filterL

```
filterL         :: (a → Bool) → [a] → [a]
filterL _ []            = []
filterL f (x:xs)        = let (p, xs') = f x ||| filterL f xs
                                        in if p then x:xs' else xs'
```

Trabajo

$$W_{filterL}(f, []) = c_0$$
$$W_{filterL}(f, x : xs) = c_1 + W_f(x) + W_{filterS}(xs)$$
$$W_{filterL} \in O(|xs| + \sum_{x}^{xs} W_f(x))$$

Profundidad

$$S_{filterL}(f, []) = c_0$$
$$S_{filterL}(f, x : xs) = c_1 + \max\{S_f(x), S_{filterL}(xs)\}$$
$$S_{filterL} \in O(|xs| + \max_{x}^{xs} S_f(x))$$

showtL

```
showtL          :: [a] → TreeView a [a]
showtL []       = EMPTY
showtL [x]      = ELT x
showtL xs       = let (l', r') = takeL xs (quot l 2) ||| dropL xs (quot l 2) in NODE l' r'
                        where l = lengthL xs
```

Trabajo

$$W_{showtL}([]) = c_0$$
$$W_{showtL}([x]) = c_1$$
$$W_{showtL}(xs) = c_2 + W_{takeL}(xs, |xs|/2) + W_{dropL}(xs, |xs|/2) + W_{lengthL}(xs)$$
$$W_{showtL} \in O(|xs|)$$

Profundidad

$$S_{showtL}([]) = c_0$$
$$S_{showtL}([x]) = c_1$$
$$S_{showtL}(xs) = c_2 + \max\{S_{takeL}(xs, |xs|/2), S_{dropL}(xs, |xs|/2)\} + S_{lengthL}(xs)$$
$$S_{showtL} \in O(|xs|)$$

reduceL

```
reduceL         :: (a → a → a) → a → [a] → a
reduceL f b []          = b
reduceL f b [x]         = f b x
reduceL f b (x:xs)      = reduceL f b (combineL f xs)
```

Trabajo

$$W_{reduceL}(f, []) = c_0$$
$$W_{reduceL}(f, [x]) = c_1 + W_f(x)$$
$$W_{reduceL}(f, xs) = c_2 + W_{reduceL}(f, xs/2) + W_{combineL}(f, xs)$$
$$W_{reduceL}(f, xs) \in O(|xs| + \sum_{x}^{xs} W_f(x))$$

Profundidad

$$S_{reduceL}(f, []) = c_0$$
$$S_{reduceL}(f, [x]) = c_1 + S_f(x)$$
$$S_{reduceL}(f, xs) = c_2 + S_{reduceL}(f, xs/2) + S_{combineL}(f, xs)$$
$$S_{reduceL}(f, xs) \in O(|xs| + \max_{x}^{xs} S_f(x))$$

scanL

```
scanL          :: (a → a → a) → a → [a] → ([a], a)
scanL f b []           = ([], b)
scanL f b (x:xs)       = let (xs', b') = scanL f (f b x) xs in (b:xs', b')
```

Trabajo

$$W_{scanL}(f, []) = c_0$$
$$W_{scanL}(f, x : xs) = c_2 + W_f(x) + W_{scanL}(xs)$$
$$W_{scanL}(f, xs) \in O(|xs| + \sum_{x}^{xs} W_f(x))$$

Profundidad

$$S_{scanL}(f, []) = c_0$$
$$S_{scanL}(f, x : xs) = c_2 + S_f(x) + S_{scanL}(xs)$$
$$S_{scanL} \in O(|xs| + \sum_{x}^{xs} S_f(x))$$

filterA

```
filterA        :: (a → Bool) → A.Arr a → A.Arr a
filterA f xs = A.flatten (mapA (λx → if f x then singletonA x else emptyA) xs)
```

Trabajo

$$W_{filterA}(f, xs) = c_0 + W_{flatten}(xs) + W_{mapA}(f, xs) =$$
$$= c_0 + O(|xs|) + O(\sum_{x}^{xs} W_\lambda(x))$$
$$W_{filterA} \in O(|xs| + \sum_{x}^{xs} W_f(x))$$

Donde

$$W_\lambda(x) = c_0 + W_f(x) + W_{singletonA}(x) + W_{emptyA}(x)$$
$$W_\lambda(x) \in O(W_f(x))$$

Profundidad

$$S_{filterA}(f, xs) = c_0 + S_{flatten}(xs) + S_{mapA}(f, xs) =$$
$$= c_0 + O(lg|xs|) + O(\max_{x}^{xs} S_\lambda(x)) =$$
$$S_{filterA} \in O(lg|xs| + \max_{x}^{xs} S_f(x))$$

