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# Electing the Pope

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Research Centre for Economic and Regional Studies
HUNGARIAN ACADEMY OF SCIENCES

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LÁSZLÓ Á. KÓCZY - BALÁZS SZIKLAI

INSTITUTE OF ECONOMICS, RESEARCH CENTRE FOR ECONOMIC AND REGIONAL STUDIES,
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Electing the Pope

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# Electing the Pope

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Abstract

Few elections attract so much attention as the Papal Conclave that elects the religious leader of over a billion Catholics worldwide. The Conclave is an interesting case of qualified majority voting with many participants and no formal voting blocks. Each cardinal is a well-known public gure with publicly available personal data and well-known positions on public matters. This provides excellent grounds for a study of spatial voting: In this brief note we study voting in the Papal Conclave after the resignation of Benedict XVI. We describe the method of the election and based on a simple estimation of certain factors that seem to influence the electors' preferences we calculate the power of each cardinal in the conclave as the Shapley-Shubik index of the corresponding voting game over a convex geometry.

Keywords: Papal Conclave, game over convexge-ometry, Shapley-Shubik index

JEL classification: C71, D72

# A pápa választása

Kóczy Á. László - Sziklai Balázs


## Összefoglaló

Kevés választás kap akkora figyelmet, mint a pápaválasztó konklávé, mely világszerte több mint egy milliárd katolikus vallási vezetőjét választja meg. A konklávé érdekes példája a sok résztvevős, formális szavazói tömbök nélküli minősített többségi szavazásnak. Ráadásul minden bíboros jól ismert közéleti személyiség, nyilvános személyes adatokkal és közismert állásponttal egyházi, társadalmi és morális kérdésekben. Mindez kitűnő alap a konklávé térbeli szavazásként való vizsgálatához. Ebben a rövid tanulmányban a XVI. Benedek pápa lemondása okán összehívott konklávét vizsgáljuk. Leírjuk a szavazás menetét és a szavazó bíborosok preferenciáit vélhetően befolyásoló faktorok egyszerű becslése alapján meghatározzuk az egyes bíborosok szavazási befolyását, mint a – megfelelően definiált konvex geometria felett értelmezett szavazási játékban elért – Shapley-Shubik indexüket.



Tárgyszavak: Pápaválasztó konklávé, egy konvex geometria felett értelmezett játék, Shapley–Shubik-index

JEL kódok: C71, D72

# Electing the Pope*

László Á. Kóczy[†] Balázs Sziklai[‡]

**Abstract**

Few elections attract so much attention as the Papal Conclave that elects the religious leader of over a billion Catholics worldwide. The Conclave is an interesting case of qualified majority voting with many participants and no formal voting blocks. Each cardinal is a well-known public figure with publicly available personal data and well-known positions on public matters. This provides excellent grounds for a study of spatial voting: In this brief note we study voting in the Papal Conclave after the resignation of Benedict XVI. We describe the method of the election and based on a simple estimation of certain factors that seem to influence the electors' preferences we calculate the power of each cardinal in the conclave as the Shapley-Shubik index of the corresponding voting game over a convex geometry.

**Keywords and phrases**: Papal Conclave, game over convex geometry, Shapley-Shubik index.

**JEL codes**: C71, D72

## 1 Introduction

The election of Pope Francis has attracted much attention among Catholics and non-Catholics alike. As the religious leader of some 1.2 billion Catholics worldwide, the Pope has an enormous influence on world politics – far more than what being the head of the smallest ministate would imply. The media hype is enhanced by the complex voting procedure entwined with old traditions and secretive elements. We are, however interested in the Papal Conclave – the electing body – for another reason. The voting power literature studies examples that are either instances of qualified weighted

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majority voting, such as that in the European Union (Kóczy, 2011) where each of the voters have a give weight or examples of voting in a parliament or a similar voting body. In the latter case the voters are all individuals with equal weights, but they form parties or other voting blocks. Party discipline tends to be high, so that party members almost always vote the same. If this is always true we might as well consider a weighted voting where the party leader chooses a position and all party members follow. In practice there may be absent, or rebel voters making the analysis somewhat more complex (Kóczy and Pintér, 2011). In the Papal Conclave we only have a set of cardinals with certain policy preferences, possibly with suspected voting blocks, but no formal parties. Especially not with parties that could employ sanctions on rebel voters. In sum: each cardinal chooses his position independently of others.

