximum change agents made to take advantage of the subsidy.

$$\Delta h = \overline{h} - h \tag{9}$$

Intuition. The counterfactual is the distribution that would exist in the subsidy's absence. I calculate it fitting a flexible polynomial to the observed density and excluding the observations close to the cutoff. The differences between the counterfactual distribution and the observed distribution reflect the behavioral responses to the subsidy scheme's discontinuous incentives.

Estimation. To estimate f_{h^*} and f_{h_0} , I rely on standard techniques from the bunching literature. To estimate the empirical distribution \hat{f}_{h^*} , I calculate the share of units in each bin h_b of size $2 \cdot \epsilon$,

$$h_b = \frac{1}{N} \sum_{l=1}^{N} \mathbb{1} \left[h_l \in (b - \epsilon, b + \epsilon) \right]$$
 (10)

The estimated observed equilibrium distribution is

$$\hat{f}_{h^*}(h) = h_b$$

To estimate the counterfactual distribution, \hat{f}_{h_0} , I predict the observed values for h_b using a flexible polynomial, $l\left(h_b\right) = \sum_{p=0}^T \iota_p h_b^p$ and excluding a region around the cutoff The function $o\left(h_b; L, H\right)$ includes all the indicator variables for the bins between L and H, the lower and the upper bound, respectively, of the excluded area. $o\left(h_b; L, H\right) = \sum_{k=L}^H \mathbb{1}\left[h_k = h_b\right] h_b$

$$h_b = l(h_b) + o(h_b; L, H) + v_b$$
 (11)

Counterfactual distribution. The counterfactual distribution is the predicted density using only the flexible polynomial.

$$\hat{f}_{h_0} = \hat{l}(h_b) = \sum_{p=0}^{T} \hat{\iota}_p h_b^p$$
 (12)

Bunching. Using the estimated distributions, I can get an expression for bunching or excess mass at \underline{h} , and calculate the maximum behavioral change induced by the subsidy Δh :

$$\hat{T}\left(\underline{h}\right) = \hat{f}_{h^*}\left(\underline{h}\right) - \hat{f}_{h_0}\left(\underline{h}\right) \tag{13}$$

Equation 13 is the difference between the observed distribution and the counterfactual distribution at the discontinuity point, \underline{h} , and it represents the share of individuals who would consume $h \in (\underline{h}, \overline{h})$ in the absence of the subsidy, but consume h in a subsidy scenario.

Maximum behavioral response. The maximum behavioral response, \overline{h} , is obtained when the counterfactual and observed distributions coincide:

$$\overline{h} = min[h: h > \underline{h} \text{ and } \hat{f}_{h_0}(\underline{h}) - \hat{f}(h_b) = 0]$$

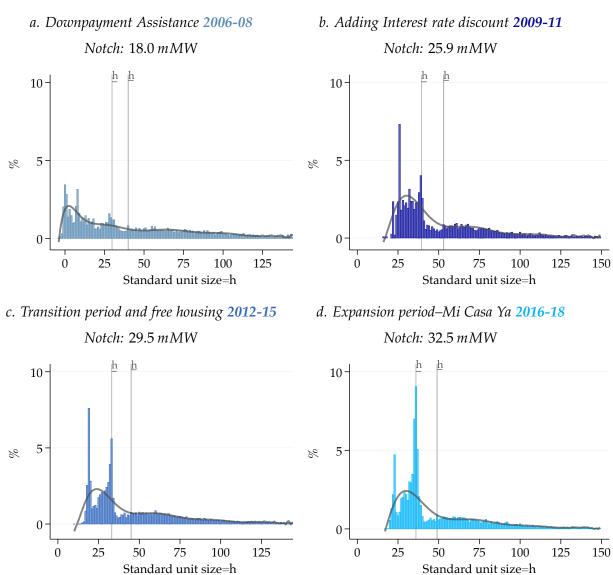
Behavioral Responses \overline{h}

The main behavioral response I explore in this paper is the trade-off between subsidy benefits' and housing unit amenities, in particular, housing size. The excess mass $\hat{T}(\underline{h})$ at the cutoff \underline{h} is the share of households that reduce their housing consumption to get the subsidy. Because this behaviour results in an equilibrium outcome, this corresponds to the response on the supply side. Developers adjust the type of housing they build to meet policy-induced changes in demand.

I exploit the changes in the policy over time to show how as the notch at the cutoff increases, the bunching increases. This introduces additional convincing evidence that the housing market responds to subsidies. The bunching responses are more pronounced as the government expenditure on subsidies increases. Figure 8 shows the distribution of the standardized units for the periods when subsidies change, and a solid grey line for the counterfactual distribution. Table 3 shows the estimates of the relevant behavioral responses. The subsidy creates a small bunching when only downpayment assistance is available (2006-08). With the addition of the interest rate discount (2009-11), there is an increase in the bunching. As expected, the transition years (2012-15) have a large bunching response at the cutoff defining the housing units that were completely subsidized. Finally, during the subsidy expansion the behavioral responses are more pronounced (2016-18), the bunching at the cutoff is 2.02 percent of the market share, compared to 4.02 percent of the market share in the transition period, and 7 percent at the end of the period when the subsidies are

highest.

Figure 8: Equilibrium Density by \tilde{h}



Note: This figure shows the market share or distribution of units by standard unit size by period. The different periods represent a different subsidy scheme and the notch increased over time. The average size of the notch (in mMW) is above each figure. The dashed line vertical line corresponds to the maximum size of the standard unit qualifying for the subsidy, \underline{h} . Each observed unit is converted into a standard unit and the size adjusted according to the estimated willingness to pay for each characteristic. The solid gray line represents the counterfactual distribution. i.e., the distribution that would exist in the absence of the subsidy.

Table 3: Behavioral Responses Estimates'

	06-08	09-11	12-15	16-18
$\int_{h_{min}}^{\underline{h}^{-}} T(h) \mathrm{d}h$	1.03	0.86	3.83	7.28
$\hat{T}\left(\underline{h}\right)$	0.50	2.02	4.02	6.97
$\int_{\underline{h}_{\underline{m}in}}^{\underline{h}} T(h) \mathrm{d}h$	1.53	2.88	7.85	14.2
$\int_h^{\overline{h}} T(h) dh$	-0.096	-6.25	-4.13	-3.42
$h_{h^0}^-\left(\underline{h}\right)$	0.72	1.28	1.06	1.44
h_{min}	26	37	29	32
<u>h</u>	29.8	39.4	33.0	36.0
\overline{h}	40	53	45	49

Note: This Table shows the statistics summarizing the behavioral responses induced by the subsidy. Each column represents different periods. \underline{h} , is the biggest housing unit qualifying for the subsidy, is the size of the unit size \overline{h} , above which households do not reduce there housing consumption to take advantage of the subsidy. h_{min} is the lower point (close to \underline{h}) where the observed distribution is above the counterfactual distribution. T(h) is the difference between the observed and counterfactual distribution. The three different integrals show the excess or missing mass across more than one bin. For example, $\int_{\underline{h}}^{\overline{h}} T(h) dh$ is the missing mass between \underline{h} and \overline{h} , $\int_{h_{min}}^{\underline{h}} T(h) dh$ is the excess mass excluding the excess at the cutoff $T(\underline{h})$, All the excess mass is calculated by $\int_{h_{min}}^{\underline{h}} T(h) dh$.

Section takeaways. This section provided compelling evidence that the Colombian housing market responded to the discontinuous incentives generated by the subsidy scheme. The suggestive evidence supports the view taken in this paper that these responses come from households buying smaller housing units to take advantage of the subsidy. How does the supply side adjust to this change? How is the equilibrium price set? Is the subsidy to developers necessary to prevent housing rationing?

IV. Competitive Housing Market Equilibrium Model

This section introduces a housing market equilibrium model. The objectives of the model are, first, to provide a framework to rationalize the observed equilibrium and understand the economic behavior of the agents. Second, it describes the equilibrium conditions for the model and the role of the hedonic price function. Third, it motivates a novel identification approach to recover the behavioral parameters of households and developers.

A. Model Setup

The proposed model introduces the discontinuous incentives produced by the subsidy scheme into a standard hedonic equilibrium model or sorting model.²⁷

Housing. The model allows for housing to be a vertically differentiated product. In this case, all units are standardized as proposed in the setup in section III.B and units differ in size h. The price of the housing unit P depends on size h, and it is described by the hedonic price function P(h).

Households. Households looking to buy a *new* housing unit are indexed by *i*. They are heterogeneous in income $Y_i \sim F_Y$.²⁸ Households decide how much housing to buy, h_i , and how much to consume of other goods, C_i , to optimize utility $U(C_i, h_i; \theta)$, where *θ* is a preference parameter to be estimated.

Developers. Developers are indexed by j and heterogeneous in their productivity $A_j \sim G_A$. They decide the size of the housing units they build, h_j , to maximize profits. The number of units, Q_j , are determined exogenously by the function $Q_j = Q(h_j; \alpha)$. They face building costs $B(h_j, Q(h_j); \beta)^{29}$ where β characterizes the cost function and is the parameter to be estimated.

Simplifying assumptions. I introduced three simplifying assumptions. First, I assume that the market is perfectly competitive, that is, developers cannot individually affect prices, and P(h) is independent of Q. Second, developers only pick the unit size they build. They follow a unit supply function, which is exogenous and differentiable, $Q = Q^S(h;\alpha)$. Third, the building costs depend on $Q(h;\alpha)$ and h, that is, $B = B(Q(h;\alpha), h, A_j; \beta)$. The last two simplifying assumptions make it straightforward to specify functional forms for the profit function and offer curves. Allowing for an endogenous choice of Q could be a better characterization, but would make

²⁷This is a simplified version of a model with heterogeneous households and developers buying housing units of different sizes. For ease of exposition, I simplify it by assuming that a single variable describes the housing. For some examples of these types of models, without a notch in the budget set, see S. Rosen (1974), Epple (1987), Heckman et al. (2010) or Epple et al. (2020). The literature based on this models is summarized by Kuminoff et al. (2013) and Greenstone (2017). For a survey of the empirical applications see Palmquist (2006).

 $^{^{28}}$ I call Y_i income for simplicity, but it also contains wealth, assets, and their returns, transfer, and so forth. F_Y is the cdf describing the income's distribution

²⁹The cost function $B(Q, h, A_j; \beta)$ is derived from minimizing the production constraints related to producing Q units of products with characteristics h. A_j reflects underlying variables in the cost minimization, that is, factor prices and production function parameters. Different values of A express different factor prices or productivity among developers. For some discussion on this, see S. Rosen (1974) p.43

it difficult to obtain a functional form for the offer curve, which is essential in the identification approach. Relaxing this assumption and allowing for imperfect competition is feasible but beyond the scope of this paper.