Donde

$$S_\lambda(x) = c_0 + S_f(x) + S_{singletonA}(x) + S_{emptyA}(x)$$
$$S_\lambda(x) \in O(S_f(x))$$

showtA

```
showtA          :: A.Arr a → TreeView a (A.Arr a)
showtA xs       | l == 0        = EMPTY
                | l == 1        = ELT (xs ! 0)
                | otherwise     = NODE (takeA xs (quot l 2)) (dropA xs (quot l 2))
                        where l = A.length xs
```

Trabajo

$$W_{showtA}(xs) = c_2 + W_{takeA}(xs, |xs|/2) + W_{dropA}(xs, |xs|/2) + W_{length}(xs)$$

$$W_{showtA} \in O(1)$$

Profundidad

$$S_{showtA}(xs) = c_2 + \max\{S_{takeA}(xs, |xs|/2), S_{dropA}(xs, |xs|/2)\} + S_{lengthL}(xs)$$

$$S_{showtA} \in O(1)$$

reduceA

```
reduceA         :: (a → a → a) → a → A.Arr a → a
reduceA f b xs  | l == 0        = b
                | l == 1        = f b (xs ! 0)
                | otherwise     = reduceA f b (combineA f xs)
                        where l = A.length xs
```

Trabajo

$$W_{reduceA}(f, xs) = c_0 + W_{reduceA}(f, xs/2) + W_{combineA}(f, xs) =$$

$$= c_0 + W_{reduceA}(f, xs/2) + O(\sum_{x,y}^{x:y:ys} (W_f(x, y))$$

$$W_{reduceA}(f, xs) \in O(|xs| + \sum_{x,y}^{x:y:ys} W_f(x, y))$$

Profundidad

$$S_{reduceA}(f, xs) = c_0 + S_{reduceA}(f, xs/2) + S_{combineA}(f, xs)$$

$$= c_0 + S_{reduceA}(f, xs/2) + O(\max_{x,y}^{x:y:ys} W_f(x, y))$$

$$S_{reduceA}(f, xs) \in O(lg|xs| * \max_{x,y}^{x:y:ys} S_f(x, y))$$

scanA

```
scanA           :: (a → a → a) → a → A.Arr a → (A.Arr a, a)
scanA f b xs    | A.length xs == 0      = (emptyS, b)
                | A.length xs == 1      = (singletonS b, f b (xs ! 0))
                | otherwise             = (A.tabulate (λi → if mod i 2 == 0
                                                then xs' ! (quot i 2)
                                                else f (xs' ! (quot i 2)) (xs ! (i - 1))) (A.length xs), b')
                        where   (xs', b') = scanA f b (combineA f xs)
```

Trabajo

$$W_{scanA}(f, xs) = c_0 + W_{tabulate}(\lambda, xs) + W_{length}(xs) + W_{scanA}(f, xs/2) + W_{combineA}(f, xs) =$$

$$= c_0 + O(\sum_{i}^{|xs|} W_\lambda(i)) + W_{length}(xs) + W_{scanA}(f, xs/2) + O(\sum_{x,y}^{x:y:ys} W_f(x, y))$$

$$W_{scanA}(f, xs) \in O(|xs| + \sum_{x}^{xs} W_f(x', x))$$

Donde

$$W_\lambda(i) \in O(W_f(x'_{i/2}, x_{i-1}))$$

Profundidad

$$S_{scanA}(f, xs) = c_0 + S_{tabulate}(\lambda, xs) + S_{length}(xs) + S_{scanA}(f, xs/2) + S_{combineA}(f, xs) =$$

$$= c_0 + O(\sum_{i}^{|xs|} S_\lambda(i)) + S_{length}(xs) + S_{scanA}(f, xs/2) + O(\max_{x,y}^{x:y:ys} W_f(x, y))$$

$$S_{scanA} \in O(lg|xs| * \max_{x}^{xs} S_f(x', x))$$

Donde

$$S_\lambda(i) \in O(S_f(x'_{i/2}, x_{i-1}))$$

takeL

```
takeL           :: [a] → Int → [a]
takeL xs 0              = []
takeL [] n              = error "Invalid index."
takeL (x:xs) n  = x:(takeL xs (n - 1))
```

Trabajo

$$W_{takeL}(x : xs) = c_1 + W_{takeL}(xs)$$
$$W_{takeL}(xs) \in O(|xs|)$$

Profundidad

$$S_{takeL}(x : xs) = c_1 + S_{takeL}(xs)$$
$$S_{takeL}(xs) \in O(|xs|)$$

dropL

```
dropL           :: [a] → Int → [a]
dropL xs 0              = xs
dropL [] n              = error "Invalid index."
dropL (x:xs) n  = dropL xs (n - 1)
```

Trabajo

$$W_{dropL}(x : xs) = c_1 + W_{dropL}(xs)$$
$$W_{dropL}(xs) \in O(|xs|)$$

Profundidad

$$S_{dropL}(x:xs) = c_1 + S_{dropL}(xs)$$
$$S_{dropL}(xs) \in O(|xs|)$$

combineL

```
combineL                :: (a → a → a) → [a] → [a]
combineL _ []           = []
combineL _ [x]          = [x]
combineL f (x:y:ys)     = let (x', xs') = f x y ||| combineL f ys in x':xs'
```

