Fault-tolerant Control of Networked Control Systems with Packet Dropouts: Switched System Approach

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Abstract: This paper focuses on the fault-tolerant control (FTC) of networked control systems (NCSs) with packet dropouts using switched system approach. The basic idea is to model such NCSs as a discrete-time switched system. Thus, the FTC of such NCSs can be reduced to that of switched systems. Sufficient conditions are presented about the stabilization of NCSs. Stabilizing switched state/output feedback controllers can be constructed by solving a set of linear matrix inequalities (LMIs), which can be easily tested with efficient LMI algorithms. The maximum allowable dropout bound can be obtained by solving an optimization problem.

Key Words: Networked control systems (NCSs), discrete-time switched systems, packet dropouts, switched state feedback, switched output feedback, linear matrix inequality (LMI).

1 Introduction

Networked control systems (NCSs) are feedback control systems whose feedback paths are closed through a feedback network. The defining feature of an NCS is that information (reference input, plant output, control input, etc.) is exchanged using a network among control system components (sensors, controller, actuators, etc.). Compared with traditional control systems, NCSs can reduce system wiring, expedite system diagnosis and maintenance, and improve system efficiency, flexibility, and reliability. These advantages have made the NCS popular in many applications, such as traffic control, satellite clusters, mobile robotics, etc. Meanwhile, as pointed out in [5], the introduction of communication network in the feedback loop makes the analysis and design of an NCS complex. The network-induced delay problem, packet-dropout problem, multiple-packet transmission problem are three main problems emerging in NCSs and have received much attention in the past decades (see [1-24] and the references therein).

Network data transmission paths are unreliable because of limited bandwidth and large amount of data packet transmitted over one line, packets can be lost during transmission and affect the stability and performance of the NCS due to the critical real-time requirements in control systems. At present, there have been many papers concerning NCS with dropouts (see [1-10] and the references therein). For example, the arbitrary switched system model and the time-varying discrete system discrete system model were presented, respectively, in [2] and [7]. Whereas, in [3]-[5], dropouts are modelled as asynchronous dynamical systems (ADS). They require a Lyapunov-type function to decrease along state trajectories of the asynchronous system. [6,10] models dropouts as a Markov chain, and uses past control signals or estimated lost data to compute the new control signals. Recently, [7] models the NCS as a linear time-varying systems $x_{k+1} = A_k x_k$, and studies its quadratic stability by searching a **common** Lyapunov function, which is a restrict constraint.

Because of the complexity induced by a network, an automated control system based on it may be vulnerable to faults. The faults in sensors, actuators, or in the process itself, or within the controller may be amplified by the NCS and affect the system's stability and performance. Hence, it is necessary to study the FTC of NCS, which has great theoretical and applied significance, and is a growing research field at present.

On the other hand, switched systems are a class of hybrid systems consisting of several subsystems and a switching law specifying which subsystem will be activated along the system trajectory at each instant of time. In the last decade, there have been many studies for switched systems, primarily on stability/performance analysis and design. Recently, switched system approach is presented as an efficient method to study NCSs (see [8,22-25] and the references therein). The basic idea of this approach is to model the NCSs as a switched system; then, the behavior of the NCSs is converted into that of switched systems; thus, we can apply the rich theory of switched systems to analyze NCSs.

Motivated by the above discussions, in this paper, we focus on investigating the FTC of networked control systems with dropouts using switched system approach. The main contributions of our paper are as follows:
(1) Compared with [7], we directly model the NCSs with

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dropouts as a switched system and use the theory of switched systems to study its FTC. Whereas [7] model the NCS as a time-varying system. Our results complement those results of [7] in a sense.
(2) The controllers designed in this paper are not linear time-invariant, but depend on the switching signal, i.e., the number of data packet dropout in our NCSs (so called switched state/output feedback controller). In doing so, we can reduce the conservativeness of the results.
(3)Based on multiple Lyapunov function method, LMI-based asymptotical stabilizability criteria of NCSs are established. Switched state/output feedback controllers can be constructed by solving a set of linear matrix inequalities (LMIs), which can be easily tested with efficient LMI algorithms.

The outline of this paper is as follows. Section 2 models the NCS as a switched system and formulates our main problem. Our main results are given in Section 3. Section 4 concludes this paper.