Equilibrium. When developers and households decide the type of units they buy, they are in practice choosing the developer type from which to buy. Then, the equilibrium is an implicit price making the densities of the housing units demanded and produced match.

B. OPTIMAL CHOICES

Prices

Section II explained that given the subsidy scheme, there are three relevant prices, market household, and developer prices.

Market:
$$P(h; \rho)$$
 (14)

Household:
$$P^{\tau}(h,\tau;\boldsymbol{\rho}) = P(h;\boldsymbol{\rho}) - \tau \cdot \mathbb{1}\left[P(h;\boldsymbol{\rho}) \leq \lambda\right]$$
 (15)

Developer:
$$P^{\delta}(h, \delta; \rho) = P(h; \rho) (1 + \delta \cdot \mathbb{1} [P(h; \rho) \leq \lambda])$$
 (16)

Differences in prices. A household getting a subsidy pays P^{τ} $(h, \tau; \rho)$ instead of $P(h; \rho)$ which is the market price, and developers who build low-cost housing can get back the VAT taxes paid for materials. The cutoff, λ , to get the subsidies is the same on the supply and demand sides, $P(h, \rho) < \lambda = 135 \, mMW$. The reimbursement of VAT taxes cannot exceed a value δ =4 percent of the value of the house. In other settings where the price can increase and the limit is set in terms of size, the market equilibrium could be achieved by an increase in price, and δ would represent a premium to build low-cost housing.

The price function $P(h; \rho)$ can be a continuous and differentiable function for all $h \in \mathcal{H}$, but the developer and household price functions, $P^{\delta}(h, \delta; \rho)$, and $P^{\tau}(h, \tau; \rho)$, are non differentiable at $P(h; \rho) = \lambda$.

Decision Problem

Households. A household $i \in N$ maximizes its utility given its income level Y_i . It solves the following optimization problem:

$$\max_{h,C} \quad U\left(h,C;\theta\right)$$
 subject to:
$$Y_{i} = P^{\tau}\left(h,\tau;\rho\right) + C,$$

$$h \geq 0.$$

Developers. Developer's profits $\pi\left(Q(h), A_j; \boldsymbol{\rho}\right)$ are determined by the total revenue minus costs.

$$\max_{h} \pi \left(Q(h), A_{j}; \boldsymbol{\rho}, \delta \right)$$

$$\pi = Q(h) \cdot P^{\delta} \left(h; \delta, \boldsymbol{\rho} \right) - B \left(Q(h), A_{j}; \boldsymbol{\beta} \right)$$

Bid functions (or indifference curves). The bid functions, $\varphi_D(h, Y, \bar{U}; \theta)$, represent all the combinations of prices P and unit size h that provide the same level of utility \bar{U} to a household with income Y_i . So, φ_D is such that,

$$\bar{U} = U(h, Y_i - \varphi_D; \theta) \tag{17}$$

Offer function (or iso-profits) The offer function represents the indifference surface for all possible combinations of prices and size h providing the same profits. φ_j^S represents the price developers are willing to accept at different unit sizes to get a constant level of profits $\bar{\pi}_j$. To define the offer function, I replace the developers' price, $P^\delta(h; \delta, \rho)$, by φ_j^s , profits by $\bar{\pi}$, and solve for φ_j^s ,

$$\varphi_{j}^{s} = \frac{B\left(Q^{s}\left(h;\alpha\right), A_{j}; \boldsymbol{\beta}\right) + \bar{\pi}}{Q^{s}\left(h;\alpha\right)} \tag{18}$$

Tangency Conditions

Households. On the demand side, households choose their housing size h to maximize their utility. Because of the notch in the budget set, the standard tangency conditions do not correspond to the optimal choice for all households. I define the

tangency conditions,30

$$\frac{\partial P(h, \rho)}{\partial h} = \frac{\frac{\partial U(h, C; \theta)}{\partial h}}{\frac{\partial U(h, C; \theta)}{\partial C}}$$
(19)

Assuming that equation 19 has a unique solution and using the budget constraint, $P^{\tau}(h,\tau;\rho) - Y_i = C_i$, we can *solve* for h^* , the housing choice satisfying the *tangency* conditions.³¹

$$h^{*}(Y_{i}, \tau; \theta, \rho, \lambda) = \begin{cases} h(Y_{i} + \tau; \theta, \rho) & \text{if } P(h; \rho) \leq \lambda \\ h(Y_{i}; \theta, \rho) & \text{if } \lambda < P(h; \rho) \end{cases}$$
(20)

Developers. On the supply side, the design satisfying the optimality conditions, $h^*(A_j, \beta, \rho)$ for a given price function $P(h, \rho)$ is achieved when developers maximize profits subject to the developers' price being equal to the offer curve. The unit size satisfying the tangency conditions $h^*(A_j, \beta, \rho)$ and optimal profits $\bar{\pi}(A_j, \beta, \rho)$ are achieved when the price and offer curves are tangent

$$\frac{\partial \varphi^{s}\left(h, A_{j}; \boldsymbol{\beta}, \bar{\pi}\right)}{\partial h} = \begin{cases}
\frac{\partial P\left(h, \boldsymbol{\rho}\right)}{\partial h} \cdot (1 + \delta) & \text{if } P\left(h; \boldsymbol{\rho}\right) \leq \lambda \\
\frac{\partial P\left(h, \boldsymbol{\rho}\right)}{\partial h} & \text{if } \lambda < P\left(h; \boldsymbol{\rho}\right)
\end{cases} \tag{21}$$

We can solve 21 for h, and obtain an expression for the tangency conditions,

$$h^{*}\left(A_{j}, \delta; \boldsymbol{\beta}, \boldsymbol{\rho}, \lambda\right) = \begin{cases} h\left(A_{j}, \delta; \boldsymbol{\beta}, \boldsymbol{\rho}\right) & \text{if } P\left(h; \boldsymbol{\rho}\right) \leq \lambda \\ h\left(A_{j}; \boldsymbol{\beta}, \boldsymbol{\rho}\right) & \text{if } \lambda < P\left(h; \boldsymbol{\rho}\right) \end{cases}$$
(22)

Definition of Optimizer Types.

The demand and supply for housing in this case do not correspond to the optimality conditions in this setting because there is a subset of households for which it is optimal to sacrifice housing consumption to get the subsidy. For developers, it is also beneficial to produce a smaller housing unit to benefit from the tax refund.

³⁰This follows by defining a Lagrangian and taking first-order conditions with respect to h and C and taking the ratio. I am assuming that the composite good has a price $p_c = 1$

³¹It has been discussed in the literature that a sufficient condition for this to hold is to assume a Spence-Mirrlees type single crossing condition. See for example Heckman et al. (2010) p.1573 or Kuminoff et al. (2013) for an overview.

There are four types of households and developers.

1. Threshold optimizer $Y_i = \underline{Y}, A_i = \underline{A}$

$$h^*(Y_i, \tau; \theta, \rho, \lambda) = \underline{h} \text{ and } h^*(A_i, \delta; \beta, \rho, \lambda) = \underline{h}$$
 (23)

These are agents who receive the subsidies and whose tangency point is located exactly at the *size threshold* \underline{h} .

2. Marginally subsidized $Y_i \in (\underline{Y}, \overline{Y})$, $A_j \in (\underline{A}, \overline{A})$

$$U\left(Y_{i}-P^{\tau}\left(h^{*}\left(Y_{i},\tau;\theta,\boldsymbol{\rho},\lambda\right),\tau;\rho\right),h^{*}\left(Y_{i},\tau;\theta,\boldsymbol{\rho},\lambda\right);\theta\right) < U\left(Y_{i}-P^{\tau}\left(\underline{h},\tau;\boldsymbol{\rho}\right),\underline{h}:\theta\right)$$
(24)

$$\pi\left(Q(h^*\left(A_j,\delta;\boldsymbol{\beta},\boldsymbol{\rho},\lambda\right)),\overline{A};\boldsymbol{\rho},\delta\right) < \pi\left(Q(\underline{h}),\overline{A};\boldsymbol{\rho},\delta\right)$$
(25)

These agents buy and produce different housing units than the ones satisfying the optimality conditions to take advantage of the subsidies.

3. Marginal bunchers: $Y_i = \overline{Y}$

$$h^{*}\left(\overline{Y},\tau;\theta,\boldsymbol{\rho},\lambda\right) = \overline{h} \iff U\left(\overline{Y} - P^{\tau}\left(\overline{h},\tau;\boldsymbol{\rho}\right),\overline{h};\theta\right) = U\left(\overline{Y} - P^{\tau}\left(\underline{h},\tau;\boldsymbol{\rho}\right),\underline{h}:\theta\right)$$
(26)

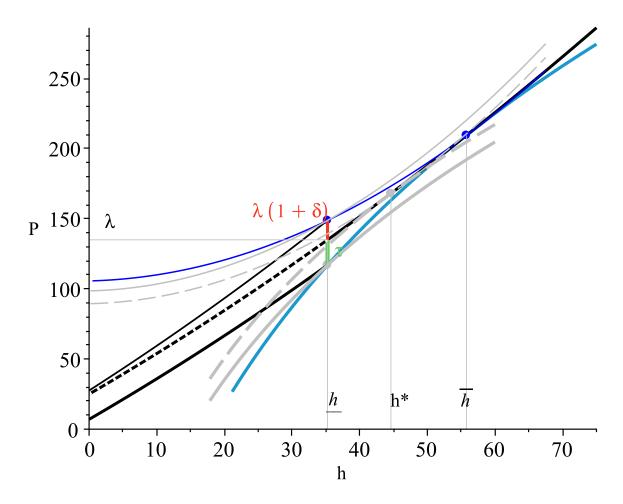
$$h^{*}\left(A_{j}, \delta; \boldsymbol{\beta}, \boldsymbol{\rho}\right) = \overline{h} \iff$$

$$\pi\left(Q(\underline{h}; \alpha), \overline{A}; \boldsymbol{\rho}, \delta\right) = \pi\left(Q(\overline{h}; \alpha), \overline{A}; \boldsymbol{\rho}, \delta\right) \tag{27}$$

These are agents who are indifferent between getting or not the policy incentives.

4. *Subsidized and non-subsidized.* These are agents whose optimal choices are achieved when the *tangency conditions* are reached.

