PROYECTO AUTOMATAS

Juan Pablo Sibecas
juan.sibecas@gmail.com
Matias Gaviño
matias.linares.g@gmail.com
Autómatas y Control Discreto, Facultad de Ingeniería,
Universidad Nacional de Cuyo,
Mendoza, Argentina

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Resumen

1. Introducción

2. Desarrollo

2.1. Modelo del Sistema Físico

2.1.1. Subsistema de Izaje

Segunda ley de Newton del lado tambor:

$$J_{hd+hEb}\frac{d\omega_{hd}}{dt} = T_{hd}(t) + T_{hEb}(t) - b_{hd}\omega_{hd}(t) - T_{hdl}(t) \tag{1}$$

Segunda ley de Newton del lado motor:

$$J_{hm+hb}\frac{d\omega_{hm}}{dt} = T_{hm}(t) + T_{hb}(t) - b_{hm}\omega_{hm}(t) - T_{hml}(t) \tag{2}$$

relacion de transmision

$$i_h = \frac{\omega_{hm}(t)}{\omega_{hd}(t)} = \frac{T_{hd}(t)}{T_{hml}(t)} \tag{3}$$

si reemplazo 3 en 2 y despejo $T_{hd}(t)$

$$T_{hd}(t) = J_{hm+hb}\frac{d\omega_{hd}}{dt}{i_h}^2 - b_{hm}\omega_{hd}(t){i_h}^2 + i_h(T_{hm}(t) + T_{hb}(t)) \tag{4}$$

reemplazando en 1 y operando se obtiene

$$(J_{hd+hEb} + J_{hm+hb}i_h^2)\frac{d\omega_{hd}}{dt} = -(b_{hd} + b_{hm}i_h^2)\omega_{hd}(t) + i_h(T_{hm}(t) + T_{hb}(t)) + T_{hEb}(t) - T_{hdl}(t) \tag{5}$$

como $T_{hdl}(t) = F_{hw}(t) * r_{hd}$, $2V_h = r_{hd} * \omega_{hd}(t)$ y $V_h = -\frac{dl_h(t)}{dt}$ y dividiendo por r_{hd}:

$$2\frac{(J_{hd+hEb} + J_{hm+hb}i_h^2)}{r_{hd}^2}\frac{d^2l_h(t)}{dt^2} = -2\frac{(b_{hd} + b_{hm}i_h^2)}{r_{hd}^2}\frac{dl_h(t)}{dt} - \frac{i_h}{r_{hd}}(T_{hm}(t) + T_{hb}(t)) - \frac{T_{hEb}(t)}{r_{hd}} + F_{hw}(t) \tag{6}$$

Reemplazando por parametros equivalentes:

$$M_{Eh}\ddot{l_h}(t) = -b_{Eh}\dot{l_h}(t) - \frac{i_h}{r_{hd}}(T_{hm}(t) + T_{hb}(t)) - \frac{T_{hEb}(t)}{r_{hd}} + F_{hw}(t) \tag{7}$$

Donde

$$M_{Eh} = 2\frac{(J_{hd+hEb} + J_{hm+hb}i_h^2)}{r_{hd}^2} \tag{8}$$

$$b_{Eh} = 2\frac{(b_{hd} + b_{hm}i_h^2)}{r_{hd}^2} \tag{9}$$

$$\tag{10}$$

2.1.2. Subsistema Carro

Segunda ley de Newton del lado tambor:

$$J_{td}\frac{d\omega_{td}(t)}{dt} = T_{td}(t) - b_{td}\omega_{td}(t) - T_{tdl}(t) \tag{11}$$

Segunda ley de Newton del lado motor:

$$J_{tm+tb}\frac{d\omega_{tm}(t)}{dt} = T_{tm}(t) + T_{tb}(t) - b_{tm}\omega_{tm}(t) - T_{tml}(t) \tag{12}$$

relacion de transmision

$$i_t = \frac{\omega_{tm}(t)}{\omega_{td}(t)} = \frac{T_{td}(t)}{T_{tml}(t)} \tag{13}$$

si reemplazo 13 en 12 y despejo $T_{td}(t)$

$$T_{td}(t) = J_{tm+tb}\frac{d\omega_{td}(t)}{dt}{i_t}^2 - b_{tm}\omega_{td}(t){i_t}^2 + i_t(T_{tm}(t) + T_{tb}(t)) \tag{14}$$

