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Kinematics Analysis of a Six-Wheeled Mobile Robot

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Abstract **- The paper analysis a kinematic model for a wheeled mobile robot (WMR) traversing uneven terrain. A new form of the kinematics for wheeled mobile robot is deduced, through analyzing Jacobian matrices of individual wheel and rearranging the variables. The performance and characters of the kinematic formulation are explained using physical conception. A new method is proposed to set up the kinematics formulation for wheeled mobile robots. After analyzing the actuation kinematics, simulation results are provided to validate the motion of wheeled mobile robot over a special terrain.**

I. INTRODUCTON

This paper deals with the kinematic analysis of a wheeled mobile robot (WMR) moving on uneven terrain. Early research in the area of mobile robot kinematics has been essentially limited to robots moving on a flat and smooth surface [1]-[4]. These models are based on motions in the 2-dimensional x-y plane, and rotation about z-axis. Muir and Neuman proposed the methodology for a WMR kinematics. They applied the Sheth-Uicker convention to model the higher-pair relationship between each wheel of WMR and the floor. Problems associated with high mobility rovers operating in uneven terrain have not been addressed. And they adjoin the equations of motion of all of the wheels to formulate, solve, and interpret the solutions of the composite robot equation. This kinematic methodology has been extensive used for the kinematic modeling of WMRs [5]-[7]. Recently, there have been great research efforts on WMRs that have sophisticated mobility systems for enabling their traversal over uneven terrain. For example, Rocky series rovers developed at JPL include Rocky 4, Sojourner and Rocky 7. They have versatile mobility system, consisting of adjustable rocker-bogie which enable the rovers adapt the uneven terrain [8]. But most of researches in the area of kinematic get the similar form of the kinematics. They introduce wheel jacobian matrix to relate the motions of each wheel to the motions of the robot. The resulting equations of the individual wheel motions are then combined to form the composite equation for the rover motion [7]-[9]. Mahmoud and Gregory propose a method for developing a complete kinematics model of a WMR and its interaction with the terrain. But the kinematics they proposed does not give us a deep insight about the parameters in equations. In this paper, we present a general kinematic formulation deduced from individual wheel Jacobian matrices. The physical explanation and the special characters of the formulation are interpreted in detail. A simple method to establish kinamatics formulation for such robot is proposed. And the application of the formulation in actuation is analyzed. The simulation results show that both the actuation kinematics and the general kinematic formulation is valid and reasonable .

In Section II, we describe the mechanism configure of our rover and design the coordinates for the rover. The kinematic formulation will be deduced and the analysis about it will be discussed in Section III. The simulation result will be given in Section IV. Finally, Section V provides the conclusions of the work.

II. MECHANICAL CONFIGURATION

The mobile robot consists of six independently driven wheels using a rocker-bogie design as shown in fig 1. Two main rockers are hinged to the sides of body. At the end of front rocker and rear of bogie there are two steerable wheels respectively, and there are two non-steerable wheels in between. The rover is capable of locomotion over uneven terrain by rolling of the wheels and adjusting its joints, and the only contact with the terrain is at the wheel surfaces.

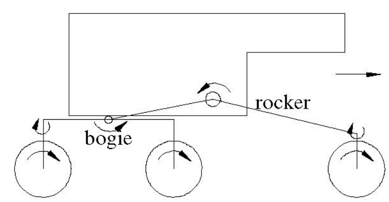
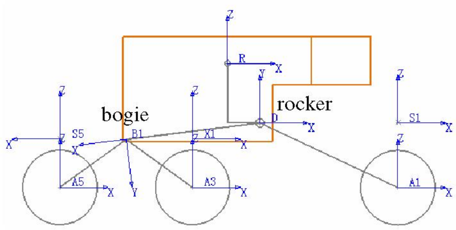


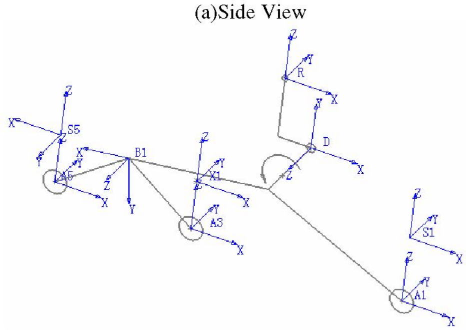
Fig.1 Schematic of rover configuration

We begin by defining a set of coordinate frames for the rover. These coordinate frames are illustrated in Fig. 2 for the right side of the rover consisting of wheels 1, 3 and 5. The left side (not shown) is assigned similar frames and consists of wheels 2, 4 and 6. The subscript for the coordinate frames is as follows: R refers to the rover frame (passing through its center of gravity), D refers to differential, Si refers to the Steering of wheel i (i=1, 3, 4, 6), Ai denotes the wheel i axle (i=1, 2, …, 6), B_1 and B_2 are left and right bogie joint frames, and X_1 and X_2 are auxiliary frames. Each coordinate frame represents one step in the kinematic chain from the rover's reference frame to a wheel.

The Denavit-Hartenberg (D-H) parameters γ, d, a and α for the rover are given in Table 1 with units rad, centimeter, centimeter and degree respectively. The parameters a_i, d_i (i=1, 2, 3, 4) are constants representing the dimensions of the rover configure. ρ_1 and ρ_2 are right and left bogie angle, The rockers are connected to the main body with a differential so that the rocker angles β_1 and β_2 can be represented by a single joint angle β, where $\beta_1 = -\beta_2 = \beta$.



(a)Side View



(b)Perspective View

Fig. 2 Coordinate frames for rover's right side

After defining the coordinate frames homogeneous transformation can be written using D-H convention.