Trabajo

$$W_{combineL}(f, []) = c_0$$
$$W_{combineL}(f, [x]) = c_1$$
$$W_{combineL}(f, x:y:ys) = c_2 + W_f(x,y) + W_{combineL}(f, ys)$$
$$W_{combineL}(f, xs) \in O(|xs| + \sum_{x,y}^{x:y:xs} W_f(x,y))$$

Profundidad

$$S_{combineL}(f, []) = c_0$$
$$S_{combineL}(f, [x]) = c_1$$
$$S_{combineL}(f, x:y:ys) = c_2 + \max\{S_f(x,y), S_{combineL}(f, ys)\}$$
$$S_{combineL}(f, xs) \in O(|xs| + \max_{x,y}^{x:y:xs} W_f(x,y))$$

mapA

```
mapA            :: (a → b) → A.Arr a → A.Arr b
mapA f xs = A.tabulate (λi → f (xs!i)) (A.length xs)
```

Trabajo

$$W_{mapA}(f, xs) = c_0 + W_{tabulate}(\lambda, |xs|) + W_{length}(xs) =$$
$$= c_0 + O(\sum_i^{|xs|} W_\lambda(i)) + W_{length}(xs) =$$
$$= c_0 + O(\sum_i^{|xs|}(W_f(x_i) + W_!(xs, i))) + W_{length}(xs)$$
$$W_{mapA}(f, xs) \in O(\sum_x^{xs} W_f(x))$$

Profundidad

$$S_{mapA}(f, xs) = c_0 + S_{tabulate}(\lambda, |xs|) + S_{length}(xs) =$$
$$= c_0 + O(\sum_i^{|xs|} S_\lambda(i)) + S_{length}(xs) =$$
$$= c_0 + O(\sum_i^{|xs|}(S_f(x_i) + W_!(xs, i))) + S_{length}(xs)$$
$$S_{mapA}(f, xs) \in O(\max_x^{xs} S_f(x))$$

takeA

```
takeA          :: A.Arr a → Int → A.Arr a
takeA xs n = A.subArray 0 n xs
```

Trabajo

$$W_{takeA}(xs) = c_0 + W_{subArray}(xs)$$
$$W_{takeA}(xs) \in O(1)$$

Profundidad

$$S_{takeA}(xs) = c_0 + S_{subArray}(xs)$$
$$S_{takeA}(xs) \in O(1)$$

dropA

```
dropA          :: A.Arr a → Int → A.Arr a
dropA xs n = A.subArray n ((A.length xs) - n) xs
```

Trabajo

$$W_{dropA}(xs) = c_0 + W_{subArray}(xs) + W_{length}(xs)$$
$$W_{dropA}(xs) \in O(1)$$

Profundidad

$$S_{dropA}(xs) = c_0 + S_{subArray}(xs) + S_{length}(xs)$$
$$S_{dropA}(xs) \in O(1)$$

combineA

```
combineA               :: (a → a → a) → A.Arr a → A.Arr a
combineA f xs   | l == 0        = emptyA
                | mod l 2 == 0  = A.tabulate (λi → f (xs ! (2 * i)) (xs ! ((2 * i) + 1))) (quot l 2)
                | otherwise     = A.tabulate (λi → if i == quot l 2 then xs ! (2 * i) else f (xs ! (2 *
i)) (xs ! ((2 * i) + 1))) ((quot l 2) + 1)
                                    where l = A.length xs
```

Trabajo

$$W_{combineA}(f, []) = c_0$$
$$W_{combineA}(f, xs) = c_1 + W_{tabulate}(\lambda, [xs]/2) + W_{length}(xs) =$$
$$= c_1 + O(\sum_{i}^{|xs|/2} W_\lambda(i)) + W_{length}(xs) =$$
$$= c_1 + O(\sum_{i}^{|xs|/2} (W_f(x_{2i}, x_{2i+1}) + W_!(xs, 2i) + W_!(xs, 2i+1))) + W_{length}(xs)$$
$$W_{combineA}(f, xs) \in O(\sum_{x,y}^{x:y:ys} W_f(x, y))$$

Profundidad

$$S_{combineA}(f, []) = c_0$$

$$S_{combineA}(f, xs) = c_1 + S_{tabulate}(\lambda, [xs]/2) + S_{length}(xs) =$$

$$= c_1 + O(\max_{i}^{|xs|/2} W_\lambda(i)) + S_{length}(xs) =$$

$$= c_1 + O(\max_{i}^{|xs|/2}(S_f(x_{2i}, S_{2i+1}) + S_!(xs, 2i) + S_!(xs, 2i+1))) + S_{length}(xs)$$

$$S_{combineA}(f, xs) \in O(\max_{x,y}^{x:y:ys} S_f(x, y))$$