Figure 9: Marginally Subsidized and Marginal Buncher Agents' Choices



Note: This figure shows the optimal choices for the marginal buncher household and developer. The figures presents the intuition for the identification idea. The gray offer and bid functions represent the indifference curves for the marginally subsidized agents. These are the ones who can increase their profits or utility by increasing or reducing h to take advantage of the subsidy and tax incentives.

Figure explanation. Figure 9 shows an example of developers' and households' equilibrium choices. The price function is the envelope of the offer curves when developers produce their optimal unit size and the assigned number of units. The figure shows a representative marginal buncher household and developer. It also shows in gray marginally subsidized households and developers, which are the agents that change their behaviour to take advantage of the subsidy. A developer type A_j matches with a household type Y_i in terms of their optimal choice of h when the dashed lines meet. However this is not an equilibrium choice because both developers and households can be better off if they reduce size h. Figure B.5a, shows the

case of subsidized households and developers. Below \bar{h} , developers receive $P(1+\delta)$ and households pay $P-\tau$. Developers and households increase their utility and profits as a result. The marginal bunching agents are indifferent between getting the subsidy or not. The identification approach in this paper relies on these agents and therefore the main identification strategy is conveyed in Figure 9. The idea is that the bunching in the observed equilibrium distribution allows me to recover \bar{h} . Therefore, I can observe two points, \underline{h} and \bar{h} , on the same indifference curves and offer function and recover their shape. Figure B.5 shows the examples of the optimal choices for other developer types and household types.

Individual-Level Supply and Demand

If $Y_i \in (\underline{Y}, \overline{Y})$ and $A_j \in (\underline{A}, \overline{A})$, households and developers pick \underline{h} , which is lower than their marginal willingness to pay but allows them to get the subsidy. When $Y_i \notin (\underline{Y}, \overline{Y})$ and $A_j \notin (\underline{A}, \overline{A})$, households choose h such that their marginal willingness to-pay for bigger units is equal to the marginal implicit price. This happens when the bid and price functions are tangent. Therefore, the individual supply and demand functions are

$$h^{\mathrm{D}}(Y_{i};\tau,\theta,\boldsymbol{\rho},\lambda) = \begin{cases} h^{*}(Y_{i},\tau;\theta,\boldsymbol{\rho},\lambda) & \text{if } Y_{i} \leq \underline{Y} \\ \underline{h} & \text{if } \underline{Y} < Y_{i} < \overline{Y} \\ h^{*}(Y_{i},\tau;\theta,\boldsymbol{\rho},\lambda) & \text{if } \overline{Y} \leq Y_{i} \end{cases}$$
(28)

$$h^{S}(A_{j}, \delta; \boldsymbol{\beta}, \boldsymbol{\rho}, \lambda) = \begin{cases} h^{*}(A_{j}, \delta; \boldsymbol{\beta}, \boldsymbol{\rho}, \lambda) & \text{if } A_{j} \leq \underline{A} \\ \underline{h} & \text{if } \underline{A} < A_{j} < \overline{A} \\ h^{*}(A_{j}, \delta; \boldsymbol{\beta}, \boldsymbol{\rho}, \lambda) & \text{if } \overline{A} \leq A_{j} \end{cases}$$
(29)

C. Market-Level Supply and Demand

Productivity and Income Mapping to Housing Size.

Because households only differ by income level Y_i , and under the assumption that $h^*(Y_i, \tau; \theta, \rho, \lambda)$ is strictly monotone, it is possible to know the household income

from the choice of housing satisfying the optimality conditions.³²

$$Y_i = \tilde{Y}(h, \tau; \theta, \rho, \lambda) = h^{*-1}(h_i, \tau; \theta, \rho, \lambda)$$
(30)

Similarly, the productivity level determines the unit size.

$$A_{j} = \tilde{A}(h; \boldsymbol{\beta}, \delta, \boldsymbol{\rho}) = h^{*-1}(A_{j}, Q(h); \boldsymbol{\beta}, \delta, \boldsymbol{\rho})$$
(31)

From distribution of income and productivity to a size distribution. The proportion of households choosing a housing unit of size h determines the market-level demand and supply densities. The income and productivity levels uniquely determine the optimal level of housing. This monotonic relationship, and the change of variable formula, allows an expression for the distribution of h^* using the distribution of income f_Y , and productivity g_A . For households the density is $f_h^*(h; \tau, \rho, \theta, \lambda)$ and for developers it is $g_{h^*}(h; \beta, \delta, \rho)$,

$$f_{h^*} = \begin{cases} f_Y \left(\tilde{Y} \left(h, \tau \neq 0; \theta, \rho, \lambda \right); \gamma \right) \frac{\mathrm{d}}{\mathrm{d}h} \tilde{Y} \left(h, \tau \neq 0; \theta, \rho, \lambda \right) & \text{if } h < \underline{h} \left(\rho, \lambda \right) \\ f_Y \left(\tilde{Y} \left(h, \tau = 0; \theta, \rho, \lambda \right); \gamma \right) \frac{\mathrm{d}}{\mathrm{d}h} \tilde{Y} \left(h, \tau = 0; \theta, \rho, \lambda \right) & \text{if } \underline{h} \left(\rho, \lambda \right) < h \end{cases}$$
(32)

$$g_{h^*} = \begin{cases} g_A \left(\tilde{A} \left(h; \boldsymbol{\beta}, \delta \neq 0, \boldsymbol{\rho} \right) \right) \frac{d\tilde{A} \left(h; \boldsymbol{\beta}, \delta \neq 0, \boldsymbol{\rho} \right)}{dh} & \text{if } h < \underline{h} \left(\boldsymbol{\rho}, \lambda \right) \\ g_A \left(\tilde{A} \left(h; \boldsymbol{\beta}, \delta = 0, \boldsymbol{\rho} \right) \right) \frac{d\tilde{A} \left(h; \boldsymbol{\beta}, \delta = 0, \boldsymbol{\rho} \right)}{dh} & \text{if } \underline{h} \left(\boldsymbol{\rho}, \lambda \right) < h \end{cases}$$
(33)

Densities

The distributions f_{h^*} and g_{h^*} and the demand and supply functions, $h^D(Y_i; \tau, \theta, \rho, \lambda)$ and $h^S(A_j, \delta; \beta, \rho, \lambda)$, allows to get the market-level demand density function, $D_h(h; \rho, \tau, \theta, \gamma, \lambda)$, and the market level supply function $S_h(h, \beta, \delta, \rho)$.

Aggregate Demand density. The demand for housing at the size limit \underline{h} contains the demand for the threshold maximizing households, $f_{h^*}(\underline{h};\tau,\rho,\theta,\gamma)$, and the marginally subsidized households $\int_{\underline{h}}^{\overline{h}} f_{h^*}(h;\tau,\rho,\theta,\lambda,\gamma) dh$. Finally there is no demand for housing

³²This mapping from housing consumption to income is a consequence of the assumption $\theta_i = \theta \, \forall i$. If I allow heterogeneity in θ , the same demand for housing h can come from different combinations of Y_i , θ_i .

units with $h \in (\underline{h}, \overline{h})$.

$$D_{h} = \begin{cases} f_{h^{*}}(h; \tau, \rho, \theta, \lambda, \gamma) & dh & \text{if } h < \underline{h}(\rho, \lambda) \\ f_{h^{*}}(\underline{h}; \tau, \rho, \theta, \lambda, \gamma) & dh & \text{if } \underline{h}(\rho, \lambda) = h \\ + \int_{\underline{h}}^{\overline{h}} f_{h^{*}}(h; \tau, \rho, \theta, \lambda, \gamma) & dh & \text{if } \underline{h}(\rho, \lambda) = h \\ 0 & \text{if } \underline{h}(\rho, \lambda) < h < \overline{h}(\tau, \theta, \rho, \lambda) \\ f_{h}^{*}(h; \tau, \rho, \theta, \lambda, \gamma) & dh & \text{if } \overline{h}(\tau, \theta, \rho, \lambda) \leq h \end{cases}$$

$$(34)$$

Aggregate Supply density. This distribution and the unit supply function, $Q(h; \alpha)$, allows us to get the market level supply density function, $S_h(h, \beta, \delta, \rho)$.

$$S_{h} = \begin{cases} g_{h^{*}}(h; \boldsymbol{\beta}, \delta, \boldsymbol{\rho}) \cdot Q(h; \alpha) & \text{if } h < \underline{h}(\boldsymbol{\rho}, \lambda) \\ g_{h^{*}}(h; \boldsymbol{\beta}, \delta, \boldsymbol{\rho}) \cdot Q(h; \alpha) \\ + \int_{\underline{h}}^{\overline{h}} g_{h^{*}}(h; \boldsymbol{\beta}, \delta, \boldsymbol{\rho}) dh \cdot Q(h; \alpha) & \text{if } \underline{h}(\boldsymbol{\rho}, \lambda) = h \\ 0 & \text{if } \underline{h}(\boldsymbol{\rho}, \lambda) < h < \overline{h}(\tau, \boldsymbol{\theta}, \boldsymbol{\rho}, \lambda) \\ g_{h^{*}}(h; \boldsymbol{\beta}, \delta, \boldsymbol{\rho}) dh \cdot Q(h; \alpha)(h) & \text{if } \overline{h}(\tau, \boldsymbol{\theta}, \boldsymbol{\rho}, \lambda) \leq h \end{cases}$$

$$(35)$$

D. Market Equilibrium

The housing market achieves an equilibrium E when market level demand and supply are the same for all values of h at a given price scheme $P(h; \rho)$:

$$E = \left\{ P(h; \boldsymbol{\rho}) \in \mathcal{P} : D(h; \tau, \theta, \boldsymbol{\rho}, \boldsymbol{\gamma}, \lambda) = S(h; A_j, \boldsymbol{\rho}, \boldsymbol{\phi}) \, \forall h \in \mathcal{H} \right\}$$
(36)

where, \mathcal{P} is the space of all price functions and \mathcal{H} is the space of all possible housing units. The equilibrium market price function make the densities $S\left(h;A_{j},\rho,\phi\right)$ and $D\left(h;\tau,\theta,\rho,\gamma,\lambda\right)$ to be equal. The equilibrium price function allows the match between types of households and developers that clears the market.

