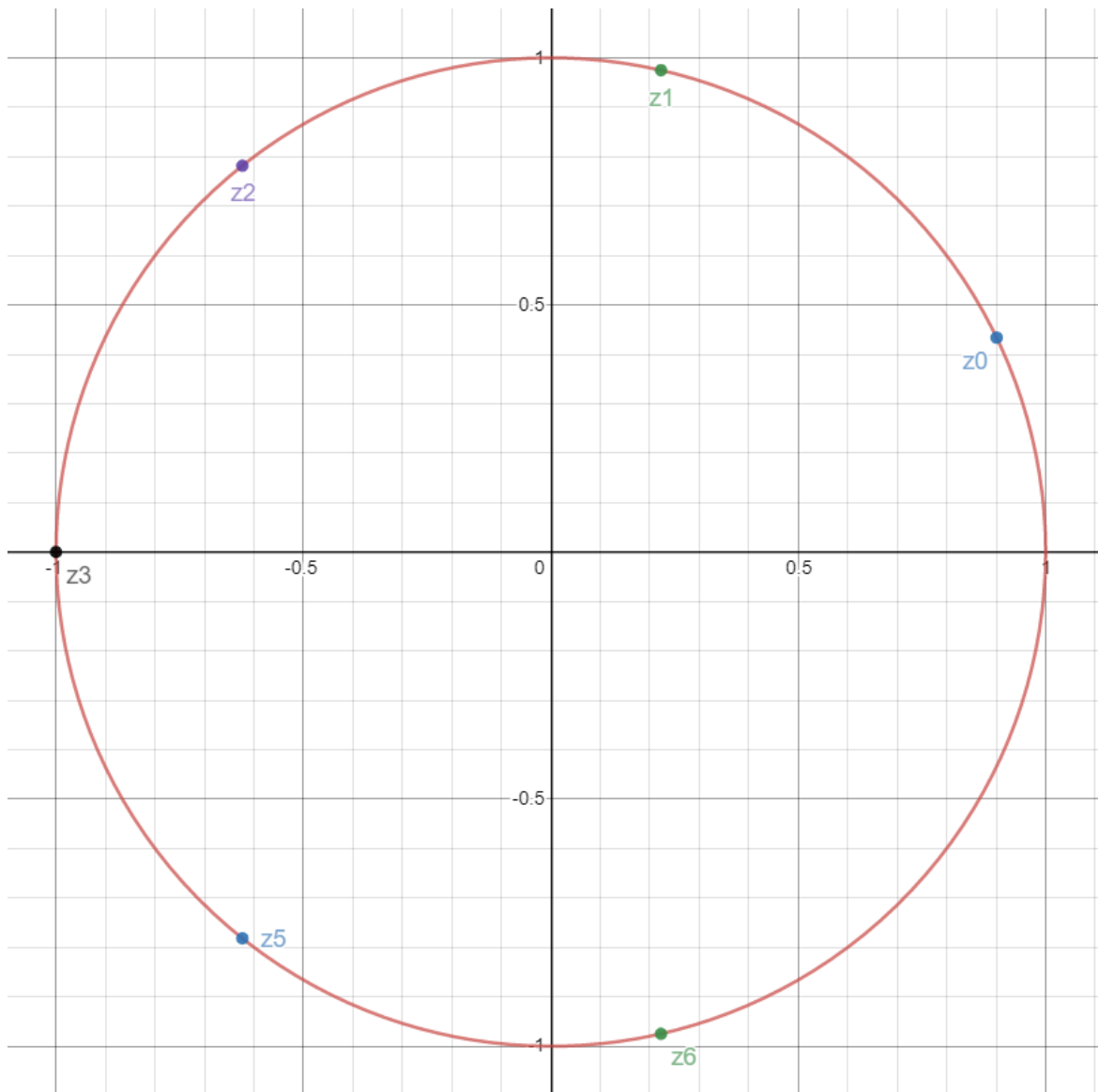


Homework #1

1.