Notations: We use standard notations throughout this paper. M^T is the transpose of the matrix M. $M > 0$ ($M < 0$) means that M is positive definite (negative definite). $\mathbb{R}^+, \mathbb{Z}^+$ denote the nonnegative real and integer number, respectively. $\mathbb{R}^n$ is the set of n-dimensional real Euclidean space. The symbol $*$ will be used to denote a symmetric structure in a matrix, that is, $\begin{bmatrix} L & N \\ * & R \end{bmatrix} = \begin{bmatrix} L & N \\ N^T & R \end{bmatrix}$. $diag\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2 Problem Formulations and Preliminaries

In this section, we first model the NCSs as a switched system; then we formulate our main problems and establish some lemmas as the preliminaries of our paper.

The model of the NCSs used in this paper is shown in Figure 1 and Figure 2 below.
The plant can be modelled as a continuous-time system as follows:

$$\begin{cases} \dot{x}(t) = A^c x(t) + B^c u(t), \ \ t \in \mathbb{R}^+ \\ y(t) = C^c x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the control output.

For simplicity, but without loss of generality, all the time delay and packet dropout effects are combined into the sensor to controller path and assume that the controller and the actuator communicate ideally. In this paper, we just consider the case that there are no transmission delays between the sensor and the combined nodes.

The plant output node (sensor) is assumed to be time-driven, whereas the controller and actuator are event-driven. The controller, as a receiver, has a receiving buffer which contains the most recently received data packet from the sensors (the overflow of the buffer may be dealt with as packet dropouts). Whenever there are new data in the buffer, according to the data transmission process between sampler and controller, the decision maker instantly decides which controller will be activated, which will calculate the new control signal and transmit it to the actuator. Upon the arrival of the new control signal, the actuator updates the output of the zero-order-hold (ZOH) to the new value.

Suppose $0 = k_0 < k_1 < k_2 < \cdots < k_l < \cdots < \infty$ be the successive update instants that control input do arrive to the actuator of the plant. We refer to the time interval between k_l and k_{l+1} as one transmission period. In this pattern of transmission, the states of the NCS at the update steps can be described as follows:

$$\begin{cases} x(k_{l+1}) = \Phi_{d_l} x(k_l) + \Gamma_{d_l} u(k_l), \\ y(k_l) = C x(k_l), l \in \mathbb{Z}. \end{cases} \tag{2}$$

where $d_i = k_{i+1} - k_i - 1 \in \mathcal{I} = \{0, 1, 2, \cdots, m\}, i \in \mathbb{Z}^+$, is defined as be the number of packet dropout during the time interval $[k_l, k_{l+1}]$. $\Phi_{d_l} = e^{A^c(d_l+1)h}, \Gamma_{d_l} = \int_0^{(d_l+1)h} e^{A^c\eta} B^c d\eta, C_{d_l} = C^c$.
Define

$$x[0] = x(0), x[1] = x(k_1), \cdots, x[l] = x(k_l), \cdots,$$

and

$$u[0] = u(0), u[1] = u(k_1), \cdots, u[l] = u(k_l), \cdots.$$

Then, (2) can be rewritten as

$$\begin{cases} x[l+1] = \Phi_{d_l} x[l] + \Gamma_{d_l} u[l], l = 0, 1, 2, \cdots \\ y[l] = C x[l]. \end{cases} \tag{3}$$

Thus, the FTC of NCS (1) with dropouts can be equivalently converted into that of system (3), which can be regarded as a typical switched systems. Here, the function $d_l : \{0, 1, \cdots\} \to \mathcal{I}$ can be regarded as the switching signal to be designed. Moreover, $d_l = i$ means that the subsystem (Φ_i, Γ_i, C) is activated.