III. Kinematic Model Development

A. Individual Wheel Jacobian Matrices

We will deduce the individual wheel Jacobian matrices using the method in Ref. [7] [9]. In our analysis, a single, rigid, continuous contact point between each wheel and the terrain is assumed. In order to capture the wheel motion a contact coordinate frame Ci (i=1,…,6) is defined at each wheel contact point as illustrated in Fig. 3, where its x-axis is tangent to the terrain at the point of contact and its z-axis is normal to the terrain. The contact angle δ_i is the angle between the z-axes of the wheel i axle and contact coordinate frames

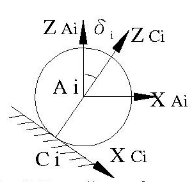


Fig. 3 Coordinate frames for terrain contact

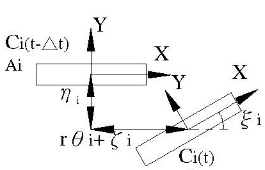


Fig. 4 Wheel slip model

In addition, we include the rolling and slip motion for wheels, we consider the instantaneous contact frames $C_i(t-\Delta t)$ and $C(t)$, where Δt is a time increment as show in Fig.4. The coordinate transformation from $C_i(t-\Delta t) \equiv \overline{C}_i$ to $C_i(t) \equiv C_i$ is

Table 1 D-H parameters for primary coordinated frames

Frame	θ	D	a	α
D	0	-d1	a1	90
S1	β	d2	a2	-90
A1	ψ_1	-d3	0	0
B1	P+ β	d2	a3	0
X1	Q+ ρ_1	0	a4	-90
A3.	0	-d4	0	0
S5	W+ ρ_1	0	a4	90
A5.	$\pi + \psi_5$	-d4	0	0
S2	$-\beta$	-d2	a2	-90
A2.	ψ_2	-d3	0	0
B2	P- β	-d2	a3	0
X2	Q+ ρ_2	0	a4	-90
A4.	0	-d4	0	0
S6	W+ ρ_2	0	a4	90
A6	$\pi + \psi_6$	-d4	0	0

a1=0 cm – horizontal offset between R and D
d1=0 cm – vertical offset between R and D
a2=58.8 cm – length of link from D to S1 ,S4
d2=30 cm – horizontal distance from D to wheels
a3=46.96 cm – length of link from rocker joint to bogie joint
d3=22.5 cm – vertical offset between Si and Ai (i=1,4)
a4=23.1 cm – length from bogie joint to forward/rear bogie
d4=16.7 cm – height of bogie joint from wheel axles
P=187.1*pi/180 rad - angle of link between rocker and bogie joint
Q=172.9*pi/180 rad – angle of link between Bi and Xi
W=-7.1*pi/180 rad – angle of link between bogie joint and steering axis of rear wheel.

defined for wheel slip model including wheel rolling $r\theta_i+\zeta_i$) along the x axis (where ζ_i is the rolling slip and θ_i is wheel angular rate), wheel side slip η_i along the y-axis, and a rotation slip ξ_i about the z axis.

Using the similar method in Ref.[9], we can deduced each wheel Jacobian matrix which we refer as forward kinematics. The matrix $T_{(\overline{R},R)}$ can be written as $T_{(\overline{R},R)} = T_{(\overline{R},\overline{C}_i)} \cdot T_{(\overline{C}_i,R)}$. Since $T_{(\overline{R},\overline{C}_i)}$ is constant matrix for its independence of time, the derivative of $T_{(\overline{R},R)}$ is $\mathring{T}_{(\overline{R},R)} = T_{(\overline{R},\overline{C}_i)} \cdot \mathring{T}_{(\overline{C}_i,R)}$. Equating the elements on both sides of equation, we can deduce each wheel Jacobian matrix which we refer as forward kinematics [7][9].

$$\begin{bmatrix} \mathring{X} \\ \mathring{Y} \\ \mathring{Z} \\ \mathring{\phi}_x \\ \mathring{\phi}_y \\ \mathring{\phi}_z \end{bmatrix} = \begin{bmatrix} 0 & J_{12} & R\cdot J_{17} & J_{14} & J_{15} & J_{16} & J_{17} \\ 0 & J_{22} & R\cdot J_{27} & J_{24} & J_{25} & J_{26} & J_{27} \\ 0 & J_{32} & R\cdot J_{37} & J_{34} & J_{35} & J_{36} & J_{37} \\ 0 & J_{42} & 0 & J_{44} & J_{45} & 0 & 0 \\ 1 & 0 & 0 & J_{54} & J_{55} & 0 & 0 \\ 0 & J_{62} & 0 & J_{64} & J_{65} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathring{\beta} \\ \mathring{\psi}_1 \\ \mathring{\theta}_1 \\ \mathring{\delta}_1 \\ \mathring{\xi}_1 \\ \mathring{\eta}_1 \\ \mathring{\zeta}_1 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \mathring{X} \\ \mathring{Y} \\ \mathring{Z} \\ \mathring{\phi}_x \\ \mathring{\phi}_y \\ \mathring{\phi}_z \end{bmatrix} = \begin{bmatrix} 0 & J_{12} & R\cdot J_{17} & J_{14} & J_{15} & 0 & J_{17} \\ 0 & 0 & 0 & 0 & J_{25} & 1 & 0 \\ 0 & J_{13} & R\cdot J_{37} & J_{34} & J_{35} & 0 & J_{37} \\ 0 & 0 & 0 & 0 & J_{45} & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_{65} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathring{\beta} \\ \mathring{\rho}_1 \\ \mathring{\theta}_3 \\ \mathring{\delta}_3 \\ \mathring{\xi}_3 \\ \mathring{\eta}_3 \\ \mathring{\zeta}_3 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} \mathring{X} \\ \mathring{Y} \\ \mathring{Z} \\ \mathring{\phi}_x \\ \mathring{\phi}_y \\ \mathring{\phi}_z \end{bmatrix} = \begin{bmatrix} 0 & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} & J_{17} & J_{18} \\ 0 & 0 & J_{23} & J_{24} & J_{25} & J_{26} & J_{27} & J_{28} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} & J_{37} & J_{38} \\ 0 & 0 & J_{43} & 0 & J_{45} & J_{46} & 0 & 0 \\ 1 & 1 & 0 & 0 & J_{55} & J_{56} & 0 & 0 \\ 0 & 0 & J_{63} & 0 & J_{65} & J_{66} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathring{\beta} \\ \mathring{\rho}_1 \\ \mathring{\psi}_5 \\ \mathring{\theta}_5 \\ \mathring{\delta}_5 \\ \mathring{\xi}_5 \\ \mathring{\eta}_5 \\ \mathring{\zeta}_5 \end{bmatrix} \tag{3}$$

