Juan Tarrat

HOMEWORK 3

1. $X(s) = \frac{1}{(s+1)^2}$

   $x(t) = \frac{1}{2}\, e^{-t} \sin(2t)$

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s = sym('s')
Xs = 1/((s+1)^2+4)
xt = ilaplace(Xs)
fplot(xt)
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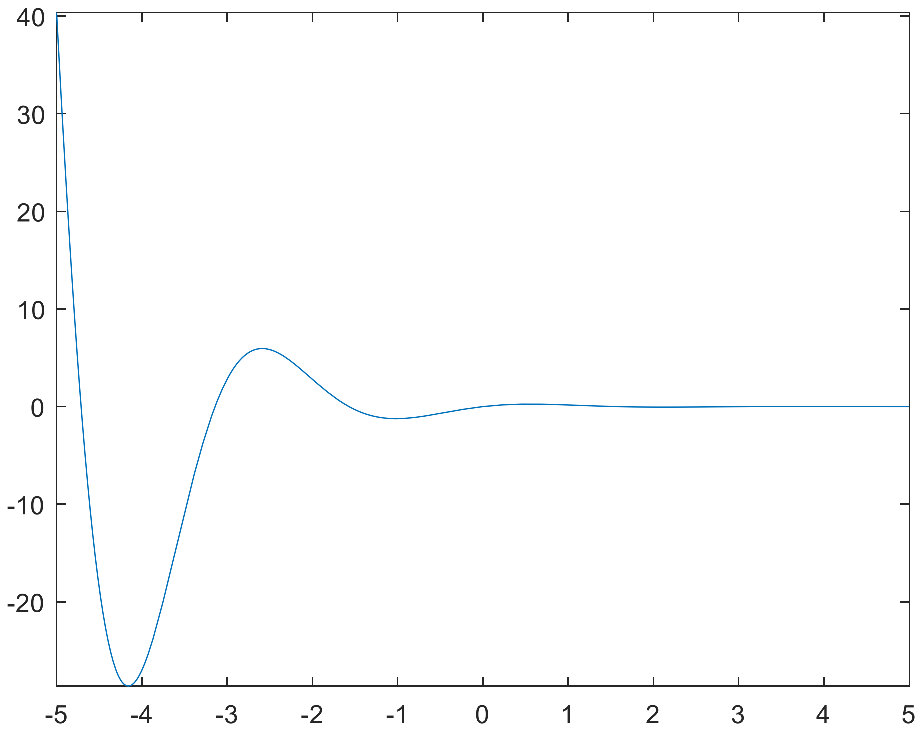


2. $(s^2+3s+2)Y(s) = X(s); H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2}; h(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2+3s+2}\right];$

   $\frac{1}{s^2+3s+2} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{1}{s+1} - \frac{1}{s+2}\ ; \boldsymbol{h(t) = (e^{-t} - e^{-2t})u(t)}$

   $x(t) = u(t) \to X(s) = U(s) \to Y(s) = H(s)X(s) = \frac{1}{s^2+3s+2} * \frac{1}{s};$

   $\frac{1}{s^2+3s+2} * \frac{1}{s} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s} = -\frac{1}{s+1} + \frac{1}{s+2} + \frac{1}{2s}$

   $$\boldsymbol{y(t) = \left(\frac{e^{-2t}}{2} - e^{-t} + \frac{1}{2}\right)u(t)}$$

3. m = 1kg; g = 10m/s^2; L = 20cm; l = -10cm (raised 10 cm from initial position)

   $m * a = k * L \to k = \frac{m * a}{L} = \frac{50N}{m}$

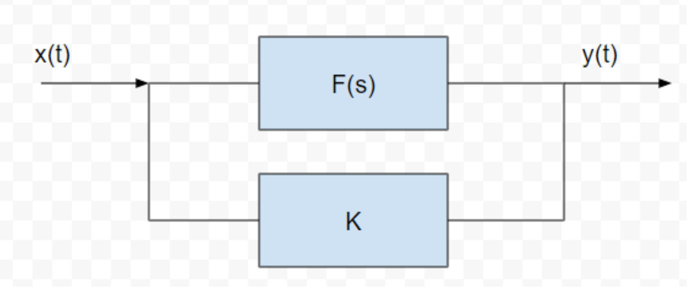
   $\mathcal{L}\left[m\frac{d^2x}{dt^2} + kx\right] = (s^2 + 50s)X(s) + 0.1s = 0 \to X(s) = \frac{-0.1s}{s^2+50} \to$

