PHY324: Hall Effect Experiment

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1 Introduction

The purpose of this lab was to observe the Hall effect that occurred in a chromium and in a silver sample. The Hall effect occurs when a current-carrying conductor is placed in a transverse magnetic field. The Lorentz force on the moving charges produces a potential difference (voltage) inside the conductor, which is perpendicular to both the magnetic field and the electric current[1]. There exist then some relation between the voltage that is generated and the corresponding magnetic field that is present.

In this lab we explore this relation as follows. We set up our axis such that $\vec{B}$ points along the z-axis, $\vec{J}$ points in the x-axis and $\vec{E}$ points in the y-axis. The three measured quantities in this lab were: the magnitude of B, the magnitude of I and the generated Hall Voltage (V_H). Using these the following parameters were obtained: Hall coefficient (R_H), density of charge carriers (n), drift velocity (v_d), conductive mobility (μ), and electric conductivity (σ). The following equations were relevant for doing so and we will be listed here and referenced throughout the rest of the report.

$$V_H = -E_y w \tag{1}$$

Where w is the distance across the conductor over which the Hall Voltage is generated

$$R_H = \frac{E_y}{J_x B} \tag{2}$$

$$R_H = \frac{1}{ne} \tag{3}$$

$$\rho = R\frac{A}{l} = \frac{1}{\sigma} \tag{4}$$

Where R is the resistance, A is the cross sectional area of the conductor, and l is the length of the conductor

$$\mu = \sigma R_H \tag{5}$$

$$\mu = \frac{v_d}{E} \tag{6}$$

$$\frac{fringestep}{fringeseparation} \times 2945\mathring{A} = t \tag{7}$$

Where t is the thickness of the conductor, and fringe step and fringe separation are obtained from interferometric photographs.

2 Materials & Methods

- HP multimeter
- Banana wires (2)
- Controllable magnetic field source (coiled wires with controlled current)
- DC power supply with variable voltage
- Variable resistors
- Silver conductor probe
- Chromium conductor probe
- Gaussmeter
- Fluke high impedance null detector
- Interferometric images of the conductors
- Traveling microscope

1. Measure the resistance of the conductor by using a four-wire method as shown in Fig. 1 below. Record this value for later use. Note the multimeter will send a test current through one set of wires and measure the test resistance, while the voltage across will be measured by the second set of wires.

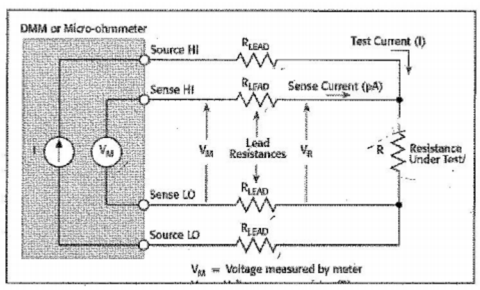

Figure 1: 4 wire method to measure resistance of conductor. The multimeter used is the HP multimeter[1]

2. Using the traveling microscope and the inferometric images of the conductor measure the thickness of the conductor using Eq. 7. Using the traveling microscope also measure the length of the conductor inside the probe.
3. Set up the listed materials according to Fig. 2 bellow.

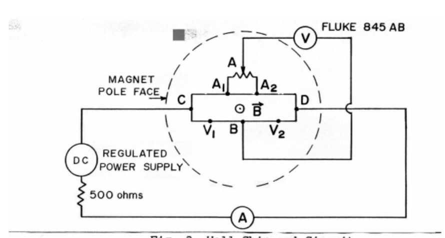

Figure 2: Set up for measuring Hall Voltage[1]

4. The above set up will be used to obtain R_H by measuring B, V_H, J_x and fitting this data to Eqn. 2. Record the values of I (covert this to J using the fact that $J = \frac{I}{A}$ where A is the cross sectional area) and the values of V_H at while varying the voltage from the DC supply and keeping B fixed. Before any measurements ensure the B field is switched off and that your voltage at this point is zero. This can be adjusted using the resistor knob. Repeat this zeroing process every time you change the voltage from the DC power supply.

5. Repeat the above step for three different values of B. These values should be measured using the Gaussmeter and recorded for later use.

6. Repeat steps 1-5 for both conductors. You should have 6 data sets all together (a set of data for each of the three B fields for each of the two conductors).

