Math 301: Function Theory

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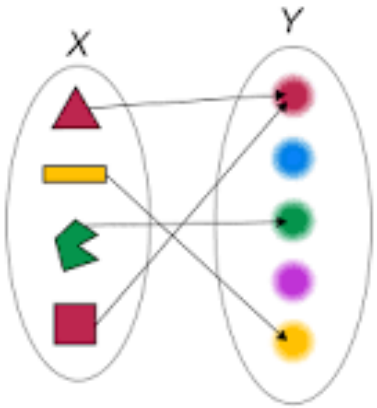
What Are Functions?

Definition

- In mathematics, a function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output.



- General format of functions:

$$f(x) = x^2$$

Why Functions?

- They are an integral part of mathematics and computational world
  - Theory is foundation for functional programming paradigm seen in languages such as Haskell and Python
- Allow for use of rules more than once in an expression
- Allow for different parameter values
- Useful for recognizing similarities in a set of observations

Components – Domain and Range

Functions can be seen as relations that uniquely associate members of one set with members of another set. *A function from* $A \rightarrow B$ is an object f such that every $\alpha \in A$ is uniquely associated with an object $f(\alpha) \in B$. A function is therefore a many-to-one relation.

- The set A of values at which a function is defined is called its domain
- The set $f(A) \subset B$ of values that the function can produce is called the *range*. It is denoted as follows:

$$D(f) = \{x \in \mathbb{R}; f(x)\}$$

- In other words, range of a function is the set of all values obtained by substituting arguments from its domain

Components — Real Functions

A function is defined as "Real" when $f : A \subset \mathbb{R} \in \mathbb{R}$

- Domain elements in real functions are mapped to unique range elements
  - Can be expressed graphically by performing the *vertical line test*
- They are the most important type of *mapping*.
  - Can be thought of as any mapping from some subset of the set of real numbers to the set of real numbers

Basic Notions of Real Functions

- Can be thought of as a perscription which assigns values to arguments
  - For example: $y = f(x)$ means that to the value x of the argument, the function f assigns the value y
  - Second notation: $f : x \to y$ implies that the function f sends the value x to y
- Usual way of specifying assignments is that the function value y can be obtained by substituting x to a specific formula:

$$f(x) = 2x + 3$$

Substitution and Visualization

$$f(a) = f|_a = f|_{x=a}$$

- Used when function needs adjustment, but not yet ready for substitution. Example:

$$\left.\frac{x^2-1}{x-1}\right|_{x=3} = \left.\frac{(x-1)(x+1)}{x-1}\right|_{x=3} = 4.$$

- Graphs are common way of visuaizing functions in mathematics.
- Used to mark in the two-dimensional plane (x, y) all couples $x, f(x)$

Types of Functions

Functions are primary objects of study in calculus, consisting of various types:

- linear
- polynomial
- trigonometric
- logarithmic

Linear Functions

Allow us to approximate more complicated functions in differential calculus. There are three standad forms of linear functions:

$$f(x) = mx + b \qquad \text{(Slope-intercept form)}$$

$$y - y_o = m(x - x_0) \qquad \text{(The "point-slope" form)}$$

$$Ax + By = C \qquad \text{(The "general" form)}$$

Linear Functions

Frequently used to solve for the following calculations:

- **Intercepts**
  - Finding the slope, x-intercept, and y-intercept of the line given its equation
- **Intersections**
  - Finding the intersection of two lines from their equations
- **Equations**
  - Finding the equation of the line through a series of points given one of the following three pairs of data:
    - the slope of the line and the y-intercept
    - the slope of the line and a point (x_0, y_0) on the line
    - the coordinates of two points on the line

File Structure

File is separated into frames:

Inside the document:

```
\begin{frame}\frametitle{Title}

Contents

\end{frame}
```

Compile to PDF! (DVI has . . . issues)

Spacing

- Text is automatically centered vertically.
- Starts flush left.
- No separation or indentation between paragraphs.

He's a stiff! Bereft of life, he rests in peace! If you hadn't nailed him to the perch he'd be pushing up the daisies! His metabolic processes are now history! He's off the twig! He's kicked the bucket, he's shuffled off his mortal coil, run down the curtain and joined the bleeding choir invisible!
He's an ex-parrot!

Spacing

Vertical spacing: give blank lines before and after the command

- Manual vertical spacing: `\vspace{amount in units}`
- Automatic spacing: `\vfill`

Horizontal spacing:

- Manual vertical spacing: `\hspace{amount in units}`
- Automatic spacing: `\hfill`

Earlier example

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With vfill

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vspace

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Overlays

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Two general methods . . .

Pause: Quick and Dirty Method

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- Insert where the overlay should occur.

