# Fault-tolerant Control Research on Networked Control Systems with Multiple-Packet Transmission

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**Abstract:** Implementing a control system over a communication network induces inevitable time delays and data packet dropout that degrade system performance and can even cause instability. In this paper, A kind of networked control system with multiple-packet transmission is considered, and the fault-tolerant control design of an NCS with actuators failures is analyzed based on fault-tolerant control theory. After detailed theoretical analysis, the paper also provides the simulation results, which further demonstrates the proposed scheme.

Key Words: Networked control systems, Fault Tolerant Control, Packet

#### 1 INTRODUCTION

A major trend in modern industrial and commercial systems is to integrate computing, communication, and control into different levels of machine/factory operations information processes. The traditional solution information exchanging and control signals point-to-point communication, which is a wire connects the central control computer with each sensor or actuator point and has been successfully implemented for decades. The point-to-point wiring is complex and expensive and the whole system is difficult to maintain and diagnose due to large number of connectors and cables. With the development of network technology, there is a trend in factory, home and automotive equipment toward distributed networking. This trend can be inferred from many proposed or emerging network standards, such as controller area network (CAN, ANSI/ISO ANSI/ISO 11519-2) for automotive and industrial automation, BACnet (ANSI/ASHRAE 135) for building automation, and ProfibBus (EN 50170) and WorldFIP (EN 50170) feildbus for process control.

In manufacturing plants, HVAC systems, vehicles, aircraft and spacecraft, serial communication networks are employed to exchange information and control signals between spatially distributed system components, such as supervisory computers, controllers, and intelligent I/O devices (e.g., smart sensors and actuators). Each of the system components connected to the network directly is denoted as a node. When a control loop is close via a serial communication channel (wireline or wireless), it is labeled as a networked control system.

According to [1, 2], feedback control systems wherein the control loops are closed through a real-time network are called networked control systems (NCS). The defining feature of an NCS is that information (reference input, plant output, control input, etc.) is exchanged using a network among control system components (sensors, controller, actuators, etc.). Fig.1 illustrates a typical setup and the information flows of an NCS. Compared with conventional point-to-point interconnected control systems, the primary advantages of an NCS are modular and flexible system design (e.g., distributed processing and interoperability), simple and fast implementation (e.g., reduced system wiring and powerful configuration tools), ease of system diagnosis and maintenance, and increased system agility. Consequently, there is a growing theoretical interest in the field of networked control systems (NCS).

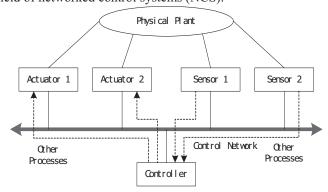


Fig. 1. Typical NCS setup and information flows.

The insertion of the communication network in the feedback control loop makes the analysis and design of an NCS complex. Conventional control theories with many ideal assumptions, such as synchronized control and non-delayed sensing and actuation, must be reevaluated before they can be applied to NCS. Many scholars have worked on the analysis, design, modeling and control of NCS. The primary objective of NCS analysis and design is to efficiently use the finite bus capacity while maintaining good closed-loop control system performance.

This work is supported by Hohai University Natural Science Foundation(2008426311), Hohai University Doctor Scientific Research Foundation(2084/40601136), the National High Technology Research and development "863"Program(2007AA05Z445-2) and National Science Foundation of China Grant(60574088).

Furthermore, research in NCS is different from that in traditional time-delay systems where time delays are assumed constant or bounded. Because of the variability of networked-induced time delays, the NCS may be time-varying systems, making analysis and design more challenging. The paper aims at to provide an overview of the state of arts of NCS, and give the further research field of NCS.

With the development of network technology, networked control systems (NCS) are more and more popular in most engineering practices [1-3]. Networks can be viewed as unreliable data transmission paths, where congestion and node failure can occasionally occur. When NCS transmissions are non-ideal, not only network-induced delay, but also data packet dropout must be considered [4-7]. The closed-loop may hide a fault from being observed until a situation is reached in which a failure is inevitable. The consequence could be damage to technical parts of the plant, to personnel or the environment. A cost effective way to obtain increased availability and reliability in networked control system is to introduce fault-tolerant control. Therefore, the research on fault-tolerant control of networked control system has great theoretical and applied significance, and it is a new research field at present [8].

In this paper the networked control systems with multiple-packet transmission is considered and the fault-tolerant control design of the NCS with actuators failures is analyzed based on fault tolerant control theory. The NCS with networked-induced delay and packet dropout is considered. The dropout probability is taken as a measure of the network quality of service (QoS). So a direct way of linking control system performance to the network's QoS is provided.

