# PROYECTO AUTOMATAS

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**Resumen**

# 1. Introducción

# 2. Desarrollo

## 2.1. Modelo del Sistema Físico

### 2.1.1. Subsistema de Izaje

Segunda ley de Newton del lado tambor:

$$J_{hd+hEb}\frac{d\omega_{hd}}{dt} = T_{hd}(t) + T_{hEb}(t) - b_{hd}\omega_{hd}(t) - T_{hdl}(t) \tag{1}$$

Segunda ley de Newton del lado motor:

$$J_{hm+hb}\frac{d\omega_{hm}}{dt} = T_{hm}(t) + T_{hb}(t) - b_{hm}\omega_{hm}(t) - T_{hml}(t) \tag{2}$$

relacion de transmision

$$i_h = \frac{\omega_{hm}(t)}{\omega_{hd}(t)} = \frac{T_{hd}(t)}{T_{hml}(t)} \tag{3}$$

si reemplazo 3 en 2 y despejo $T_{hd}(t)$

$$T_{hd}(t) = J_{hm+hb}\frac{d\omega_{hd}}{dt}{i_h}^2 - b_{hm}\omega_{hd}(t){i_h}^2 + i_h(T_{hm}(t) + T_{hb}(t)) \tag{4}$$

reemplazando en 1 y operando se obtiene

$$(J_{hd+hEb} + J_{hm+hb}i_h^2)\frac{d\omega_{hd}}{dt} = -(b_{hd} + b_{hm}i_h^2)\omega_{hd}(t) + i_h(T_{hm}(t) + T_{hb}(t)) + T_{hEb}(t) - T_{hdl}(t) \tag{5}$$

como $T_{hdl}(t) = F_{hw}(t) * r_{hd}$, $2V_h = r_{hd} * \omega_{hd}(t)$ y $V_h = -\frac{dl_h(t)}{dt}$ y dividiendo por $r_{hd}$:

$$2\frac{(J_{hd+hEb} + J_{hm+hb}i_h^2)}{r_{hd}^2}\frac{d^2l_h(t)}{dt^2} = -2\frac{(b_{hd} + b_{hm}i_h^2)}{r_{hd}^2}\frac{dl_h(t)}{dt} - \frac{i_h}{r_{hd}}(T_{hm}(t) + T_{hb}(t)) - \frac{T_{hEb}(t)}{r_{hd}} + F_{hw}(t) \tag{6}$$

Reemplazando por parametros equivalentes:

$$M_{Eh}\ddot{l_h}(t) = -b_{Eh}\dot{l_h}(t) - \frac{i_h}{r_{hd}}(T_{hm}(t) + T_{hb}(t)) - \frac{T_{hEb}(t)}{r_{hd}} + F_{hw}(t) \tag{7}$$

Donde

$$M_{Eh} = 2\frac{(J_{hd+hEb} + J_{hm+hb}i_h^2)}{r_{hd}^2} \tag{8}$$

$$b_{Eh} = 2\frac{(b_{hd} + b_{hm}i_h^2)}{r_{hd}^2} \tag{9}$$

$$\tag{10}$$

### 2.1.2. Subsistema Carro

Segunda ley de Newton del lado tambor:

$$J_{td}\frac{d\omega_{td}(t)}{dt} = T_{td}(t) - b_{td}\omega_{td}(t) - T_{tdl}(t) \tag{11}$$

Segunda ley de Newton del lado motor:

$$J_{tm+tb}\frac{d\omega_{tm}(t)}{dt} = T_{tm}(t) + T_{tb}(t) - b_{tm}\omega_{tm}(t) - T_{tml}(t) \tag{12}$$

relacion de transmision

$$i_t = \frac{\omega_{tm}(t)}{\omega_{td}(t)} = \frac{T_{td}(t)}{T_{tml}(t)} \tag{13}$$

si reemplazo 13 en 12 y despejo $T_{td}(t)$

$$T_{td}(t) = J_{tm+tb}\frac{d\omega_{td}(t)}{dt}{i_t}^2 - b_{tm}\omega_{td}(t){i_t}^2 + i_t(T_{tm}(t) + T_{tb}(t)) \tag{14}$$

Reemplazo 14 en 11 y reordeno:

$$(J_{td} + J_{tm+tb} * i_t^2)\frac{d\omega_{td}(t)}{dt} = i_t(T_{tm}(t) + T_{tb}(t)) - (b_{td} + b_{tm}{i_t}^2)\omega_{td}(t) - T_{tdl}(t) \tag{15}$$