3 Results

3.1 Chromium

Following what is outlined in step 4 in the previous section the values for B, V_H, J_x were obtained and fitted to Eqn. 2 to obtain R_H. More specifically using the measured V_H, E_y was obtained following Eqn. 1 and a plot of E_y vs J_x (at fixed B) were generated and curve fitted to obtain R_H.

A subset of the data collected for trial 1 is shown below. Although not shown the data collected for trial 2 and 3 are similar and can be found in the `.txt` files attached to this report. Note that the text files do not have headings for the data as `numpy` does not allow mixing of floats and strings in the same array, but the data follows the same structure as the table below. Each trial is a set of 10 points and so can be separated in `.txt` files by taking every n to $n + 9$ rows for $n = 1, 2, 3$. Additionally the import commands in the Python script will indicate which variable each column represents, and which rows correspond to which trial.

Table 1			
Current (mA)	Uncertainty in Current (mA)	Hall Voltage (mV)	Uncertainty in Hall Voltage (mV)
5.95	±0.01	14	±1
8.11	±0.01	17	±1
11.69	±0.01	24	±1
...	...	...	...
20.50	±0.01	43	±1
22.18	±0.01	45	±1
24.25	±0.01	50	±1

Table 1: Hall Voltage and Current Measured for trial 1 of the chromium sample

The slope of the the line generated by plotting E_y vs J_x is known to be $R_H B$ by Eqn. 2 and so taking the slope of this line and dividing by B we could find R_H. These plots and the generated curve fit are shown below:

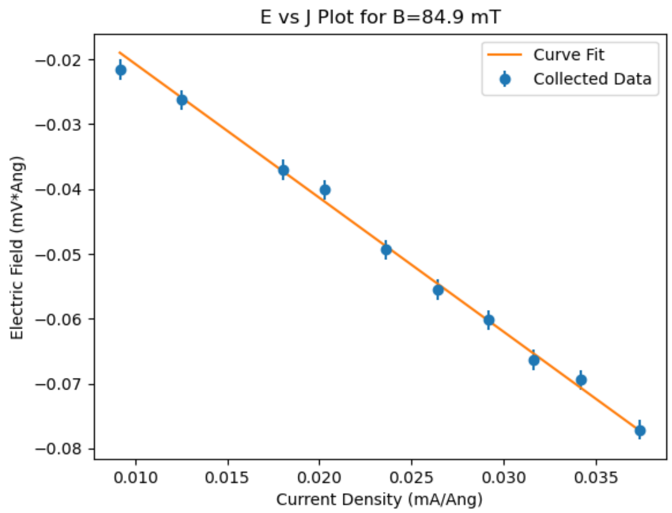

Figure 3: Trial 1 for Chromium

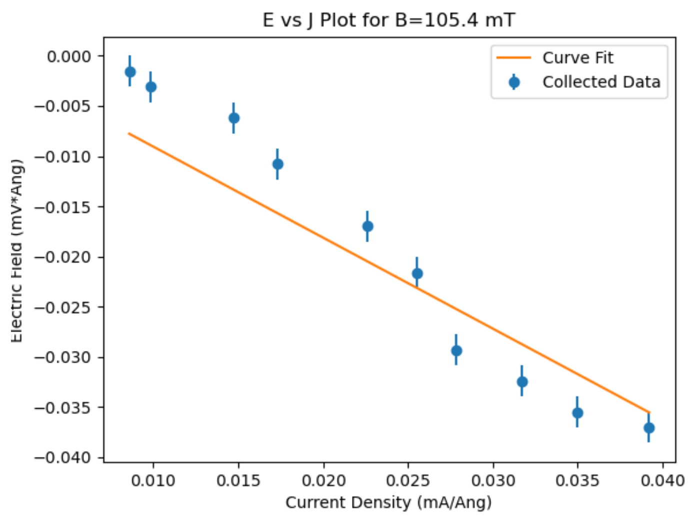

Figure 4: Trial 2 for Chromium

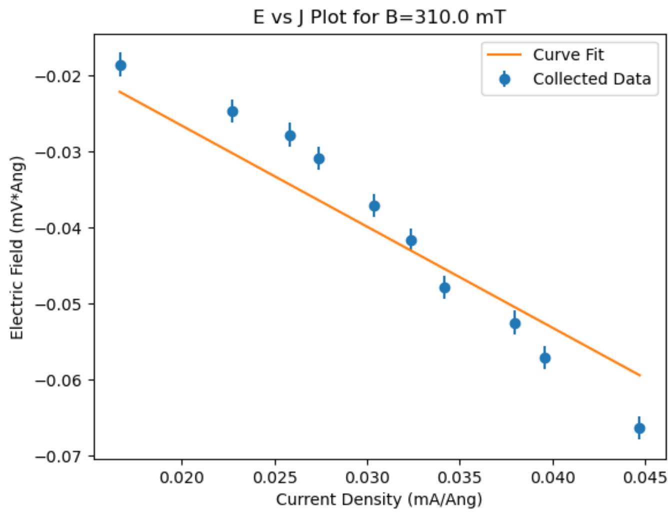

Figure 5: Trial 3 for Chromium

The χ^2_{red} of each trial and the returned parameter R_H from each curve fit is shown in the table below:

Table 2			
Trial	Parameter($\frac{mV}{mAmT}$)	Uncertainty in Parameter ($\frac{mV}{mAmT}$)	χ^2_{red}
1	$R_H = -2.43 \times 10^{-2}$	$\pm 2.2 \times 10^{-3}$	12940
2	$R_H = -8.59 \times 10^{-3}$	$\pm 1.8 \times 10^{-3}$	925
3	$R_H = -4.28 \times 10^{-3}$	$\pm 4.9 \times 10^{-4}$	4677

Table 2: Curve fit data

Note that in the above table the errors are not necessarily what is returned by `curve_fit`, but rather by how much we could vary the parameter and still have the line cross some of the points accounting for their error bars.

3.2 Silver

Using the exact same procedure as was outlined with the chromium sample the Hall effect was studied on a silver sample. A table analogous to table 1 but containing the data from the silver sample is shown below:

Table 3			
Current (mA)	Uncertainty in Current (mA)	Hall Voltage (μV)	Uncertainty in Hall Voltage (μV)
8.18	$\pm$0.01	5	$\pm$0.5
11.13	$\pm$0.01	8.5	$\pm$0.5
15.02	$\pm$0.01	10	$\pm$0.5
...	...	...	...
24.26	$\pm$0.01	18	$\pm$0.5
25.28	$\pm$0.01	20	$\pm$0.5
28.21	$\pm$0.01	23	$\pm$0.5

Table 3: Hall Voltage and Current Measured for trial 1 of the silver sample

The plots below show E_y vs J_x for each of the three trials at fixed B values:

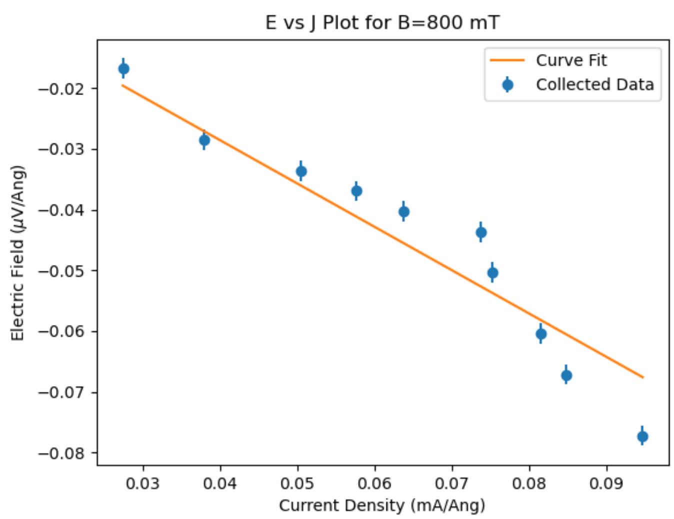

Figure 6: Trial 1 for Silver

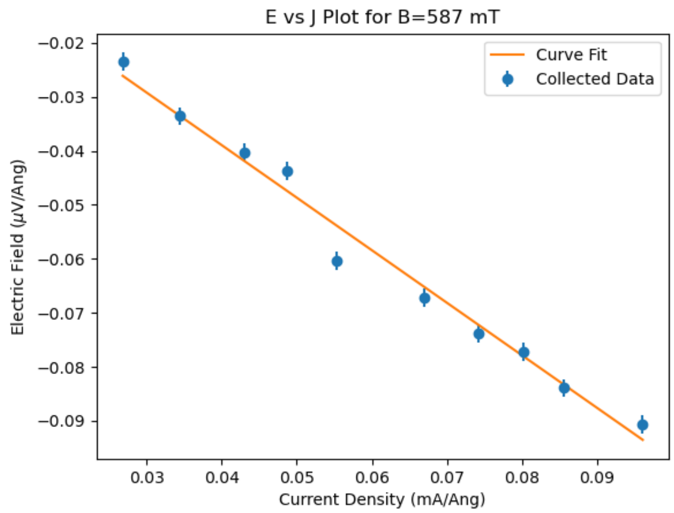

Figure 7: Trial 2 for Silver

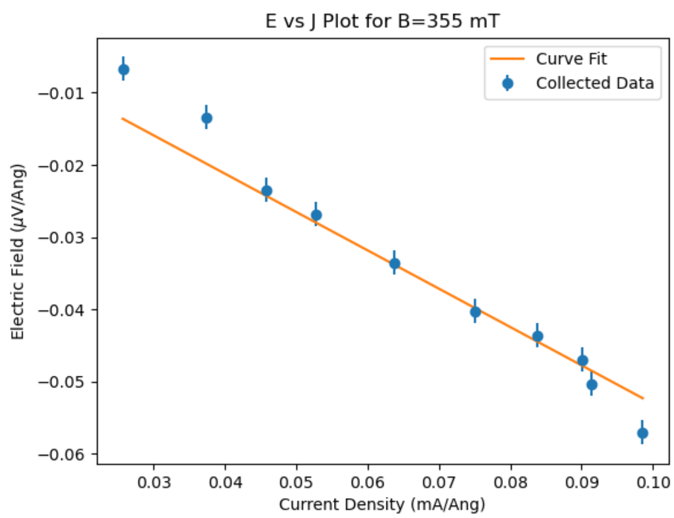

Figure 8: Trial 3 for Silver

The χ^2_{red} of each trial and the returned parameter R_H from each curve fit is shown in the table below:

Table 4			
Trial	Parameter($\frac{\mu V}{mAmT}$)	Uncertainty in Parameter ($\frac{\mu V}{mAmT}$)	χ^2_{red}
1	$R_H = -8.92 \times 10^{-4}$	$\pm 9.76 \times 10^{-5}$	2753
2	$R_H = -1.65 \times 10^{-3}$	$\pm 1.4 \times 10^{-4}$	925
3	$R_H = -1.49 \times 10^{-3}$	$\pm 2.1 \times 10^{-4}$	4677

Table 4: Curve fit data

Note that in the above table the errors are not necessarily what is returned by `curve_fit`, but rather by how much we could vary the parameter and still have the line cross some of the points accounting for their error bars.

4 Discussion

In this lab the voltage across a charge carrying conductor was observed when it was placed in a magnetic field. This transverse voltage is known as the Hall voltage and I will now briefly discuss why it occurs. The idea is that while the charge carriers, in this case electrons, are moving through the conductor the magnetic field will make them bunch up towards one side of the conductor. This is due to the Lorentz force that these charge carries would experience as they move through the conductor with a magnetic field present. This accumulation of charge on one side of the conductor creates a potential difference across the conductor which is the voltage that is observed (the Hall voltage).

4.1 Chromium

For each of the three trials we see that the χ^2_{red} of the fits were fairly large, meaning it was hard to fit the collected data to a linear model. This is highly due to the fact to how sensitive the whole experimental set up was, and especially how sensitive the fluke detector was. Any small change to the either orientation of the probe or the magnetic field strength would cause it to fluctuate and it would have to be re-zeroed. Additionally, it was so sensitive that turning on the other magnet in the room would have an effect on the measurement, despite it being far away enough where we would assume its effect to be negligible. This likely factors into why the `curve_fit` function had trouble fitting the data linearly. As for the uncertainties in the measured Hall voltage they were taken to be half of one dial measurement as is the custom for analog instruments. The device is very sensitive accurate which would lead us to believe the measurements should have a low uncertainty, but due the sensitive nature of the experiment these uncertainties are likely higher than expected, which also explains the magnitude of the χ^2_{red} values.