Reemplazo 14 en 11 y reordeno:

$$(J_{td} + J_{tm+tb} * i_t^2)\frac{d\omega_{td}(t)}{dt} = i_t(T_{tm}(t) + T_{tb}(t)) - (b_{td} + b_{tm}{i_t}^2)\omega_{td}(t) - T_{tdl}(t) \tag{15}$$

Como $\omega_{td}(t)r_{td} = V_{td}(t)$, $F_{tw}(t)r_{td} = T_{tdl}(t)$ y $V_{td}(t) = \frac{dx_{td}}{dt}$ y dividiendo por r_{td}:

$$\frac{(J_{td} + J_{tm+tb} * i_t^2)}{r_{td}^2}\frac{d^2x_{td}(t)}{dt^2} = -\frac{(b_{td} + b_{tm}{i_t}^2)}{r_{td}^2}\frac{dx_{td}(t)}{dt} + \frac{i_t}{r_{td}}(T_{tm}(t) + T_{tb}(t)) - F_{tw}(t) \tag{16}$$

Reemplazando por parametros equivalentes se obtiene la ecuacion del tambor del subsistema carro:

$$M_{Etd}\ddot{x_{td}}(t) = -b_{Etd}\dot{x_{td}}(t) + \frac{i_t}{r_{td}}(T_{tm}(t) + T_{tb}(t)) - F_{tw}(t) \tag{17}$$

La ecuacion de movimiento del carro es:

$$M_t\ddot{x}_t(t) = -b_t\dot{x}_t(t) + F_{tw}(t) + 2F_{hw}(t)\sin\theta_l(t) \tag{18}$$

Y la fuerza transmitida por el cable del subsistema carro es:

$$F_{tw}(t) = K_{tw}(x_{td}(t) - x_t(t)) + b_{tw}(\dot{x_{td}}(t) - \dot{x}_t(t)) \tag{19}$$

seria un sistema acoplado? preguntar si se resuelve asi

2.2. Diseño del controlador

$$T'_m(t) = b_a e_\omega(t) + K_{sa}e_\theta(t) + K_{sia}\int e_\theta(t)dt \tag{20}$$

Por lo tanto, por Laplace:

$$T_m(s) = G(s)[b_a E_\omega(s) + K_{sa}\frac{1}{s} + K_{sia}\frac{1}{s^2}]E_\theta(s) \tag{21}$$

Donde $G_T(s)$ es la función de transferencia del modulador de torque que, como se supone ideal, es igual a 1.

Para obtener la expresión que nos permita obtener las constante que definen al controlador se remplaza la ecuacion 20 en la ecuacion de movimiento del izaje y del carro, se obtiene: Para el izaje, reemplazando 20 en 7 y transformandola con Laplace, se obtiene:

$$M_{Eh}\ddot{L_h}(s) = -b_{Eh}sL_h(s) - \frac{i_h}{r_{hd}}[G(s)[b_a E_\omega(s) + K_{sa}\frac{1}{s} + K_{sia}\frac{1}{s^2}]E_\theta(s)] + F_{hw}(s) \tag{22}$$

despejando

2.2.1. Control de balanceo

Acontinuación se derivan las ecuaciones que modelan el sistema carro-pendulo. Se utilizará el metodo de Lagrange definiendo las cordenadas generalizadas x_t, θ y l. Donde x_t es la posición del carro, θ es el angulo del pendulo respecto a la vertical y l es la longitud del pendulo.