Como $\omega_{td}(t)r_{td} = V_{td}(t)$, $F_{tw}(t)r_{td} = T_{tdl}(t)$ y $V_{td}(t) = \frac{dx_{td}}{dt}$ y dividiendo por $r_{td}$:

$$\frac{(J_{td} + J_{tm+tb} * i_t^2)}{r_{td}^2}\frac{d^2x_{td}(t)}{dt^2} = -\frac{(b_{td} + b_{tm}{i_t}^2)}{r_{td}^2}\frac{dx_{td}(t)}{dt} + \frac{i_t}{r_{td}}(T_{tm}(t) + T_{tb}(t)) - F_{tw}(t) \tag{16}$$

Reemplazando por parametros equivalentes se obtiene la ecuacion del tambor del subsistema carro:

$$M_{Etd}\ddot{x_{td}}(t) = -b_{Etd}\dot{x_{td}}(t) + \frac{i_t}{r_{td}}(T_{tm}(t) + T_{tb}(t)) - F_{tw}(t) \tag{17}$$

La ecuacion de movimiento del carro es:

$$M_t\ddot{x}_t(t) = -b_t\dot{x}_t(t) + F_{tw}(t) + 2F_{hw}(t)\sin\theta_l(t) \tag{18}$$

Y la fuerza transmitida por el cable del subsistema carro es:

$$F_{tw}(t) = K_{tw}(x_{td}(t) - x_t(t)) + b_{tw}(\dot{x_{td}}(t) - \dot{x}_t(t)) \tag{19}$$

seria un sistema acoplado? preguntar si se resuelve asi

## 2.2. Diseño del controlador

$$T_m'(t) = b_a e_\omega(t) + K_{sa}e_\theta(t) + K_{sia}\int e_\theta(t)dt \tag{20}$$

Por lo tanto, por Laplace:

$$T_m(s) = G(s)[b_a E_\omega(s) + K_{sa}\frac{1}{s} + K_{sia}\frac{1}{s^2}]E_\theta(s) \tag{21}$$

Donde $G_T(s)$ es la función de transferencia del modulador de torque que, como se supone ideal, es igual a 1.

Para obtener la expresión que nos permita obtener las constante que definen al controlador se remplaza la ecuacion 20 en la ecuacion de movimiento del izaje y del carro, se obtiene: Para el izaje, reemplazando 20 en 7 y transformandola con Laplace, se obtiene:

$$M_{Eh}\ddot{L_h}(s) = -b_{Eh}sL_h(s) - \frac{i_h}{r_{hd}}[G(s)[b_a E_\omega(s) + K_{sa}\frac{1}{s} + K_{sia}\frac{1}{s^2}]E_\theta(s)] + F_{hw}(s) \tag{22}$$

despejando

### 2.2.1. Control de balanceo

Se deducen las ecuaciones de movimiento del sistema carro-péndulo, se obtiene:
Planteando el equilibrio dinámico de los torques en el anclaje del péndulo:

$$\sum \tau = I\ddot{\theta} \tag{23}$$

$$ml^2\ddot{\theta} = -mg\sin\theta + m\cos\theta\ddot{x}_t \tag{24}$$

despejando $\ddot{\theta}$:

$$\ddot{\theta} = \frac{\cos\theta\ddot{x}_t}{l} - \frac{g\sin\theta}{l} \tag{25}$$

También:

$$x_l = \sin(\theta)l + x_t \tag{26}$$

$$\dot{x}_l = \cos(\theta)\dot{\theta}l + \dot{x}_t \tag{27}$$

Se definen el vector de estado como:

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \tag{28}$$

$$u = \ddot{x}_t \tag{29}$$

$$y = \dot{x}_l \tag{30}$$

Por lo tanto se expresa el modelo del sistema en el espacio de estados no lineal:

$$\begin{cases} \dot{x} = f(x,u,t); x(0) = x_0 \\ y = h(x,u,t) \end{cases} \tag{31}$$

Donde:

$$f(x,u,t) = \begin{bmatrix} \dot{\theta} \\ \frac{\cos\theta u}{l} - \frac{g\sin\theta}{l} \end{bmatrix} \tag{32}$$

$$h(x,u,t) = \cos\theta\dot{\theta}l \tag{33}$$

Se ignora $\dot{x}_t$ dado que buscaremos el incremendo de velocidad que debemos aplicar al carro para que el péndulo se mantenga en equilibrio.

