Least squares

Sample Subtitle

Juan V. Vía

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Example

Showing why least squares

We have a variable y. We know that it's dependent of another variable x in some way. But we don't know how, exactly.
So we go to the field and measure certain points. Those that we can reach. Six of them.

$$(2, 5), (5, 5), (7, 8), (11, 7), (14, 9), (18, 7)$$

That is: at $x = 2$ we measure $y = 5$, at $x = 5$ we measure $y = 5$ again, but at $x = 7$ we got $y = 8$, and so on.

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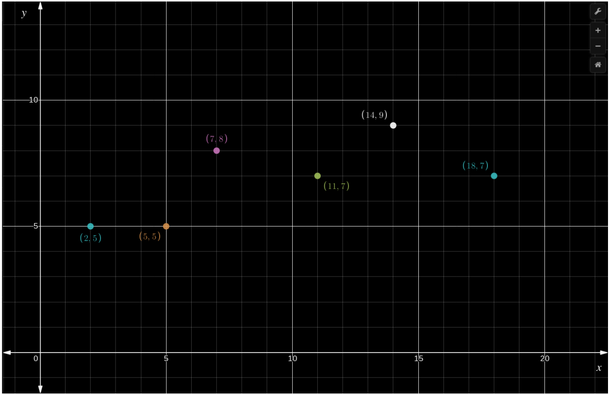
Next step, obviously, is to plot these points.



Example

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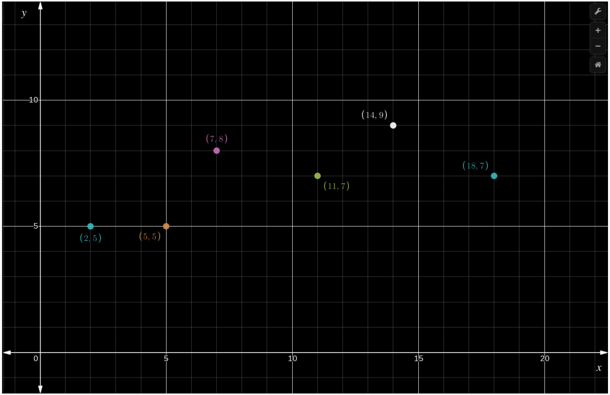
Let's start by tracing a line wich "best fit" that data



Example

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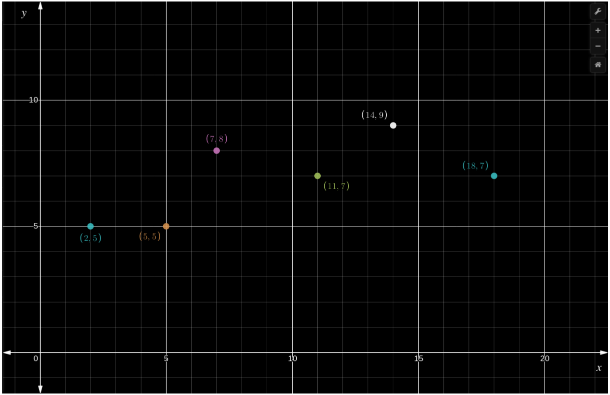
What line? How to choose that line?



Example

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A very common way to express a line is using the equation (a polinomial function)

$$y = mx + b$$

Using vector notation:

$$y = \begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix}$$

Or, switching terms:

$$\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = y$$

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Ok. Fine. That's it. This form

$$\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = y$$

will be a useful one because we know x and y in six points. For example take the first point $(2, 5)$

$$\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = 5$$

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And yes, your guess is true. We can incorporate the second point $(5, 5)$ and get

$$\begin{bmatrix} 2 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

...and the third point...and the fourth...

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Reaching the end this is the result

$$\begin{bmatrix} 2 & 1 \\ 5 & 1 \\ 7 & 1 \\ 11 & 1 \\ 14 & 1 \\ 18 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 8 \\ 7 \\ 9 \\ 7 \end{bmatrix}$$

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Time to name things.

Example

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Call A to the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 1 \\ 7 & 1 \\ 11 & 1 \\ 14 & 1 \\ 18 & 1 \end{bmatrix}$$

Example

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Call x to the column vector

$$x = \begin{bmatrix} m \\ b \end{bmatrix}$$

Example

Showing why least squares

Call b to the column vector

$$b = \begin{bmatrix} 5 \\ 5 \\ 8 \\ 7 \\ 9 \\ 7 \end{bmatrix}$$

Example

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Thus, the equation

$$\begin{bmatrix} 2 & 1 \\ 5 & 1 \\ 7 & 1 \\ 11 & 1 \\ 14 & 1 \\ 18 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 8 \\ 7 \\ 9 \\ 7 \end{bmatrix}$$

becomes

$$Ax = b$$

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