### NCS MODEL WITH MULTIPLE-PACKET TRANSMISSION

The process to be controlled is assumed to be of the form  $\dot{x}(t) = Ax(t) + Bu(t)$ 

$$v(t) = Cx(t)$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$ ,  $v(t) \in \mathbb{R}^p$  are system state, control input and system output respectively. A, B, C are of compatible dimensions.

There are essentially two kinds of delays in NCS: communication delay between the sensor and the controller  $\tau_{sc}$  , communication delay between the controller and the actuator  $au_{ca}$  . For fixed control law, the sensor-to-controller delay and controller-to-actuator delay can be lumped together as  $\tau_{k} = \tau_{sc} + \tau_{ca}$  for analysis purposes [1]. We consider the setup with clock-driven sensors, an event-driven controller and event-driven actuators. In this paper, we only consider the delay of each sample  $\tau_k$  is constant and less than one sampling period.

Consider the following discrete controller

$$x_{k+1}^c = A_k^c x_k^c + B_k^c \overline{y}_k$$

$$u_k = C_k^c x_k^c + D_k^c \overline{y}_k$$

where  $x_k^c$  is the state of the controller,  $\overline{y}_k \in \mathbb{R}^p$  is the most recent information of the plant outputs received by controller.

$$y_k = (y_k^1, y_k^2, \dots, y_k^M)^T,$$
$$\overline{y}_k = (\overline{y}_k^1, \overline{y}_k^2, \dots, \overline{y}_k^M)^T.$$

When the sensor j transmits the data, we have

$$\overline{y}_k^i = \begin{cases} y_k^i & i = j \\ \overline{y}_k^i & i \neq j \end{cases}$$

thus,  $\overline{y}_k$  can be expressed<sup>[1,9]</sup>

$$\overline{y}_k = P_i y_k + Q_i \overline{y}_{k-1}$$

$$i = 1, 2, \dots, M$$
where  $P = diag(P) P = 1 P$ 

where 
$$P_i = diag(P_{jj}), P_{ii} = 1, P_{jj} = 0 (j \neq i),$$
  
 $Q_i = diag(q_{jj}), q_{jj} = 1 (j \neq i), q_{ii} = 0.$ 

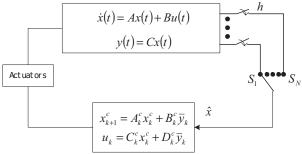


Fig.2. NCS setup with data packet dropout

Sampling the plant with period h we obtain

$$\begin{aligned} x_{k+1} &= A_d x_k + B_{d0} u_k + B_{d1} u_{k-1} \\ y_k &= C_d x_k \end{aligned}$$
 where  $A_d = e^{Ah}$ ,  $B_{d0} = \left(\int_0^{h-\tau} e^{As} ds\right) B$ ,

$$B_{d1} = \left( \int_{1-\tau}^{h} e^{As} ds \right) B, C_{d} = C.$$

The augmented closed-loop system is

$$Z_{k+1} = \widetilde{\Phi}_i Z_k \quad i = 1, 2, \dots, M$$

where 
$$Z_k = \begin{pmatrix} x_k^T & (x_k^c)^T & \overline{y}_{k-1}^T & u_{k-1}^T \end{pmatrix}^T$$

$$\widetilde{\Phi}_i = \begin{bmatrix} A_d + B_{d0} D_k^c P_i C_d & B_{d0} C_k^c & B_{d0} D_k^c Q_i & B_{d1} \\ B_k^c P_i C_d & A_k^c & B_k^c Q_i & 0 \\ P_i C_d & 0 & Q_i & 0 \\ D_k^c P_i C_d & C_k^c & D_k^c Q_i & 0 \end{bmatrix}$$
 $i = 1, 2, \dots, M$ .

#### 3 WITH FTC **DESIGN FOR** NCS MULTIPLE-PACKET TRANSMISSION

Fault tolerant control (FTC) strives to make the system stable and retain acceptable performance under the system faults. Integrity design is one of important means for the fault tolerant control. Integrity design is that system can keep asymptotically stable under some sensors failures or actuators failures.

