# Linear Algebra: Coursera

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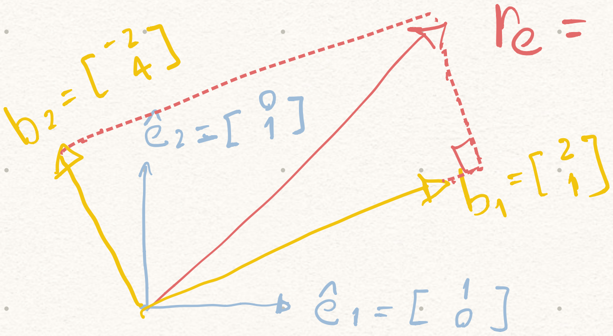
## Week 1 - Changing basis on vectors

ê-basis



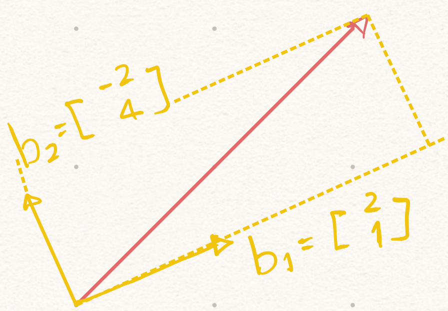

$$r_e = 3\hat{e}_1 + 4\hat{e}_2 = \begin{bmatrix}3\\4\end{bmatrix}$$

$$r_{\hat{e}} = 3\begin{bmatrix}1\\0\end{bmatrix} + 4\begin{bmatrix}0\\1\end{bmatrix}$$

- So r in b-basis

$$r_b = ?$$

- So changing basis:




Remember proyection:

1) $$Proy_v U = \frac{(U \cdot V)}{|V|^2} V$$

2) Escalar

$$\frac{U \cdot V}{|V|^2}$$

$$\frac{r_e \cdot b_1}{|b_1|^2} = \frac{3\cdot 2 + 4\cdot 1}{(\sqrt{4+1})^2} = \frac{6+4}{5} = \frac{10}{5} = 2$$

$$Proy_{b_1} r_e = 2 b_1 = 2\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix}$$

$$\frac{r_e \cdot b_2}{|b_2|^2} = \frac{3\cdot -2 + 4\cdot 4}{(\sqrt{4+16})^2}$$

$$= \frac{-6+16}{20} = \frac{10}{20} = \frac{1}{2}$$

$$\mathrm{Proj}_{b_2}\, r_e = \frac{1}{2}\, b_2 = \frac{1}{2}\begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

In summary:

$$r_{\hat{b}_1} + r_{\hat{b}_2} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\boxed{\begin{aligned} r_b &= \begin{bmatrix} 2 \\ 1/2 \end{bmatrix} \\ &= 2\,\hat{b}_1 + \frac{1}{2}\,\hat{b}_2 \end{aligned}}$$

# Cuestionario

1.) $v = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$, $b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$b_1 \perp b_2$

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(1) $$\frac{v \cdot b_1}{|b_1|^2} = \frac{5 \cdot 1 + -1 \cdot 1}{(\sqrt{2})^2} = \frac{5-1}{2} = \frac{4}{2} = 2$$

$$\text{Proy}_{b_1} v = 2 b_1 = 2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(2) $$\frac{v \cdot b_2}{|b_2|^2} = \frac{5 \cdot 1 + -1 \cdot -1}{(\sqrt{2})^2} = \frac{5+1}{2} = \frac{6}{2} = 3$$

$$\text{Proy}_{b_2} v = 3 b_2$$

$$v_{\hat{b}} = 2 \hat{b_1} + 3 b_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

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2.) $v = \begin{bmatrix} 10 \\ -5 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $b_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

$$\frac{v \cdot b_1}{|b_1|^2} = \frac{10 \cdot 3 + -5 \cdot 4}{(\sqrt{9+16})^2} = \frac{30-20}{25} = \frac{10}{25} = \frac{2}{5}$$

$\frac{2}{5} b_1$

$$\frac{v \cdot b_2}{|b_2|^2} = \frac{10 \cdot 4 + -5 \cdot -3}{(\sqrt{16+9})^2} = \frac{40+15}{25} = \frac{55}{25}$$

$$= \frac{11}{5} \rightarrow \frac{11}{5} b_2$$

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3.) $v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, b_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$$\frac{v \cdot b_1}{|b_1|^2} = \frac{2 \cdot -3 + 2 \cdot 1}{(\sqrt{9+1})^2} = \frac{-6+2}{10} = \frac{-4}{10} = -\frac{2}{5}$$

$-\frac{2}{5} b_1$

$$\frac{v \cdot b_2}{|b_2|^2} = \frac{2 \cdot 1 + 2 \cdot 3}{(\sqrt{10})^2} = \frac{8}{10} = \frac{4}{5}$$

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4.) $v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$

$b_3 = \begin{bmatrix} -1 \\ 2 \\ -5 \end{bmatrix}$

$$\frac{v \cdot b_1}{|b_1|^2} = \frac{1 \cdot 2 + 1 \cdot 1 + 1 \cdot 0}{(\sqrt{4+1+0})^2} = \frac{3}{5} \Rightarrow \frac{3}{5} b_1$$

$|b_1|$

5.) Given vectors

$$V = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0 \\ 2 \\ -1 \\ 0 \end{bmatrix}$$

$$b_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \quad b_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\frac{V \cdot b_1}{|b_1|^2} = \frac{1 \cdot 1 + 0}{1} = 1 \Rightarrow 1 \cdot b_1$$

$$\frac{V \cdot b_2}{|b_2|^2} = \frac{\cancel{1 \cdot 0} + 1 \cdot 2 + 2 \cdot -1 + \cancel{3 \cdot 0}}{(\sqrt{4+1})^2} = \frac{2-2}{5} = 0$$