   $\boldsymbol{x(t) = \mathcal{L}^{-1}[X(s)] = -\frac{1}{10}(\cos(5\sqrt{2}\, t))}$

   At t = 2.5s→x(2.5) = -0.03884m = -3.9cm or 3.9 cm above equilibrium position

4.

$$F(s) = \frac{1}{2 \cdot s - 3}$$



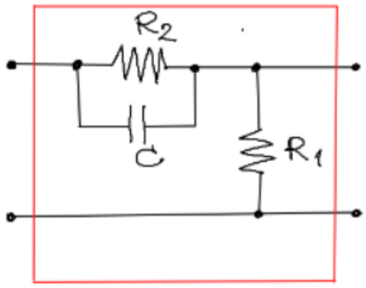
$$F(s)\big(X(s) - KY(s)\big) = Y(s) \to Y(s) = \frac{F(s)}{1 + F(s)K}X(s) = \frac{1}{2s - 3 + K}X(s)$$

For the system to be BIBO stable the part of the denominator not multiplied by s must be greater than 0 so that the function of t does not go to infinity.

$-3 + K > 0 \to K > 3 \to K = 4, 5, 6$ ... for part b we let K = 4

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{2s + 1} \to h(t) = \mathcal{L}^{-1}\left[\frac{1}{2s + 1}\right] = \boldsymbol{\frac{1}{2}e^{-\frac{1}{2}t}u(t)}$$

5.



$$Z_{R_2,C} = \frac{R_2}{R_2Cs+1}; Z_{R_1,Z_{R_2,C}} = \frac{R_1}{R_1+\frac{R_2}{R_2Cs+1}} = \frac{R_1R_2Cs+R_1}{R_1R_2Cs+R_1+R_2}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{R_1R_2Cs + R_1}{R_1R_2Cs + R_1 + R_2}$$

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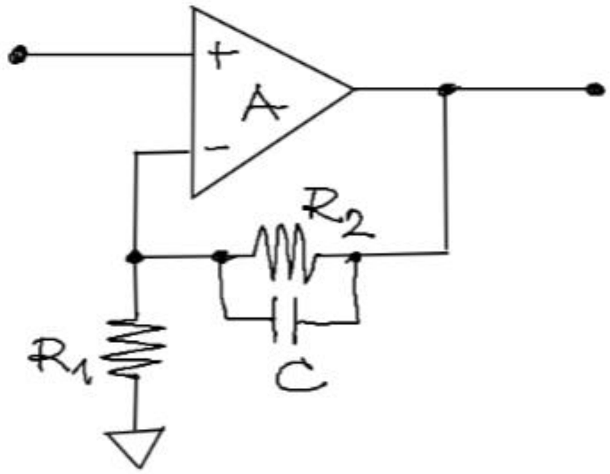

$$Y(s) = \frac{A}{1+AG(s)}X(s)$$ Where G(s) is H(s) from Part a and A = ∞

$$H(s) = \frac{1}{G(s)} = \frac{1}{\frac{R_1R_2Cs + R_1}{R_1R_2Cs + R_1 + R_2}} = \frac{R_1R_2Cs + R_1 + R_2}{R_1R_2Cs + R_1}$$

c)

$$X(s) = U(s) \rightarrow Y(s) = \frac{R_1R_2Cs + R_1 + R_2}{R_1R_2Cs + R_1} * \frac{1}{s}$$

$$s(t) = \left(3 - 2e^{\frac{-t}{2}}\right)u(t)$$

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r1 = 1
r2 = 2
c = 1
s = sym('s')
unitResponse = (r1*r2*c*s+r2+r1)/(r1*r2*c*s+r1)*1/s
fplot(ilaplace(unitResponse),[0,15])
```