If system (1) is under actuators failures, introducing switching matrix  $L_i$ , where  $L_i = diag\{l_1, l_2, \cdots, l_n\}$ . Let

$$l_{j} = \begin{cases} 1 & \text{no actuator fault} \\ 0 & \text{jth actuator fault} \end{cases}, \quad j = 1, 2, \cdots, n \;, \quad L_{i} \neq 0 \;.$$

The system input under actuators failures can be described as  $L_i u_k$  and then the close-loop control system can be written as

$$Z_{k+1} = \widetilde{\Phi}_i Z_k$$
,  $i = 1, 2, \dots, M$   
where

$$\widetilde{\Phi}_{i} = \begin{bmatrix} A_{d} + B_{d0}L_{i}D_{k}^{c}P_{i}C_{d} & B_{d0}L_{i}C_{k}^{c} & B_{d0}L_{i}D_{k}^{c}Q_{i} & B_{d1} \\ B_{k}^{c}P_{i}C_{d} & A_{k}^{c} & B_{k}^{c}Q_{i} & 0 \\ P_{i}C_{d} & 0 & Q_{i} & 0 \\ L_{i}D_{k}^{c}P_{i}C_{d} & L_{i}C_{k}^{c} & L_{i}D_{k}^{c}Q_{i} & 0 \end{bmatrix}$$

An NCS with multiple-packet transmission can be modeled as an asynchronous dynamical system (ADS) with rate constraints on event. ADS, like hybrid systems, are systems that incorporate continuous and discrete dynamics. The continuous dynamics are governed by differential or difference equations, where as the discrete dynamics are governed by finite automata that are driven asynchronously by external discrete events with fixed rates<sup>[11]</sup>.

We consider simplified ADS with rate constraints that can be described by a set of difference equations

$$x(k+1) = f_s(x(k)), s = 1, 2, \dots, N$$

with continuous-valued state  $x(k) \in \mathbb{R}^n$ . Here,  $1,2,\cdots,N$  represents the set of discrete states, which has a corresponding set of rates  $r_1,r_2,...r_n$ . These rates represent the fraction of time that each discrete state occurs; thus

$$\sum_{i=1}^{N} r_i = 1.$$

**Lemma 1**<sup>[11]</sup> (Stability of ADS) Given an ADS as defined above. If there exist a Lyapunov function  $V(x(k)): R^n \to R_+$  and scalars  $\alpha_1, \alpha_2, \cdots \alpha_N$  corresponding to each rate such that

$$\alpha_1^{r_1}\alpha_2^{r_2}\cdots\alpha_N^{r_N}>\alpha>1$$

and

$$V(x(k+1)) - V(x(k)) \le (\alpha_s^{-2} - 1)V(x(k)),$$
  
 $s = 1, 2, \dots, N$ 

then the ADS remains exponentially stable, with decay rate greater than  $\, \alpha \,$  .

In this paper, we assume the delay of each sample  $\tau_k$  is constant, such as token ring and token bus, in this case the rate  $r_1 = r_2 = \dots = r_M = \frac{1}{M}$ , so we have the following results.

**Theorem 1** For the above system setup, if there exist a lyapunov function  $V(z(kh)) = z^T(kh)Pz(kh)$  and scalars  $\alpha_1, \alpha_2, \cdots, \alpha_M$ , and the controller parameters such that

$$\frac{1}{M} \left( \log \alpha_1 + \log \alpha_2 + \dots + \log \alpha_M \right) > 0$$

$$\widetilde{\Phi}_i^T P \widetilde{\Phi}_i \le \alpha_i^{-2} P \quad i = 1, 2, \dots, M$$

then the system (1) has the ability of fault tolerant against actuators failures  $L_i \in \Omega$ .

From Lemma 1, the proof of Theorem 1 is easy.

**Theorem 2** Suppose the controller receives the sensors data in regular sequence  $y_1, y_2, \dots, y_M$ , and if the

controller parameters satisfy the  $\prod_{i=1}^{M} \widetilde{\Phi}_{i}$  is Schur, then the

system (1) has the ability of fault tolerant against actuators failures  $L_i \in \Omega$ .

#### Proof

$$\begin{split} z_{kM} &= \widetilde{\Phi}_M z_{kM-1} = \widetilde{\Phi}_M \widetilde{\Phi}_{M-1} z_{kM-2} = \cdots = \\ \left( \prod_{i=M}^1 \widetilde{\Phi}_i \right) & z_{(k-1)M} = \left( \prod_{i=M}^1 \widetilde{\Phi}_i \right)^k z_0 \\ \text{thus, if } \prod_{i=M}^1 \widetilde{\Phi}_i \text{ is Schur, } i = \prod_{1}^{i=M} \widetilde{\Phi}_i \text{ is Schur} \\ \text{and } z_k &\to 0 \big( k \to \infty \big). \end{split}$$

## The proof is completed.

# SIMULATION ANALYSIS

Consider the state-space plant model  $\dot{x}(t) = \begin{bmatrix} -3 & 1 \\ 1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} u(t)$ 

$$y(t) = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -2 \end{bmatrix} x(t)$$

and the discrete controller

$$u(k) = \begin{bmatrix} 0.0625 & 0 \\ 0 & 0.115 \end{bmatrix} y(k)$$

the sampling period is h = 0.3s,  $\tau = 0.1s$ .

