Practice Problems 3

$x_1 = (1, 2)$ $x_2 = (3, 1)$ $x_3 = (0, 1)$

$k(x, y) = (\langle x, y \rangle + 2)^2$

1) $\langle x, y \rangle = x \cdot y = (x_1 y_1 + x_2 y_2)$ $\mathbb{R}^2 \to \mathbb{R}$?

$k(x, y) = (x_1 y_1 + x_2 y_2 + 2)^2$

$= (x_1 y_1 + x_2 y_2 + 2)(x_1 y_1 + x_2 y_2 + 2)$

$= x_1^2 y_1^2 + x_1 y_1 x_2 y_2 + 2 x_1 y_1 + x_1 y_1 x_2 y_2 + x_2^2 y_2^2 + 2 x_2 y_2$

$+ 2 x_1 y_1 + 2 x_2 y_2 + 4$

$= (2 x_1 y_1 x_2 y_2 + 4 x_1 y_1 + 4 x_2 y_2 + x_1^2 y_1^2 + x_2^2 y_2^2 + 4)$ 6

$\langle x, y \rangle = x \cdot y = (x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4 + x_5 y_5 + x_6 y_6)$

$x \in \mathbb{R}^n, y \in \mathbb{R}^n \to \langle x, y \rangle \in \mathbb{R}^n$ $\in \mathbb{R}^6$

$a = (\sqrt{2} a_1 a_2, 2 a_1, 2 a_2, a_1^2, a_2^2, 2) \in \mathbb{R}^6$

2) $\langle \phi(x_1), \phi(x_2) \rangle_F = K(x_1, x_2) = 49$

$= (\langle x_1, x_2 \rangle + 2)^2$

$= (\langle (1, 2), (3, 1) \rangle + 2)^2 = (3 + 2 + 2)^2 = 49$

$=$

$= \langle (\sqrt{2}2, 2, 4, 1, 4, 2), (\sqrt{2}3, 6, 2, 9, 1, 2) \rangle$

$= 12 + 12 + 8 + 9 + 4 + 4 = 49$

3) $\| \phi(x_1) - \phi(x_2) \|_F^2 = K(x_1, x_1) + K(x_2, x_2) - 2K(x_1, x_2)$

$\| \phi(x_1) - \phi(x_2) \|_F^2 = 49 + 144 - 2(49) = 95$ 95

$\| \phi(x_1) - \phi(x_3) \|_F^2 = 49 + 9 - 2(16) = 26$ 26

$\| \phi(x_2) - \phi(x_3) \|_F^2 = 144 + 9 - 2(9) = 135$ 135