These equations are for wheel 1, 3, and 5. The equations for the wheels of left side of rover are similar with the right ones. Elements of the jacobian matrix for all wheels are given in the Appendix.

B. Development of kinematic for the model

We will give a new kinematic formulation for wheeled mobile robot by deducing from the individual wheel Jacobian matrics without doing any assumptions.

Explicit expression and interpretation will be given. We introduce the process of deducing using wheel 3.

$$\dot{X} + K_1 \cdot \dot{\phi}_y + d_2 \cdot \dot{\phi}_z - K_1 \cdot \dot{\beta} - (J_{12} + K_1) \cdot \dot{\rho}_1 = R \cdot J_{18} \cdot \dot{\theta}_5 + J_{17} \cdot \dot{\eta}_5 + J_{18} \cdot \dot{\zeta}_5$$

$$\dot{Z} + K_2 \cdot \dot{\phi}_y - d_2 \cdot \dot{\phi}_x - K_2 \cdot \dot{\beta} - (J_{32} + K_2) \cdot \dot{\rho}_1 = R \cdot J_{38} \cdot \dot{\theta}_5 + J_{37} \cdot \dot{\eta}_5 + J_{38} \cdot \dot{\zeta}_5$$

$$\dot{Y} - K_1 \cdot \dot{\phi}_x - K_2 \cdot \dot{\phi}_z = R \cdot J_{28} \cdot \dot{\theta}_5 + J_{27} \cdot \dot{\eta}_5 + J_{28} \cdot \dot{\zeta}_5$$

where $K_1 = a_3 \cdot s(W - \beta) - d_4 \cdot c(\beta + \rho_1) - a_4 \cdot s(\beta + \rho_1)$

$K_2 = a_3 \cdot c(W - \beta) - d_4 \cdot s(\beta + \rho_1) + a_4 \cdot c(\beta + \rho_1)$

By writing the above equations in a matrix form, a new form is found

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} + \begin{bmatrix} 0 & -K_1 & d_2 \\ K_1 & 0 & K_2 \\ -d_2 & -K_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\phi}_x \\ \dot{\phi}_y \\ \dot{\phi}_z \end{bmatrix} + \begin{bmatrix} K_1 \\ 0 \\ K_2 \end{bmatrix} \dot{\beta} + \begin{bmatrix} d_4 \cdot c(\beta + \rho_1) + a_4 \cdot s(\beta + \rho_1) \\ 0 \\ d_4 \cdot s(\beta + \rho_1) - a_4 \cdot c(\beta + \rho_1) \end{bmatrix} \dot{\rho}_1$$

$$= \begin{bmatrix} J_{18} \\ J_{28} \\ J_{38} \end{bmatrix} \cdot R \cdot \dot{\theta}_5 + \begin{bmatrix} J_{17} \\ J_{27} \\ J_{37} \end{bmatrix} \cdot \dot{\eta}_5 + \begin{bmatrix} J_{18} \\ J_{28} \\ J_{38} \end{bmatrix} \cdot \dot{\zeta}_5 \tag{4}$$

Equation (1) for wheel 1 can be deduced to obtain the same form similar with (4). So does equation (2).

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} + \begin{bmatrix} 0 & -K_1 & d_2 \\ K_1 & 0 & K_2 \\ -d_2 & -K_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\phi}_x \\ \dot{\phi}_y \\ \dot{\phi}_z \end{bmatrix} + \begin{bmatrix} K_1 \\ 0 \\ K_2 \end{bmatrix} \cdot \dot{\beta} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \dot{\rho}_1 \quad \text{(wheel 1)} \tag{5}$$

$$= \begin{bmatrix} J_{17} \\ J_{27} \\ J_{37} \end{bmatrix} \cdot R \cdot \dot{\theta}_1 + \begin{bmatrix} J_{16} \\ J_{26} \\ J_{36} \end{bmatrix} \cdot \dot{\eta}_1 + \begin{bmatrix} J_{17} \\ J_{27} \\ J_{37} \end{bmatrix} \cdot \dot{\zeta}_1$$

where

$K_1 = -a_2 \cdot s\beta + d_3 \cdot c\beta$

$K_2 = a_2 \cdot c\beta + d_3 \cdot s\beta$

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} + \begin{bmatrix} 0 & -K_1 & d_2 \\ K_1 & 0 & K_2 \\ -d_2 & -K_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\phi}_x \\ \dot{\phi}_y \\ \dot{\phi}_z \end{bmatrix} + \begin{bmatrix} K_1 \\ 0 \\ K_2 \end{bmatrix} \cdot \dot{\beta} + \begin{bmatrix} d_4 c(\beta + \rho_1) - a_4 s(\beta + \rho_1) \\ 0 \\ d_4 c(\beta + \rho_1) + a_4 s(\beta + \rho_1) \end{bmatrix} \cdot \dot{\rho}_1$$