The new pope is elected by the Papal Conclave consisting of the cardinals under 80 using a two-thirds majority. In the election of Pope Francis this meant 117 cardinals – 115 participating. The voting rules are very elaborate where, until the required majority is reached, several voting rounds might take place. With this process the positions of the participating cardinals might and do change. In our simple analysis, however we ignore the changes of preferences, the strategic games that may be played. We seek to determine the more influential cardinals in the voting process based on their positions. Our assumption is that the elected Pope will then be similar to these powerful players, they themselves are likely candidates, although some would likely use their influence to elect someone else.

Since Shapley and Shubik (1954) adopted the Shapley value to measure a priori voting power, we use simple games to model voting situations. In these games there are no actual payments, *power* itself is the payoff of the game. A coalition is either *winning* and has a payoff of 1, or *losing* getting 0. We are interested in swing voters who can turn a losing coalition into a winning one. Due to the symmetry of the voting situation, the swing is always the 77th supporting cardinal; in the theory of the Banzhaf power index (Banzhaf, 1965), where we look for *critical* players, the same player is critical for the coalition.

If we assume that any combination of 77 cardinals is equally likely, we are very remote from reality. There are very strong interest groups based on location (Italy, America, Europe) and progressiveness. We assume that a candidate that is acceptable to both a liberal and a conservative cardinal will also be acceptable to a moderate cardinal. These, so-called games on convex geometries (Edelman, 1997) presume that only convex coalitions may form. Both the Shapley-Shubik index (Bilbao and Edelman, 2000) and the Banzhaf index (Bilbao, Jiménez, and López, 1998) has been extended to such games.

In the following we introduce our notation, explain the geometry of the game and present our *papabili*.

## 2 Power indices

### 2.1 Simple games

First we introduce the usual terminology and notation. Let $N = \{1, \ldots, n\}$ be the set of the players. $v : 2^N \to \mathbb{R}$, where $v(\emptyset) = 0$, is a transferable utility (TU) cooperative game (henceforth *game*) with player set $N$. For any player $i$ and any coalition $S$: $v'_i(S) = v(S \cup \{i\}) - v(S)$, that is $v'_i(S)$ is player $i$'s marginal contribution to coalition $S$ in game $v$. Let $v'_i$ stand for player $i$'s marginal contribution function in game $v$. Player $i$ is a null-player in game $v$ if $v'_i = 0$. Finally, $|A|$ is for the cardinality of set $A$.

A voting situation or voting game is a pair $(N, \mathcal{W})$, where the players are the voters and $\mathcal{W}$ denotes the set of *winning coalitions*. We consider *simple voting games* where

1. $\emptyset \notin \mathcal{W}$ and $N \in \mathcal{W}$,
2. if $C \subseteq D$ and $C \in \mathcal{W}$, then $D \in \mathcal{W}$,
3. if $S \in \mathcal{W}$ and $T \in \mathcal{W}$, then $S \cap T \neq \emptyset$.

Condition 3 requires the game to be *proper*, that is, a motion and its opposite cannot be approved simultaneously.

Let $\bar{\Gamma}_N$ denote the set of *proper simple voting games over the player set* $N$.

We can also write a simple voting game in the form of a transferable utility game $v$, where $v(S) = 1$ if $S \in \mathcal{W}$ and 0 otherwise. The term "simple" comes from having coalitions with payoffs 0 or 1 only.

We study the players' ability to change decisions. If, by joining a losing coalition, a player can turn it winning, we call the player swing. *Voting power* then refers to this ability to change decisions.

Given a game $v$ of $\bar{\Gamma}_N$, an *a priori measure of voting power* or *power measure* $\kappa : \bar{\Gamma}_N \to \mathbb{R}^N_+$ assigns to each player $i$ a non-negative real number $\kappa_i(v)$, its *power* in game $v$; if for any game $v$ of $\bar{\Gamma}_N$: $\sum_{i \in N} \kappa_i(v) = 1$, then it is also a power *index*.