Se linealiza el sistema en torno a un punto de trabajo $x(t), u(t)$ y se obtiene:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{34}$$

Donde:

$$A_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{x(t),u(t)} \tag{35}$$

$$B_{ij} = \left.\frac{\partial f_i}{\partial u_j}\right|_{x(t),u(t)} \tag{36}$$

$$C_{ij} = \left.\frac{\partial h_i}{\partial x_j}\right|_{x(t),u(t)} \tag{37}$$

$$D_{ij} = \left.\frac{\partial h_i}{\partial u_j}\right|_{x(t),u(t)} \tag{38}$$

Se obtiene:

$$A = \begin{bmatrix} 0 & 1 \\ -\cos\theta\frac{g}{l^2} - \sin\theta\frac{\ddot{x}_t}{l^2} & 0 \end{bmatrix} \tag{39}$$

$$B = \begin{bmatrix} 0 \\ \frac{\cos\theta}{l^2} \end{bmatrix} \tag{40}$$

$$C = \begin{bmatrix} -\sin\theta\dot{\theta}l & \cos\theta l \end{bmatrix} \tag{41}$$

$$D = 0 \tag{42}$$

Se propone un controlador PD para el sistema:

$$u = K_p(y^* - y) + K_d\frac{d}{dt}(y^* - y) \tag{43}$$

$$\dot{x} = Ax + B(K_p(y^* - y) + K_d\frac{d}{dt}(y^* - y)) \tag{44}$$

$$\dot{x} = Ax + B(K_p(y^* - Cx) + K_d\frac{d}{dt}(y^* - Cx)) \tag{45}$$

$$\ddot{\theta} = A_{21}\dot{\theta} + B_2(K_p(\dot{x}_l^* - C_1\theta - C_2\dot{\theta}) + K_d\frac{d}{dt}(\dot{x}_l^* - C_1\theta - C_2\dot{\theta})) \tag{46}$$

$$\ddot{\theta} = A_{21}\theta + B_2K_p\dot{x}_l^* - B_2K_pC_1\theta - B_2K_pC_2\dot{\theta} + \frac{d}{dt}\left(B_2K_d\dot{x}_l^* - B_2K_dC_1\theta - B_2K_dC_2\dot{\theta}\right) \tag{47}$$

Utilizando la transformada de Laplace:

$$s^2\Theta = A_{21}\Theta + B_2K_p\dot{X}_l^* - B_2K_pC_1\Theta - B_2K_pC_2s\Theta + B_2K_d\dot{X}_l^*s - B_2K_dC_1\Theta s - B_2K_dC_2\Theta s^2 \tag{48}$$

Despejando $\theta/\dot{X}_l^*$:

$$\Theta(s^2(B_2K_dC_2) + s(B_2K_pC_2 - B_2K_dC_1) - A_{21} + B_2K_pC_1) = \dot{X}_l^*(B_2K_p + B_2K_ds) \tag{49}$$

$$\frac{\theta}{\dot{X}_l^*} = \frac{B_2K_p + B_2K_ds}{s^2(B_2K_dC_2) + s(B_2K_pC_2 - B_2K_dC_1) - A_{21} + B_2K_pC_1} \tag{50}$$

Se obtinen las constantes $K_p$ y $K_d$ de forma que el denominado de 50 cumpla $s^2 + s2\eta\omega + \omega^2 = 0$

$$\begin{cases} 2\eta\omega = \frac{B_2K_pC_2 - B_2K_dC_1}{B_2K_dC_2} \\ \omega^2 = \frac{A_{21} - B_2K_pC_1}{B_2K_dC_2} \end{cases} \tag{51}$$

$$\begin{cases} 2\eta\omega = \frac{K_p}{K_d} - \frac{C_1}{C_2} \\ \omega^2 = \frac{A_{21}}{B_2K_dC_2} - \frac{K_pC_1}{K_dC_2} \end{cases} \tag{52}$$

$$\begin{cases} 2\eta\omega K_d = K_p - K_d\frac{C_1}{C_2} \\ \omega^2K_d = \frac{A_{21}}{B_2C_2} - K_p\frac{C_1}{C_2} \end{cases} \tag{53}$$