$$= \begin{bmatrix} J_{17} \\ 0 \\ J_{37} \end{bmatrix} \cdot R \cdot \dot{\theta}_3 + \begin{bmatrix} 0 \\ J_{26} \\ 0 \end{bmatrix} \cdot \dot{\eta}_3 + \begin{bmatrix} J_{17} \\ 0 \\ J_{37} \end{bmatrix} \cdot \dot{\zeta}_3 \quad \text{(wheel 2)} \tag{6}$$

where

$K_1 = a_3 \cdot s(Q - \beta) + d_4 \cdot c(\beta + \rho_1) - a_4 \cdot s(\beta + \rho_1)$

$K_2 = a_3 \cdot c(Q - \beta) + d_4 \cdot s(\beta + \rho_1) + a_4 \cdot c(\beta + \rho_1)$

So the new general expression of kinematic equations is given below

$$\dot{V} + D_{1i}\dot{\Phi} + D_{1i}\dot{\beta} + D_{2i}\dot{\rho} = M_i R \dot{\theta}_i + Q\dot{W}_i \tag{7}$$

Where $\dot{V} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix}$, $D_{1i} = \begin{bmatrix} 0 & -d_5 & b_i \cdot d_2 \\ d_5 & 0 & d_6 \\ -b_i \cdot d_2 & -d_6 & 0 \end{bmatrix}$, $\dot{\Phi} = \begin{bmatrix} \dot{\phi}_x \\ \dot{\phi}_y \\ \dot{\phi}_z \end{bmatrix}$

$\dot{\beta} = \begin{bmatrix} 0 \\ \mp b_i \cdot \dot{\beta} \\ 0 \end{bmatrix}$, $D_{2i} = \begin{bmatrix} d_7 \\ 0 \\ d_8 \end{bmatrix}$, $Q = \begin{bmatrix} M_i \\ N_i \end{bmatrix}^T$, $\dot{W} = \begin{bmatrix} \dot{\varsigma}_i \\ \dot{\eta}_i \end{bmatrix}$,

($d_5 = K_1$, $d_6 = K_2$, $d_7 = -J_{12} + K_1$, $d_8 = -J_{32} + K_2$

and $b_i = (-1)^{i-1}$, i=1,2…6)

$$M_i = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} J_{17} \\ J_{27} \\ J_{37} \end{bmatrix}, \quad N_i = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} J_{16} \\ J_{26} \\ J_{36} \end{bmatrix} \quad \text{(i=1,2,3,4)}$$

$$M_i = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} J_{18} \\ J_{28} \\ J_{38} \end{bmatrix}, \quad N_i = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} J_{17} \\ J_{27} \\ J_{37} \end{bmatrix} \quad \text{(i=5,6)}$$

$\dot{V}$ is the vector of rover body linear velocity and $\dot{\Phi}$ is the vector of rover body angular velocity. We defined the D_{1i} as the rotation matrix of the rover with three radiuses namely d_2, d_5, d_6. D_{1i} is a skew-symmetric matrix, where d_2 defined in section Ⅱ is the offset distance between wheel center and the origin of rover frame along the y axis of frame R, and similarly, d_5 and d_6 are distance along z axis and x axis as shown in Fig. 5. The rover exerts its rotation influence on wheel central velocity by multiplying rotation rate and corresponding radius. It should be noticed that for both sides d_2 is constant representing half width of rover. d_5 and d_6, are variables with the change of the rover configuration.

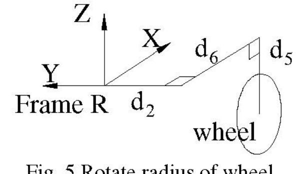


Fig. 5 Rotate radius of wheel

As for the variables $\dot{\beta}$ and $\dot{\rho}_i$, they represent the angular velocity of rocker and bogie and they express the rotation effect causing by the passive links rotation. The coefficients of $\dot{\beta}$ and $\dot{\phi}_y$ are identical because they have the same center of rotation. The elements of D_{2i} represent corresponding radiuses from bogie joint to wheel centre to frame R, similarly with D_{1i}. The first element d_7 and the last one d_8 are the offset distances in z direction and x direction in R. The second one represents there is no effect on wheel velocity in y direction causing by the rotation of bogie. The rotation effect can be calculated by multiplying the link rotation rate by its corresponding distance.

M_i is a unit vector in x orientation of frame C_i with respect to frame R and N_i is unit vector in y orientation. ($M_i \cdot \mathrm{R} \cdot \dot{\theta}_i$) represents the wheel center velocity in x direction of frame R. $M_i \cdot \dot{\zeta}_i$ and $N_i \cdot \dot{\eta}_i$ represent the wheel center slip velocity in x and y direction of frame R.

Now we can get the knowledge of the equation (7). Both sides express the wheel center velocity in frame R. As an example of the first raw in (4), the left side states wheel center velocity that is the sum of two parts. One is the rover body velocity $(\dot{\mathrm{X}})$, the other is the velocity causing by body rotation rates $(\mathrm{K}_1 \cdot \dot{\phi}_\mathrm{x} + \mathrm{d}_2 \cdot \dot{\phi}_\mathrm{z})$, the rocker rotation rate $(\text{-}\mathrm{K}_1 \cdot \dot{\beta})$ and bogie rotation rate $((\mathrm{d}_4 \cdot \mathrm{c}(\beta+\rho_1) + \mathrm{a}_4 \cdot \mathrm{s}(\beta+\rho_1)) \cdot \dot{\rho}_1)$. The right side is the sum of velocity components in the x axis of frame R, including rolling rate ($\dot{\theta}_5$), side slip ($\dot{\eta}_5$) and rolling slip rate ($\dot{\zeta}_5$). The components of velocity on both sides are completely identical.