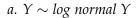
Existence of hedonic equilibrium. The existence of a hedonic equilibrium has received comparatively less attention than the identification of this type of model. S. Rosen (1974) and Epple (1987) show that under some specified utility function, cost functions, and distributions for the unobserved heterogeneity, a closed-form solution for the equilibrium price function exists. Heckman et al. (2010) explicitly describe how the equilibrium price function depends on the distributions of observable characteristics of firms and workers. Ekeland (2010) shows an existence proof and provides a particular example for an equilibrium. Moreover, Bajari and Benkard (2005) prove that in equilibrium, the price of a differentiated product will be a function of its characteristics if utility is continuously differentiable, monotonic in numeraire, and Lipschitz continuous. Under some particular functional forms, the model presented in this paper can have an analytical solution.³³

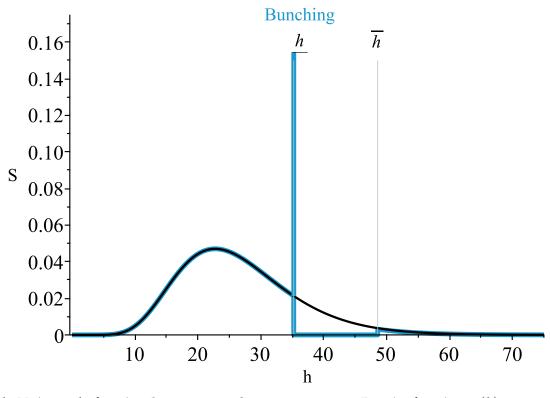
Graphical Representation

Figure 10c shows the product space or developer density, and the exogenous unit supply function. Figure 10a shows an example of the equilibrium density when f_Y and g_A follow a log-normal distribution. The equilibrium price makes the product of the functions in figures 10c and 10b to match the demand density in Figure 10c. Appendix Figure A.4 shows the price function and densities that correspond to the observed equilibrium densities of the standardized unit by size. Additionally, appendix Figure A.1 represents the empirical analog of Figure 10c. The observed density function suggests that the market equilibrium has a discontinuous density and that this stylized model can explain the equilibrium outcomes presented in Figure 8. Because I standardized all housing units, the differences in housing characteristics disappear and standardized units only vary along the dimensions of price and size.

 $^{^{33}\}mathrm{A}$ particular example is available upon request.

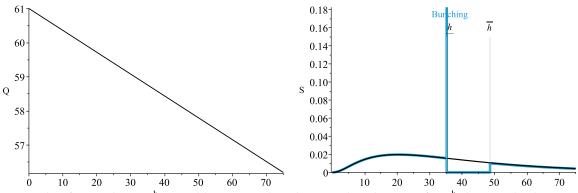
Figure 10: Equilibrium Density, Developer's Choice Density and the Unit Supply Function





b. Unit supply function $Q_i = \alpha_0 + \alpha_1 \cdot h_i$

c. Density function $g_h(h)$



Note: This figure shows the equilibrium market share or distribution of units by standard unit size for a given income density f_y following a log-normal distribution. The empirical analogs of this figure, are in Figure 8. The bottom 2 figures show the share of developers choosing to build at each unit size and the unit supply function. Appendix Figure A.1 and Figure 7 and numbers in Table 2 are the empirical analogs for these figures for the different periods.

Missing Mass between h and \overline{h}

The model predicts that no housing units should be between \underline{h} and \overline{h} . However, in my empirical estimates, I only observe a partial missing mass in the distribution. This partial missing mass is common in bunching analysis using notches (Best et al., 2019; Kleven & Waseem, 2013). This is usually attributed to at least two potential factors, optimization frictions or heterogeneity on the the behavioral parameters θ . Some households may not be aware of the subsidies or the application costs may be too high. In my setting, there is a limited number of subsidies and not all households that qualify actually receive the subsidy. It is also the case that some households get the downpayment and interest rate, but others get only one of the two. This means that the notch may vary across individuals due to different type of frictions. Moreover, it could be the case that households that are eligible do not really see the benefits. Buying and living in a low-cost housing unit could create stigma and households may have a large dis-utility related to that.

If there is heterogeneity across preferences, in this case θ may differ across cities. If that is the case, Best et al. (2019) suggest that the behavioral response can be interpreted as the average marginal response in such cases.

V. IDENTIFICATION

The behavioral parameters to estimate are θ , β , which describe the curvature of the bid and offer curves. This section explains the identification and the estimation.

A. Marginal Buncher

Marginal Buncher Indifference conditions. The identification argument in this paper follows the approach used by Best et al. (2019) and more generally the one suggested by Bertanha et al. (2021) and Blomquist et al. (2021). The idea is that the existence of the marginal buncher allows observing two points in the same bid and offer function.³⁴ Following the hedonic literature, the estimation approach follows a two-step procedure. In the first step, I use the analysis in sections II and III, to get the hedonic price function, the notch, and the behavioral responses. In the second step, I use the marginal bunching condition to solve for the two parameters of interest. From equations 26 and 27, I recover the marginal buncher household's and

³⁴Best et al. (2019) proposition 1 and Bertanha et al. (2021) Theorem 1 both prove identification using the same identification idea as in this paper. The identification in my setting follows the same principal conditional on getting consistent parameters in the first step.

developer's conditions:

$$V_{d}\left(\theta|\underline{h},\overline{h},\boldsymbol{\rho},\boldsymbol{\tau},\overline{Y}\right) = U\left(\overline{Y} - P\left(\overline{h};\boldsymbol{\rho}\right),\overline{h};\theta\right) - U\left(\overline{Y} - P^{\tau}\left(\underline{h};\boldsymbol{\rho},\boldsymbol{\tau}\right),\underline{h};\theta\right) = 0$$

$$(37)$$

$$V_{s}\left(\beta|\underline{h},\overline{h},\boldsymbol{\rho},\boldsymbol{\alpha},\delta,\overline{A}\right) = \pi\left(Q(\overline{h};\boldsymbol{\alpha}),\overline{A};P\left(\overline{h};\boldsymbol{\rho}\right);\beta\right) - \pi\left(Q(\underline{h};\boldsymbol{\alpha}),\overline{A};P^{\delta}\left(\underline{h};\boldsymbol{\rho},\delta\right);\beta\right) = 0$$

$$(38)$$

The parameters, θ and β are estimated as the solutions of these two equations. Given that the estimates of \overline{h} , \underline{h} , $P\left(\overline{h};\rho\right)$, τ , δ , α and the expressions for \overline{Y} and \overline{A} , β and θ are the only unknowns in equations 37 and 38. I do not observe \overline{Y} and \overline{A} , but I use the optimality conditions in equations 29, 25 to express \overline{Y} and \overline{A} in terms of observable characteristics \overline{h} , \underline{h} , $P\left(\overline{h};\rho\right)$, τ , δ , and α .

Bertanha et al. (2021) describe the identification assumptions under which we can recover the structural parameters from observed bunching. They argue that notches allow for the identification of elasticities, while kinks need additional assumptions about the unobserved heterogeneity. Blomquist et al. (2021) show the conditions under which elasticities can be identified under notches and kinks. They illustrate their approach using Saez (2010) setting. In contrast to Blomquist et al. (2021) who assume the pdf of heterogeneity is monotone, Bertanha et al. (2021) derive partial identification bounds by assuming the pdf has a bounded slope. Using censored regression models, covariates, and semi-parametric assumptions on the distribution of heterogeneity, they provide point estimation for kink points.

B. ESTIMATION

Table 4 shows the functional forms that I use to recover the behavioral parameters θ and β . The estimation approach follows a two-step procedure. The first step relies on the estimated parameters presented in the first part of the paper, \overline{h} , \underline{h} , $P(\overline{h}; \rho)$, τ , δ , α . The second step uses the estimates of these parameters and the marginal buncher condition to recover the behavioral parameters describing the utility and cost functions, β and θ .

Table 4: Functional Form and Identification Equations.

Marginal Buncher Condition			
Household $V_D = U$	$\overline{\left(\overline{Y}-P\left(\overline{h}\right),\overline{h};\theta\right)-U\left(\overline{Y}-P^{\tau}\left(\underline{h}\right),\underline{h};\theta\right)}=0$		
Developer $V_S = \pi \left(Q \right)$	$(\overline{Y} - P(\overline{h}), \overline{h}; \theta) - U(\overline{Y} - P^{\tau}(\underline{h}), \underline{h}; \theta) = 0$ $(\overline{h}), \overline{A}, P(\overline{h}); \beta) - \pi(Q(\underline{h}), \overline{A}; P^{\delta}(\underline{h}); \beta) = 0$		
Optimality Conditions			
Income	$\overline{Y} = \widetilde{Y}\left(\overline{h}; \theta, P(h), \lambda\right)$		
Productivity	$\overline{Y} = \tilde{Y}\left(\overline{h}; \theta, P(h), \lambda\right)$ $\overline{A} = \tilde{A}\left(\overline{h}; \boldsymbol{\beta}, P(h), \lambda\right)$		
Functional Forms			
Implicit Price	$P = \rho_0 + \rho_1 \cdot h + \rho_2 \cdot h^2$		
Utility	$U = \left[\frac{1}{2} \cdot C^{\theta} + \frac{1}{2} \cdot h^{\theta}\right]^{\frac{1}{\theta}}$		
Unit Supply	$Q = \alpha_0 + \alpha_1 h$		
Cost	$B = A_j \cdot Q \cdot h^{oldsymbol{eta}}$		

Note: This Table summarizes the functional forms used for the estimation of β and θ . The functional forms for the optimality conditions are in the appendix A.2.

Observed and Unobserved Objects

The observable objects are developer density, g_h , and the equilibrium distribution of housing size f_h . The equilibrium distribution f_h is g_h multiplied by the number of units built Q(h), which is observable. This allows me to estimate Q(h|X) and get α_0 and α_1 . The equilibrium price P and housing characteristics X_{lt} allow me to estimate the implicit price function for housing characteristics, P(h|X). With this, I recover the parameters ρ_0 , ρ_1 , ρ_2 and use the implied substitution pattern to transform every house into a standard housing unit of size h. The policy parameters, $\tau = \tau^m + \tau^r$, δ correspond to the average subsidy τ and refund rate δ .

My data do not have information on income Y or productivity A, and the functional forms for P(h), U(.), B(.), Q(.) are unknown. The estimation approach relies on specified functional forms for utility, unit supply, cost, and implicit price functions. Table 4 shows the particular functional and main identification conditions I use to recover the structural parameters.

First Step: Parameter Estimates

The reduced-form evidence and analysis correspond to the first step. In particular, The policy parameters τ , δ , correspond to the notches presented in Figure 2 and Table 1. The implicit price function and implied size threshold are recovered from the hedonic price function presented in Figure 6. The estimates from the hedonic price function allow me to get the empirical content for the standardized housing size h. Using the estimates from the hedonic price function, each housing unit is transformed into a standard unit of size h h, P $(\bar{h}; \rho)$ The bunching estimates allow me to recover \bar{h} and the relationship between size and number of units allows me to recover the parameters α_0 and α_1 .