$$\begin{cases} K_p + K_d(-\frac{C_1}{C_2} - 2\eta\omega) = 0 \\ K_p\frac{C_1}{C_2} + K_d\omega^2 = \frac{A_{21}}{B_2C_2} \end{cases} \tag{54}$$

$$\begin{cases} K_p = \frac{\frac{A_{21}}{B_2C_2}}{\omega^2 - (-\frac{C_1}{C_2} - 2\eta\omega)\frac{C_1}{C_2}} \\ K_d = \frac{-\frac{A_{21}}{B_2C_2}(-\frac{C_1}{C_2} - 2\eta\omega)}{\omega^2 - (-\frac{C_1}{C_2} - 2\eta\omega)\frac{C_1}{C_2}} \end{cases} \tag{55}$$

$$\begin{cases} K_p = \frac{\frac{A_{21}}{B_2C_2}}{\omega^2 + (\frac{C_1}{C_2} + 2\eta\omega)\frac{C_1}{C_2}} \\ K_d = \frac{\frac{A_{21}}{B_2C_2}(\frac{C_1}{C_2} + 2\eta\omega)}{\omega^2 + (\frac{C_1}{C_2} + 2\eta\omega)\frac{C_1}{C_2}} \end{cases} \tag{56}$$

### 2.2.2. Control de balanceo

A continuación se derivan las ecuaciones que modelan el sistema carro-pendulo. Se utilizará el metodo de Lagrange definiendo las cordenadas generalizadas $x_t$ y $\theta$ . Donde $x_t$ es la posición del carro y $\theta$ es el angulo del pendulo respecto a la vertical. A modo de simplificaion se toma $l$ como un parametro y no como una funcion del tiempo. El sistema se modela siguiento el modelo físico de la figura 3 del enunciado.

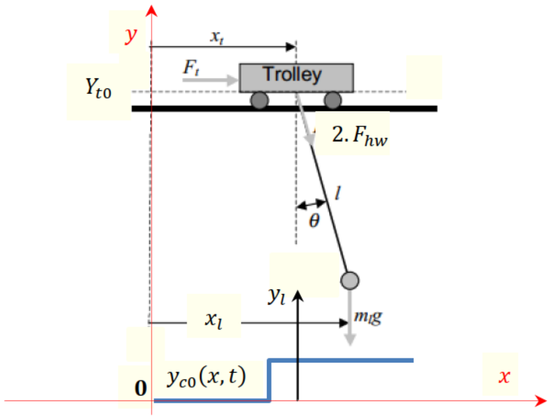


Figura 1: Modelo físico simplificado del subsistema Carro – Cable – Carga y Perfil de Obstáculos

$$K = K_t + K_{lx} + K_{ly} \tag{57}$$

$$x_l = x_t + l\sin\theta \tag{58}$$

$$\dot{x}_l = \dot{x}_t + l\cos\theta\dot{\theta} \tag{59}$$

$$y_l = Y_{t0} - l\cos\theta \tag{60}$$

$$\dot{y}_l = -l\sin\theta\dot{\theta} \tag{61}$$

$$K = \frac{1}{2}m_t\dot{x_t}^2 + \frac{1}{2}m_l\dot{x_l}^2 + \frac{1}{2}m_l\dot{y_l}^2 \tag{62}$$

$$K = \frac{1}{2}m_t\dot{x_t}^2 + \frac{1}{2}m_l(\dot{x}_t + l\cos\theta\dot{\theta})^2 + \frac{1}{2}m_l(-l\sin\theta\dot{\theta})^2 \tag{63}$$

$$K = \frac{1}{2}m_t\dot{x_t}^2 + \frac{1}{2}m_l(\dot{x_t}^2 + l^2\cos^2\theta\dot{\theta}^2 + 2l\dot{x}_t\cos\theta\dot{\theta}) + \frac{1}{2}m_l l^2\sin^2\theta\dot{\theta}^2 \tag{64}$$

$$U = -m_l g l\cos\theta \tag{65}$$

$$L = K - U \tag{66}$$

$$L = \frac{1}{2}m_t\dot{x_t}^2 + \frac{1}{2}m_l(\dot{x_t}^2 + l^2\cos^2\theta\dot{\theta}^2 + 2l\dot{x}_t\cos\theta\dot{\theta}) + \frac{1}{2}m_l l^2\sin^2\theta\dot{\theta}^2 + m_l g l\cos\theta \tag{67}$$

$$L = \frac{1}{2}m_t\dot{x_t}^2 + \frac{1}{2}m_l\dot{x_t}^2 + \frac{1}{2}m_l l^2\dot{\theta}^2 + m_l\dot{x}_t l\cos\theta\dot{\theta} + m_l g l\cos\theta \tag{68}$$