C. Special Character

The kinematic formulation (7) can be easily written out using physical conception. The left side of (7) can be written out according to the parameters of rover structure. After we determine the rover structure we can write out the expression of velocity for each end part (wheel) according to the classical mechanical theories as analyzed before. The velocity is equal to the linear velocity add to the arithmetic product of multiplying angular velocity and corresponding radius. The right side of (7) also can be written out if the transformation matrix $T_{(R,Ci)}$ is given. The transformation matrix can be easily got after we defined the coordinates for the rover and using the D-H notation. If the transformation $T_{(R,Ci)}$ is given, the up corner 3x3 rotation matrix is the unit matrix representing the orientation of coordinate frame C_i with respect to the coordinate frame R. And we have pointed out before that are the unit vectors of frame C_i with respect to frame R. The unit vectors M_i and N_i deduced from individual wheel Jacobin matrix are completely identical with the vectors in transformation matrix $T_{(R,Ci)}$. The vectors both are expressed in frame R.

It is convenient to write out the simple kinematics formulation if the parameters of the rover and transformation matrix are given. We can obtain the simple expression without deducing the complicated differential equations. And we have introduced this method to deduce the kinematic formulation for other vehicles including three-wheeled, four-wheeled mobile robots and rocky 7. The results are completely right.

D. Application in actuation kinematics

The merit of the formulation is the application in inverse solution which we refer as actuation kinematics. The kinematic formulation can be used to determine proper actuation of the wheels.

For the case of actuation kinematics, the desired rover motions such as rover velocity ($\dot{\mathrm{X}}_\mathrm{d}\ \dot{\phi}_\mathrm{zd}$), the initial conditions of the rover including joints angles (β, ρ_i), the contact angles δ_i must be specified or sensed and the slip should be neglected ($\dot{\zeta}_i = \dot{\eta}_i = 0$).

Firstly, we deal with the steering actuation kinematics. Using (7) steering angles can be calculated assumed that the rover moves on a plane ($\beta = \dot{\beta} = \rho_i = \dot{\rho}_i = \delta_i = \dot{\phi}_\mathrm{x} = \dot{\phi}_\mathrm{y} = 0$) and the slip should be neglected ($\dot{\zeta}_i = \dot{\eta}_i = 0$). The result is a initial angle of steering wheel. And the error causing by the rover moving on uneven plate will be discussed later. Steering angles are calculated by simplifying (7).

$$\begin{bmatrix} \dot{\mathrm{X}} + \mathrm{d}_2 \cdot \dot{\phi}_z = \mathrm{c}\psi_1 \cdot \mathrm{R} \cdot \dot{\theta}_1 \\ \dot{\mathrm{Y}} - \mathrm{a}_2 \cdot \dot{\phi}_z = \mathrm{s}\psi_1 \cdot \mathrm{R} \cdot \dot{\theta}_1 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \dot{\mathrm{X}} + \mathrm{d}_2 \cdot \dot{\phi}_z = \mathrm{c}\psi_5 \cdot \mathrm{R} \cdot \dot{\theta}_5 \\ \dot{\mathrm{Y}} - (\mathrm{a}_3 \cdot \mathrm{c(W)} + \mathrm{a}_4) \cdot \dot{\phi}_z = \mathrm{s}\psi_5 \cdot \mathrm{R} \cdot \dot{\theta}_5 \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \dot{\mathrm{X}} - \mathrm{d}_2 \cdot \dot{\phi}_z = \mathrm{c}\psi_2 \cdot \mathrm{R} \cdot \dot{\theta}_2 \\ \dot{\mathrm{Y}} + \mathrm{a}_2 \cdot \dot{\phi}_z = \mathrm{s}\psi_2 \cdot \mathrm{R} \cdot \dot{\theta}_2 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} \dot{\mathrm{X}} - \mathrm{d}_2 \cdot \dot{\phi}_z = \mathrm{c}\psi_6 \cdot \mathrm{R} \cdot \dot{\theta}_6 \\ \dot{\mathrm{Y}} - (\mathrm{a}_3 \cdot \mathrm{c(W)} + \mathrm{a}_4) \cdot \dot{\phi}_z = \mathrm{s}\psi_6 \cdot \mathrm{R} \cdot \dot{\theta}_6 \end{bmatrix} \tag{11}$$

Equation (8) and (11) both having three unknowns can be solved if $\dot{\mathrm{Y}}$ has been assumed. If we make such assumption $\dot{\mathrm{Y}} = 0$, the steering angle can be calculated. Then, if the assumption is adopted there will be a contradiction for wheel 3 and wheel 4.

$$\begin{array}{ll} \dot{\mathrm{Y}} + \mathrm{K}_2 \cdot \dot{\phi}_z = \dot{\eta}_3 & \text{(wheel 3)} \\ \dot{\mathrm{Y}} + \mathrm{K}_2^{'} \cdot \dot{\phi}_z = \dot{\eta}_4 & \text{(wheel 4)} \end{array} \tag{12}$$