The parameters that were calculated using the recorded data for chromium, as outlined in the introduction of this report, are shown in the table below:

Table 5			
Trial	Density of Charge Carrier n $(\frac{mVs}{mT})$	Conductive Mobility μ $(\frac{1}{mT})$	Drift Velocity v_d $(\frac{mV}{mT})$
1	2.53×10^{17} $\pm 2.13 \times 10^{15}$	238 ± 2	11.97 ± 0.26
2	7.1×10^{17} $\pm 1.53 \times 10^{16}$	84 ± 5	1.63 ± 0.03
3	1.44×10^{18} $\pm 1.64 \times 10^{16}$	42 ± 3	1.69 ± 0.04

Table 5: Calculated parameters using results

Note that in the table above absolute values were taken since most parameters depend on R_H which would make them negative. This however would not make physical sense for things like density, so the absolute value is taken knowing that the negative simply tells us the charge of the charge carries (in this case negative since they are electrons). It is also worth noting that v_d will vary for each measurement of V_H (as it affects the value of E), but the table only shows the means of these values to get an overview of v_d. All v_d values can be seen in the provided Python script and we see a clear linear relation between E and v_d as is expected from Eqn. 6.

4.2 Silver

The results for the silver trials are similar to those of the chromium sample. We see a similar trend in the fitted data, likely for the same reason discussed above. A table analogous to table 5, but for the silver samples are shown below:

Table 6			
Trial	Density of Charge Carrier n ($\frac{\mu V s}{mT}$)	Conductive Mobility μ ($\frac{1}{mT}$)	Drift Velocity v_d ($\frac{\mu V}{mT}$)
1	6.91×10^{18} $\pm 7.5 \times 10^{16}$	270 ± 3	12.29 ± 0.33
2	3.72×10^{18} $\pm 3.1 \times 10^{16}$	502 ± 2	29.83 ± 0.72
3	4.13×10^{18} $\pm 5.9 \times 10^{16}$	453 ± 4	15.50 ± 0.63

Table 6: Calculated parameters using results

As mentioned below table 5 the same conditions apply to the values seen in table 6.

4.3 Explanation of Calculated Parameters

We can now clearly see that the conduction of the silver sample is much higher than that of the chromium sample. This is evident when just comparing their measured resistance (31.554Ω vs 2.217Ω), but is also verified by comparing their Hall coefficients. We can see from tables 2 and 4 that the chromium had Hall coefficient's that were around three-four orders of magnitude bigger (when accounting for the units of voltage). This is to be expected because electrons moving through a conductor with a higher resistance will have a longer exposure time to the external magnetic fields (the electron is forced across it slower), and so we can expect to see a bigger magnitude for the Hall Voltage as we will have a larger build up of charge. We know from combining Eqn. 1 and Eqn. 2 that R_H is directly proportional to V_H, and so it makes sense that the metal with the higher resistance has the higher Hall coefficient. We can use this to also explain why the v_d values of the chromium sample are lower than those of the silver sample (see last column of tables 5 and 6). Again, this is to be expected as the electrons travel slower in the presence of higher resistance. This also explains the difference in the conductive mobility values (μ in the second column of tables 5 and 6), as this is essentially a measure of how freely/fast the electrons can flow through the material. Finally, we see again that the density of the charge carries is also higher for chromium than for silver which makes sense as already discussed we expect the electrons to bunch of more in a metal with a higher resistance than in one with a lower resistance. While the difference in the magnitudes between all these discussed parameters may not be perfect, the overall trend seen between the chromium and silver sample make physical sense.

5 Conclusion

In conclusion the Hall effect was observed in a sample of chromium and silver metal. The three following quantities were measured in this lab: the magnitude of B, the magnitude of I and the generated Hall Voltage (V_H). Using these the following parameters were obtained for both metal samples: Hall coefficient (R_H), density of charge carriers (n), drift velocity (v_d), conductive mobility (μ), and electric conductivity (σ). It was found that metals with a higher resistance have a more pronounced Hall effect, in the sense that they produce Hall voltages with bigger magnitudes.

References

[1] Ruxandra Serbanescu. *The Hall Effect.*