$$Z^7 + 1 = 0 \quad z_k = e^{\frac{j(2k+1)\pi}{7}} \quad k = 0, 1, 2, 3, 4, 5, 6$$

$$z_0 = e^{j\frac{\pi}{7}}; z_1 = e^{j\frac{3\pi}{7}}; z_2 = e^{j\frac{5\pi}{7}}; z_3 = e^{j\frac{7\pi}{7}}; z_4 = e^{j\frac{9\pi}{7}}; z_5 = e^{j\frac{11\pi}{7}}; \\ z_6 = e^{j\frac{13\pi}{7}}$$



2.

$$\text{a) } 1 + j = \sqrt{2}e^{j\frac{\pi}{4}}$$

$$\text{b) } 1 - j = \sqrt{2}e^{-j\frac{\pi}{4}}$$

$$\text{c) } 5e^{j210^\circ} = -\frac{5\sqrt{3}}{2} - \frac{5}{2}j$$

$$\text{d) } 5e^{-j210^\circ} = -\frac{5\sqrt{3}}{2} + \frac{5}{2}j$$

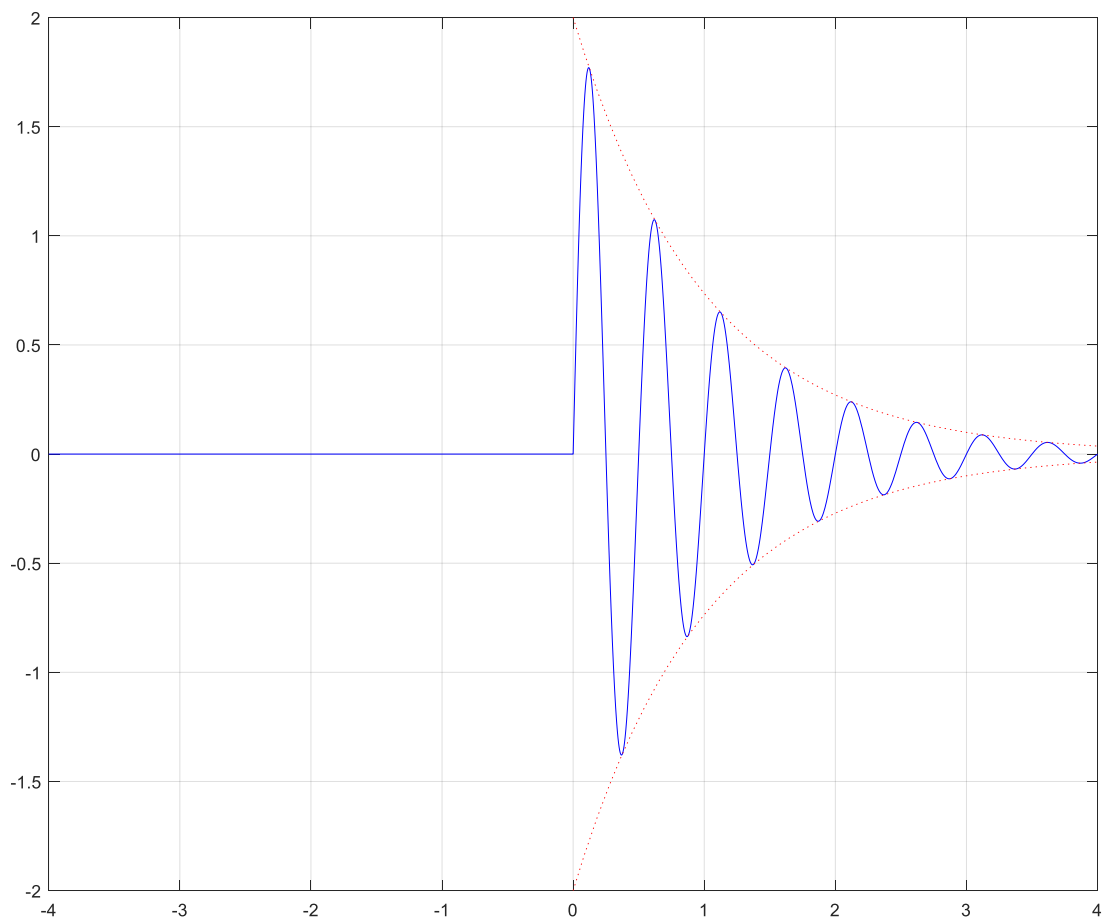
$$\text{e) } zz^* = |z|^2$$

3.