Second Step: Estimation of θ , and β

For given functional forms and estimates of the observable parameters describe in section IV, I can solve for θ and β . The marginal buncher functions are highly nonlinear and it is not possible to get a closed-form solution. I use numerical methods to find the values of θ and β . I present the estimates separately for each subperiod with specific subsidy schemes.

VI. STRUCTURAL ESTIMATES AND WELFARE

The main objective of the paper is to evaluate how the subsidy affects households and developers. The first part of the paper presented compelling empirical evidence suggesting that the subsidy affects the housing market equilibrium. Section IV introduced an economic model to rationalize such responses. In this section, I use the economic model and the estimated parameters to illustrate the incidence of this policy. The economic model allows me to quantify the gains and losses induced by the subsidy and evaluate its relative importance.

Behavioral Parameter Estimates

Table 5 summarizes the results from the first step and presents the structural parameters recovered in the second step for the four different periods. The parameter $\sigma = 1/(1-\theta)$ is the constant elasticity of substitution utility function (CES). It represents how the relative consumption of housing varies when the relative price changes.

Households' parameters. The estimate for the elasticity of substitution between housing and consumption of other goods is around 0.9. It is slightly lower in the first period and higher in the second period. But the fact that the estimated parameters are similar across years is reassuring, considering that these are economic fundamentals and therefore very unlikely to fluctuate over time.

An elasticity of 1 corresponds to a Cobb-Douglas elasticity; therefore my estimates suggest that a Cobb-Douglas utility function would not be a bad representation. A negative value of θ which corresponds to an elasticity of substitution σ less than one, means that housing and other goods are gross complements. If θ is positive, the elasticity of substitution is greater than one, and housing and consumption of other goods would be gross complements.

Table 5: Structural and First Stage Parameters.

	06-08	09-11	12-15	16-18
Pric	ce Functi	on		
$\overline{\rho_0}$	17.0	-300.0	-243.5	-240.6
$ ho_1$	2.70	4.75	4.48	4.66
$ ho_2$	0.90	0.32	0.73	0.60
Policy Parameters				
τ	18.0	25.9	29.5	32.6
Bunchers Interval				
\overline{h}	40	53	45	49
<u>h</u>	29.8	39.4	33.0	36.0
Unit Supply Function				
α_0	70.5	12.7	81.1	33.3
α_1	-0.068	-0.020	-0.020	-0.042
Structural Parameters				
β	2.53	1.67	1.77	1.70
σ	0.85	0.97	0.90	0.90
θ	-0.17	-0.028	-0.11	-0.11

Note: This table summarizes the relevant estimates from the first part of the paper and is part of the estimation, in particular, the hedonic price functions; the notch τ ; the behavioral response \underline{h} ; the policy threshold \underline{h} ; that is, the maximum size of a standard unit that gets the subsidy and the parameters of the unit supply function α_1 and α_0 . The elasticity of substitution implied by the CES utility function is $\sigma = 1/(1-\theta)$.

Bayer et al. (2007) present an approach that integrates the hedonic insights into a discrete choice framework. As pointed out by Yinger (2015), their approach im-

plicitly assumes a linear utility function, which violates the strict quasi-concavity postulate. In other approaches in the urban economics literature, the utility function is assumed to be Cobb-Douglas. In my setting, I allow for a less restrictive functional form, but my estimates suggest that the Cobb-Douglas utility function would be a close approximation.

Developers' parameters. On the developer side, the estimated parameter β , changes as the new subsidies are introduced. In the first period, β is 2.53, this decreases to around 1.7 in the following periods. This change means that the costs of building bigger houses decreased over time.

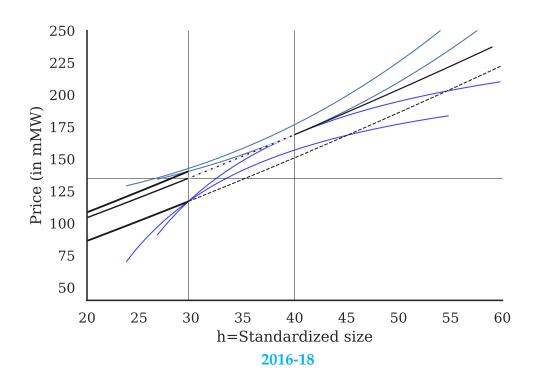
Benefits and Efficiency Losses on the Demand Side

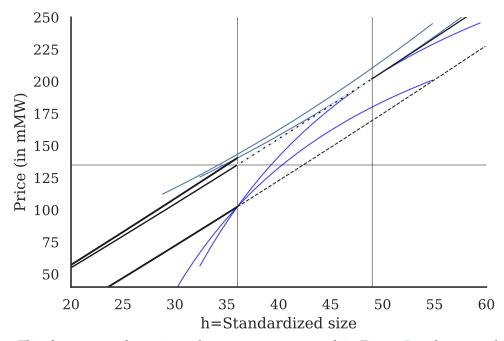
To evaluate the benefits of the subsidy, I focus on the effect on marginally subsidized households. The price limit creates efficiency losses if, like I am assuming, the response on the supply side is a reduction in price and housing size. Figure 11 illustrates how a representative marginally subsidized household benefits from the subsidy. By reducing its household consumption and getting the subsidy, the household's utility increase. The indifference curve moves from the dashed line to the solid line. Without the existence of the price limit and if the marginally subsidized household gets the subsidy without reducing its consumption, households can be better off.

Welfare if only price changes. This paper interprets the bunching as changes in housing characteristics. However, I acknowledge that it could be the case that there are only changes in housing prices without adjustments in characteristics. If this is the case, the welfare analysis is very different. What I call a welfare loss would be in contrast a welfare increase induced by the price limit. Having conclusive evidence on whether housing characteristics change or not is crucial to determine welfare benefits. However, the framework presented in this paper can interpret the two scenarios.

Figure 11: Equilibrium Choices using the estimated parameters

2006-08





Note: This figure uses the estimated parameters presented in Figure 5 and creates the empirical analog of Figure 9 for the marginally subsidized households and developers. The figure represents the equilibrium choices and bid and offer functions estimated at the beginning and end of the study period. The figure motivates the counterfactual exercises presented in Table 11.

Table 6: Evaluating the effects of the policy using the estimated parameters

	Households			
	06-08 09-11 12-15 16-18			
2/				
$\frac{Y}{Y}$	215.7	324.0	275.6	291.5
$\frac{Y_*}{}$	248.7	392.5	333.7	355.6
Y	282.8	461.7	393.2	421.1
$U\left(\underline{Y}-P\left(\underline{h}\right),\underline{h}\right)$	47.9	85.6	66.3	72.9
$U(\underline{Y}-P^{\tau}(\underline{h}),\underline{h})$	52.5	91.2	72.4	79.5
$U\left(Y_{*}-P\left(h^{*}\right),h^{*}\right)$	56.8	100.7	78.9	86.6
$U\left(Y_{*}-P^{\tau}\left(\underline{h}\right),\underline{h}\right)$	59.6	104.3	82.7	90.7
$U\left(Y_{*}-P^{\tau}\left(h^{*}\right),h^{*}\right)$	61.6	106.4	85.2	93.5
$U\left(\overline{Y}-P\left(\overline{h}\right),\overline{h}\right)$	65.9	116.0	91.7	100.5
	Developers			
	06-08	09-11	12-15	16-18
<u>A</u>	0.0068	0.26	0.19	0.25
A_*	0.0054	0.23	0.17	0.22
\overline{A}	0.0045	0.21	0.15	0.20
$MgC(\underline{h}, Q(\underline{h}), \underline{A})$	214.4	59.5	398.2	160.7
$MgC(\underline{h}, Q(\underline{h}), A_*)$	219.0	59.0	404.4	161.2
$MgC\left(h^{\ast},Q\left(h^{\ast}\right) ,A_{\ast}\right)$	172.4	53.7	356.0	144.9
$MgC(\overline{h},Q(\overline{h}),\overline{A})$	223.4	58.6	410.6	161.7
$\pi\left(Q\left(\underline{h}\right),\underline{A};P\left(\underline{h}\right)\right)$	6725.5	205.8	3449.4	881.6
$\pi\left(Q\left(\underline{h}\right),A_{*};P^{\delta}\left(\underline{h}\right)\right)$	7588.2	406.1	4668.9	1388.9
$\pi\left(Q\left(\underline{h}\right),A_{*};P\left(\underline{h}\right)\right)$	7218.6	341.6	4234.4	1217.2
$\pi\left(Q\left(h^{*}\right),A_{*};P^{\delta}\left(h^{*}\right)\right)$	7740.0	439.8	4889.1	1478.6
$\pi\left(Q\left(h^{*}\right),A_{*};P\left(h^{*}\right)\right)$	7326.4	359.9	4359.1	1266.4
$\pi\left(Q\left(\overline{h}\right),\overline{A};P\left(\overline{h}\right)\right)$	7930.5	512.2	5262.2	1646.0

Note: This table shows estimates of the implied income Y and productivity level of three representative households and developers. It shows the threshold optimizer, with income \underline{Y} and \underline{A} , a representative marginally subsidized household with income Y_* and A_* , and the marginal buncher, with income \overline{Y} and \overline{A} . For households, the table shows different utility levels, $U(Y_i - P(h_i), h_i)$ evaluated at different choices of h_i , paying the market price $P(h_i)$ or receiving the subsidy $P^{\tau}(h_i)$. For developers, the table shows the marginal costs MgC(h,Q(h),A) and profits $\pi\left(Q\left(h_j\right),A_j;P\left(h_j\right)\right)$, evaluated at different levels of h_j getting the sales price only $P(h_j)$ or getting the tax incentives too, $P^{\delta}(h_i)$.

Estimates. Table 6 uses the estimated parameters to evaluate the utility levels at the observed choices and the relevant counterfactual scenarios. In particular the table shows the implied income Y for the threshold optimizer household, a representative marginally subsidized household, and the marginal buncher. It evaluates the utility of the representative marginally subsidized household at three points: the counterfactual scenarios where the household consumes a housing unit with size satisfying the *tangency conditions*, h^* at the observed price scheme P(h); the utility for the observed consumption; and the utility if the household gets the subsidy but does not have to reduce housing consumption to qualify for it.

How Important Are the Supply Subsidies?