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  - Nothing to appear then disappear;
  - Entire environments at a time.

onslide: far more robust

Recall the Taylor series for sin and cos:

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n!}$$

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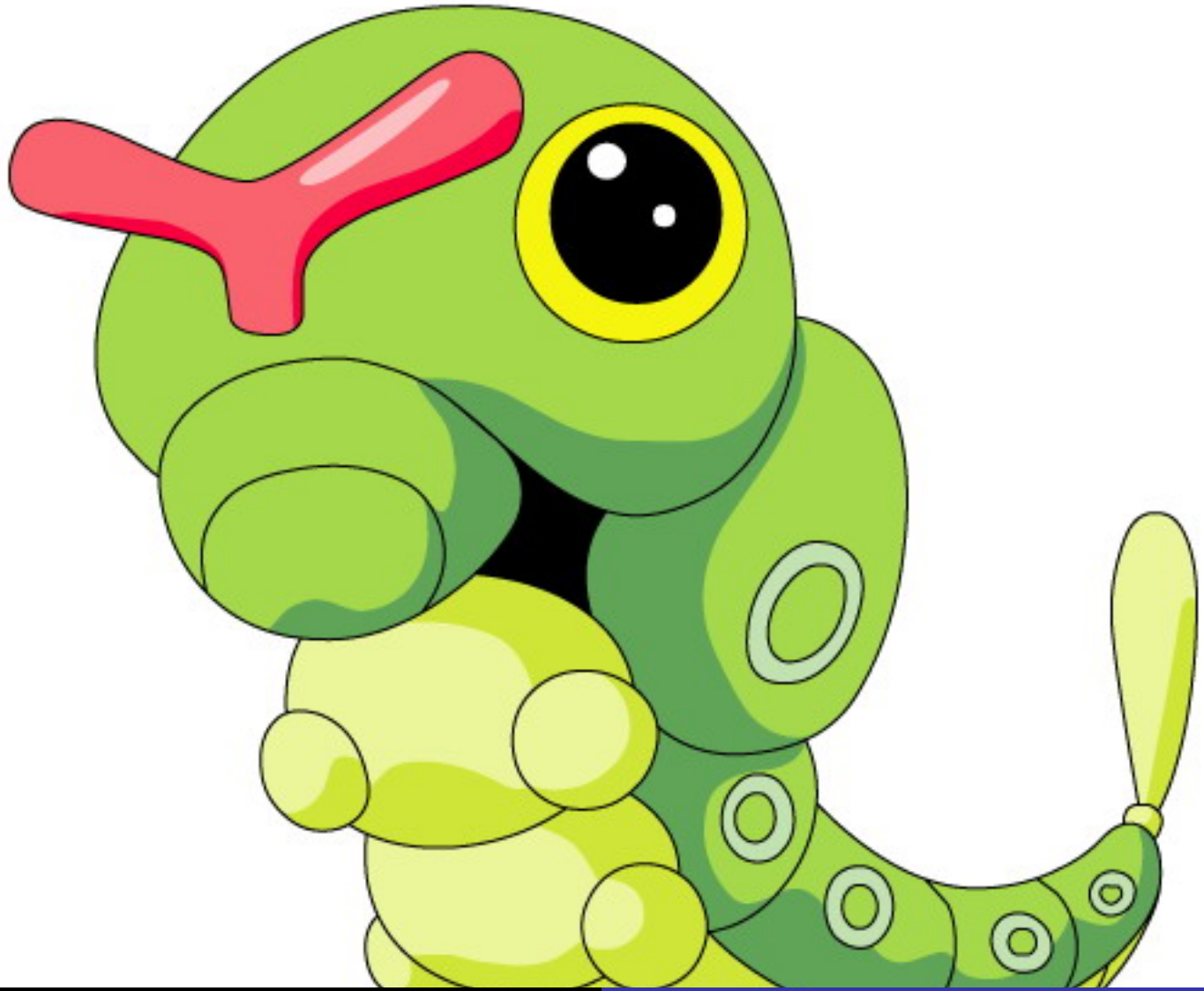
onslide: syntax

- Initial frame is considered slide 1.
- Content from slide n onward: `\onslide<n->{ Content }`
- Content on slides $m - n$: `\onslide<m-n>{ Content }`
- Content on slides i, j, k: `\onslide<i,j,k>{Content}`
- More options . . .

Graphics

Here is a picture . . .



Here's how you get it . . .

- In preamble: `\usepackage{graphicx}` (accept no substitute)
- Generate graphic in appropriate format: pdf, jpg, gif, eps
- Command: `\includegraphics[options]{filename}`
- Common options:
  - scale=*scaling factor*
  - rotate=*counterclockwise rotation in degrees*
  - many others

A Word about Compilation Errors...

Get an error message while compiling,

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it might as well say

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Bad Karma.