In the following we explain some of the well-known indices. The *Shapley-Shubik index* $\phi$ (Shapley and Shubik, 1954) applies the Shapley value (Shapley, 1953) to simple games: Voters arrive in a random order; if and when a coalition turns winning the full credit is given to the last arriving, the *pivotal* player. A player's power is given as the proportion of orderings where it is pivotal, formally for any simple voting game $v$ player $i$'s Shapley-Shubik index in game $v$ is as follows

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} v'_i(S) \ ,$$

where $s = |S|$.

The *Banzhaf measure* $\psi$ (Penrose, 1946; Banzhaf, 1965) is the vector of probabilities that a party is *critical* for a coalition, that is, the probabilities that it can turn winning coalitions into losing ones. Formally, for any simple voting game $v$ player $i$'s Banzhaf-measure in game $v$ is as follows

$$\psi_i(v) = \frac{\eta_i(\mathcal{W})}{2^{n-1}} ,$$

where $\eta_i(\mathcal{W})$ is the number of coalitions in $\mathcal{W}$ in which $i$ is critical. When normalized to 1, we get the *Banzhaf index* $\beta$ (Coleman, 1971). Formally, for any simple voting game $v$ player $i$'s Banzhaf index in game $v$ is as follows

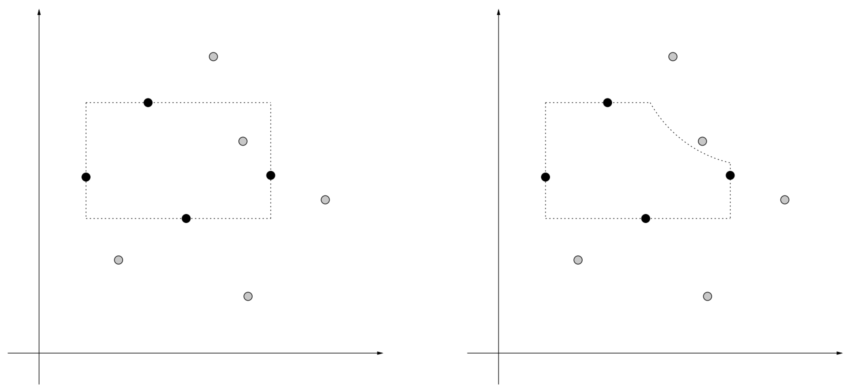
$$\beta_i(v) = \frac{\eta_i(\mathcal{W})}{\sum\limits_{j \in N} \eta_j(\mathcal{W})} .$$

## 2.2 Games over convex geometries

The set $\mathcal{L} \subseteq 2^N$ of sets of coalitions is a convex geometry (Edelman and Jamison, 1985) if two properties are satisfied:

1. $\varnothing \in \mathcal{L}$ and $\mathcal{L}$ is closed under intersection.
2. if $S \in \mathcal{L}$ and $S \neq N$ then there exists $i \in N \setminus S$ such that $S \cup i \in \mathcal{L}$.

The set $\mathcal{L}$ collects convex coalitions. We call $i \in S$ for $S \in \mathcal{L}$ an extremal point if $S \setminus \{i\} \in \mathcal{L}$. We denote the extremal points of $S$ by $\delta(S)$. For games over convex geometries we can also define winning and losing coalitions.



(a) Convex rectangle (b) Concave rectangle

Figure 1: Examples of coalitions

We say that $i$ is a convex swing in coalition $S$ if $S \in \mathcal{L}, i \in \delta(S), v(S) = 1$, but $v(S \setminus \{i\}) = 0$. Note that if $i \in \delta(S)$, $S \setminus \{i\} \in \mathcal{L}$ this property only tells about the transition between feasible coalitions. If these extremal players continue to find the coalition acceptable, a situation where internal players would like to leave the coalition is not conceivable. Therefore no power can be derived from turning a convex coalition into a non-feasible one.