Remark 1. *Compared NCS (1) with switched systems (3), we can easily get their relationship: the number of dropouts corresponds to the value of switching signal. The change of the number of dropouts corresponds to the switching of the corresponding subsystems.*

Remark 2. *The controller adopted in this paper is not linear time-invariant, but depends on the switching signal (the so-called switched state/output feedback controller). In doing so, we can reduce the conservativeness of the results.*

In this paper, we consider the FTC of NCS (2), i.e, design the switched state feedback controller

$$u[k] = K_{d_k} x[k] \tag{4}$$

or the switched output feedback controller

$$u[k] = K_{d_k} y[k] \tag{5}$$

such that the corresponding closed-loop system

$$x[k+1] := (\Phi_{d_k} + \Gamma_{d_k}\bar{L}K_{d_k})x[k], \quad (6)$$

or

$$x[k+1] := (\Phi_{d_k} + \Gamma_{d_k}\bar{L}K_{d_k}C)x[k], \quad (7)$$

can keep exponentially stable under any actuator faults $\bar{L} \in \Omega$, where $\bar{L} = diag\{l_1, \cdots, l_m\}$. The j-th diagonal element l_j is a binary indicator, i.e., $l_j = \begin{cases} 1 & \text{no actuator fault,} \\ 0 & \text{jth actuator fault,} \end{cases}$ $j = 1, 2, \cdots, m, \bar{L} \neq 0$. $\Omega := \{\bar{L} = diag\{l_1, \cdots, l_m\} | l_i = 0$ or $l_i = 1$and $\bar{L} \neq 0\}$ denotes the set of all possible actuator fault switching matrices.

Remark 3. *It is easy to compute that there are $2^m - 1$ elements in the set of actuator fault switching matrices. Denote them as $L_1, L_2, \cdots, L_q$, respectively, where $q = 2^m - 1$. Denote $\mathcal{I} := \{1, 2, \cdots, 2^m - 1\}$.*

Here, we first establish an important lemma, which will play an important role in the proof of our main results in Section 3.

Lemma 1. *For system (6), suppose that there exist a set of functions $V_i : \mathbb{R}^n \to \mathbb{R}, i \in \mathcal{I}$, positive numbers α, μ satisfying $0 < \alpha < 1, \mu > 1, (1-\alpha)\mu < 1, \gamma_1 > 0, \gamma_2 > 0$, such that*
$(a) \gamma_1 ||x||^2 \leq V_i(x) \leq \gamma_2 ||x||^2$,
$(b)\ \Delta V_{d_l}(x[l]) := V_{d_l}(x[l+1]) - V_{d_l}(x[l]) \leq -\alpha V_{d_l}(x[l]), \forall\, l \in \mathbb{Z}^+$,
$(c) V_{d_{l+1}}(x[l+1]) \leq \mu V_{d_l}(x[l+1])$,
then system (6) is exponentially stable for arbitrary switching signal $d_l \in \mathcal{I}$, any actuator fault $L_q \in \Omega$.

Proof. $\forall\, l \in \mathbb{Z}^+$, by condition (b)(c), we get

$$\begin{array}{rcl} V_{d_l}(x[l+1]) & \leq & (1-\alpha)V_{d_l}(x[l]) \\ & \leq & (1-\alpha)\mu V_{d_{l-1}}(x[l]) \\ & \leq & (1-\alpha)^2\mu V_{d_{l-1}}(x[l-1]) \\ & \vdots & \\ & \leq & (1-\alpha)^{l+1}\mu^l V_{d_0}(x[0]) \end{array}$$

Then, by condition (a), we get

$$\gamma_1 ||x[l+1]||^2 \leq V_{d_l}(x[l+1]), V_{d_0}(x[0]) \leq \gamma_2 ||x[0]||^2.$$

Thus,

$$||x[l+1]|| \leq M\lambda^{l+1}||x[0]||,$$

where $M = \sqrt{\dfrac{\gamma_2}{\gamma_1\mu}}, \lambda = \sqrt{(1-\alpha)\mu}$.
Obviously, by $0 < \lambda < 1$, we can easily get that NCS (6) is exponentially stable. Hence, this lemma holds. □

Remark 4. *It is easy to prove that Lemma 1 also holds for system (7).*

3 Stabilization of NCSs with data packet dropout

Based on Lemma 1 above, in this section, we aim to establish the LMI-based stabilizability criteria of NCS (3).

3.1 State feedback stabilization

In this subsection, the FTC criterion of NCS (3) via switched state feedback is established (Fig. 1). Moreover, the switched state feedback controller can be designed by solving a set of LMIs.

