PROBLEM SOLUTION

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Statement

Show that among twelve composite numbers selected from the first 1200 natural numbers, there will always be two that have a common factor greater than 1.

Definition 1. *A composite number is a positive integer with at least one positive divisor other than one or the number itself.*

Each integer in the list below is every possible square from 9 to 1200 whose *square root* is a number with multiples of 1 and itself. In other words, the factors of every number above are a prime number and *one*.

$$\{9, 25, 49, 121, 169, 289, 361, 529, 841, 961\}$$

The factorization for every integer $\{1 \ldots n\}$ can be simplified to *prime numbers*. Since the numbers above cover every number whose square root $n\prime$ is a prime number, and the square of $n\prime$ is a composite number where $n\prime^2 \leq 1200$, every prime factor for any number $n \leq 1200$ is present as the square root of every number in the list above, except the prime number 2.

A worst-case scenario is shown below:

$$\{4, 8, 9, 25, 49, 121, 169, 289, 361, 529, 841, 961\}$$

Since we have any two other numbers n (where $n \leq 1200$) yet to add to the list above, adding two more composite numbers whose factors are $2, 1$, and itself, will make the newly added pair fit still our initial statement. Since any other prime factors are covered, for any number $n \leq 1200$, it can safely be stated that among any twelve composite numbers selected from the first 1200 natural numbers, there will always be at least one pair with a common factor greater than 1.

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