# 3 The Papal Conclave

## 3.1 A brief history

Despite the importance of the bishop of Rome in the Catholic Church, relatively little is known about the first popes. The Bible itself provides no rules "how bishops were selected, but the process of choosing the seven deacons detailed in Acts (6:1–6) influenced the way they were." (Baumgartner, 2003, p 4) Peter probably appointed Linus as his successor, while the the first account of an election is that of Fabian in 236, although records for the time before 400 are considered unreliable. Over the centuries the election of the new Pope has changed a lot. Until the late renaissance times Papal elections have often been bitter, even violent conflicts between strong candidates coming from strong Roman families, between parties supporting Rome versus the emperor (first the Byzantine then the Holy Roman emperor) or candidates loyal to the king of France or to Italy. Initially the voting game was not even proper: multiple popes could be elected simultaneously creating a temporary schism in the Church. To eliminate this awful situation the Third Lateran Council (1179) prescribed a two-third majority of all voting cardinals (Baumgartner, 2003, p 32). The idea was that no faction can elect its "own" Pope, who is, on the other hand resented by the opposition. The two third majority implied that no 'strong' candidates could be elected. In practice, however, making it to the election was difficult and often only a very small portion of cardinals were present. In 1191 only 8 cardinals voted of the 31 cardinals then alive. Such elections undermined the legitimacy of the pope and cardinals lived increasingly in Rome to be available for elections.

In 1241 the cardinals have been locked up in a building and the word conclave, coming from *cum clave* ("with keys") can since then be used for the electing body. While the number of cardinals have been kept low, for a long time it was limited to 24, reaching a compromise was not always trivial. According to the records the decision to 40 months to be reached in 1268-72 (Baumgartner, 2003, p 37). To avoid similar situations the elected Gregory X issued the bull *Ubi Periculum* that specified some the rules of the conclave in great detail. In 1294 the hermite Pietro de Murronne was elected to become Celestine V. While becoming the pope has been a dream for many, he considered it a burden and wanted to abdicate. Before tha actual abdication he announced to apply the conclave's rules "however a vacancy might occur in the papacy". (Baumgartner, 2003, p 45). The vote after his abdication is the first instance when the actual voting records have survived. The voting has not been anonymous and a two-third majority plus one vote was required "to ensure that a cardinal's vote for himself did not provide the margin of victory" (Baumgartner, 2003, p 46). For a long time yet it was common that cardinals name more than one candidate on their ballots, so that the voting was actually approval voting with a qualified majority.

Since then the rules have changed little. John-Paul II (1996) introduced the clause that after 33 unsuccessful rounds simple majority suffices to elect the pople. This rule has been criticised that it does not force an agreement upon the cardinals making a simple majority enough to elect a pope if the electors hold out for long enough. Benedict XVI (2007) essentially reverted to the original requirement of a two-third majority.

## 3.2 The cardinals

While the election is at the influence of the Holy Spirit, those are ultimately the cardinals who cast their votes and in the past a number of objective factors have clearly been good predictors of the final outcome. Magister (2013) lists four characteristics of the cardinals – besides their name: nationality, age, who made them cardinal and whether they belong (or have belonged) to the Curia. It would be beyond the scope of this paper to review the discussions why these might play a role, but a few points may be noted: The bishop of Rome has been more often than not Italian; whether a pope is Italian or not, is a central question. Currently Latin America is the continent with the largest number of Catholics and despite having only 19 cardinals together with cardinals of North America and some European Latin countries have a strong voting block. Ultimately the question is wether the pope is European or non-European. After the long reign of John Paul II many favoured an older pope; now the feelings are at least ambiguous. Cardinals at the curia are well known and well connected in the Church but lack pastoral experience and may be more affected by the so-called Vatileaks scandals. This trade-off might play an important role, too. We consider two characteristics: physical distance from Rome and a measure of conservatism. The first is a one-dimensional proxy for nationality and the associated "games", the second has become important since the Church faces serious challenges from the rapidly changing society.

We measure distance by the distance, denoted $d_i$ of the birthplace of the cardinal (obtained from the cardinals' Wikipedia pages) from Rome. For conservatism we use a very simple Google-metric: of the number of hits for the searches *Is "X" conservative?* and *Is "X" liberal?* we take the percentage of the first. While this metric, denoted $c_i$ for cardinal $i$ is admittedly imperfect when compared with comments by Vatican insiders, cardinals known for their conservatism tend to score high on our chart, while known liberals usually score low. Note that a liberal cardinal may still be seen very conservative by the society. These two scores uniquely determine the location of a cardinal on the policy space.