Theorem 1. *Given positive constants α, μ satisfying $0 < \alpha < 1, \mu > 1$, and $(1-\alpha)\mu < 1$, then the NCS (3) is exponentially stabilizable for any $d_l \in \mathcal{I}$, any actuator failures $L_q \in \Omega$, if there exist matrices $Q_i > 0, \Theta_i, \forall\, i \in \mathcal{I}$ such that*

$$\begin{bmatrix} -(1-\alpha)Q_i & Q_i\Phi_i^T + \Theta_i^T L_q^T \Gamma_i^T \\ \Phi_i Q_i + \Gamma_i \Theta_i & -Q_i \end{bmatrix} < 0, \forall\, i, q \in \mathcal{I}, \quad (8)$$

$$Q_j \leq \mu Q_i, \forall\, i, j \in \mathcal{I}, \quad (9)$$

Moreover, the state feedback controller is given by $K_i = \Theta_i Q_i^{-1}$.

Proof. (i) Choose $V_{d_l}(x[l]) = x^T[l]P_{d_l}x[l]$, where $P_{d_l} = Q_{d_l}^{-1}$. Obviously, **condition (a)** in Lemma 1 holds.
(ii) By Schur complement,

$$\begin{array}{rcl} Q_j - \mu Q_i \leq 0 & \Leftrightarrow & \begin{bmatrix} -\mu Q_i & I \\ I & -Q_j^{-1} \end{bmatrix} \leq 0 \\ & \Leftrightarrow & -Q_j^{-1} + \dfrac{1}{\mu}Q_i^{-1} \leq 0 \\ & \Leftrightarrow & Q_i^{-1} - \mu Q_j^{-1} \leq 0,\ \forall\ (i,j) \in \mathcal{I} \times \mathcal{I}, \end{array}$$

i.e.,

$$P_i - \mu P_j \leq 0, \text{ where } P_i = Q_i^{-1},\ \forall\ (i,j) \in \mathcal{I} \times \mathcal{I}.$$

This guarantees

$$x^T[l+1]p_{d_{l+1}}x[l+1] \leq \mu x^T[l+1]p_{d_l}x[l+1],$$

i.e.,

$$V_{d_{l+1}}(x[l+1]) \leq \mu V_{d_l}(x[l+1]).$$

Hence, **condition (c)** in Lemma 1 holds.
(iii) From $K_i = \Theta_i Q_i^{-1}$, we get $\Theta_i = K_i Q_i$. Replacing Θ_i with $K_i Q_i$, Q_i with P_i^{-1} in (8), we obtain

$$\begin{bmatrix} -(1-\alpha)P_i^{-1} & P_i^{-1}\Phi_i^T + P_i^{-1}K_i^T L_q^T \Gamma_i^T \\ * & -P_i^{-1} \end{bmatrix} < 0, \forall\, i, q \in \mathcal{I}, \quad (10)$$

Multiplying both sides of (10) by $diag\{P_i, P_i\}$, we get

$$\begin{bmatrix} -(1-\alpha)P_i & \Phi_i^T P_i + K_i^T L_q^T \Gamma_i^T P_i \\ * & -P_i \end{bmatrix} < 0, \forall\, i, q \in \mathcal{I},$$

i.e.,

$$\begin{bmatrix} -(1-\alpha)P_i & (\Phi_i + \Gamma_i L_q K_i)^T P_i \\ * & -P_i \end{bmatrix} < 0, \forall\, i, q \in \mathcal{I},$$

i.e.,

$$(\Phi_i+\Gamma_i L_q K_i)^T P_i(\Phi_i+\Gamma_i L_q K_i)-(1-\alpha)P_i < 0, \forall\, i, q \in \mathcal{I}.$$

This guarantees that

$$V_{d_l}(x[l+1]) \leq (1-\alpha)V_{d_l}(x[l]).$$

Thus, **condition (b)** in Lemma 1 is satisfied.
In conclusion, by Lemma 1, system (3) is exponentially stabilizable for any $d_l \in \mathcal{I}$, any actuator fault $L_q \in \Omega$. □

3.2 Output feedback stabilization

In this subsection, for NCS (3), we consider the synthesis problem of an switched output feedback ensuring that the closed-loop switched system (7) is exponentially stable (See Fig. 2 for the NCSs model).