Equation (12) will contradict if we make the assumptions of $\dot{\mathrm{Y}} = 0$ and $\dot{\zeta}_i = \dot{\eta}_i = 0$ ($\mathrm{K}_2 \neq \mathrm{K}_2^{'}$). So we assume that, $\dot{\mathrm{Y}} = 0$, $\dot{\eta}_3 = \dot{\eta}_4 \neq 0$ and $\dot{\eta}_1 = \dot{\eta}_2 = \dot{\eta}_5 = \dot{\eta}_6 = 0$. The point we done such assumption is that middle two wheels without steering axle can't make change to fit the environment but the front and rear steering wheels can fit the field by steering. We define the increment of steering angle $\Delta\psi_i$ for each steering wheel to modify the result got from (8)-(11). Substituting the parameters to the elements of (7) the result of $M_i \cdot \mathrm{R}\dot{\theta}_i$ is

$$\dot{V} + D_{1i}\dot{\Phi} + D_{1i}\dot{\beta} + D_{2i}\dot{\rho} = M_{i0} \cdot \mathrm{R}\dot{\theta}_\mathrm{i} + N_i \mathrm{c}\delta_\mathrm{i} \cdot \Delta\psi_\mathrm{i} \cdot \mathrm{R}\dot{\theta} \tag{13}$$

(for i=1, 3, 4 and 6)

$$\begin{pmatrix} \mathrm{c}\psi_i = \mathrm{c}(\psi_{i0} + \Delta\psi_i) = \mathrm{c}\psi_{i0} - \mathrm{s}\psi_{i0} \cdot \Delta\psi_i \\ \mathrm{s}\psi_i = \mathrm{s}(\psi_{i0} + \Delta\psi_i) = \mathrm{s}\psi_{i0} + \mathrm{c}\psi_{i0} \cdot \Delta\psi_i \end{pmatrix}$$

The angle of $\psi_{\mathrm{i}0}$ is the angle calculated by (8)-(11). The initial angle of $\psi_{\mathrm{i}0}$ can be modified by $\Delta\psi_\mathrm{i}$. In the next section of simulation we will modified the $\psi_{\mathrm{i}0}$ by adding $\Delta\psi_\mathrm{i}$

repeatedly during each time-interval until $\Delta\psi_i$ less than given number.

In (7) each wheel has three equations and six wheels will build a system of equations with eighteen equations. For actuation the system has eighteen unknown quantities including rover linear velocity and rotate rate ($\dot{Z},\dot{\phi}_x,\dot{\phi}_y$), wheel rolling rate ($\dot{\theta}_{1...}\dot{\theta}_6$), rocker and bogie rotate rate ($\dot{\beta},\dot{\rho}_1,\dot{\rho}_2$),wheel 3, 4 slip rate ($\dot{\eta}_3,\dot{\eta}_4$), and four increments for steering angle ($\Delta\psi_i, i=1,2,5,6$). $\dot{X},\dot{Y},\dot{\phi}_z$ and the initial steering angles ($\psi_{10},\psi_{20},\psi_{50},\psi_{60}$) are known conditions. So the system of equations can be solved to find unique arithmetic solution including actuating rolling rate ($\dot{\theta}_{1...}\dot{\theta}_6$) and steering angle ($\psi_1,\psi_2,\psi_5,\psi_6$) to accomplish the desired body motion ($\dot{X},\dot{Y},\dot{\phi}_z$) Note that steering angles ψ_i can be obtained by adding $\Delta\psi_i$ to ψ_{i0} . In addition, the shape of terrain should be known. The effect of contact angle (δ_i) and how to thinking about it will be discussed in future. We can calculate all values including joints angles and rates, rolling rate, rover rotate angles and rates. We can use the results to estimate the rover orientation and configuration changing.

Ⅳ. Simulation And Results

We design a simulation of rover motion over a special terrain. Such a terrain is designed including two inclined plates staggered arrangement with the same slope, as shown in Fig.6. The rover climb the inclined plate straight without any turning ($\dot{\phi}_z = 0$). We are concerned here only the wheel rolling rate, steering angle and configuration changing. not surface conditions and the rover position.

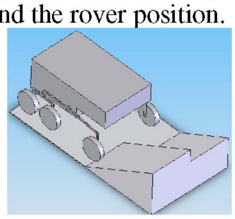


Fig.6 Rover-terrain interaction model

We determine the initial values of rover joints angles ($\beta = \rho_i = 0$). $\delta_i (i = 2,3,4,5,6) = 0$ and $\delta_1 = -\mathrm{pi}/6$. For initial state wheel 1 touches the slope. Here we just give the result of thinking about such terrain. When wheel i touch the inclined plated the changing angle ($\Delta\delta_i$) of the slope will be add to the previous angle (δ_{i0}). The rover velocity ($\dot{X} = 0.05 \quad \dot{Y} = 0$) and yaw rate ($\dot{\phi}_z = 0$) are constants during the simulation. At each step the computation result can be used to calculate joints angles (β,ρ_i), contact angle (δ_i) and steering angle (ψ_i) both needed for the next step. The following is the results of simulation and analysis about it.

Fig. 7 shows the contact angle of each wheel during the simulation. The angles occur abrupt changes when the wheels touch the inclined plate. Each contact angle trends to zero after he climbed on the slope. And the contact angles should be zeros when all the wheels climb on the slopes. The result has calculation error.