$$K = K_t + K_{lx} + K_{ly} \tag{23}$$

$$x_l = x_t + l\sin\theta \tag{24}$$

$$\dot{x_l} = \dot{x_t} + l\cos\theta\dot{\theta} + \dot{l}\sin\theta \tag{25}$$

$$y_l = Y_{t0} - l\cos\theta \tag{26}$$

$$\dot{y_l} = l\sin\theta\dot{\theta} \tag{27}$$

$$K = \frac{1}{2}m_t\dot{x_t}^2 + \frac{1}{2}m_l\dot{x_l}^2 + \frac{1}{2}m_l\dot{y_l}^2 \tag{28}$$

$$K = \frac{1}{2}m_t\dot{x_t}^2 + \frac{1}{2}m_l(\dot{x_t} + l\cos\theta\dot{\theta} + \dot{l}\sin\theta)^2 + \frac{1}{2}m_l(l\sin\theta\dot{\theta})^2 \tag{29}$$

$$\begin{aligned} K &= \frac{1}{2}m_t\dot{x_t}^2 \\ &+ \frac{1}{2}m_l(\dot{x_t}^2 + l^2\cos^2\theta\dot{\theta}^2 + \dot{l}^2\sin^2\theta + 2l\dot{x}_t\cos\theta\dot{\theta} + 2l\dot{l}\sin\theta\cos\theta\dot{\theta} + 2\dot{l}\dot{x}_t\sin\theta) \\ &+ \frac{1}{2}m_l l^2\sin^2\theta\dot{\theta}^2 \end{aligned} \tag{30}$$

$$U = -m_l g l\cos\theta \tag{31}$$

$$L = K - U \tag{32}$$

$$\begin{aligned} L &= \frac{1}{2}m_t\dot{x_t}^2 \\ &+ \frac{1}{2}m_l(\dot{x_t}^2 + l^2\cos^2\theta\dot{\theta}^2 + \dot{l}^2\sin^2\theta + 2l\dot{x}_t\cos\theta\dot{\theta} + 2l\dot{l}\sin\theta\cos\theta\dot{\theta} + 2\dot{l}\dot{x}_t\sin\theta) \\ &+ \frac{1}{2}m_l l^2\sin^2\theta\dot{\theta}^2 + m_l g l\cos\theta \end{aligned} \tag{33}$$

$$\begin{aligned} L = \frac{1}{2}m_t\dot{x_t}^2 + \frac{1}{2}m_l\dot{x_t}^2 + \frac{1}{2}m_l l^2\cos^2\theta\dot{\theta}^2 + \frac{1}{2}m_l\dot{l}^2\sin^2\theta + m_l\dot{x}_t l\cos\theta\dot{\theta} + m_l\dot{l}l\sin\theta\cos\theta\dot{\theta} \\ + m_l\dot{l}\dot{x}_t\sin\theta + \frac{1}{2}m_l l^2\sin^2\theta\dot{\theta}^2 + m_l g l\cos\theta \end{aligned} \tag{34}$$

Se define el sistema de ecuaciones de Euler-Lagrange:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{35}$$

Para $q_i = x_t$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_t}\right) - \frac{\partial L}{\partial x_t} = Q_t \tag{36}$$

$$\frac{\partial L}{\partial \dot{x}_t} = (m_t + m_l)\dot{x}_t + m_l l\cos\theta\dot{\theta} + m_l\dot{l}\sin\theta \tag{37}$$

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_t}\right) = \\ &(m_t + m_l)\ddot{x}_t + m_l\dot{l}\cos\theta\dot{\theta} + m_l l\cos\theta\ddot{\theta} - m_l l\sin\theta\dot{\theta}^2 + m_l\ddot{l}\sin\theta + m_l\dot{l}\cos\theta\dot{\theta} \end{aligned} \tag{38}$$

$$\frac{\partial L}{\partial x_t} = 0 \tag{39}$$

Entonces:

$$(m_t + m_l)\ddot{x}_t + m_l\dot{l}\cos\theta\dot{\theta} + m_l l\cos\theta\ddot{\theta} - m_l l\sin\theta\dot{\theta}^2 + m_l\ddot{l}\sin\theta + m_l\dot{l}\cos\theta\dot{\theta} = F_t(t) - b_{eqt}\dot{x}_t \tag{40}$$

Para $q_i = \theta$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = Q_\theta \tag{41}$$

$$\frac{\partial L}{\partial \dot{\theta}} = m_l l^2 \cos\theta\dot{\theta} + m_l\dot{x}_t l\cos\theta + m_l l\dot{l}\sin\theta\cos\theta + m_l l^2\sin^2\theta\dot{\theta} \tag{42}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \tag{43}$$

3. Resultados

4. Conclusión