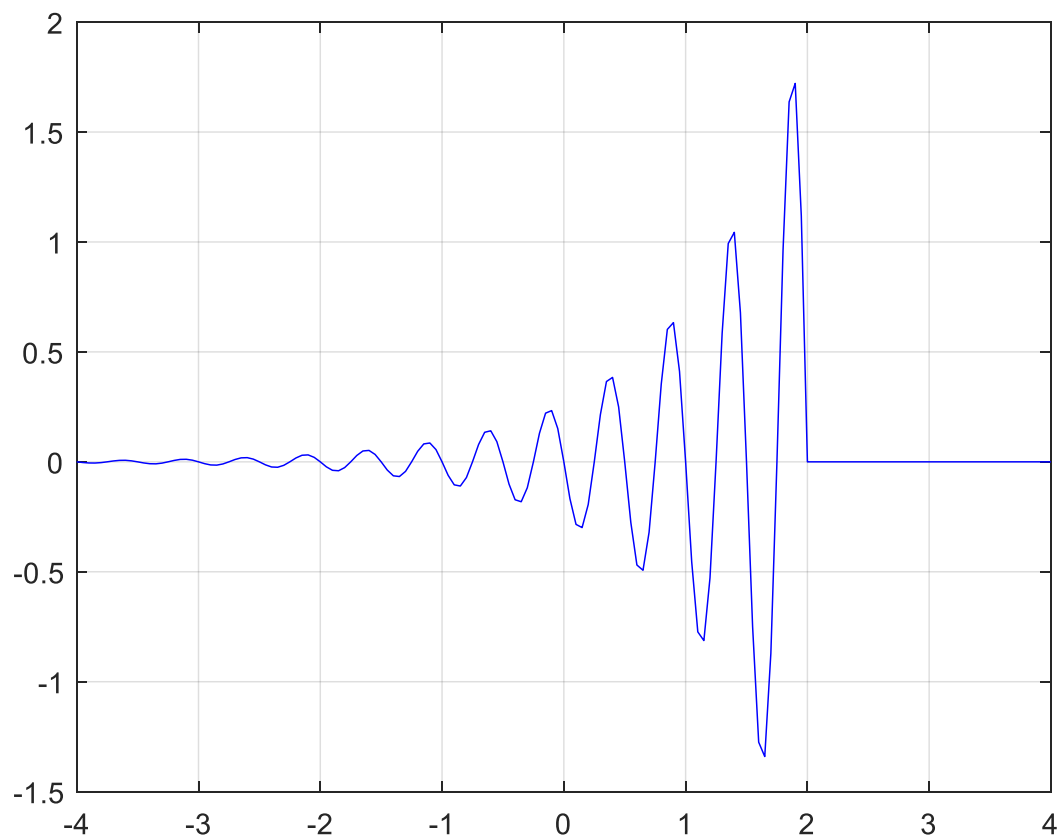
$$\begin{aligned} \sin(\theta + \pi) &= \frac{e^{j(\theta+\pi)} - e^{-j(\theta+\pi)}}{2j} = \frac{1}{2j} (e^{j(\theta+\pi)} - e^{-j(\theta+\pi)}) \\ &= \frac{1}{2j} (e^{j(\theta)} e^{j(\pi)} - e^{-j(\theta)} e^{-j(\pi)}) \\ &= \frac{1}{2j} (e^{j(\theta)} (\cos(\pi) + j\sin(\pi)) - e^{-j(\theta)} (\cos(\pi) - j\sin(\pi))) \\ &= \frac{1}{2j} (e^{j(\theta)} \cos(\pi) + e^{j(\theta)} j\sin(\pi) - e^{-j(\theta)} \cos(\pi) \\ &\quad + e^{-j(\theta)} j\sin(\pi)) \\ &= \frac{1}{2j} ((e^{j(\theta)} - e^{-j(\theta)})\cos(\pi) + (e^{j(\theta)} + e^{-j(\theta)})j\sin(\pi)) \\ &= \sin(\theta) \cos(\pi) + \cos(\theta) \sin(\pi) \end{aligned}$$