Se define el sistema de ecuaciones de Euler-Lagrange:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{69}$$

Para $q_i = x_t$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_t}\right) - \frac{\partial L}{\partial x_t} = Q_t \tag{70}$$

$$\frac{\partial L}{\partial \dot{x}_t} = (m_t + m_l)\dot{x}_t + m_l l \cos\theta\dot{\theta} \tag{71}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_t}\right) = (m_t + m_l)\ddot{x}_t + m_l l \cos\theta\ddot{\theta} - m_l l \sin\theta\dot{\theta}^2 \tag{72}$$

$$\frac{\partial L}{\partial x_t} = 0 \tag{73}$$

Entonces:

$$(m_t + m_l)\ddot{x}_t + m_l l \cos\theta\ddot{\theta} - m_l l \sin\theta\dot{\theta}^2 = F_t(t) - b_{eqt}\dot{x}_t \tag{74}$$

Para $q_i = \theta$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = Q_\theta \tag{75}$$

$$\frac{\partial L}{\partial \dot{\theta}} = m_l \dot{x}_t l \cos\theta + m_l l^2 \dot{\theta} \tag{76}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = m_l \ddot{x}_t l \cos\theta - m_l \dot{x}_t l \sin\theta\dot{\theta} + m_l l^2 \ddot{\theta} \tag{77}$$

$$\frac{\partial L}{\partial \theta} = -m_l \dot{x}_t l \sin\theta\dot{\theta} - m_l g l \sin\theta \tag{78}$$

Entonces:

$$m_l \ddot{x}_t l \cos\theta - m_l \dot{x}_t l \sin\theta\dot{\theta} + m_l l^2 \ddot{\theta} + m_l \dot{x}_t l \sin\theta\dot{\theta} + m_l g l \sin\theta = 0 \tag{79}$$

Finalmente, el sistema de ecuaciones que define el modelo del sistema carro-pendulo es:

$$\begin{cases} (m_t + m_l)\ddot{x}_t + m_l l \cos\theta\ddot{\theta} - m_l l \sin\theta\dot{\theta}^2 = F_t(t) - b_{eqt}\dot{x}_t \\ m_l \ddot{x}_t l \cos\theta - m_l \dot{x}_t l \sin\theta\dot{\theta} + m_l l^2 \ddot{\theta} + m_l \dot{x}_t l \sin\theta\dot{\theta} + m_l g l \sin\theta = 0 \end{cases} \tag{80}$$

$$\boxed{\begin{cases} (m_t + m_l)\ddot{x}_t + m_l l \cos\theta\ddot{\theta} - m_l l \sin\theta\dot{\theta}^2 = 0 \\ \ddot{x}_t \cos\theta + l\ddot{\theta} + g\sin\theta = 0 \end{cases}} \tag{81}$$

Para representar lo en el espacio de estados se definen las siguientes variables de estado $x$ y entradas $u$:

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \tag{82}$$

$$u = \ddot{x}_l \tag{83}$$

Entonces, se obtiene el siguiente modelo en el espacio de estados no lineal:

$$\begin{cases} \dot{x} = f(x, u, t); x(0) = x_0 \\ y = h(x, u, t) \end{cases} \tag{84}$$

### 2.2.3. Control de balanceo

$$U = \ddot{X}_t = (b_a s + k_{sa})(X_l^* - X_l) \tag{85}$$

$$(X_l^* - X_l) = \frac{1}{s}\left(\dot{X}_l^* - \dot{X}_l\right) \tag{86}$$

$$U = \ddot{X}_t = \left(b_a + k_{sa}\frac{1}{s}\right)\left(\dot{X}_l^* - \dot{X}_l\right) \tag{87}$$

$$s^2\Theta = A_{21}\Theta + B_2\left(\left(b_a + k_{sa}\frac{1}{s}\right)\left(\dot{X}_l^* - C_1\Theta - C_2\Theta s - \dot{X}_t\right)\right) \tag{88}$$