### 3.3 The majority

Given the distribution of cardinals over this two-dimensional space we look for coalitions that (i) are convex and (ii) have the required majority. Observe that coalitions with supermajority, that is with more members than the required quota are not interesting for us, since here none of the cardinals are critical, no-one can push his own policy. In coalitions with the (exact) majority each cardinal on the borderline of the coalition is swing, that is, his presence is essential for the coalition. What are the convex coalitions in this setting? A cardinal $c$ with position $(c_x, c_y)$ belongs to a coalition $S$ if and only if there exist four (not necessarily distinct) cardinals $e, f, g, h$ in $S$, such that $e_x \leq c_x \leq g_x$ and $f_y \leq c_y \leq h_y$.

In order to find the influence of the individual cardinals we must first identify all such coalitions. These coalitions are really rectangles spanned by 2 or more positions in the policy space. For each rectangle we must identify the cardinals on the borderline – each of these cardinals get a point for being swing/critical. The value of each cardinal is given as the total points collected, normalized to 1. Note that due to the fact that cardinals only differ in their positions, the game is symmetric. In particular, the probability of forming any of the minimal winning coalitions is the same. In such a game the Shapley-Shubik index coincides with the Banzhaf index we calculated.

### 3.4 The algorithm for identifying the minimal winning coalitions

The cardinals, denoted by $c^1, c^2, \ldots, c^{115}$ are treated as points in the plane where the horizontal axis $(x)$ represents the distance from Rome and the vertical axis $(y)$ represents the conservatism of the cardinals.

In the following we calculate the influence of each cardinal by finding all rectangles corresponding to minimal winning coalitions and checking how frequently a particular cardinal is placed on the outline of such a rectangles. First we provide the general outline of our method and then provide a more efficient algorithm

1. Select the left, right, bottom, and top side of our rectangle: Chose an arbitrary not necessarily different quadruple of cardinals $S = \{l, r, b, t\} \subset$

$N$, such that

$$\begin{aligned} l_x &= \min_{k \in S} k_x \\ r_x &= \max_{k \in S} k_x \\ b_y &= \min_{k \in S} k_y \\ t_y &= \max_{k \in S} k_y \end{aligned}$$

2. Check the number of cardinals with positions in the closed rectangle spanned by $S$. If it is exactly 77, $S$ spans a minimal winning coalition.

3. Check that the rectangle has not been considered before.

4. Give points to cardinals on the borderline.

5. Rank according to the points given.

Since the points need not be disjoint, there are more than $\binom{|N|}{4}$ or in our case about 7 million rectangles to look at.

A more systematic search is more economical. The detailed algorithm we have used can be found in Appendix A.

# 4 Results

In Table 1 we present a list of the cardinals together with their coordinates and the number of minimal winning coalitions where they are critical. Distance is given in kilometers, while conservatism in basis points.

Table 1: Cardinals ranked by the number of minimal winning coalitions where they are critical

| rank | cardinal | $c_x$ (km) | $c_y$ (bp) | score |
|---|---|---|---|---|
| 1 | George Pell | 15902 | 4477 | 599 |
| 2 | Francisco Javier Errázuriz Ossa | 11921 | 4617 | 572 |
| 3 | Jorge Bergoglio | 11162 | 4507 | 546 |
| 4 | Leonardo Sandri | 11162 | 4967 | 536 |
| 5 | Théodore-Adrien Sarr | 4181 | 185 | 517 |
| 6 | Juan Luis Cipriani Thorne | 10873 | 4247 | 497 |
| 7 | Telesphore Placidus Toppo | 6039 | 7264 | 481 |
| 8 | Agostino Vallini | 34 | 4992 | 461 |
| 8 | Jean-Pierre Ricard | 603 | 6396 | 461 |
| 10 | James Michael Harvey | 7686 | 734 | 459 |
| 11 | Francisco Robles Ortega | 10563 | 3134 | 455 |
| 12 | Dominik Duka | 961 | 6394 | 444 |