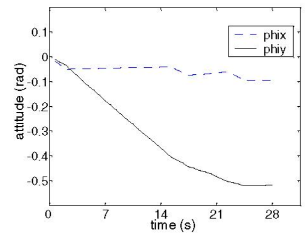


Fig. 8 Rover roll and pitch angles

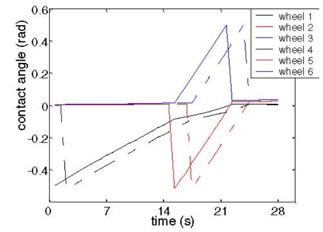


Fig.7 Contact angles

Fig. 8 shows the rover roll and pitch angles as the results of combined motion of six wheels. When the rover climb, the pitch angle trends to slope angle of the plate (-pi/6=0.52rad) and the roll angle is smaller because the perpendicular fall (L) between the two inclined plated is smaller with respect to rover body width (W) of the rover (-tan(roll) $\approx$ L/W). Rover pitch and roll reach the maximum when the rover entirely moves onto inclined plate.

Fig. 9 shows the rover joint angles including the rocker angle (β), and two bogie angles (ρ_1,ρ_2) . The rocker angle does not change so much because the perpendicular fall of the two inclined plated is not so big with respect to the width of rover body. But the bogie angles changes bigger when the rover climbs because bogie dimension is relative smaller. The angles trend to zero at last because the rover reconfigures to the initial states.

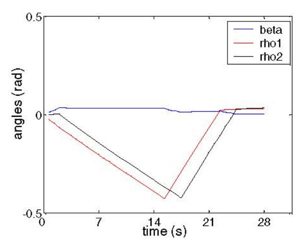


Fig.9 Rover joint angles

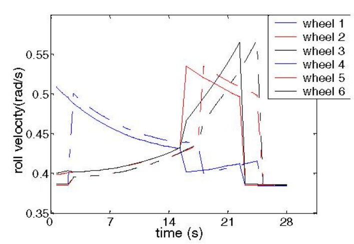


Fig.10 Wheel rotation velocity

Fig. 10 shows the rotation velocities of the wheels to accomplish the desired body motion. Abrupt changing takes place when the wheel touched the slope. The reason is there is no transitional surface between flat and slope surfaces, and contact angles are not continuous due to abrupt change of contact points between the wheel and terrain.

Fig.11 shows the steering angle changing during climbing which has bigger value when wheel touch the slopes. Similar, the abrupt change occurs due to no transitional surface and abrupt changing of contact angles.

The results of the simulation validate assumptions and analysis about the actuation of the rover over such terrain.

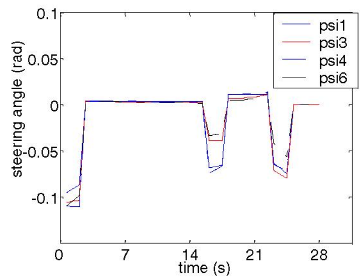


Fig. 11 Rover steering angles

V. CONCLUSION

A new form kinematics has presented for developing kinematic models of wheeled mobile robots. The brevity of the new formulation deduced from the individual wheel Jacobian matrix gives visual representation of the physical conception about the parameters defined by the configure of the rover. And the superiority is that the formulation can be write out easy if the transformation matrix from rover to the contact point.

The formulation can be used to determine proper actuation of the wheels to achieve desired rover motion. By feasible analysis we can get the unique solution which can help us get the knowledge about the angle or rate of joins and rover. In other words we can do know the changing configure during the motion if rover rate ($\dot{X}, \dot{Y}, \dot{\phi}_z$) and the contact angles (δ_i) are specified.

Finally, there are a number of assumptions at first about the conditions about contact between wheels and terrain. The assumption of a single, rigid, continuous contact will not fit the reality and we will go on investigating to find proper method to make modification. We should make further research on the formulation.

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Appendix

Elements of Jacobian matrix for all the wheels are given here. The followings are for wheel 1, 2. (where $b=(-1)^{i-1}$, i=1,2)

$J_{14} = -bd_3 \cdot c\psi_i \cdot c\beta + ba_2 c\psi_i \cdot s\beta - bd_2 s\psi_i \cdot s\beta$

$J_{15} = -b \cdot d_2 \cdot c\beta \cdot c\delta_i - b \cdot a_2 s\psi_i \cdot s\delta_i \cdot s\beta - b \cdot d_2 c\psi_i \cdot s\delta_i \cdot s\beta + d_3 s\delta_i \cdot s\psi_i \cdot c\beta$

$J_{17} = c\delta_i \cdot c\psi_i \cdot c\beta + bs\delta_i \cdot s\beta \qquad J_{24} = -d_3 \cdot s\psi_i \qquad J_{27} = c\delta_i \cdot s\psi_i$

$J_{25} = -d_3 s\delta_i \cdot c\psi_i - a_2 c\delta_i$

$J_{34} = -b \cdot d_3 \cdot s\beta \cdot c\psi_i - a_2 \cdot c\beta \cdot c\psi_i + bd_2 \cdot c\beta \cdot s\psi_i$

$J_{35} = b \cdot d_3 s\delta_i \cdot s\psi_i \cdot s\beta - d_2 \cdot c\delta_i \cdot s\beta + bd_2 c\psi_i \cdot s\delta_i \cdot c\beta + a_2 s\psi_i \cdot s\delta_i \cdot c\beta$

$J_{37} = b \cdot c\delta_i \cdot c\psi_i \cdot s\beta - c\beta \cdot s\delta_i$

$J_{44} = s\psi_i \cdot c\beta \qquad J_{45} = s\delta_i \cdot c\psi_i \cdot c\beta - b \cdot c\delta_i \cdot s\beta \qquad J_{54} = -c\psi_i$

$J_{55} = s\delta_i \cdot s\psi_i \qquad J_{64} = b \cdot s\psi_i \cdot s\beta \qquad J_{65} = c\delta_i \cdot c\beta + bs\delta_i \cdot c\psi_i \cdot s\beta$