$$s^2\Theta = A_{21}\Theta + B_2\left(b_a\left(\dot{X}_l^* - C_1\Theta - C_2\Theta s + \dot{X}_t\right) + k_{sa}\frac{1}{s}\left(\dot{X}_l^* - C_1\Theta - C_2\Theta s - \dot{X}_t\right)\right) \tag{89}$$

$$\begin{aligned} s^2\Theta = A_{21}\Theta + B_2\Bigg(& b_a\dot{X}_l^* - b_aC_1\Theta - b_aC_2\Theta s + k_{sa}\dot{X}_l^*\frac{1}{s} - k_{sa}C_1\Theta\frac{1}{s} - k_{sa}C_2\Theta \\ & -\dot{X}_t\left(b_a + k_{sa}\frac{1}{s}\right)\Bigg) \end{aligned} \tag{90}$$

$$\begin{aligned} s^2\Theta = A_{21}\Theta + B_2b_a\dot{X}_l^* - B_2b_aC_1\Theta - B_2b_aC_2\Theta s + B_2k_{sa}\dot{X}_l^*\frac{1}{s} - B_2k_{sa}C_1\Theta\frac{1}{s} - B_2k_{sa}C_2\Theta \\ -B_2\dot{X}_t\left(b_a + k_{sa}\frac{1}{s}\right) \end{aligned} \tag{91}$$

$$\begin{aligned} \Theta s^3 = A_{21}\Theta s + B_2b_a\dot{X}_l^* s - B_2b_aC_1\Theta s - B_2b_aC_2\Theta s^2 + B_2k_{sa}\dot{X}_l^* - B_2k_{sa}C_1\Theta - B_2k_{sa}C_2\Theta s \\ -B_2\dot{X}_t\left(b_a s + k_{sa}\right) \end{aligned} \tag{92}$$

$$\begin{aligned} 0 = \Theta\left(s^3 + s^2\left(B_2b_aC_2\right) + s\left(-A_{21} + B_2b_aC_1 + B_2k_{sa}C_2\right) + \left(B_2k_{sa}C_1\right)\right) \\ -\dot{X}_l^*\left(B_2b_a s + B_2k_{sa}\right) + \dot{X}_t\left(B_2b_a s + B_2k_{sa}\right) \end{aligned} \tag{93}$$

$$\begin{aligned} \Theta = & \frac{B_2b_a s + B_2k_{sa}}{s^3 + s^2\left(B_2b_aC_2\right) + s\left(-A_{21} + B_2b_aC_1 + B_2k_{sa}C_2\right) + \left(B_2k_{sa}C_1\right)}\dot{X}_l^* \\ & - \frac{B_2b_a s + B_2k_{sa}}{s^3 + s^2\left(B_2b_aC_2\right) + s\left(-A_{21} + B_2b_aC_1 + B_2k_{sa}C_2\right) + \left(B_2k_{sa}C_1\right)}\dot{X}_t \end{aligned} \tag{94}$$

$$p_r(s) = (s + \omega_n)(s^2 + 2\zeta\omega_n s + \omega_n^2) \tag{95}$$

$$p_r(s) = s^3 + \omega_n(2\zeta + 1)s^2 + \omega_n^2(2\zeta + 1)s + \omega_n^3 \tag{96}$$

$$p_r(s) = s^3 + \omega_n\eta s^2 + \omega_n^2\eta s + \omega_n^3 \tag{97}$$

$$\begin{cases} \omega_n^3 = B_2k_{sa}C_1 \\ \omega_n^2\eta = -A_{21} + B_2b_aC_1 + B_2k_{sa}C_2 \\ \omega_n\eta = B_2b_aC_2 \end{cases} \tag{98}$$

SI $C_1 = 0$.

$$\begin{aligned} \Theta = & \frac{B_2b_a s + B_2k_{sa}}{s\left(s^2 + s\left(B_2b_aC_2\right) + \left(-A_{21} + B_2k_{sa}C_2\right)\right)}\dot{X}_l^* \\ & - \frac{B_2b_a s + B_2k_{sa}}{s\left(s^2 + s\left(B_2b_aC_2\right) + \left(-A_{21} + B_2k_{sa}C_2\right)\right)}\dot{X}_t \end{aligned} \tag{99}$$