Table 1: Cardinals ranked by the number of minimal winning coalitions where they are critical

| rank | cardinal | $c_x$ (km) | $c_y$ (bp) | score |
|---|---|---|---|---|
| 13 | Antonio María Rouco Varela | 1653 | 817 | 440 |
| 14 | Baselios Cleemis | 7183 | 6391 | 426 |
| 15 | Odilo Scherer | 10408 | 4399 | 424 |
| 16 | Luis Antonio Tagle | 10403 | 2568 | 420 |
| 17 | Béchara Boutros Raï | 2214 | 877 | 419 |
| 17 | Norberto Rivera Carrera | 10384 | 2973 | 419 |
| 19 | Rubén Salazar Gómez | 9392 | 113 | 410 |
| 20 | André Vingt-Trois | 1107 | 925 | 397 |
| 21 | Juan Sandoval Íñiguez | 10376 | 1560 | 391 |
| 22 | John Tong Hon | 9279 | 6064 | 387 |
| 23 | Antonio Maria Vegliò | 213 | 4649 | 385 |
| 23 | Jorge Urosa | 8365 | 1491 | 385 |
| 25 | Lluís Martínez Sistach | 859 | 1190 | 383 |
| 26 | Velasio de Paolis | 83 | 5183 | 379 |
| 26 | Crescenzio Sepe | 176 | 3364 | 379 |
| 28 | Roger Mahony | 10201 | 5863 | 377 |
| 29 | Francesco Monterisi | 323 | 5013 | 375 |
| 30 | Raffaele Farina | 220 | 4609 | 366 |
| 31 | Giuseppe Betori | 119 | 2871 | 365 |
| 32 | John Onaiyekan | 3843 | 5956 | 362 |
| 33 | Angelo Amato | 351 | 2524 | 355 |
| 34 | Raúl Eduardo Vela Chiriboga | 10234 | 5526 | 352 |
| 35 | Josip Bozanić | 414 | 2616 | 347 |
| 36 | Jean-Louis Tauran | 1105 | 5643 | 341 |
| 37 | Justin Francis Rigali | 10201 | 5295 | 337 |
| 38 | Carlo Caffarra | 390 | 3734 | 332 |
| 38 | João Braz de Aviz | 9911 | 3119 | 332 |
| 40 | George Alencherry | 6857 | 5589 | 325 |
| 41 | Seán Patrick O'Malley | 8984 | 5300 | 311 |
| 42 | Julio Terrazas Sandoval | 10325 | 216 | 308 |
| 43 | Raymundo Damasceno Assis | 9062 | 2676 | 298 |
| 44 | Carlos Amigo Vallejo | 1449 | 2273 | 297 |
| 45 | Mauro Piacenza | 401 | 4179 | 292 |
| 46 | Manuel Monteiro de Castro | 1755 | 2327 | 279 |
| 47 | William Levada | 10224 | 4244 | 276 |
| 48 | Rainer Woelki | 1092 | 5499 | 271 |
| 49 | Domenico Calcagno | 432 | 3643 | 267 |
| 50 | Nicolás de Jesús López Rodríguez | 10836 | 6034 | 266 |
| 51 | Ennio Antonelli | 99 | 5542 | 262 |
| 52 | José Policarpo | 1834 | 2447 | 261 |

Table 1: Cardinals ranked by the number of minimal winning coalitions where they are critical