According to Theorem1, we can get the feasible solution of linear matrix inequality (LMI).

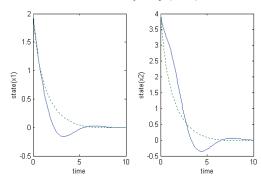


Fig. 3. The response of state in actuator failures

The simulation result is shown in Fig.3. The solid line denotes in the actuator failures situation, and dash line denotes the state in normal situation. In view of simulation results, the system can keep steady and when actuator failures happen. Simulation results further demonstrated the proposed scheme.

According to Theorem 2, we can obtain the following system matrix parameters under actuators failures

Through validation,  $\Phi_1 \bullet \Phi_2$ ,  $\overline{\Phi}_1 \bullet \overline{\Phi}_2$ ,  $\widetilde{\Phi}_1 \bullet \widetilde{\Phi}_2$  are Schur respectively. The closed loop system is asymptotical stabilization with actuators failures. Simulation results further demonstrated the proposed scheme.

#### 5 CONCLUSION AND PROSPECT

In NCS, the performance of the control loops not only depends on the design of the control algorithms but also on the scheduling of the shared network resource. Protocol redesign for deterministic transmission and optimal scheduling of message transmission can be further studied to effectively utilizes network bandwidth and control medium access for NCS applications. Especially wit respect to the device-level control system, real-time communication with guaranteed and deterministic transmission is important for system stability and performance. The system performance should be evaluated based on the requirements of control applications as well as the network architecture itself.

In this paper we consider a kind of networked control system with multiple-packet transmission, and the fault-tolerant control design of an NCS with actuators failures is analyzed based on fault-tolerant control theory. The research presented in this paper provides a foundation for future research in NCS fault-tolerant control. The paper also provides the simulation results, which further demonstrated the proposed scheme. In future research work, more complex networked control system, in which network effects on system performance should be further considered.

#### REFERENCES

- [1] W. Zhang, "Stability Analysis of Networked Control Systems," Ph.D. dissertation, Dept. Elect. Eng., Case Western Reserve University, 2001.
- [2] G.C. Walsh, H Ye, "Scheduling of Networked Control Systems," IEEE Control Systems Magazine, vol. 21 pp. 57-65, Jan. 2001
- [3] M.S. Branicky, S.M. Phillips, W. Zhang, "Stability of Networked Control Systems: Explicit Analysis of Delay," in Proc the American Control Conference, Chicago, 2000, pp. 2352-2357
- [4] G.C. Walsh, H. Ye, L. Bushnell, "Stability Analysis of Networked Control Systems," in Proc the American Control Conference, San Diego, 1999, pp. 2876-2880.
- [5] Lian Fengli. Network Design Consideration for Distributed Control Systems. IEEE Trans. Control Systems Technology, vol. 10, pp. 297–307, Feb. 2002.
- [6] W. Zhang, M.S. Branicky, S.M. Phillips, "Stability of Networked Control System," IEEE Control System Magazine, 2001, vol. 21, pp. 84-99, Jan. 2001.
- [7] J Nilsson, "Real-time Control Systems with Delays," Ph.D. dissertation, Lund Institute of Technology, Lund, Sweden, 1998
- [8] Y. Zheng, "Fault Diagnosis and Fault Tolerant Control of Networked Control System," Ph.D. dissertation, Huazhong University of Science and Technology, Wuhan, 2003
- [9] Zhigang Sun, Li Xiao, Deseng Zhu, "Analysis of Networked Control systems with Multiple-Packet Transimission," in Proc. 37th Proceeding of the 5th World Congress on Intelligent Control and Automation, China, 2004, pp. 1358-1360
- [10] A. Hassibi, S.P. Boyd, and J.P. How, "Control of Asynchronous dynamical Systems with Rate Constraints on Events," in Proc. 38th IEEE Conf on Decision and Control, USA, 1999, pp.1345-1351.

[11] D.Theilliol, H.Noura, D.Sauter, "Fault-tolerant Control Method for Actuator and Component Faults," in Proc. 37th IEEE Conf on Decision and Control, USA, 1998, pp. 604-608.