### 2.2.4. Control de balanceo

$$U = \ddot{X}_t = \left(b_a s + k_{sa}\right)\left(\dot{X}_l^* - \dot{X}_l\right) \tag{100}$$

$$s^2\Theta = A_{21}\Theta + B_2\left(\left(b_a s + k_{sa}\right)\left(\dot{X}_l^* - C_1\Theta - C_2\Theta s - \dot{X}_t\right)\right) \tag{101}$$

$$s^2\Theta = A_{21}\Theta + B_2\left(b_a s\left(\dot{X}_l^* - C_1\Theta - C_2\Theta s + \dot{X}_t\right) + k_{sa}\left(\dot{X}_l^* - C_1\Theta - C_2\Theta s - \dot{X}_t\right)\right) \tag{102}$$

$$
\begin{aligned}
s^2\Theta = A_{21}\Theta + B_2\Big(& b_a\dot{X}_l^* s - b_a C_1\Theta s - b_a C_2\Theta s^2 + k_{sa}\dot{X}_l^* - k_{sa}C_1\Theta - k_{sa}C_2\Theta s \\
& -\dot{X}_t\left(b_a s + k_{sa}\right)\Big)
\end{aligned} \tag{103}
$$

$$
\begin{aligned}
s^2\Theta = A_{21}\Theta + B_2 b_a\dot{X}_l^* s - B_2 b_a C_1\Theta s - B_2 b_a C_2\Theta s^2 + B_2 k_{sa}\dot{X}_l^* - B_2 k_{sa}C_1\Theta - B_2 k_{sa}C_2\Theta s \\
-B_2\dot{X}_t\left(b_a + k_{sa}\frac{1}{s}\right)
\end{aligned} \tag{104}
$$

$$
\begin{aligned}
0 = \Theta\big(s^2\left(1 + B_2 b_a C_2\right) + s\left(B_2 b_a C_1 + B_2 k_{sa} C_2\right) + \left(B_2 k_{sa} C_1\right)\big) \\
-\dot{X}_l^*\left(B_2 b_a s + B_2 k_{sa}\right) + \dot{X}_t\left(B_2 b_a s + B_2 k_{sa}\right)
\end{aligned} \tag{105}
$$

$$
\begin{aligned}
\Theta = \frac{B_2 b_a s + B_2 k_{sa}}{s^2\left(1 + B_2 b_a C_2\right) + s\left(B_2 b_a C_1 + B_2 k_{sa} C_2\right) + \left(B_2 k_{sa} C_1\right)}\dot{X}_l^* \\
-\frac{B_2 b_a s + B_2 k_{sa}}{s^2\left(1 + B_2 b_a C_2\right) + s\left(B_2 b_a C_1 + B_2 k_{sa} C_2\Theta\right) + \left(B_2 k_{sa} C_1\right)}\dot{X}_t
\end{aligned} \tag{106}
$$

$$
p_r(s) = s^2 + s\frac{B_2 b_a C_1 + B_2 k_{sa} C_2}{1 + B_2 b_a C_2} + \frac{B_2 k_{sa} C_1}{1 + B_2 b_a C_2} \tag{107}
$$

$$
p_r(s) = (s^2 + 2\zeta\omega_n s + \omega_n^2) \tag{108}
$$

$$
\begin{cases}
2\zeta\omega_n = \frac{B_2 b_a C_1 + B_2 k_{sa} C_2}{1 + B_2 b_a C_2} \\
\omega_n^2 = \frac{B_2 k_{sa} C_1}{1 + B_2 b_a C_2}
\end{cases} \tag{109}
$$

$$
\begin{cases}
(1 + B_2 b_a C_2)2\zeta\omega_n = B_2 b_a C_1 + B_2 k_{sa} C_2 \\
(1 + B_2 b_a C_2)\omega_n^2 = B_2 k_{sa} C_1
\end{cases} \tag{110}
$$

$$
\begin{cases}
2\zeta\omega_n + 2\zeta\omega_n B_2 b_a C_2 = B_2 b_a C_1 + B_2 k_{sa} C_2 \\
\omega_n^2 + B_2 b_a C_2 \omega_n^2 = B_2 k_{sa} C_1
\end{cases} \tag{111}
$$

$$
\begin{cases}
b_a(2\zeta\omega_n B_2 C_2 - B_2 C_1) & +k_{sa}(-B_2 C_2) = -2\zeta\omega_n \\
b_a(\omega_n^2 B_2 C_2) & +k_{sa}(-B_2 C_1) = -\omega_n^2
\end{cases} \tag{112}
$$

## 3. Resultados

## 4. Conclusión