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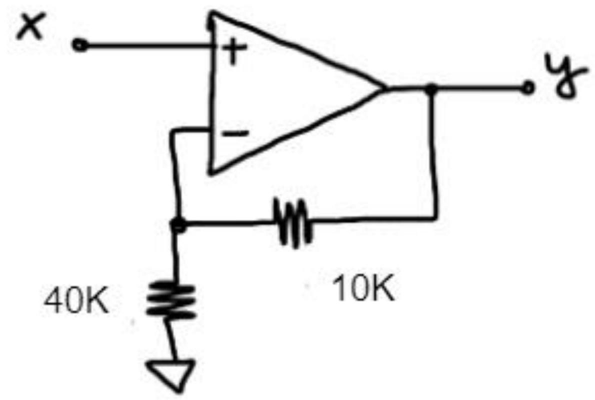
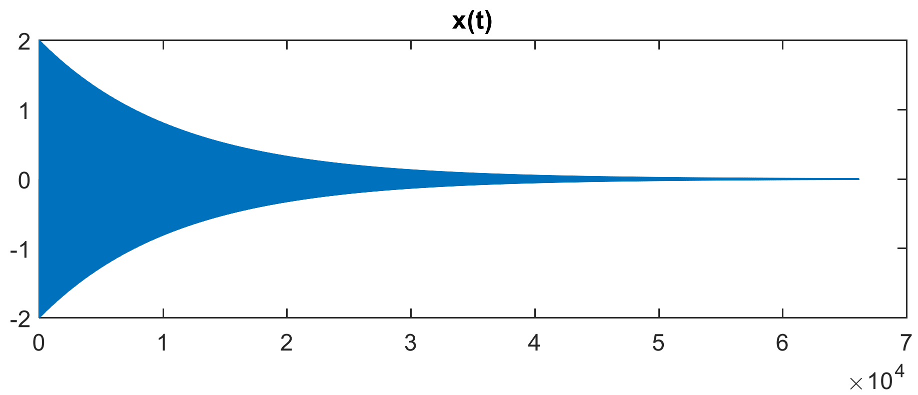
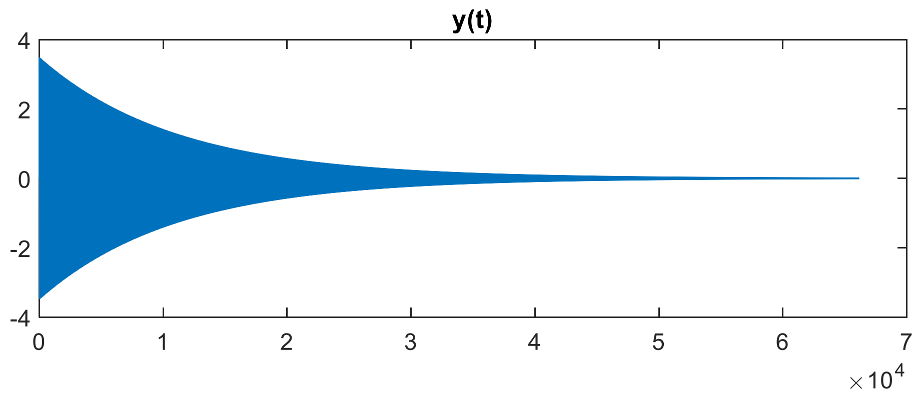
HOMEWORK 2

1. $y(t) = \left(1 + \frac{R2}{R1}\right) x(t) = \left(1 + \frac{10}{40}\right) x(t) = 1.25x(t) \rightarrow \boldsymbol{y(t) = 1.25x(t)}$

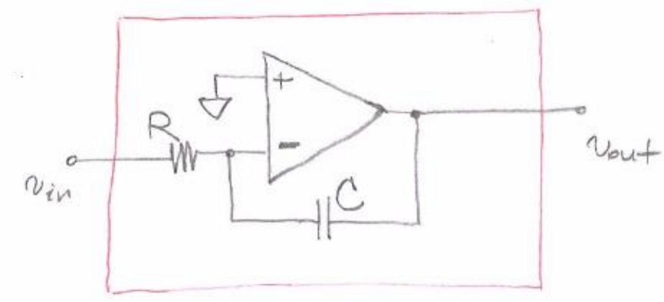


2. Since we have that F_s = 110025Hz that means we will have a sample of the wave every $1/F_s$ = 90.7us. If we sample the wave for 6 seconds, we will have a total of 66151 samples for both signals. In the figure x(t) is the original signal and y(t) is the amplified signal. Code is at the end. As a fact when we use sound(y(t)) in MATLAB we hear the same signal but louder.

