

Tallos  $A \cap A^c = \emptyset$ ,  $\mathcal{S}: A \neq \emptyset$  se cumple.

Sea  $N(A) = \{x \in X : \mu_A(x) = 1\}$  (núcleo) Sea  $A$  un conj. difuso (cd)

tal que  $\exists x \in X$  t.q.  $\mu_A(x) \in (0, 1)$ .

Supongamos que  $A \cap A^c = \emptyset$

Como  $\mu_A(x) \in (0, 1) \Rightarrow \mu_{A^c}(x) = 1 - \mu_A(x) \in (0, 1)$

Por lo tanto,  $\mu_{A \cap A^c}(x) = \min\{-, -\} > 0$ .

Con lo que  $A \cap A^c \neq \emptyset$ .

b. Sea  $A$  en cd en  $X$  t.q.  
 $\exists x \in X$  t.q.  $\mu_A(x) \in (0, 1)$ .

$\Rightarrow \mu_{A^c}(x) \in (0, 1) \Rightarrow \mu_{A \cup A^c}(x) = \max\{-, -\} < 1$ .

luego  $N(A \cup A^c) \neq X$ .

c. Tener en cuenta  
 $\min\{x, x\} = \max\{x, x\} = x$ .

d. Sea  $A$  cd en  $X$ .

Sea  $x \in X$ .  $\mu_{A \cup A^c}(x) = 1 - \mu_A(x)$   
 $= 1 - (1 - \mu_A(x)) = \mu_A(x)$   $\square$

e. Por conmutatividad del  $\max$   
 $\min$

f. Sea  $A$  cd en  $X$ .

Sea  $x \in X$ .

1)  $\mu_{A \cap B}(x) = \min\{\min\{\mu_A(x), \mu_B(x)\}, \mu_C(x)\}$   
2)  $\mu_A(x) \leq \mu_B(x) \leq \mu_C(x)$   
 $\Delta = \min\{\mu_A(x), \mu_C(x)\}$   
 $= \mu_A(x)$   
 $= \min\{\mu_A(x), \mu_C(x)\}$   
 $= \min\{\mu_A(x), \min\{\mu_B(x), \mu_C(x)\}\}$   
Análogo,  $\mu_B(x) \leq \mu_C(x) \leq \mu_A(x)$

Análogamente se hace  $\mu_B(x) \leq \mu_A(x) \leq \mu_C(x)$

3)  $\mu_C(x) \leq \mu_A(x) \leq \mu_B(x)$   
 $\Delta = \min\{\mu_A(x), \mu_C(x)\}$   
 $= \min\{\mu_A(x), \min\{\mu_B(x), \mu_C(x)\}\}$

4)  $\mu_C(x) \leq \mu_B(x) \leq \mu_A(x)$   
 $\Delta = \min\{\mu_B(x), \mu_C(x)\}$   
 $= \mu_C(x)$   
 $= \min\{\mu_B(x), \mu_C(x)\}$   
 $= \min\{\mu_B(x), \min\{\mu_A(x), \mu_C(x)\}\}$

Análogo para  $\cup$

g.  $(A \cup B) \cap (B \cup C) = \min\{\max\{\mu_A, \mu_B\}, \max\{\mu_B, \mu_C\}\}$

$\mu_{A \cup B \cap C}(x) = \max\{\mu_A, \min\{\mu_B, \mu_C\}\}$

1)  $\mu_A \leq \mu_B \leq \mu_C$

$\Delta = \max\{\mu_A, \mu_B\}$

$= \mu_B$

$= \min\{\mu_B, \mu_C\}$

$= \min\{\max\{\mu_A, \mu_B\}, \max\{\mu_B, \mu_C\}\}$

Análogo a  $\mu_B \leq \mu_C \leq \mu_A$

2)  $\mu_C \leq \mu_B \leq \mu_A$

$\Delta = \max\{\mu_A, \mu_B\}$

$= \mu_A$

$= \min\{\mu_A, \mu_B\}$

$= \min\{\max\{\mu_A, \mu_B\}, \max\{\mu_B, \mu_C\}\}$

$= \min\{\max\{\mu_A, \mu_B\}, \max\{\mu_A, \mu_C\}\}$

Análogo a  $\mu_B \leq \mu_A \leq \mu_C$

3)  $\mu_B \leq \mu_C \leq \mu_A$

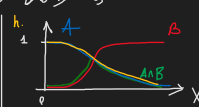
$\Delta = \max\{\mu_A, \mu_B\}$

$= \mu_A$

$= \min\{\mu_A, \mu_B\}$

$= \min\{\max\{\mu_A, \mu_B\}, \max\{\mu_B, \mu_C\}\}$

Análogo a  $\mu_C \leq \mu_B \leq \mu_A$



$A \cup (A \cap B)$

1)  $\mu_A \leq \mu_B$

$\max\{\mu_A, \min\{\mu_A, \mu_B\}\} = \max\{\mu_A, \mu_A\}$

$= \mu_A$

2)  $\mu_A \geq \mu_B$

$\max\{\mu_A, \min\{\mu_A, \mu_B\}\} = \max\{\mu_A, \mu_B\} \geq \mu_A$

3) Supongamos que  $\mu_{A \cup (A \cap B)}(x) = \mu_{A \cup B}(x)$

1)  $\mu_A \leq \mu_B$ ,  $1 - \mu_A \leq \mu_B$

$\max\{\mu_A, \min\{\mu_A, \mu_B\}\} = \max\{\mu_A, \mu_A\} \neq \max\{\mu_A, \mu_B\}$

luego  $A \cup (A^c \cap B) \neq A \cup B$

2)  $\mu_{A \cup B}(x) = \mu_{A \cap B}(x)$

$\mu_A \leq \mu_B$

$\mu_{A \cup B}(x) = 1 - \mu_{A \cap B}(x)$

$= 1 - \max\{\mu_A, \mu_B\}$

$= 1 - \mu_B$

$= \min\{1 - \mu_B, 1 - \mu_A\}$

$= \mu_{A \cap B}$