| rank | cardinal | $c_x$ (km) | $c_y$ (bp) | score |
|---|---|---|---|---|
| 53 | Angelo Bagnasco | 423 | 4488 | 258 |
| 54 | Stanisław Dziwisz | 1029 | 5378 | 253 |
| 55 | Cláudio Hummes | 10317 | 7841 | 245 |
| 56 | Severino Poletto | 424 | 3931 | 243 |
| 57 | Angelo Comastri | 108 | 1206 | 239 |
| 58 | Ivan Dias | 6183 | 3299 | 237 |
| 59 | Paolo Sardi | 453 | 4891 | 235 |
| 60 | Timothy M. Dolan | 8148 | 5217 | 230 |
| 61 | Giovanni Lajolo | 503 | 5010 | 228 |
| 62 | Óscar Andrés Rodríguez Maradiaga | 9758 | 4327 | 205 |
| 63 | Fernando Filoni | 463 | 4759 | 200 |
| 63 | John Njue | 9509 | 4911 | 200 |
| 65 | Dionigi Tettamanzi | 498 | 4979 | 195 |
| 66 | Edwin Frederick O'Brien | 6879 | 5247 | 191 |
| 67 | Pham Minh Man | 9557 | 4600 | 188 |
| 68 | Francesco Coccopalmerio | 466 | 3855 | 184 |
| 69 | Giovanni Battista Re | 487 | 4631 | 168 |
| 70 | Vinko Puljić | 516 | 3387 | 163 |
| 71 | Gianfranco Ravasi | 490 | 3961 | 153 |
| 72 | Antonios Naguib | 2258 | 5087 | 143 |
| 73 | Seán Brady | 1978 | 5054 | 129 |
| 74 | Franc Rodé | 503 | 4388 | 126 |
| 74 | Giuseppe Versaldi | 503 | 4657 | 126 |
| 76 | Geraldo Majella Agnelo | 9119 | 3995 | 124 |
| 77 | Antonio Cañizares Llovera | 1182 | 3195 | 116 |
| 78 | Attilio Nicora | 527 | 3602 | 113 |
| 79 | Oswald Gracias | 6183 | 5309 | 104 |
| 80 | Angelo Scola | 506 | 4424 | 91 |
| 81 | Philippe Barbarin | 1093 | 5004 | 90 |
| 82 | Jaime Lucas Ortega y Alamino | 8653 | 4770 | 88 |
| 83 | Polycarp Pengo | 5937 | 5000 | 78 |
| 84 | Santos Abril y Castelló | 1144 | 3401 | 73 |
| 85 | Paolo Romeo | 526 | 4277 | 69 |
| 86 | Reinhard Marx | 1126 | 3447 | 64 |
| 86 | Wilfrid Napier | 8215 | 4204 | 64 |
| 88 | Daniel DiNardo | 7382 | 4941 | 55 |
| 89 | Giuseppe Bertello | 532 | 3851 | 51 |
| 90 | Kazimierz Nycz | 1140 | 3629 | 46 |
| 91 | Marc Ouellet | 7911 | 4605 | 45 |
| 92 | Kurt Koch | 665 | 3779 | 42 |

Table 1: Cardinals ranked by the number of minimal winning coalitions where they are critical

| rank | cardinal | $c_x$ (km) | $c_y$ (bp) | score |
|---|---|---|---|---|
| 92 | Donald Wuerl | 7334 | 4927 | 42 |
| 92 | Raymond Leo Burke | 7822 | 4813 | 42 |
| 95 | Tarcisio Bertone | 538 | 4696 | 39 |
| 96 | Stanisław Ryłko | 1033 | 3675 | 31 |
| 97 | Francis George | 7749 | 4579 | 28 |
| 98 | Karl Lehmann | 736 | 3836 | 24 |
| 99 | Paul Josef Cordes | 1077 | 4852 | 15 |
| 99 | Malcolm Ranjith | 7643 | 4509 | 15 |
| 101 | Thomas Christopher Collins | 7153 | 3790 | 14 |
| 102 | Walter Kasper | 778 | 4599 | 13 |
| 103 | Wim Eijk | 1293 | 4842 | 10 |
| 104 | Péter Erdő | 811 | 4008 | 9 |
| 104 | Audrys Bačkis | 1671 | 3826 | 9 |
| 106 | Christoph Schönborn | 971 | 4139 | 1 |
| 107 | Joachim Meisner | 1083 | 4698 | 0 |
| 107 | Godfried Danneels | 1231 | 4043 | 0 |
| 107 | Zenon Grocholewski | 1335 | 3877 | 0 |
| 107 | Anthony Olubunmi Okogie | 4047 | 4727 | 0 |
| 107 | Gabriel Zubeir Wako | 4098 | 4474 | 0 |
| 107 | Robert Sarah | 4165 | 4077 | 0 |
| 107 | Peter Turkson | 4319 | 3887 | 0 |
| 107 | Laurent Monsengwo Pasinya | 5004 | 3936 | 0 |
| 107 | Jean-Claude Turcotte | 6588 | 4632 | 0 |
| | Total | | | 28192 |