It is important to know how important the supply subsidies are to maintain the equilibrium in this market. What would happen if there were not supply subsidies? Figure 11, illustrates how tax refunds increase developers' profits, when they reduce the unit size to meet the increase in demand at the price threshold. If they do not get that incentive, they would have to reduce their profits to stay in the market.

Estimates. Table 6 shows estimates of marginal costs and profits using the estimated parameters and economic model. As with the demand side, I show marginal cost estimates for three relevant developer types.

Context and relevance. An argument to support housing subsidies is that by giving a subsidy to buy new housing, developers also benefit.³⁵ Moreover, this is an important policy debate as in Colombia the construction sector argues that eliminating the housing subsidies will create a rationing problem.³⁶

Differences from a Standard Model with Homogeneous Agents and Housing Units.

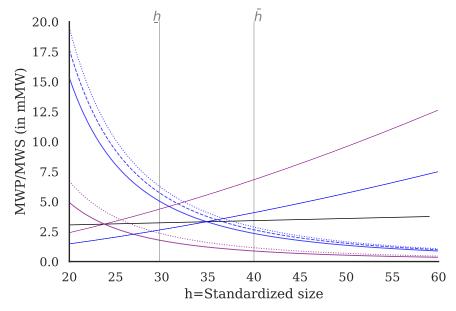
In this subsection I represent the equilibrium choices graphically in an alternative way to show how the economic analysis differs from a basic framework, rejected

³⁵For example "Another explanation for the existence of low-income housing subsidies is political. An in-kind subsidy tends to help not only the beneficiary, but also the producers of the favored commodity. Thus, a transfer program that increases the demand for housing will tend to benefit the building industry, which will then lend its support to a coalition in favor of the program. As indicated in Section 4.1 below, housing programs for the poor have focused on the construction of new units, thus benefit the housing industry rather directly." H. S. Rosen (1985)

³⁶If these items are repealed, in Valle del Cauca we would go from having an offer of SH and sales of 23,000 homes, average year, to one of sales of 4,600 homes El Tiempo (2021)

by the data, assuming homogeneous goods and agents. To illustrate this, Figure ?? shows an alternative graphical representation of the equilibrium choices. It shows the demand and supply functions in terms of the marginal price. If there were only one type of housing and homogeneous agents, a single point where the demand and supply match would characterize the equilibrium. The welfare evaluation of the subsidy would be straightforward. Under the case studied here, multiple goods are treated in equilibrium. However, there is an additional caveat, that is, if developers all have the same cost function, a single function will characterize the supply side, which in this case is the black line in the figure. In contrast, with heterogeneous developers, the black line is the marginal equilibrium price where different developers and households match. The effect of the subsidies on developers will be very different in the two scenarios. Moreover, if we do not allow for nonlinearities in the implicit price function, the black line would be constant, implying a perfectly elastic supply. In Figure 12, the different lines show three developer and household types. The figure makes clear how to evaluate the effects of the subsidies, that is, it is not enough to recover the shape of the implicit price function. We need the demand and supply curves. Without estimates of the primitives, it is impossible to do nonmarginal welfare analysis. In this paper, I recover them using the marginal buncher condition.

a. Downpayment Assistance 2006-08



b. Expansion period Mi Casa Ya 2016-18

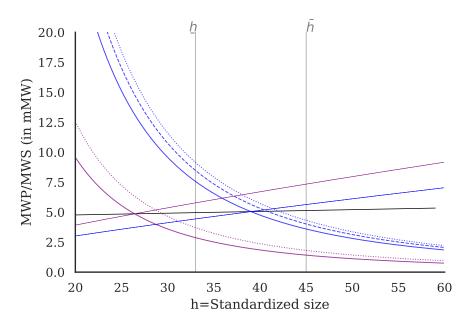


Figure 12: Demand, supply function and marginal equilibrium implicit price

Note: This figure shoes the compensated demand for size and supply. It is alternative representation of the equilibrium choices.

VII. Conclusions

This paper presents compelling evidence of the market responding to subsidies. I rely on detailed data on the universe of new housing, data on subsidies to both households and developers, the policy cutoff inducing discontinuous incentives, and the variation of the subsidy over time. I used the behavioral responses induced by the subsidy and introduce a novel identification approach to estimate a hedonic housing market equilibrium with heterogeneous agents and housing that rationalizes the observed responses. The model-guided estimation approach translates the bunching reflecting the behavioral responses and the reduced form estimates into parameters of both household utility and developers' production function. I used those estimates to illustrate the type of welfare analysis that the estimation approach allows.

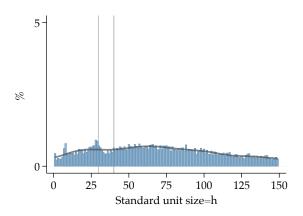
I found that households and developers changed their housing consumption to take advantage of the policy. The price cap, which could be important if the response does not induce a change in housing consumption, induced some welfare losses. Households would have been better off if they received the subsidy without reducing their housing consumption. The welfare analysis also suggested that in a world with developers' heterogeneity, subsidizing the demand side of the market may be insufficient. Developers need to be compensated to produce low-cost housing, which they can produce but at a higher marginal cost. The type of welfare analysis allowed by this approach goes beyond the examples presented here. Because I recovered the income and productivity levels of households and developers together with parameters describing their preferences and costs, the approach allows evaluating different housing policies. The method could apply to other markets with vertical differentiation and price caps, such as labor markets. The findings of this paper suggest that a careful evaluation of the market structure matters for policy design. Understanding how the policy affects the housing market's incentives is crucial to understanding how the observed equilibrium outcomes inform us about the effects of the policy. In this case, the policy induced a change in the type of housing bought and built. The housing stock accumulated smaller housing units purchased by households who would prefer a bigger house. Considering that housing is a durable asset that affects urban structure and city planning, this could translate into significant consequences for cities reaching a suboptimal equilibrium.

APPENDICES

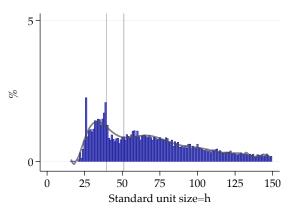
Appendix A. Additional Figures and Tables

Figure A.1: Share of developers by unit size of the Standardized housing

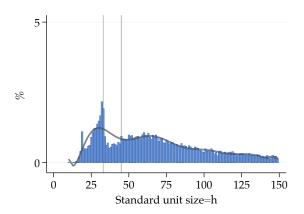




b. Adding Interest rate discount **09-11**



c. Transition period and free housing 12-15



d. Expansion period-Mi Casa Ya 16-18

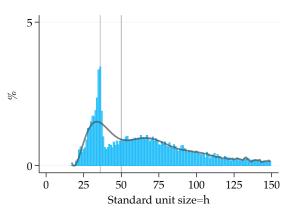


Table A.1: Behavioral Responses Estimates'

	06-08	09-11	12-15	16-18
$\int_{h_{min}}^{\underline{h}^{-}} T(h) \mathrm{d}h$	0.75	0.61	1.81	3.61
$\hat{T}\left(\underline{h}\right)$	0.29	0.85	0.72	1.94
$\int_{\underline{h}_{\underline{m}in}}^{\underline{h}} T(h) \mathrm{d}h$	1.05	1.46	2.53	5.56
$\int_{h}^{\overline{h}} T(h) dh$	-0.40	-1.79	-3.68	-4.48
$g_{h^0}(\underline{h})$	0.57	1.01	1.01	1.18
h_{min}	24	37	27	30
\underline{h}	29.8	39.4	33.0	36.0
\overline{h}	40	51	45	50

Figure A.2: Quantile-to-Quantile Plots of Housing Size: Low versus High Subsidy Periods

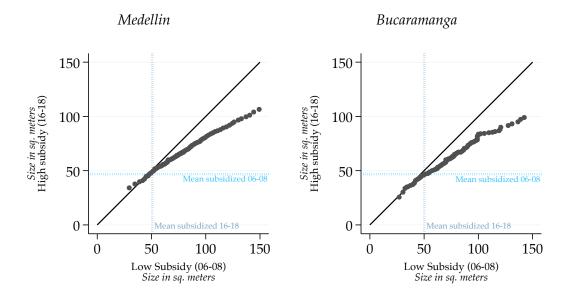


Figure A.3: Observed Equilibrium Densities and Hedonic Price Observed Size

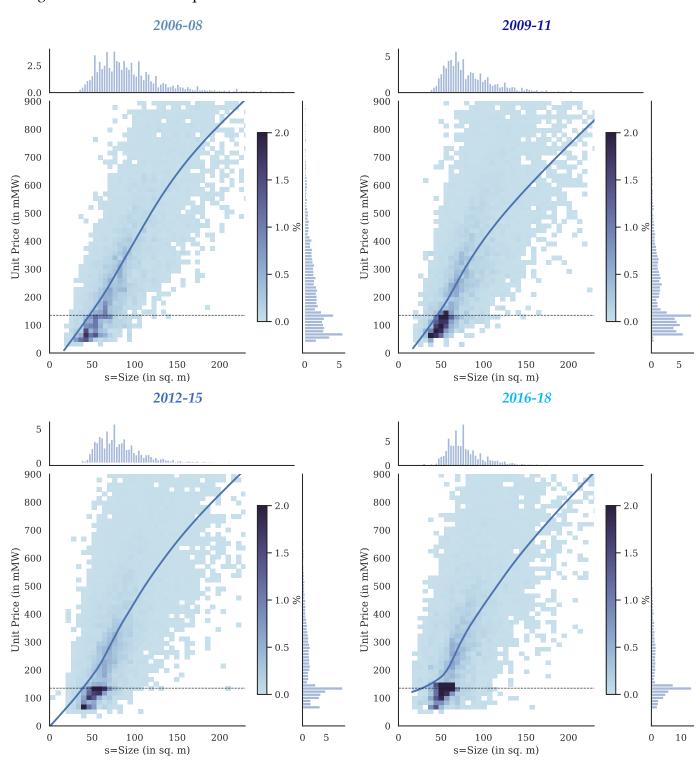


Figure A.4: Observed Equilibrium Densities and Hedonic Price Standard Unit Size

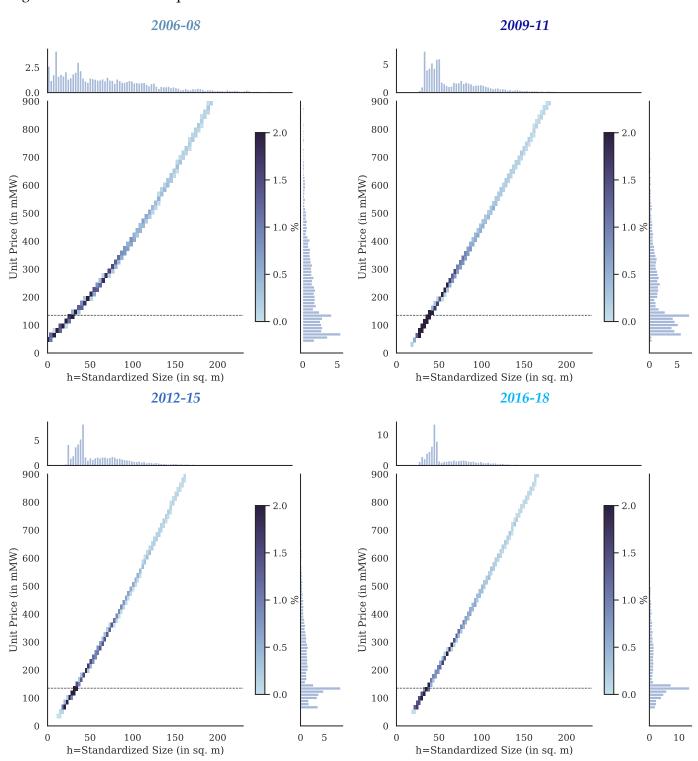


Table A.2: Functional Form and Identification Equations.

