Cálculo Integral

Clase # 33-2

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Contenidos:

- Ejercicios de repaso Unidad IV (continuación)

Ejercicio 8: Hallar una representación en series de potencias para la función $f(x) = \dfrac{1}{1-x}$ centrada en $x = 3$

$$\sum_{n=0}^{\infty} a_n (x-c)^n \Longleftrightarrow \sum_{n=0}^{\infty} a_n (x-3)^n$$

$$\frac{1}{1-x} = \frac{1}{\cancel{1} - x - \cancel{3} + 3} = \frac{1}{-2 - x + 3} = \frac{1}{-2-(x-3)}$$

$$= \frac{1}{-2\left[1 + \frac{(x-3)}{2}\right]} = -\frac{1}{2} \cdot \frac{1}{1 + \frac{(x-3)}{2}}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$\frac{1}{1+\frac{x}{2}} = \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{2}\right)^n \qquad \left|\frac{x}{2}\right| < 1$$

$$\frac{1}{1+\frac{x-3}{2}} = \sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{2^n}$$

$$\left|\frac{x-3}{2}\right| < 1$$

$$|x-3| < 2$$

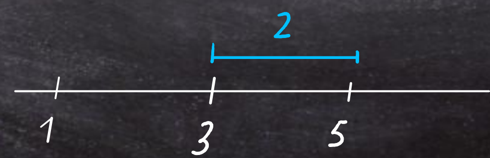
$$-\frac{1}{2}\,\frac{1}{1+\frac{x-3}{2}} = -\frac{1}{2}\sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{2^n}$$

$$-2 < x-3 < 2$$

$$1 < x < 5$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2^{n+1}}\cdot(x-3)^n$$

$$I = (1, 5) \qquad R = 2.$$



Ejercicio 9: Hallar una representación en series de potencias para la función $f(x) = \dfrac{4x}{x^2 + 2x - 3}$ centrada en $x = 0$

$$\frac{4x}{x^2 + 2x - 3} = \frac{4x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$4x = A(x-1) + B(x+3)$$

$$x = 1: \quad 4 = B(4) \quad \therefore \quad B = 1$$

$$x = -3: \quad -12 = A(-4) \quad \therefore \quad A = 3$$

$$\frac{4x}{x^2+2x-3} = \frac{3}{x+3} + \frac{1}{x-1}$$

1 2

$$\frac{3}{x+3} = \frac{3}{3+x} = \frac{3}{3\left(1+\frac{x}{3}\right)} = \frac{1}{1+\frac{x}{3}}$$

1 $$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \iff \frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$\frac{1}{1+\frac{x}{3}} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{3^n} \qquad \left|\frac{x}{3}\right| < 1$$

2 $\dfrac{1}{x-1} = \dfrac{1}{-1+x} = \dfrac{1}{-(1-x)} = -\dfrac{1}{1-x}$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$-\frac{1}{1-x} = -\sum_{n=0}^{\infty} x^n$$

$$\frac{4x}{x^2+2x-3} = \frac{3}{x+3} + \frac{1}{x-1}$$

$$\left|\frac{x}{3}\right| < 1 \iff |x| < 3$$

$$-3 < x < 3$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{3^n} + \left(-\sum_{n=0}^{\infty} x^n\right)$$

$$= \sum_{n=0}^{\infty} \left(\frac{(-1)^n}{3^n} - 1\right) x^n \qquad I = (-3, 3).$$

Ejercicio 10: Usando series de potencias, calcular $\int \operatorname{sen}(x^3)\,\mathrm{d}x$

$$\operatorname{sen} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\operatorname{sen}(x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n+1}}{(2n+1)!}$$

$$\operatorname{sen}(x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+3}}{(2n+1)!}$$

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$$\operatorname{sen}(x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+3}}{(2n+1)!}$$

$$\operatorname{sen}(x^3) = x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} + \cdots$$

$$\int \operatorname{sen}(x^3)\,dx = \frac{x^4}{4} - \frac{x^{10}}{10\cdot 3!} + \frac{x^{16}}{16\cdot 5!} - \cdots + C$$

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Ejercicio 11: Con los primeros cinco términos distintos de cero de la serie de Maclaurin para la función $f(x) = e^{-x^2}$, aproximar el valor de la integral definida $\int_0^1 e^{-x^2}\,dx$

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}$$

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$$e^{-x^2} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}$$

$$e^{-x^2} = 1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \cdots$$

$$\int_0^1 e^{-x} dx = x - \frac{x^3}{3 \cdot 1!} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!} + \frac{x^9}{4! \cdot 9} - \cdots \Big|_0^1$$

$$= \left(1 - \frac{1}{1! \, 3} + \frac{1}{5 \cdot 2!} - \frac{1}{7 \cdot 3!} + \frac{1}{9 \cdot 4!} - \cdots \right) - (0)$$

$$\simeq$$