Without the loss of generality, the system matrix C is assumed to be of full row rank. This assumption is reasonable since it can be achieved by discarding redundant measurement components of the output y[k]. On the basis of Lemma 1, we aim to establish the LMI-based exponential stabilisability criterion.

Theorem 2. *Given positive constants α, μ satisfying $0 < \alpha < 1, \mu > 1$, and $(1-\alpha)\mu < 1$, then the NCS (3) is exponentially stabilizable for any $d_l \in \mathcal{I}$, any actuator failures $L_q \in \Omega$, if there exist matrices $Q_i > 0, \Theta_i, \Pi_i, \forall\, i \in \mathcal{I}$ such that*

$$\begin{bmatrix} -(1-\alpha)Q_i & Q_i\Phi_i^T + C^T\Theta_i^T L_q^T \Gamma_i^T \\ * & -Q_i \end{bmatrix} < 0, \forall\, i \in \mathcal{I}, \quad (11)$$

$$Q_j \leq \mu Q_i, \forall\, i, j \in \mathcal{I}, \quad (12)$$

$$CQ_i = \Pi_i C. \quad (13)$$

Moreover, the switched output feedback controller is given by $K_i = \Theta_i \Pi_i^{-1}$.

Proof. Since C is of full row rank and Q_i is positive definite, it follows from (13) that Π_i is invertible.
From $K_i = \Theta_i\Pi_i^{-1}$ and (13), we get $\Theta_i C = K_i \Pi_i C = K_i C Q_i$. Replacing $\Theta_i C$ with $K_i C Q_i$ in (11), following similar discussions in the proof of Theorem 1, we can prove this theorem. □

Remark 5. *Theorems 1-2 provides a method of designing a switched state/output feedback controller to exponentially stabilize the NCS in (1) for arbitrary but finite data packet dropout.*

Remark 6. *For NCS (1), we can find the maximum allowable bound of data packet dropout by searching the largest d satisfying the condition in Theorem 1 (or Theorem 2). That is,*

$$\begin{array}{ll} \mathit{max}\ d & \\ \textit{subject to} & Q_i > 0, \Theta_i \ \textit{satisfying}\ (8)\ \textit{and}\ (9) \\ & \qquad (\mathit{or}(11)-(13)). \end{array}$$

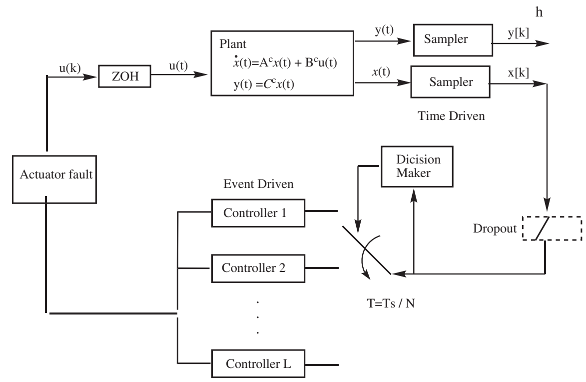

Fig. 1. Switched state feedback stabilizability of NCSs.

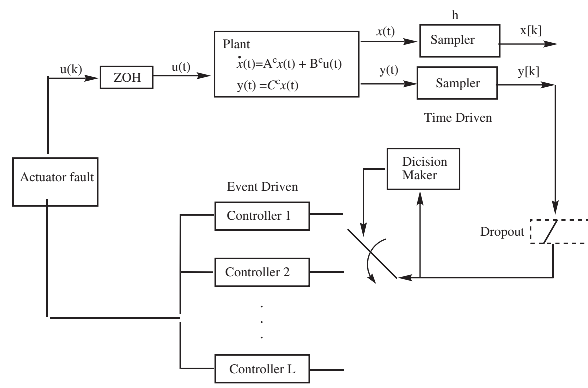

Fig.2. Switched output feedback stabilizability of NCSs.

4 Conclusions

In this paper, we model the NCS with dropouts as a switched system and use the switched system method to study its FTC. Stabilizing switched state/output feedback controllers can be constructed by solving a set of linear matrix inequalities (LMIs). The maximum allowable drop bound can be obtained by solving an optimization problem. In our future paper, we will consider FTC of NCS with dropouts and variable delays.

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