Marginal Buncher Condition	
Household	$V_D = U\left(\overline{Y} - P\left(\overline{h}\right), \overline{h}; \theta\right) - U\left(\overline{Y} - P^{\tau}\left(\underline{h}\right), \underline{h}; \theta\right) = 0$
Developer	$V_{D} = U\left(\overline{Y} - P\left(\overline{h}\right), \overline{h}; \theta\right) - U\left(\overline{Y} - P^{\tau}\left(\underline{h}\right), \underline{h}; \theta\right) = 0$ $V_{S} = \pi\left(Q(\overline{h}), \overline{A}, P\left(\overline{h}\right); \beta\right) - \pi\left(Q(\underline{h}), \overline{A}; P^{\delta}\left(\underline{h}\right); \beta\right) = 0$
Functional Forms	
Implicit Price	$P = \rho_0 + \rho_1 \cdot h + \rho_2 \cdot h^2$
Utility	$U = \left[rac{1}{2}\cdot C^{ heta} + rac{1}{2}\cdot h^{ heta} ight]^{rac{1}{ heta}}$
Unit Supply	$Q = \alpha_0 + \alpha_1 h$
Cost	$B = A_j \cdot Q \cdot h^{oldsymbol{eta}}$
Optimality Conditions	,

Income $\overline{Y} = \left(2 \cdot h \cdot \rho_2 + \rho_1\right)^{-\frac{1}{\theta - 1}} \cdot h + h^2 \cdot \rho_2 + h \cdot \rho_1 + \rho_0$ Productivity $\overline{A} = \frac{3 \cdot \left(\delta + 1\right) \cdot \left(h \cdot \alpha_1 + \alpha_0\right) \cdot \left(\left(h^2 \cdot \rho_2 + \frac{2}{3} \cdot h \cdot \rho_1 + \frac{1}{3} \cdot \rho_0\right) \cdot \alpha_1 + \frac{2 \cdot \left(h \cdot \rho_2 + \frac{\rho_1}{2}\right) \cdot \alpha_0}{3}\right)}{h^{\beta_1 + 1} \cdot \alpha_1^2 + h^{\beta_1} \cdot \alpha_0 \cdot \alpha_1 + 2 \cdot h \cdot \left(h \cdot \alpha_1 + \alpha_0\right)^2}$

Marginal Buncher Condition

$$\text{Household} \qquad 0 = \left(\frac{\underline{h}^{\theta} + \left(\bar{h}^2 \rho_2 - \rho_2 \underline{h}^2 + \bar{h} \rho_1 + \bar{h} \left(2\bar{h} \rho_2 + \rho_1\right) \frac{1}{1-\theta} - \rho_1 \underline{h} + \tau\right)^{\theta}}{2}\right)^{\frac{1}{\theta}} - \left(\frac{\left(\left(2\bar{h} \rho_2 + \rho_1\right) \frac{\theta}{1-\theta} + 1\right)\bar{h}^{\theta}}{2}\right)^{\frac{1}{\theta}}$$

$$\text{Developer} \qquad 0 = \frac{\overline{h}^{\beta_1 + 1} \alpha_1 - \underline{h}^{\beta_1 + 1} \alpha_1 + \alpha_0 \left(\overline{h}^{\beta_1} - \underline{h}^{\beta_1}\right)}{\overline{h}^{\beta_1 + 1} \alpha_1^2 + \overline{h}^{\beta_1} \alpha_0 \alpha_1 + 2\overline{h} \left(\overline{h} \alpha_1 + \alpha_0\right)^2} - \frac{\left(-\rho_2 (1 + \delta) \underline{h}^3 - \rho_1 (1 + \delta) \underline{h}^2 - \rho_0 (1 + \delta) \underline{h} + \overline{h} \left(\overline{h}^2 \rho_2 + \overline{h} \rho_1 + \rho_0\right)\right) \alpha_1 + \alpha_0 \left(-\rho_2 (1 + \delta) \underline{h}^2 - \rho_1 (1 + \delta) \underline{h} + \overline{h}^2 \rho_2 + \overline{h} \rho_1 - \delta \rho_0\right)}{3\left(\left(\overline{h}^2 \rho_2 + \frac{2}{3}\overline{h} \rho_1 + \frac{1}{3}\rho_0\right) \alpha_1 + \frac{2\left(\overline{h} \rho_2 + \frac{\rho_1}{2}\right) \alpha_0}{3}\right) \left(\overline{h} \alpha_1 + \alpha_0\right)}$$

Note: This Table summarizes the functional forms used for the estimation of β and θ .

APPENDIX B. MODEL APPENDIX:

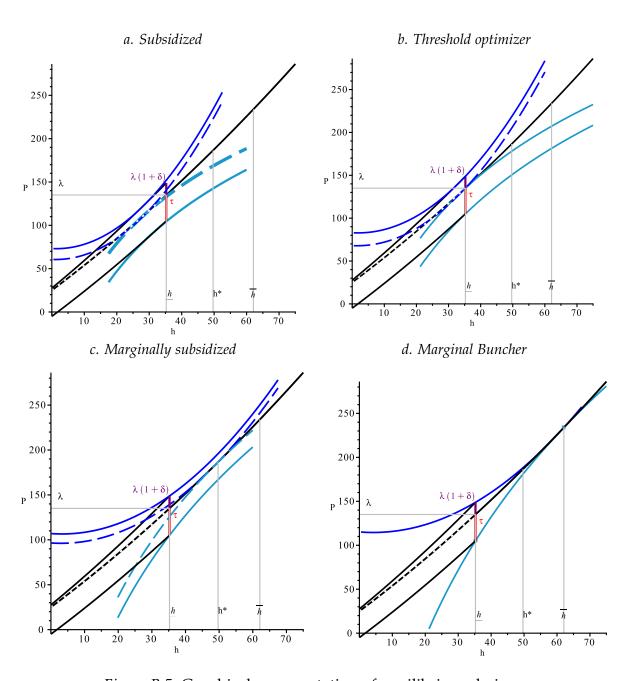


Figure B.5: Graphical representation of equilibrium choices

The offer function functional form is: $\varphi_j^S = \frac{\bar{\pi}_j \cdot h^2}{\beta_1} + A_j$. This results from a cost function $C\left(Q(h), A_j\right) = A_j * Q(h)$, and $Q(h) = \frac{\alpha}{h^2}$

APPENDIX C. COLOMBIAN HOUSING POLICY: ADDITIONAL DETAILS

MINIMUM WAGE AND INFLATION

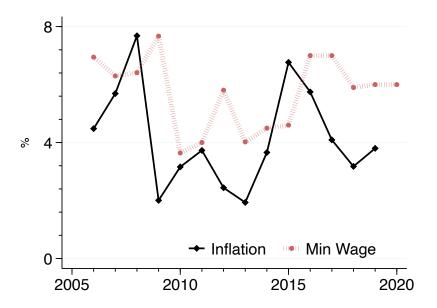
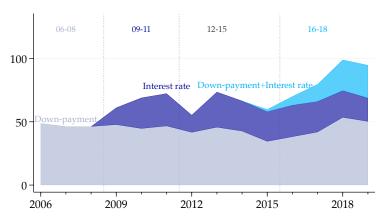


Figure C.6: Inflation and Minimum Wage Over-Time

Number of Subsidies over Time

Figure C.7: Total number of Subsidies Over Time



Source: Minvivienda, FRECH

DETAILS ON SUBSIDIES

downpayment subsidy. The down payment subsidy was introduced at the beginning of the nineties and is available to formal employees who contribute to the family compensation funds.³⁷ The **gray blue** area in Figure 1 shows the number of subsidies and total government expenditure from 2006 to 2019. The number of downpayment subsidies to formal employees was more or less stable during the study period, but the government spending increased in 2015 due to an increase in the size of the subsidy. Only formal households earning less than four times the minimum wage (mMW) are eligible for the subsidy, and the subsidy can only be used to buy a low-cost housing units.

Interest rate subsidy. In 2009, the government introduced a program to subsidize mortgage' interest rates. This program, called FRECH, started as a program to incentivize economic growth after the crisis, but it became a permanent policy. In contrast to the downpayment subsidies, interest rate subsidies were also available to households buying housing units above the $135 \times mMW$ threshold. However, the subsidy is larger if the households buy a low-cost housing unit, that is, the price is less than $135 \times mMW$. If a household receives the subsidy, the government pays the bank the corresponding amount during the first seven years of the loan. Three different schemes existed during the study period, but in all the schemes, there was a discontinuity in the subsidy at the cutoff defining low-cost housing. The **dark blue** area in Figure 1 shows the number of subsidies and total government expenditure from 2011 to 2019. The subsidies were more or less stable over time; around 20,000 households received this subsidy. This subsidy represents lower government expenditure and expenditure has slightly decreased overtime partly due to lower interest rates.

In contrast to the downpayment subsidy, interest rate subsidies were also available to households buying housing units above the $135 \times m$ -MW threshold. However, there is a notch at $135 \times m$ -MW. Figure C.8 shows the interest rate subsidies for the all the house price ranges.