# 5 Discussion

The obtained ranking contains some interesting findings. Firstly note Pope Francis at rank 3. With this our ranking method proved to be more successful to identify the next Pope than many well-informed analysts. To the best of our knowledge cardinal Bergoglio was not mentioned among the papabili, the people who are likely to become the next pope. On the other hand the papabili ended in much lower positions, Peter Turkson, a strong African candidate turned out to be a null player, that is, a player who is never critical.

A closer inspection of the results also reveals that Pope Francis was right when he mentioned that the conclave had to go to the other end of the world to find the suitable person to become the new pope: all our top candidates are from far away places, in fact our top candidates are the cardinals with the birthplaces most distant from Rome. This is hardly a surprising result. The Median Voter Theorem (Black, 1948) stating that the society will choose the median voter's choice assumes that the voters' positions are distributed unidimensionally and that decisions are taken with simple majority. As soon as a qualified majority is required, the median voter becomes a null player, while voters with more extreme positions gain influence. The results do not generalise to preferences in a multi-dimensional space (for a discussion see Caplin and Nalebuff (1991)), but the intuition is similar: Very central cardinals will rarely be critical (if at all), while cardinals with relatively extreme location or level of conservativism are likely to do well. While the list is topped by cardinals with very distant birthplaces, they are soon followed by cardinals who have extreme positions along the liberal/conservative axis. The first Italian, Augostino Vallini is also the cardinal born closest to Rome. To test if the results are due more to the majority voting with a qualified majority or due to the two-dimensional location of the cardinals. Repeating the calculations with simple majority does not result in wild changes with the top three keeping their positions.

While it would be difficult to repeat the same exercise for the election of Pope Benedict XVI, one must note that being a German cardinal Joseph Ratzinger had a near-median distance, but was considered rather a conservative. With such a position he would have likely scored high in our ranking.

# A Algorithm to find minimal winning coalitions

Let $\xi$ be the inverse of a (not necessarily unique) permutation of indices that arranges the cardinals in a non-decreasing order according to their $x$ coordinates. Hence $c^{\xi(1)}$ is the closest to Rome and $c^{\xi(115)}$ is the farthest from it. Similarly let $\eta$ be the inverse of a permutation that arranges the cardinals in a non-decreasing order according to their $y$ coordinate, where $c^{\eta(1)}$ indicates the most liberal among them.

1. Let $\mathcal{R}$ collect feasible rectangles and set $\mathcal{R} = \emptyset$ and let $\mathbf{s} \in \mathbb{Z}^{115}$ denote a score function with $\mathbf{s} = \mathbf{0}$.

2. Starting from $c^{\xi(1)}$ take the leftmost cardinal as $l$ and allow all cardinals to be considered for $r, t, b$.

3. Starting from $c^{\eta(1)}$ take the cardinal $b$ with the lowest $c_y$ satisfying $b_x \geq l_x$ and $b_y \leq l_y$ and allow all cardinals to be considered for $r, t$.

4. Starting from $c^{\xi(115)}$ take the rightmost cardinal as $r$ if it satisfies $r_x \geq b_x$ and $r_y \geq b_y$ and allow all cardinals to be considered for $t$.

5. Starting from $c^{\eta(115)}$ the the cardinal $t$ with the highest $c_y$ satisfying $t_y \geq l_y$, $t_y \geq r_y$, $t_x \geq l_x$ and $t_x \leq r_x$.

6. If the rectangle has exactly 77 cardinals, check if it is in the list of rectangles $\mathcal{R}$. If it is not
   (a) Store it in the list $\mathcal{R}$.
   (b) Give points to each cardinal on the borderline.
7. Take the next $t$ while $t_y \geq c_y^{\eta(77)}$.
8. Take the next $r$ while $r_x \geq c_x^{\eta(77)}$.
9. Take the next $b$ while $b_y \leq c_y^{\eta(39)}$.
10. Take the next $l$ while $l_x \leq c_x^{\eta(39)}$.

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