The followings are for wheel 3, 4.
(where $b=(-1)^{i-1}$, j=1 for i=3,j=2 for i=4 (i=3,4))

$J_{12} = a_3 \cdot s(Q - b \cdot \beta) \qquad J_{14} = a_3 \cdot s(b \cdot \beta - Q) + a_4 \cdot s(\rho_j + b \cdot \beta) - d_4 \cdot c(\rho_j + b \cdot \beta)$

$J_{15} = -b \cdot d_2 \cdot c(\rho_j + b \cdot \beta - \delta_2) \qquad J_{17} = c(\rho_j + b \cdot \beta - \delta_i)$

$J_{32} = a_3 \cdot c(Q - b \cdot \beta)$

$J_{25} = -a_3 \cdot c(\rho_j + Q - \delta_i) - a_4 \cdot c\delta_i - d_4 \cdot s\delta_i$

$J_{34} = -a_3 \cdot c(Q - b \cdot \beta) - a_4 \cdot c(b \cdot \beta + \rho_j) - d_4 \cdot s(b \cdot \beta + \rho_j)$

$J_{35} = -b \cdot d_2 \cdot s(\rho_j + b \cdot \beta - \delta_i)$

$J_{37} = s(\rho_j + b \cdot \beta - \delta_i) \qquad J_{45} = -s(\rho_j + b \cdot \beta - \delta_i) \qquad J_{65} = c(\rho_j + b \cdot \beta - \delta_i)$

The followings are for wheel 5,6.
(where $b=(-1)^{i-1}$, j=1 for i=5,j=2 for i=6 (i=5,6))

$J_{12} = a_3 \cdot s(b \cdot \beta - W) \qquad J_{13} = b \cdot d_2 \cdot c(b \cdot \beta + \rho_j) \qquad J_{14} = R \cdot J_{18}$

$J_{17} = -s\psi_i \cdot c(b \cdot \beta + \rho_j)$

$J_{15} = a_3 c\psi_i \cdot s(W + \rho_j) \cdot c(b\beta + \rho_j) - d_4 c\psi_i \cdot c(b\beta + \rho_j) - bd_2 s\psi_i \cdot s(b\beta + \rho_j) - a_3 c\psi_i \cdot c(W + \rho_j) \cdot s(b\beta + \rho_j) - a_4 c\psi_i \cdot s(b\beta + \rho_j)$

$J_{16} = a_3 s\psi_i \cdot s\delta_i \cdot s(b\beta - W) - bd_2 c\delta_i \cdot c(b\beta + \rho_j) - bd_2 s\delta_i \cdot c\psi_i \cdot s(b\beta + \rho_j) + (a_4 s(b\beta + \rho_j) + d_4 c(b\beta + \rho_j)) s\delta_i \cdot s\psi_i$

$J_{18} = c\delta_i \cdot c\psi_i \cdot c(b\beta + \rho_j) + s\delta_i \cdot s(b\beta + \rho_j) \qquad J_{23} = -a_3 \cdot c(W + \rho_j) - a_4$

$J_{24} = R \cdot c\delta_i \cdot s\psi_i \qquad J_{25} = a_3 s\psi_i \cdot s(W + \rho_j) - d_4 s\psi_i \qquad J_{27} = c\psi_i$

$J_{26} = a_3 c\delta_3 \cdot c(W + \rho_j) + a_4 c\delta_i + a_3 c\psi_i \cdot s\delta_i \cdot s(W + \rho_j) - d_4 c\psi_i \cdot s\delta_i$

$J_{28} = c\delta_i \cdot s\psi_i \qquad J_{32} = -a_3 \cdot c(W - b\beta) \qquad J_{33} = bd_2 \cdot s(b\beta + \rho_j)$

$J_{34} = R \cdot (c\delta_i \cdot c\psi_i \cdot s(b\beta + \rho_j) - s\delta_i \cdot c(b\beta + \rho_j))$

$J_{35} = a_3 c\psi_i \cdot s(W + \rho_j) \cdot s(b\beta + \rho_j) - d_4 c\psi_i \cdot s(b\beta + \rho_j) + a_3 c\psi_i \cdot c(W + \rho_j) \cdot c(b\beta + \rho_j) + a_4 c\psi_i \cdot c(b\beta + \rho_j) + bd_2 s\psi_i \cdot c(b\beta + \rho_j)$

$J_{36} = -bd_2 c\delta_i \cdot s(b\beta + \rho_j) - a_3 s\psi_i \cdot s\delta_i \cdot c(b\beta - W) + bd_2 s\delta_i \cdot c\psi_i \cdot c(b\beta + \rho_j) + (-a_4 c(b\beta + \rho_j) + d_4 s(b\beta + \rho_j)) s\delta_i \cdot s\psi_i$

$J_{38} = c\psi_i \cdot c\delta_i \cdot s(b\beta + \rho_j) - s\delta_i \cdot c(b\beta + \rho_j) \qquad J_{43} = s(b\beta + \rho_j)$

$J_{45} = s\psi_i \cdot c(b\beta + \rho_j) \qquad J_{46} = s\delta_i \cdot c\psi_i \cdot c(b\beta + \rho_j) - c\delta_i \cdot s(b\beta + \rho_j)$

$J_{55} = -c\psi_i \qquad J_{56} = s\psi_i \cdot s\delta_i \qquad J_{63} = -c(b\beta + \rho_j)$

$J_{65} = s\psi_i \cdot s(b\beta + \rho_j) \qquad J_{66} = c(b\beta + \rho_j) \cdot c\delta_i + s\delta_i \cdot c\psi_i \cdot s(b\beta + \rho_j)$