³⁷The subsidy is called *Subsidio Familiar de Vivienda (SFV)*, was introduced in the Law 3 of 1991, and is administered by the family compensation funds. An employee is formal if she contributes to social insurance and the compensation family fund.

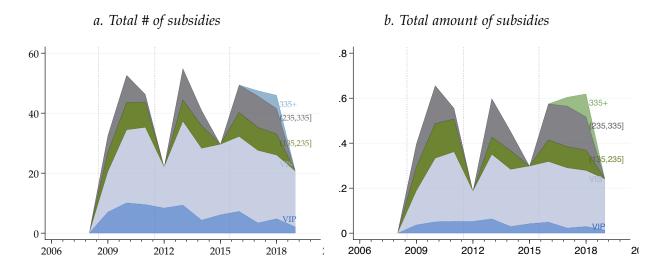


Figure C.8: FRECH subsidies for different price ranges

The subsidy expansion—Mi Casa Ya. In 2015, the government doubled the effort and introduced a new program Mi Casa Ya, (My House Now). Before this program was introduced, the down payment subsidy was only available to formal employees-contributing to the family compensation funds.³⁸ This program extended the coverage of the downpayment to noncontributing households. The households participating in this program get the downpayment subsidy and interest rate subsidy automatically with a single application. The **light blue** area in Figure 1 shows the number of subsidies and government expenditure, which is the sum of the downpayment and the total expenditures with the interest rate discount. The figure shows that the increase in the number of subsidies and government expenditure that started in 2015 was mainly driven by the introduction of this program and the increase in the down payment subsidy to formal employees.

Supply subsidy–value added tax (VAT) tax refund. A couple of years after the demand subsidies were introduced, to encourage developers to build low-cost housing, the government introduced a VAT tax refund. Developers get up to 4 percent of the sale price of each unit in the refund of taxes paid on construction materials. I include this subsidy in the analysis. Accounting for this subsidy introduces discontinous incentives on the supply side.

³⁸In theory informal household could get access to housing subsidies. However, *fonvivienda*, the institution in charge of these subsidies, assigned mostly to vulnerable populations. The vulnerable populations are displaced by armed conflict and affected by natural disasters. I include the equivalent plots for those subsidies in the Appendix C.

Other subsidies. The Colombian housing policy includes other subsidies excluded from the main analysis of this paper.³⁹ These are mainly subsidies to disadvantaged populations. These subsidies exist to follow a constitutional mandate to provide housing to people affected by forced displacement and environmental disasters. They are for cheaper housing units and households in extreme poverty. These subsidies can be used to buy priority low-cost housing, which is housing units with a market price of $70 \times mMW$ or less. The approach of using subsidies as an incentive to promote construction and purchase of housing units was mostly ineffective to provide this type of housing. As a result, in 2014, a program to build 100'000 free housing units was launched. The goal was to satisfy the constitutional mandate and provide housing to the disadvantaged population that was neglected by the previous policy approaches.A. G. Gilbert (2014) describes this program, 100 mil viviendas gratis, and evaluates its potential effectiveness. Camacho, Caputo, and Sanchez (2020) study the effect of this conditional transfer on the economic outcomes of the receiving households. The appendix Figure ?? shows the evolution of those subsidies.⁴⁰ The program of 100 thousand free housing units occurred between 2012-2015. There is a program for rural housing and subsidies for the military that I ignore in this paper.

 $^{^{39}}$ The main reason to exclude these subsidies is that they are concentrated on a lower price threshold and the market forces are less relevant. The bunching at the price threshold at $135 \times mMW$ is the most pronounced. This paper aims to explain what happens around that cutoff.

⁴⁰Other less relevant policies aim to promote low-cost housing. Most of the additional policies did not change during my study period. These policies include the following: (i) no income tax for low-cost housing unit credit (Law 546 of 1999). This is between 5 and 8 percent of the value of the credit. (ii) Long-term bonds to finance housing (Law 546 of 1999.). (iii) Tax exemption for leasing (2002). (iv) Protection for credit defaults (Access to the Fondo Nacional de Garandias). (v) New credit from the Colombian Development Bank to increase credit for new housing. Housing with a limit of 70 MW is a free housing unit. These subsidies apply to all housing units without targeting low-cost housing.

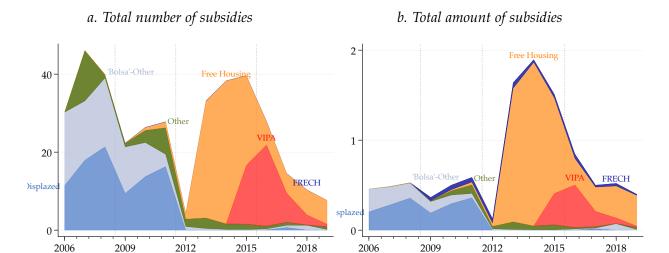


Figure C.9: Subsidies for the Vulnerable Population. Housing Units Priced Below 70 mMW.

Subsidy scheme. Figure C.10c shows the subsidy scheme for the interest rate. Three different schemes existed during my study period. Each scheme is represented in the figure by a different line. The x-axis is the monthly minimum wage and the y axis is the discount in the interest rate. If a household gets the subsidy, the government pays the bank the corresponding amount during the first seven years of the loan.

Targeting instruments. The authorities use two different tools to determine eligibility; the households' income and the total price of the housing unit. A unit can be subsidized only if the market price is below the low-cost housing threshold, 135 times the monthly minimum wage (m-MW). This arbitrary threshold is the same for all cities. Regarding income, only households earning below four times the monthly minimum wage can get the subsidy. Figure C.10a shows the subsidy scheme. Before 2015, the subsidy was decreasing on income, and the maximum possible subsidy was $22 \times \text{m-MW}$. In 2016 the generosity increased, the limit increase to $30 \times \text{m-MW}$ for individuals with income below $2 \times \text{m-MW}$ and $20 \times \text{m-MW}$ for individuals with income between $2 - 4 \times \text{m-MW}$. As the Figure 1 shows, the increase in the limit is

 $^{^{41}}$ This price limit is set by the government's National Development Plan. The price limit was the same from 1997 until 2019. With law 1467 of 2019, the price limit increased to 150 mMW for the five largest cities (including the metropolitan areas) and remained the same in the other cities. A low-cost housing unit is a house whose total price is below the threshold of 135 times the monthly minimum wage (around US\$40,000). There is an additional definition creating a similar discontinuity at a lower price cutoff. Housing units below $70 \times mMW$ (around US\$20,000) classify as priority low-cost housing. This cutoff defines eligibility for some subsidies for the extreme poor and those affected by forced displacement or natural disasters. This paper focuses mostly on the subsidies targeting the population buying low-cost housing units.

reflected in higher government expenditure. Figure C.10b the average subsidy during my study period. We can see that the average subsidies were about 20 percent before 2015 where the mean subsidy is about 26 percent.

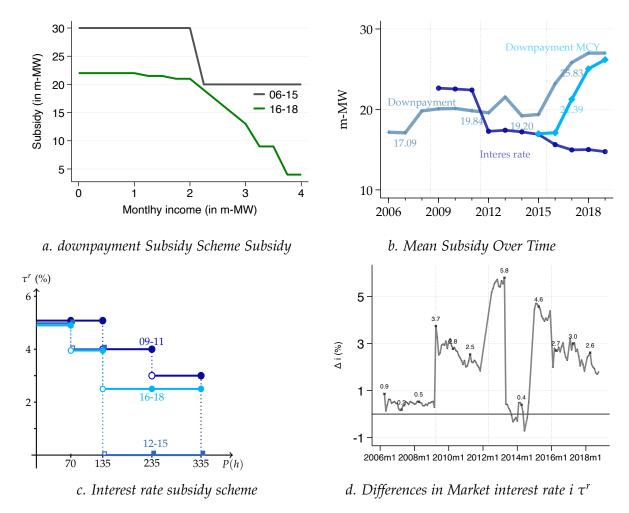


Figure C.10: Subsidy Scheme and Observed Mean Differences

Note: This figure shows the subsidy scheme and the evolution overtime of the subsidies for the interest rate and downpayment subsidy.

Mortgage terms:

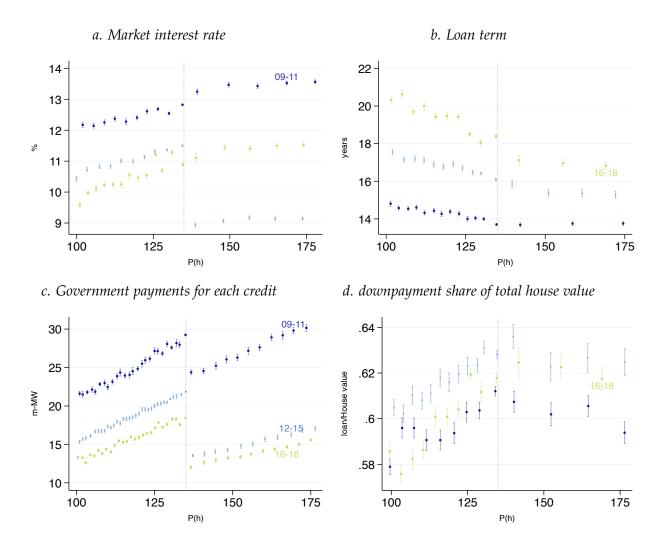


Figure C.11: Loan Terms by House Prices

Note: This figure shows the subsidy scheme and the evolution overtime of the subsidies for the interest rate and downpayment subsidy.

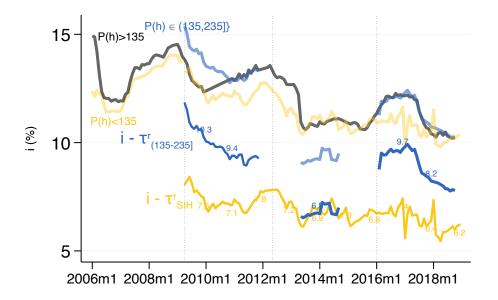


Figure C.12: Market interest rate i and subsidy τ^r Note: This figure shows the interest rates with and without the subsidy over time.

Observed differences in monthly payments. I use the administrative records of these subsidies and administrative records on all loans to check that subsidies are reflected in the lower interest rates paid by households. The administrative records for the subsidies contain relevant information about the mortgages. It has the market interest rate i, the loan L, the term n, the discount in the interest rate τ^r , and the house price P. The administrative records for all loans contain less detailed information, but I observe the interest rate of each loan and the average loan amount. I use the loans for housing, which have an indicator variable equal to 1 if the house is low-cost housing and 0 otherwise.

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