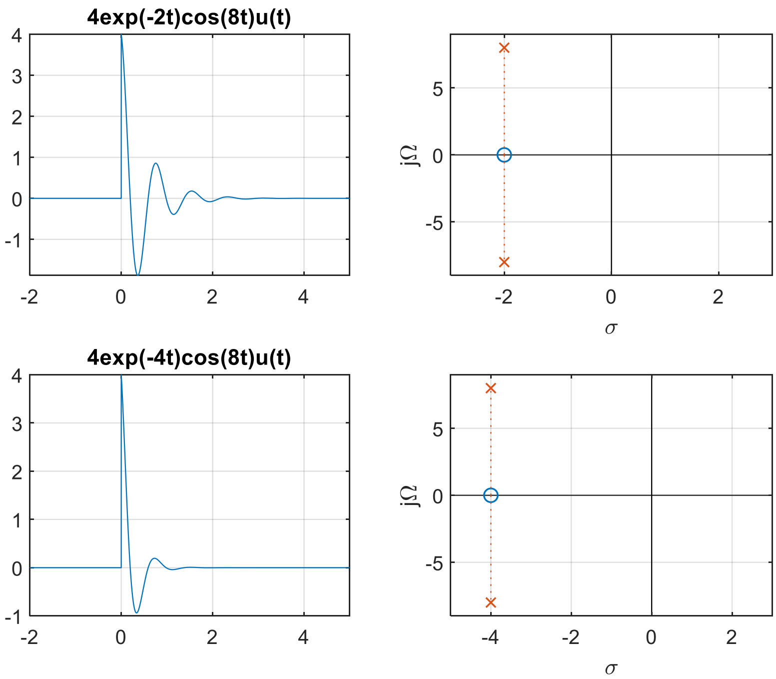




3. We first get the impulse response h(t).
   $Vc(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$
   If we let h(t) = Vc(t) and $i(\tau) = \delta(\tau)$ we get that
   $h(t) = \frac{1}{C}\int_0^t \delta(\tau)d\tau = \boldsymbol{\frac{1}{C}}$
   By knowing the relationship between the impulse response and the unit step response we get that **s(t) =** $\boldsymbol{\frac{1}{C}t}$**.**
   As we can see, the larger C gets, the smaller the slope will be. For C = 1 we have a ramp function.
4. We can get the transfer function by using the complex impedance of the resistor and the capacitor.
   $$Z_C = \frac{1}{Cs}\ ;\ Z_R = R;\ H(s) = \frac{Vout(t)}{Vin(t)} = -\frac{Z_C}{Z_R} = -\boldsymbol{\frac{1}{RCs}}$$
   For Vin = u(t) R = 10KOhm and C = 10uF we get that
   $$Vout = -\frac{1}{RC}\int_0^{0.4} u(t)dt = -\boldsymbol{4V}$$



5. Figure shows the output. Code can be seen at the end.



The second sinusoid (exp(-4t)) is more damped, so that is why the poles in the sigma axis are in -4, which corresponds, as we can see in the time domain, to a more decayng function. However, the location in the complex axis does not change because the frequency of both functions is the same.

6. The one-sided Laplace transform has the following properties:

   Functions and constants are causal. Also the one-sided Laplace transform has linearity, time and frequency shifting, expansion and contraction.

   Other properties include:

   Multiplication by t → tF(t) = - dF(s)/ds

   Derivative →sF(s) – f(0-)

   Second derivative → $s^2F(s)$- sf(0) – f(0)

   Integral→F(s)/s

7. See derivation of the formula:

$$X(s) = 0.5\mathcal{L}[e^{j(\Omega_0 t+\theta)}u(t)] + 0.5\mathcal{L}[e^{-j(\Omega_0 t+\theta)}u(t)]$$

$$X(s) = 0.5\frac{e^{j\theta}}{s-j\Omega}\frac{s+j\Omega}{s+j\Omega} + 0.5\frac{e^{-j\theta}}{s+j\Omega}\frac{s-j\Omega}{s-j\Omega} = \frac{0.5e^{j\theta}s+0.5e^{j\theta}j\Omega+0.5e^{-j\theta}s-0.5e^{-j\theta}j\Omega}{s^2+\Omega^2} = \frac{s0.5(e^{j\theta}+e^{-j\theta})-0.5\frac{1}{j}\Omega(e^{j\theta}-e^{-j\theta})}{s^2+\Omega^2} = \frac{scos(\theta)-\Omega\sin(\theta)}{s^2+\Omega^2}$$

Special cases:

$$\theta = 0 \to X(s) = \frac{s}{s^2+\Omega^2}; \theta = -\frac{\pi}{2} \to X(s) = \frac{\Omega}{s^2+\Omega^2}; = \frac{\pi}{4} \to X(s) = \frac{s-\Omega}{s^2+\Omega^2}$$

Code for 2:

```
%general data
amp = 2;
freq = 200;
sampling_freq = 11025;
%signal
t=0:1/sampling_freq:6;
y = amp.*exp(-t).*sin(2*pi*freq.*t);
y2 = 0.4*amp.*exp(-(t-0.1)).*sin(2*pi*freq.*(t-0.1));
y3 = 0.2*amp.*exp(-(t-0.4)).*sin(2*pi*freq.*(t-0.4));

ysound = y + y2 + y3;

%sound(y)
%sound(ysound)

subplot(2,1,1)
plot(y)
title("x(t)")
subplot(2,1,2)
plot(ysound)
title('y(t)')
```

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Code 5:

```
%general data
amp = 4;
freq = 200;
%signal
t = sym('t');
y = amp*exp(-2*t)*cos(8*t)*heaviside(t);
y2 = amp*exp(-4*t)*cos(8*t)*heaviside(t);
L = laplace(y)
L2 = laplace(y2)
numL=[4 8];denL=[1 4 68];
numL2=[4 16];denL2=[1 8 80];
subplot(2,2,1)
fplot(y,[-2,5]); grid
title('4exp(-2t)cos(8t)u(t)')
subplot(2,2,2)
splane(numL, denL)
subplot(2,2,3)
fplot(y2,[-2,5]); grid
title('4exp(-4t)cos(8t)u(t)')
subplot(2,2,4)
splane(numL2, denL2)
```