$$\begin{aligned}
 \cos(\theta + \pi) &= \frac{e^{j(\theta+\pi)} + e^{-j(\theta+\pi)}}{2} = \frac{1}{2}(e^{j(\theta+\pi)} + e^{-j(\theta+\pi)}) \\
 &= \frac{1}{2}(e^{j(\theta)}e^{j(\pi)} + e^{-j(\theta)}e^{-j(\pi)}) \\
 &= \frac{1}{2}(e^{j(\theta)}(\cos(\pi) + j\sin(\pi)) + e^{-j(\theta)}(\cos(\pi) - j\sin(\pi))) \\
 &= \frac{1}{2}(e^{j(\theta)}\cos(\pi) + e^{j(\theta)}j\sin(\pi) + e^{-j(\theta)}\cos(\pi) \\
 &\quad - e^{-j(\theta)}j\sin(\pi)) \\
 &= \frac{1}{2}((e^{j(\theta)} + e^{-j(\theta)})\cos(\pi) + (e^{j(\theta)} - e^{-j(\theta)})j\sin(\pi)) \\
 &= \cos(\theta)\cos(\pi) - \sin(\theta)\sin(\pi)
 \end{aligned}$$

4.



5.



6.

- a) $X = 1.5\text{V}$; $Y = 0.75\text{V}$
- b) $X = 0.75\text{V}$; $Y = 1.5\text{V}$
- c) $X = 1.5\text{V}$; $Y = 2.25\text{V}$
- d) $X = 2.25\text{V}$; $Y = 1.5\text{V}$
- e) $g_x = 0.5$; $g_y = -0.866$; $\alpha = -59.999^\circ \cong -60^\circ$
- f) $g_x = 0.866$; $g_y = 0.5$; $\alpha = 30.001^\circ \cong 30^\circ$

Appendix:

Matlab script:

```
%general data
amp = 2;
freq = 2;
sampling_freq = 20;
%original signal
t=sym('t');
ye=amp*exp(-t);
y=piecewise(t>=0,ye.*sin(2*pi*freq.*t), t<0, 0);
%shifting portion
time = 0:1/sampling_freq:6; %time data we need
negtime = -4:1/sampling_freq:0; %negative time portion
%function
env = amp.*exp(-time);
y0 = env.*sin(2*pi*freq.*time);
y1 = 0.*negtime; %constant portion
%shifting calculations
yshift = zeros(size(negtime));
y1(41:81) = y0(1:41);
yshift(1:41) = y0(41:81);
yshift(41:81) = y0(81:121);
%portion of time we want to be plotted
shifTime = 0:1/sampling_freq:4;
%original signal with envelopes
figure(1)
fplot(y,[-4,4],'b'); grid
hold on
fplot(ye,[0,4],':r');
fplot(-ye,[0,4],':r');
%shifted signal, note the use of -negtime and -
shifTime
%to mirror the signal
figure(2)
plot(-negtime, y1, 'b', -shifTime, yshift, 'b'); grid
```

Ex 2 calculations:

2.

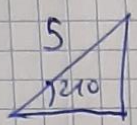
a) $1 + j = \sqrt{2} e^{j\frac{\pi}{4}}$

$|z| = \sqrt{1+1} = \sqrt{2}$ $\alpha = \arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

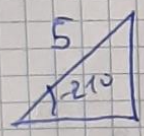
b) $1 - j = \sqrt{2}$

$|z| = \sqrt{1+1} = \sqrt{2}$ $\alpha = \arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$

c) $5 e^{j210^\circ} = -\frac{5\sqrt{3}}{2} - \frac{5}{2}j$

 $y = 5 \sin(210)$
 $x = 5 \cos(210)$

d) $5 e^{-j210} = -\frac{5\sqrt{3}}{2} + \frac{5}{2}j$

 $y = 5 \sin(-210)$
 $x = 5 \cos(-210)$

Ex6 phone drawings:

