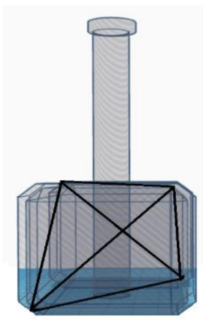
Historia que representa la ecuación

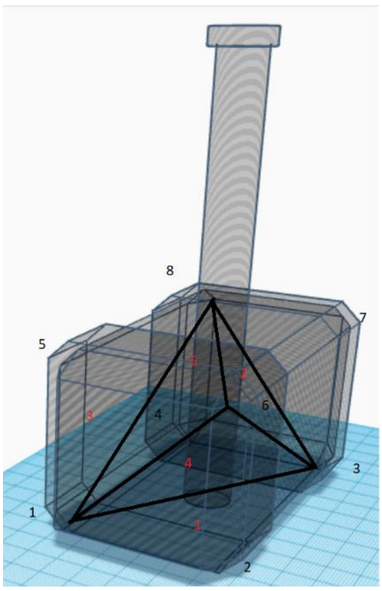
La historia del mejorado martillo “mjolnir V2” capaz de dividir los cielos, evitar hechizos, con tan solo agitar el martillo mediante el comando de su propietario. Su metal fue forjado por los hermanos enanos Sindri y Brokkr, Sindri fue quien moldeó la figura, consiguió el metal “Uru” junto a la mejora del metal “Irik” y le dio las habilidades al martillo, en altas temperaturas cercanas al sol, el metal utilizado fue obtenido en Alfheim por un trato de Sindri, mientras Brokkr al ser menos habilidoso que su hermano, fue el encargado de tallar las runas y agregar poderes mágicos al martillo.

Retomando el párrafo anterior, tendremos la siguiente ecuación: $-\eta^3\nabla\cdot(\epsilon^2\nabla X) = \eta^3 + \eta$ que representa las altas temperaturas que alcanza mjolnir cuando realiza sus distintos ataques que pueden ser rayos al momento de chocar. con el suelo, congelar al realizar una onda expansiva horizontal entre otros. Las temperaturas varían dependiendo de los ataques que se realizan con él. Así que un grupo de científicos se propuso resolver la formula para saber una aproximación de cuales temperaturas alcanza mjolnir. Representando la temperatura con la variable *X*

Mallado desde el costado del martillo



Mallado desde enfrente del martillo



Resolución de la segunda formula

$$-\eta^3\nabla * (\epsilon^2\nabla \mathrm{X}) \ = \eta^3 + \eta$$

Aproximación del modelo

Como primer paso debemos realizar la aproximación del modelo esto a través de interpolación recordando que:

$$X \ \approx \ N_1X_1 + N_2X_2 + N_3X_3 + N_nX_n$$

$$= [N_1 \quad N_2 \quad N_3]\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \ \mathbf{NX}$$

$$X \approx \mathbf{NX}$$

Así que sustituyendo en nuestra formula nos quedaría de la siguiente manera:

$$-\eta^3\nabla * (\epsilon^2\nabla(\mathbf{NX})) \ = \eta^3 + \eta$$

Definiendo el residual

Definimos el residual pasando el lado izquierdo de la formula a sumar a lado derecho

$$\mathfrak{R} \ = \eta^3 + \eta + \eta^3\nabla * (\epsilon^2\nabla(\mathbf{NX}))$$

Método de los residuos ponderados:

Aplicamos el concepto de residuos ponderados

$$\int_V \boldsymbol{w} * \mathfrak{R} \, \mathrm{dV} \ = \ 0$$

$$\int_V \boldsymbol{w} * (\eta^3 + \eta \ + \eta^3\nabla * (\epsilon^2\nabla(\mathbf{NX}))) \, \mathrm{dV} \ = \ 0$$

Método de Galerkin

Recordando que:

$$\boldsymbol{w} \ = \ \mathbf{N}^{\mathrm{t}}$$

Así que sustituyendo nos queda de la siguiente manera:

$$\int_V \mathbf{N}^{\mathrm{t}} * (\eta^3 + \eta \ + \eta^3\nabla * (\epsilon^2\nabla(\mathbf{NX}))) \, \mathrm{dV} \ = \ 0$$

Ordenamiento e interludio

Finalmente realizamos el ordenamiento y las operaciones necesarias

$$\int_V (\mathbf{N}^{\mathrm{t}}\eta^3 + \mathbf{N}^{\mathrm{t}}\eta \ + \mathbf{N}^{\mathrm{t}}\eta^3\nabla * (\epsilon^2\nabla(\mathbf{NX}))) \, \mathrm{dV} \ = \ 0$$

$$\int_V \mathbf{N}^{\mathrm{t}}\eta^3 \, dV \ + \int_V \mathbf{N}^{\mathrm{t}}\eta \, dV \ + \int_V \mathbf{N}^{\mathrm{t}}(\eta^3\nabla * (\epsilon^2\nabla(\mathbf{NX}))) \, \mathrm{dV} = \ 0$$

$$-\int_V \mathbf{N}^{\mathrm{t}}(\eta^3\nabla * (\epsilon^2\nabla(\mathbf{NX})))\,\mathrm{dV} = \int_V \mathbf{N}^{\mathrm{t}}\eta^3\, dV \; + \int_V \mathbf{N}^{\mathrm{t}}\eta\, dV$$

$$-(\int_V \mathbf{N}^{\mathrm{t}}(\eta^3\nabla * (\epsilon^2\nabla(\mathbf{NX})))\,\mathrm{dV})) = \int_V \mathbf{N}^{\mathrm{t}}\eta^3\, dV \; + \int_V \mathbf{N}^{\mathrm{t}}\eta\, dV$$

Resolución de las integrales

Finalmente llegamos a la solución de las integrales comenzamos con el lado derecho:

Lado derecho

$$\int_V \mathbf{N}^{\mathrm{t}}\eta^3\, dV \; + \int_V \mathbf{N}^{\mathrm{t}}\eta\, dV$$

Recordando que en este caso $\mathbf{N}^{\mathrm{t}}$ representa una matriz compuesta por las funciones de forma en este caso como estamos trabajando con un objecto en 3 dimensiones tomamos las funciones de forma de 3 dimensiones.

Pero claro tenemos que hacer que la integrales sea con respecto a los valores de las funciones de forma para eso nos apoyamos del jacobiano quedándonos de la siguiente forma la primera integral.

$$\int_V \mathbf{N}^{\mathrm{t}}\eta^3\, J d\epsilon d\eta d\phi$$

Sustituyendo en la $\mathbf{N}^{\mathrm{t}}$

$$J * \int_V \begin{bmatrix} 1-\epsilon-\eta-\phi \\ \epsilon \\ \eta \\ \phi \end{bmatrix} * \eta^3\, d\epsilon d\eta d\phi$$

Realizamos la operación de multiplicación con η^3

$$J * \int_V \begin{bmatrix} \eta^3-\epsilon\eta^3-\eta^4-\phi\eta^3 \\ \epsilon\eta^3 \\ \eta^4 \\ \phi\eta^3 \end{bmatrix} d\epsilon d\eta d\phi$$

Finalmente sacamos el Jacobiano(J) ya que sabemos que no son más que constantes. Ahora comenzamos con la solución de cada una de las integrales.

Resolución de $\epsilon\eta^3$:

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \epsilon\eta^3 d\epsilon d\eta d\phi$$

- **Primera integral:**

$$\int_0^{1-\eta-\phi} \epsilon\eta^3 d\epsilon d\eta d\phi$$

$$\eta^3 * \int_0^{1-\eta-\phi} \epsilon d\epsilon d\eta d\phi$$

$$\eta^3 * (\frac{e^2}{2})\ desde\ cero\ hasta\ 1-\eta-\phi$$

$$= \eta^3 * \frac{(1-\eta-\phi)^2}{2}$$

- **Segunda integral:**

$$\int_0^{1-\phi} \frac{\eta^3 * (1-\eta-\phi)^2}{2} d\eta$$

Sacamos la constante:

$$\frac{1}{2} * (\int_0^{1-\phi} \eta^3 * (1-\eta-\phi\ )^2\ d\eta)$$

Resolviendo:

$$(1-\eta-\phi)^2$$

Aplicamos leyes de los exponentes:

$$(1-\eta-\phi)(1-\eta-\phi\ )$$

Y resolvemos aplicando la regla de productos notables:

$$1*1+(1*-\eta)+(1*-\phi)+(-\eta*1)+(-\eta*-\eta)+(-\eta*-\phi)+(-\phi*1)+(-\phi*-\eta)+(-\phi*-\phi)$$

$$1-\eta-\phi-\eta+\eta^2+\eta\phi-\phi+\eta\phi+\phi^2$$

$$1-2\eta-2\phi+\eta^2+2\eta\phi+\phi^2$$

Reordenando los términos:

$$\eta^2+2\eta\phi-2\eta+\phi^2-2\phi+1$$

Volviendo a la integral nos queda de la siguiente manera:

$$\frac{1}{2} * (\int_0^{1-\phi} \eta^3 * (\eta^2+2\eta\phi-2\eta+\phi^2-2\phi+1) d\eta)$$

Multiplicamos por η^3

$$\frac{1}{2} * (\int_0^{1-\phi} \eta^5+2\eta^4\phi-2\eta^4+\eta^3\phi^2-2\eta^3\phi+\eta^3\phi d\eta)$$

Distribuimos la integral

$$\frac{1}{2} * \left(\int_0^{1-\phi} \eta^5 d\eta + \int_0^{1-\phi} 2\eta^4\phi d\eta - \int_0^{1-\phi} 2\eta^4 d\eta + \int_0^{1-\phi} \eta^3\phi^2 d\eta - \int_0^{1-\phi} 2\eta^3\phi d\eta + \int_0^{1-\phi} \eta^3 d\eta\right)$$

Resolviendo las integrales:

$$\int_0^{1-\phi} \eta^5 d\eta$$

$$\left.\frac{\eta^6}{6}\right|_0^{1-\phi} = \frac{(1-\phi)^6}{6}$$

$$\int_0^{1-\phi} 2\eta^4 \phi d\eta$$

$$2\phi \int_0^{1-\phi} \eta^4 \phi d\eta$$

$$2\phi * \left(\left.\frac{\eta^5}{5}\right|_0^{1-\phi} = \frac{(1-\phi)^5}{5}\right)$$

$$= \frac{2\phi(1-\phi)^5}{5}$$

$$-\int_0^{1-\phi} 2\eta^4 d\eta$$

$$-2 * \left(\left.\frac{\eta^5}{5}\right|_0^{1-\phi} = \frac{(1-\phi)^5}{5}\right)$$

$$= \frac{-2(1-\phi)^5}{5}$$

$$\int_0^{1-\phi} \eta^3 \phi^2 d\eta$$

$$\phi^2 * \left(\left.\frac{\eta^4}{4}\right|_0^{1-\phi} = \frac{(1-\phi)^4}{4}\right)$$

$$= \frac{\phi^2(1-\phi)^4}{4}$$

$$-\int_0^{1-\phi} 2\eta^3 \phi d\eta$$

$$-2\phi * \left(\left.\frac{\eta^4}{4}\right|_0^{1-\phi} = \frac{(1-\phi)^4}{4}\right)$$

$$= \frac{-\phi(1-\phi)^4}{2}$$

$$\int_0^{1-\phi} \eta^3 d\eta$$

$$\left.\frac{\eta^4}{4}\right|_0^{1-\phi} = \frac{(1-\phi)^4}{4}$$

Sustituyendo las integrales resueltas:

$$\frac{1}{2} * \left(\frac{(1-\phi)^6}{6} + \frac{2\phi(1-\phi)^5}{5} + \frac{-2(1-\phi)^5}{5} + \frac{\phi^2(1-\phi)^4}{4} + \frac{-\phi(1-\phi)^4}{2} + \frac{(1-\phi)^4}{4} \right)$$

Resolviendo las operaciones aritméticas y simplificando:

$$= \frac{(1-\phi)^6}{120}$$

- **Tercera integral:**

$$\int_0^1 \frac{(1-\phi)^6}{120} d\phi$$

$$\frac{1}{120} * \left(\int_0^1 (1-\phi)^6 \, d\phi \right)$$

$$\frac{1}{120} * \left(\int_0^1 (\phi^6 - 6\phi^5 + 15\phi^4 - 20\phi^3 + 15\phi^2 - 6\phi + 1) d\phi \right)$$

$$\frac{1}{120} * \left(\int_0^1 \phi^6 \, d\phi - \int_0^1 6\phi^5 \, d\phi + \int_0^1 15\phi^4 \, d\phi - \int_0^1 20\phi^3 \, d\phi + \int_0^1 15\phi^2 \, d\phi - \int_0^1 6\phi \, d\phi + \int_0^1 1 \, d\phi \right)$$

Resolviendo las integrales:

$$\int_0^1 \phi^6 \, d\phi$$

$$\left. \frac{\phi^7}{7} \right|_0^1 = \frac{1}{7}$$

$$-\int_0^1 6\phi^5 \, d\phi$$

$$-6(\left. \frac{\phi^6}{6} \right|_0^1 = \frac{1}{6})$$

$$= -1$$

$$\int_0^1 15\phi^4 \, d\phi$$

$$15(\left. \frac{\phi^5}{5} \right|_0^1 = \frac{1}{5})$$

$$= 3$$

$$-\int_0^1 20\phi^3 \, d\phi$$

$$-20(\left. \frac{\phi^4}{4} \right|_0^1 = \frac{1}{4})$$

$$= -5$$

$$\int_0^1 15\phi^2\, d\phi$$

$$15(\frac{\phi^3}{3}\Big|_0^1 = \frac{1}{3})$$

$$= 5$$

$$-\int_0^1 6\phi\, d\phi$$

$$-6(\frac{\phi^2}{2}\Big|_0^1 = \frac{1}{2})$$

$$= -3$$

$$\int_0^1 1\, d\phi$$

$$= 1$$

$$\frac{1}{120}\left(\frac{1}{7} - 1 + 3 - 5 + 5 - 3 + 1\right)$$

$$\epsilon\eta^3 = \frac{1}{840}$$

$$J * \int_V \begin{bmatrix} \eta^3 - \epsilon\eta^3 - \eta^4 - \phi\eta^3 \\ \epsilon\eta^3 \\ \eta^4 \\ \phi\eta^3 \end{bmatrix} d\epsilon d\eta d\phi$$

Resolución de η^4:

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^4 d\epsilon d\eta d\phi$$

- **Primera integral**

$$\int_0^{1-\eta-\phi} \eta^4 d\epsilon$$

$$\int_0^{1-\eta-\phi} \eta^4 d\epsilon$$

$$\eta^4 * \int_0^{1-\eta-\phi} d\epsilon$$

$$\eta^4 * (1 - \eta - \phi)$$

$$= \eta^4 - \eta^5 - \eta^4\phi$$

- **Segunda integral:**

$$\int_0^{1-\phi} \eta^4 - \eta^5 - \eta^4\phi \, d\eta$$

$$\int_0^{1-\phi} \eta^4 \, d\eta - \int_0^{1-\phi} \eta^5 \, d\eta - \phi \int_0^{1-\phi} \eta^4 d\eta$$

Resolviendo las integrales:

$$\int_0^{1-\phi} \eta^4 \, d\eta$$

$$\left(\frac{\eta^5}{5}\bigg|_0^1 = \frac{(1-\phi)^5}{5}\right)$$

$$-\int_0^{1-\phi} \eta^5 \, d\eta$$

$$-\left(\frac{\eta^6}{6}\bigg|_0^1 = \frac{(1-\phi)^6}{6}\right)$$

$$\frac{-(1-\phi)^6}{6}$$

$$-\int_0^{1-\phi} \eta^4 \phi d\eta$$

$$-\phi\left(\frac{\eta^5}{5}\bigg|_0^1 = \frac{(1-\phi)^5}{5}\right)$$

$$\frac{-\phi(1-\phi)^5}{5}$$

$$= \frac{(1-\phi)^5}{5} + \frac{-(1-\phi)^6}{6} + \frac{-\phi(1-\phi)^5}{5}$$

- **Tercera integral:**

$$\int_0^1 \left(\frac{(1-\phi)^5}{5} + \frac{-(1-\phi)^6}{6} + \frac{-\phi(1-\phi)^5}{5} \right) d\phi$$

$$\int_0^1 \frac{(1-\phi)^5}{5} d\phi + \int_0^1 \frac{-(1-\phi)^6}{6} d\phi + \int_0^1 \frac{-\phi(1-\phi)^5}{5} d\phi$$

Resolviendo las integrales:

$$\int_0^1 \frac{(1-\phi)^5}{5} d\phi$$

$$\frac{1}{5}\left(\int_0^1 (1-\phi)^5 d\phi\right)$$

$$= \frac{1}{30}$$

$$\int_0^1 \frac{-(1-\phi)^6}{6} d\phi$$

$$-\frac{1}{6}(\int_0^1 (1-\phi)^6 d\phi)$$

$$= \frac{-1}{42}$$

$$\int_0^1 \frac{-\phi(1-\phi)^5}{5} d\phi$$

$$\frac{1}{5}(\int_0^1 -\phi(1-\phi)^5 d\phi)$$

$$= \frac{-1}{210}$$

$$\frac{1}{30} + \frac{-1}{42} + \frac{-1}{210}$$

$$\eta^4 = \frac{1}{210}$$

$$J * \int_V \begin{bmatrix} \eta^3 - \epsilon\eta^3 - \eta^4 - \phi\eta^3 \\ \epsilon\eta^3 \\ \eta^4 \\ \phi\eta^3 \end{bmatrix} d\epsilon d\eta d\phi$$

Resolución de $\phi\eta^3$:

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \phi\eta^3 d\epsilon d\eta d\phi$$

- **Primera integral**

$$\int_0^{1-\eta-\phi} \phi\eta^3 d\epsilon$$

$$\phi\eta^3 * \int_0^{1-\eta-\phi} d\epsilon$$

$$\phi\eta^3 * (1-\eta-\phi)$$

$$= \phi\eta^3 - \phi\eta^4 - \phi^2\eta^3$$

- **Segunda integral**

$$\int_0^{1-\phi} \phi\eta^3 - \phi\eta^4 - \phi^2\eta^3 d\eta$$

$$\int_0^{1-\phi} \phi\eta^3 d\eta - \int_0^{1-\phi} \phi\eta^4 d\eta - \int_0^{1-\phi} \phi^2\eta^3 d\eta$$

$$\phi * \int_0^{1-\phi} \eta^3 d\eta - \phi * \int_0^{1-\phi} \eta^4 d\eta - \phi^2 * \int_0^{1-\phi} \eta^3 d\eta$$

Resolviendo las integrales:

$$\phi * \int_0^{1-\phi} \eta^3 d\eta$$

$$\phi(\frac{\eta^4}{4}\Big|_0^{1-\phi} = \frac{(1-\phi)^4}{4})$$

$$\frac{\phi(1-\phi)^4}{4}$$

$$-\phi * \int_0^{1-\phi} \eta^4 d\eta$$

$$-\phi(\frac{\eta^5}{5}\Big|_0^{1-\phi} = \frac{(1-\phi)^5}{5})$$

$$\frac{-\phi(1-\phi)^5}{5}$$

$$-\phi^2 * \int_0^{1-\phi} \eta^3 d\eta$$

$$-\phi^2(\frac{\eta^4}{4}\Big|_0^{1-\phi} = \frac{(1-\phi)^4}{4})$$

$$\frac{-\phi^2(1-\phi)^4}{4}$$

$$= \left(\frac{\phi(\phi-1)^4}{4} + \frac{-\phi(1-\phi)^5}{5} + \frac{-\phi^2(\phi-1)^4}{4}\right)$$

- **Tercera integral**

$$\int_0^{\mathbf{1}} \left(\frac{\phi(\phi-1)^4}{4} + \frac{-\phi(1-\phi)^5}{5} + \frac{-\phi^2(\phi-1)^4}{4}\right) d\eta$$

$$\int_0^{\mathbf{1}} \frac{\phi(\phi-1)^4}{4} d\eta + \int_0^{\mathbf{1}} \frac{-\phi(1-\phi)^5}{5} d\eta + \int_0^{\mathbf{1}} \frac{-\phi^2(\phi-1)^4}{4} d\eta$$

Resolviendo las integrales:

$$\int_0^{\mathbf{1}} \frac{\phi(\phi-1)^4}{4} d\eta$$

$$\frac{\mathbf{1}}{\mathbf{4}}(\int_0^{\mathbf{1}} \phi(\phi-1)^4 d\eta)$$

$$=\frac{1}{120}$$

$$\int_0^{\mathbf{1}} \frac{-\phi(1-\phi)^5}{5} d\eta$$

$$\frac{\mathbf{1}}{\mathbf{5}}(\int_0^{\mathbf{1}} -\phi(1-\phi)^5 d\eta)$$

$$=\frac{-1}{210}$$

$$\int_0^{\mathbf{1}} \frac{-\phi^2(\phi-1)^4}{4} d\eta$$

$$\frac{\mathbf{1}}{\mathbf{4}}(\int_0^{\mathbf{1}} -\phi^2(\phi-1)^4 d\eta)$$

$$=\frac{-1}{420}$$

$$=\frac{1}{120}+\frac{-1}{210}+\frac{-1}{420}$$

$$\phi\eta^3=\frac{1}{840}$$

$$J*\int_V \begin{bmatrix} \eta^3-\epsilon\eta^3-\eta^4-\phi\eta^3 \\ \epsilon\eta^3 \\ \eta^4 \\ \phi\eta^3 \end{bmatrix} d\epsilon d\eta d\phi$$

Resolución de η^3:

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^3 d\epsilon d\eta d\phi$$

- **Primera integral**

$$\int_0^{1-\eta-\phi} \eta^3 d\epsilon$$

$$\eta^3 * \int_0^{1-\eta-\phi} d\epsilon$$

$$\eta^3 * (1-\eta-\phi)$$

$$=\eta^3-\eta^4-\phi\eta^3$$

- **Segunda integral**

$$\int_0^{1-\phi} (\eta^3 - \eta^4 - \phi\eta^3) d\eta$$

Resolviendo las integrales:

$$\int_0^{1-\phi} \eta^3 d\eta - \int_0^{1-\phi} \eta^4 d\eta - \phi \int_0^{1-\phi} \eta^3 d\eta$$

$$\int_0^{1-\phi} \eta^3 d\eta$$

$$\left.\frac{\eta^4}{4}\right|_0^{1-\phi} = \frac{(1-\phi)^4}{4}$$

$$-\int_0^{1-\phi} \eta^4 d\eta$$

$$-\left(\left.\frac{\eta^5}{5}\right|_0^{1-\phi} = \frac{(1-\phi)^5}{5}\right)$$

$$= \frac{-(1-\phi)^5}{5}$$

$$-\phi \int_0^{1-\phi} \eta^3 d\eta$$

$$-\phi\left(\left.\frac{\eta^4}{4}\right|_0^{1-\phi} = \frac{(1-\phi)^4}{4}\right)$$

$$= \frac{-\phi(1-\phi)^4}{4}$$

$$\frac{(1-\phi)^4}{4} + \frac{-(1-\phi)^5}{5} + \frac{-\phi(1-\phi)^4}{4}$$

- **Tercera integral**

$$\int_{\mathbf{0}}^{\mathbf{1}} \frac{(1-\phi)^4}{4} + \frac{-(1-\phi)^5}{5} + \frac{-\phi(1-\phi)^4}{4} d\,\phi$$

$$\int_0^1 \frac{(\phi-1)^4}{4} d\phi + \int_0^1 \frac{-(1-\phi)^5}{5} d\phi + \int_0^1 \frac{-\phi(\phi-1)^4}{4} d\phi$$

Resolviendo las integrales:

$$\int_0^1 \frac{(\phi-1)^4}{4} d\phi$$

$$\frac{1}{4}(\int_0^1 (\phi-1)^4 d\phi)$$

$$=\frac{1}{20}$$

$$\int_0^1 \frac{-(1-\phi)^5}{5} d\phi$$

$$-\frac{1}{4}(\int_0^1 (1-\phi)^5 d\phi)$$

$$=\frac{-1}{30}$$

$$\int_0^1 \frac{-\phi(\phi-1)^4}{4} d\phi$$

$$-\frac{1}{4}(\int_0^1 \phi(\phi-1)^4 d\phi)$$

$$=\frac{-1}{120}$$

$$=\frac{1}{20}+\frac{-1}{30}+\frac{-1}{120}$$

$$\eta^3=\frac{1}{120}$$

$$J * \int_V \begin{bmatrix} \eta^3-\epsilon\eta^3-\eta^4-\phi\eta^3 \\ \epsilon\eta^3 \\ \eta^4 \\ \phi\eta^3 \end{bmatrix} d\epsilon d\eta d\phi$$

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^3-\epsilon\eta^3-\eta^4-\phi\eta^3 d\epsilon d\eta d\phi - \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \epsilon\eta^3 d\epsilon d\eta d\phi$$
$$- \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^4 d\epsilon d\eta d\phi - \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \phi\eta^3 d\epsilon d\eta d\phi$$

Sustituyendo los valores encontrados anteriormente

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^3-\epsilon\eta^3-\eta^4-\phi\eta^3 d\epsilon d\eta d\phi = \frac{1}{120}$$

$$-\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \epsilon\eta^3 d\epsilon d\eta d\phi = -\frac{1}{840}$$

$$-\int_{0}^{1}\int_{0}^{1-\phi}\int_{0}^{1-\eta-\phi}\eta^{4}d\epsilon d\eta d\phi\ =-\frac{1}{210}$$

$$-\int_{0}^{1}\int_{0}^{1-\phi}\int_{0}^{1-\eta-\phi}\phi\eta^{3}d\epsilon d\eta d\phi\ =-\frac{1}{840}$$

$$\frac{1}{120}-\frac{1}{840}-\frac{1}{210}-\frac{1}{840}$$

$$R=\frac{1}{840}$$

Finalmente, este sería el resultado de la primera integral sustituyendo los valores encontrados anteriormente:

$$J*\begin{bmatrix}\frac{1}{840}\\ \frac{1}{840}\\ \frac{1}{210}\\ \frac{1}{840}\end{bmatrix}$$

Ahora continuamos con la siguiente integral:

$$\int_{V}\mathbf{N}^{\mathrm{t}}\eta\ dV$$

$$J*\int_{V}\begin{bmatrix}1-\epsilon-\eta-\phi\\ \epsilon\\ \eta\\ \phi\end{bmatrix}*\eta\ d\epsilon d\eta d\phi$$

$$J*\int_{V}\begin{bmatrix}\eta-\epsilon\eta-\eta-\phi\eta\\ \epsilon\eta\\ \eta^{2}\\ \phi\eta\end{bmatrix}d\epsilon d\eta d\phi$$

Resolución de $\epsilon\eta$:

$$\int_{0}^{1}\int_{0}^{1-\phi}\int_{0}^{1-\eta-\phi}\epsilon\eta\ d\epsilon d\eta d\phi$$

- **Primera integral**

$$\int_{0}^{1-\eta-\phi}\epsilon\eta\ d\epsilon$$

$$\eta\int_{0}^{1-\eta-\phi}\epsilon\ d\epsilon$$

$$= \eta * \frac{(1-\eta-\phi)^2}{2}$$

- **Segunda integral**

$$\int_0^{1-\phi} \eta * \frac{(\mathbf{1}-\eta-\phi)^2}{2}\, d\eta$$

$$\frac{1}{2} * (\int_0^{1-\phi} \eta * \ (1-\eta-\phi)^2 d\eta)$$

Resolviendo:

$$(1-\eta-\phi)^2$$

Aplicamos leyes de los exponentes:

$$(1-\eta-\phi)(1-\eta-\phi\ )$$

Y resolvemos aplicando la regla de productos notables:

$$1*1+(1*-\eta)+(1*-\phi)+(-\eta*1)+(-\eta*-\eta)+(-\eta*-\phi)+(-\phi*1)+(-\phi*-\eta)+(-\phi*-\phi)$$

$$1-\eta-\phi-\eta+\eta^2+\eta\phi-\phi+\eta\phi+\phi^2$$

$$1-2\eta-2\phi+\eta^2+2\eta\phi+\phi^2$$

Reordenando los términos:

$$\eta^2+2\eta\phi-2\eta+\phi^2-2\phi+1$$

Sustituyendo

$$\frac{1}{2} * (\int_0^{1-\phi} \eta * \ (\eta^2+2\eta\phi-2\eta+\phi^2-2\phi+1) d\eta)$$

$$\frac{1}{2} * (\int_0^{1-\phi} \eta^3+2\eta^2\phi-2\eta^2+\phi^2\eta-2\eta\phi+\eta\ d\eta)$$

Distribuyendo la integral

$$\frac{1}{2} * (\int_0^{1-\phi} \eta^3 d\eta + \int_0^{1-\phi} 2\eta^2\phi d\eta \ - \int_0^{1-\phi} 2\eta^2 d\eta \ + \int_0^{1-\phi} \phi^2\eta d\eta \ - \int_0^{1-\phi} 2\eta\phi d\eta \ + \int_0^{1-\phi} \eta d\eta$$

Resolviendo las integrales:

$$\int_0^{1-\phi} \eta^3 d\eta$$

$$\left.\frac{\eta^4}{4}\right|_0^{1-\phi} = \frac{(1-\phi)^4}{4}$$

$$\int_0^{1-\phi} 2\eta^2\phi d\eta$$

$$2\phi * (\left.\frac{\eta^3}{3}\right|_0^{1-\phi} = \frac{(1-\phi)^3}{3})$$

$$= \frac{2\phi(1-\phi)^3}{3}$$

$$-\int_0^{\mathbf{1-\phi}} 2\eta^2 d\eta$$

$$-2 * (\frac{\eta^3}{3}\Big|_0^{1-\phi} = \frac{(1-\phi)^3}{3})$$

$$= \frac{-2(1-\phi)^3}{3}$$

$$\int_0^{\mathbf{1-\phi}} \phi^2 \eta d\eta$$

$$\phi^2 * (\frac{\eta^2}{2}\Big|_0^{1-\phi} = \frac{(1-\phi)^2}{2})$$

$$= \frac{\phi^2(1-\phi)^2}{2}$$

$$-\int_0^{\mathbf{1-\phi}} 2\eta\phi d\eta$$

$$-2\phi * (\frac{\eta^2}{2}\Big|_0^{1-\phi} = \frac{(1-\phi)^2}{2})$$

$$= -\phi(1-\phi)^2$$

$$\int_0^{\mathbf{1-\phi}} \eta d\eta$$

$$\frac{\eta^2}{2}\Big|_0^{1-\phi} = \frac{(1-\phi)^2}{2}$$

$$\frac{1}{2} * \left(\frac{(1-\phi)^4}{4} + \frac{2\phi(1-\phi)^3}{3} + \frac{-2(1-\phi)^3}{3} + \frac{\phi^2(1-\phi)^2}{2} + (-\phi(1-\phi)^2) + \frac{(1-\phi)^2}{2}\right)$$

Resolviendo las operaciones aritméticas y simplificando:

$$= \frac{(1-\phi)^4}{24}$$

- **Tercera integral**

$$\int_0^{\mathbf{1}} \frac{(1-\phi)^4}{24} d\phi$$

$$\frac{1}{24} * \left(\int_0^{\mathbf{1}} (1-\phi)^4 d\phi\right)$$

$$\frac{1}{24} * (\int_0^{\mathbf{1}} (\phi^4 - 4\phi^3 + 6\phi^2 - 4\phi + 1) d\phi)$$

$$\frac{1}{24} * \left(\int_0^{\mathbf{1}} \phi^4 d\phi - \int_0^{\mathbf{1}} 4\phi^3 d\phi + \int_0^{\mathbf{1}} 6\phi^2 d\phi - \int_0^{\mathbf{1}} 4\phi d\phi + \int_0^{\mathbf{1}} d\phi \right)$$

Resolviendo las integrales:

$$\int_0^{\mathbf{1}} \phi^4 d\phi$$

$$\left.\frac{\phi^5}{5}\right|_0^1 = \frac{1}{5}$$

$$-\int_0^{\mathbf{1}} 4\phi^3 d\phi$$

$$-4 * (\left.\frac{\phi^4}{4}\right|_0^1 = \frac{1}{4})$$

$$= -1$$

$$\int_0^{\mathbf{1}} 6\phi^2 d\phi$$

$$6 * (\left.\frac{\phi^3}{3}\right|_0^1 = \frac{1}{3})$$

$$= 2$$

$$-\int_0^{\mathbf{1}} 4\phi d\phi$$

$$-4 * (\left.\frac{\phi^2}{2}\right|_0^1 = \frac{1}{2})$$

$$-2$$

$$\int_0^{\mathbf{1}} d\phi$$

$$= 1$$

$$\frac{1}{24} * \left(\frac{1}{5} - 1 + 2 - 2 + 1\right)$$

$$\epsilon\eta = \frac{1}{120}$$

$$J * \int_V \begin{bmatrix} \eta - \epsilon\eta - \eta - \phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} d\epsilon d\eta d\phi$$

Resolución de η^2:

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^2 \, d\epsilon d\eta d\phi$$

- **Primera integral**

$$\int_0^{1-\eta-\phi} \eta^2 \, d\epsilon$$

$$\eta^2 \int_0^{1-\eta-\phi} d\epsilon$$

$$\eta^2 * (1-\eta-\phi)$$

$$= \eta^2 - \eta^3 - \phi\eta^2$$

- **Segunda integral**

$$\int_0^{\mathbf{1}-\phi} (\eta^2 - \eta^3 - \phi\eta^2) \, d\eta$$

$$\int_0^{\mathbf{1}-\phi} \eta^2 \, d\eta - \int_0^{\mathbf{1}-\phi} \eta^3 \, d\eta \; - \phi \int_0^{\mathbf{1}-\phi} \eta^2 \, d\eta$$

Resolviendo las integrales:

$$\int_0^{\mathbf{1}-\phi} \eta^2 \, d\eta$$

$$\left.\frac{\eta^3}{3}\right|_0^{1-\phi} = \frac{(\mathbf{1}-\phi)^3}{3} - \int_0^{\mathbf{1}-\phi} \eta^3 \, d\eta$$

$$\left.\frac{\eta^4}{4}\right|_0^{1-\phi} = -\frac{(\mathbf{1}-\phi)^4}{4}$$

$$-\phi \int_0^{\mathbf{1}-\phi} \eta^2 \, d\eta$$

$$-\phi * (\left.\frac{\eta^3}{3}\right|_0^{1-\phi} = \frac{(\mathbf{1}-\phi)^3}{3})$$

$$= \frac{-\phi(\mathbf{1}-\phi)^3}{3}$$

$$= \frac{(\mathbf{1}-\phi)^3}{3} - \frac{(\mathbf{1}-\phi)^4}{4} + \frac{-\phi(\mathbf{1}-\phi)^3}{3}$$

Realizando las respectivas operaciones aritméticas y reduciendo el resultado es el siguiente:

$$\frac{(\phi-1)^4}{12}$$

- **Tercera integral**

$$\int_0^{\mathbf{1}} (\frac{(\phi-1)^4}{12})\, d\phi$$

$$\frac{1}{12} * (\int_0^{\mathbf{1}} (\phi-1)^4) d\phi)$$

$$\frac{1}{12} * (\int_0^{\mathbf{1}} (\phi^4 - 4\phi^3 + 6\phi^2 - 4\phi + 1)\, d\phi)$$

$$\frac{1}{12} * \left(\int_0^{\mathbf{1}} \phi^4\, d\phi - \int_0^{\mathbf{1}} 4\phi^3\, d\phi + \int_0^{\mathbf{1}} 6\phi^2\, d\phi - \int_0^{\mathbf{1}} 4\phi\, d\phi + \int_0^{\mathbf{1}} d\phi\right)$$

Resolviendo las integrales:

$$\int_0^{\mathbf{1}} \phi^4\, d\phi$$

$$\left.\frac{\phi^5}{5}\right|_0^1 = \frac{1}{5}$$

$$-\int_0^{\mathbf{1}} 4\phi^3\, d\phi$$

$$-4 * (\left.\frac{\phi^4}{4}\right|_0^1 = \frac{1}{4})$$

$$= -1$$

$$\int_0^{\mathbf{1}} 6\phi^2\, d\phi$$

$$6 * \left(\left.\frac{\phi^3}{3}\right|_0^1 = \frac{1}{3}\right)$$

$$= 2$$

$$-\int_0^{\mathbf{1}} 4\phi\, d\phi$$

$$-4 * \left(\left.\frac{\phi^2}{2}\right|_0^1 = \frac{1}{2}\right)$$

$$= -2$$

$$\int_0^{1} d\phi$$

$$= 1$$

Sustituyendo los resultados

$$\frac{1}{12} * \left(\frac{1}{5} - 1 + 2 - 2 + 1\right)$$

$$\eta^2 = \frac{1}{60}$$

$$J * \int_V \begin{bmatrix} \eta - \epsilon\eta - \eta - \phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} d\epsilon d\eta d\phi$$

Resolución de $\phi\eta$:

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \phi\eta \, d\epsilon d\eta d\phi$$

- **Primera integral**

$$\int_0^{1-\eta-\phi} \phi\eta \, d\epsilon$$

$$\phi\eta * \int_0^{1-\eta-\phi} d\epsilon$$

$$\phi\eta * (1 - \eta - \phi)$$

$$\phi\eta - \phi\eta^2 - \phi^2\eta$$

- **Segunda integral**

$$\int_0^{1-\phi} (\phi\eta - \phi\eta^2 - \phi^2\eta) \, d\eta$$

$$\int_0^{1-\phi} \phi\eta \, d\eta - \int_0^{1-\phi} \phi\eta^2 \, d\eta - \int_0^{1-\phi} \phi^2\eta \, d\eta$$

Resolviendo las integrales:

$$\int_0^{1-\phi} \phi\eta \, d\eta$$

$$\phi * (\left.\frac{\eta^2}{2}\right|_0^{1-\phi} = \frac{(1-\phi)^2}{2})$$

$$= \frac{\phi(1-\phi)^2}{2}$$

$$- \int_0^{1-\phi} \phi\eta^2 \, d\eta$$

$$-\phi * (\frac{\eta^3}{3}\Big|_0^{1-\phi} = \frac{(1-\phi)^3}{3})$$

$$= \frac{-\phi(1-\phi)^3}{3}$$

$$-\int_0^{1-\phi} \boldsymbol{\phi^2} \eta \, d\eta$$

$$-\boldsymbol{\phi^2} * (\frac{\eta^2}{2}\Big|_0^{1-\phi} = \frac{(1-\phi)^2}{2})$$

$$= \frac{-\boldsymbol{\phi^2}(1-\phi)^2}{2}$$

El resultado de las integrales seria:

$$= \frac{\phi(1-\phi)^2}{2} + \frac{-\phi(1-\phi)^3}{3} + \frac{-\boldsymbol{\phi^2}(1-\phi)^2}{2}$$

Realizando las operaciones aritméticas y reduciendo el resultado sería el siguiente:

$$\frac{-\phi(\phi-1)^3}{6}$$

- **Tercera integral**

$$\int_0^1 (\frac{-\phi(\phi-1)^3}{6}) \, d\phi$$

$$\frac{1}{6} * (\int_0^1 (-\phi(\phi-1)^3) \, d\phi)$$

$$\frac{1}{6} * (\int_0^1 (-\phi(\phi^3 - 3\phi^2 + 3\phi - 1) d\phi)$$

$$\frac{1}{6} * (\int_0^1 (-\phi^4 + 3\phi^3 - 3\phi^2 + \phi) d\phi)$$

$$\frac{1}{6} * \left(-\int_0^1 \phi^4 d\phi + \int_0^1 3\phi^3 d\phi - \int_0^1 3\phi^2 d\phi + \int_0^1 \phi d\phi\right)$$

Resolviendo las integrales:

$$-\int_0^{\mathbf{1}} \phi^4 d\phi$$

$$-(\frac{\phi^5}{5}\Big|_0^1 = \frac{1}{5})$$

$$= \frac{-1}{5}$$

$$\int_0^{\mathbf{1}} 3\phi^3 d\phi$$

$$3*(\frac{\phi^4}{4}\Big|_0^1=\frac{1}{4})$$

$$=\frac{3}{4}$$

$$-\int_0^{1} 3\phi^2 d\phi$$

$$-3*(\frac{\phi^3}{3}\Big|_0^1=\frac{1}{3})$$

$$=-1$$

$$\int_0^{1} \phi d\phi$$

$$=\frac{\phi^2}{2}\Big|_0^1=\frac{1}{2}$$

El resultado de la integral seria:

$$\frac{1}{6}*\left(\frac{-1}{5}+\frac{3}{4}-1+\frac{1}{2}\right)$$

$$R=\frac{1}{120}$$

$$J*\int_V \begin{bmatrix} \eta-\epsilon\eta-\eta-\phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} d\epsilon d\eta d\phi$$

Resolución de η:

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi} \eta\ d\epsilon d\eta d\phi$$

- **Primera integral**

$$\int_0^{1-\eta-\phi} \eta\ d\epsilon$$

$$\eta*(1-\eta-\phi)$$

$$=\eta-\eta^2-\phi\eta$$

- **Segunda integral**

$$\int_0^{1-\phi} (\eta-\eta^2-\phi\eta)d\eta$$

$$\int_0^{1-\phi} \eta d\eta-\int_0^{1-\phi} \eta^2 d\eta-\int_0^{1-\phi} \phi\eta d\eta$$

Resolviendo las integrales:

$$\int_0^{1-\phi} \eta d\eta$$

$$\left.\frac{\eta^2}{2}\right|_0^1 = \frac{(1-\phi)^2}{2}$$

$$-\int_0^{1-\phi} \eta^2 d\eta$$

$$-(\left.\frac{\eta^3}{3}\right|_0^1 = \frac{(1-\phi)^3}{3})$$

$$= \frac{-(1-\phi)^3}{3}$$

$$-\int_0^{1-\phi} \phi\eta d\eta$$

$$-\phi(\left.\frac{\eta^2}{2}\right|_0^1 = \frac{(1-\phi)^2}{2})$$

$$\frac{-\phi(1-\phi)^2}{2}$$

El resultado de la integral seria:

$$\frac{(1-\phi)^2}{2} + \frac{-(1-\phi)^3}{3} + \frac{-\phi(1-\phi)^2}{2}$$

Realizando las operaciones aritméticas y reduciendo el resultado sería el siguiente:

$$= \frac{-(1-\phi)^3}{6}$$

- **Tercera integral**

$$\int_0^1 \left(\frac{-(1-\phi)^3}{6}\right) d\phi$$

$$\frac{1}{6} * (\int_0^1 ((1-\phi)^3) d\phi)$$

$$\frac{1}{6} * (\int_0^1 (-\phi^3 + 3\phi^2 - 3\phi + 1) d\phi)$$

$$\frac{1}{6} * \left(-\int_0^1 \phi^3 d\phi + \int_0^1 3\phi^2 d\phi - \int_0^1 3\phi d\phi + \int_0^1 d\phi\right)$$

Resolviendo las integrales:

$$-\int_0^{1} \phi^3 d\phi$$

$$-\left(\frac{\phi^4}{4}\bigg|_0^1 = \frac{1}{4}\right)$$

$$= \frac{-1}{4}$$

$$\int_0^{1} 3\phi^2 d\phi$$

$$3\left(\frac{\phi^3}{3}\bigg|_0^1 = \frac{1}{3}\right)$$

$$= 1$$

$$-\int_0^{1} 3\phi d\phi$$

$$-3\left(\frac{\phi^2}{2}\bigg|_0^1 = \frac{1}{2}\right)$$

$$= \frac{-3}{2}$$

$$\int_0^{1} d\phi$$

$$= 1$$

$$\frac{1}{6} * \left(\frac{-1}{4} + 1 + \frac{-3}{2} + 1\right)$$

$$\eta = \frac{1}{24}$$

$$J * \int_V \begin{bmatrix} \eta - \epsilon\eta - \eta^2 - \phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} d\epsilon d\eta d\phi$$

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta - \epsilon\eta - \eta^2 - \phi\eta \; d\epsilon d\eta d\phi$$

$$- \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \epsilon\eta \; d\epsilon d\eta d\phi$$

$$- \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^2 \; d\epsilon d\eta d\phi - \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \phi\eta \; d\epsilon d\eta d\phi$$

Sustituyendo los valores encontrados anteriormente:

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta - \epsilon\eta - \eta^2 - \phi\eta \; d\epsilon d\eta d\phi = \frac{1}{24}$$

$$-\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \epsilon\eta \; d\epsilon d\eta d\phi = -\frac{1}{120}$$

$$-\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^2 \; d\epsilon d\eta d\phi = -\frac{1}{60}$$

$$-\int_0^1 \int_0^{1-\phi} \int_0^{\mathbf{1}-\eta-\phi} \phi\eta \; d\epsilon d\eta d\phi = -\frac{1}{120}$$

$$\frac{1}{24} - \frac{1}{120} - \frac{1}{60} - \frac{1}{120}$$

$$= \frac{1}{120}$$

Finalmente, este sería el resultado de la primera integral:

$$J * \begin{bmatrix} \frac{1}{120} \\ \frac{1}{120} \\ \frac{1}{60} \\ \frac{1}{120} \end{bmatrix}$$

Ya tenemos ambas integrales resueltas:

$$\int_V \mathbf{N}^{\mathrm{t}}\eta^3 \; dV \; + \int_V \mathbf{N}^{\mathrm{t}}\eta \; dV$$

$$J * \begin{bmatrix} \frac{1}{840} \\ \frac{1}{840} \\ \frac{1}{210} \\ \frac{1}{840} \end{bmatrix} + J * \begin{bmatrix} \frac{1}{120} \\ \frac{1}{120} \\ \frac{1}{60} \\ \frac{1}{120} \end{bmatrix}$$

Sacando factor común obtenemos el resultado del lado derecho

$$J * \begin{bmatrix} \frac{1}{105} \\ \frac{1}{105} \\ \frac{1}{105} \\ \frac{1}{105} \end{bmatrix} = \mathbf{b} \; \rightarrow \frac{J}{105} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Lado izquierdo

$$-(\int_V \mathbf{N}^{\mathrm{t}}(\eta^3\nabla * (\epsilon^2\nabla(\mathbf{NX}))) \,\mathrm{dV}))$$

$$-(\int_V \mathbf{N}^{\mathrm{t}}(\eta^3\nabla * (\epsilon^2\nabla\mathbf{N}\,)\mathrm{dV}) \cdot \mathbf{X}$$

Aplicando integrales por partes:

$$u = \mathbf{N}^{\mathrm{t}} \;\; dv = \eta^3\nabla * (\epsilon^2\nabla\mathbf{N})$$

$$du = \nabla\mathbf{N}^{\mathrm{t}} \quad v = \eta^3\epsilon^2\nabla\mathbf{N}$$

$$-[\mathbf{N}^{\mathrm{t}}\eta^3\epsilon^2\nabla\mathbf{N}\,]v \; + \int_v \nabla\mathbf{N}^{\mathrm{t}}\eta^3\epsilon^2\nabla\mathbf{N}\, dV$$

$$\int_v \nabla\mathbf{N}^{\mathrm{t}}\eta^3\epsilon^2\nabla\mathbf{N}\, dV$$

Sabemos del proceso del MEF en 3 dimensiones que:

$$= \frac{\mathrm{k}}{J^2}\boldsymbol{B^t A^t A B}\int_{\boldsymbol{V}} \eta^3\epsilon^2 \;\boldsymbol{dV}$$

pero claro en este caso en nuestra formula no contamos con una constante k así que la formula nos queda de la siguiente manera:

$$= \frac{1}{J^2}\boldsymbol{B^t A^t A B}\int_{\boldsymbol{V}} \eta^3\epsilon^2 \;\boldsymbol{dV}$$

Así que resolvemos aplicando el Jacobiano del MEF en 3 dimensiones para resolver la integral

$$J * \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^3\epsilon^2 d\epsilon d\eta d\phi$$

- **Primera integral**

$$\int_0^{1-\eta-\phi} \eta^3\epsilon^2 d\epsilon$$

$$\eta^3 * \int_0^{1-\eta} \epsilon^2 d\epsilon$$

$$\eta^3 * (\frac{(1-\eta-\phi)^3}{3})$$

- **Segunda integral**

$$\int_0^{1-\phi} \frac{\eta^3(1-\eta-\phi)^3}{3} d\eta$$

$$\frac{1}{3} * (\int_0^{1-\phi} \eta^3(1-\eta-\phi)^3 \, d\eta)$$

Resolvemos:

$$(1-\eta-\phi)^3$$

Aplicamos leyes de los exponentes:

$$(1-\eta-\phi)(1-\eta-\phi\,)(1-\eta-\phi\,)$$

Y resolvemos aplicando la regla de productos notables:

$$1*1+(1*-\eta)+(1*-\phi)+(-\eta*1)+(-\eta*-\eta)+(-\eta*-\phi)+(-\phi*1)+(-\phi*-\eta)+(-\phi*-\phi)$$

$$1-\eta-\phi-\eta+\eta^2+\eta\phi-\phi+\eta\phi+\phi^2$$

$$1-2\eta-2\phi+\eta^2+2\eta\phi+\phi^2$$

Reordenando los términos:

$$(\eta^2+2\eta\phi-2\eta+\phi^2-2\phi+1)(1-\eta-\phi\,)$$

Realizando las operaciones y simplificando el resultado es el siguiente:

$$=\frac{-(\phi-1)^7}{420}$$

- **Tercera integral**

$$\int_0^1 \frac{-(\phi-1)^7}{420}d\phi$$

$$\frac{-1}{420}*(\int_0^1 \frac{-(\phi-1)^7}{420}d\phi)$$

$$\frac{-1}{420}*(\int_0^1 (\phi-1)^7 d\phi)$$

$$=\frac{1}{3360}$$

Finalmente, el resultado de la integral nos queda de la siguiente manera:

$$J*\frac{1}{3360}$$

Sustituyendo en la formula:

$$\frac{1}{J^2}\boldsymbol{B^t A^t A B}\cdot J*\frac{1}{3360}$$

$$\mathbf{K}=\frac{1}{J}\boldsymbol{B^t A^t A B}*\frac{1}{3360}$$

Sustituyendo en la formula principal:

$$-(\int_V \mathbf{N}^{\mathrm{t}}(\eta^3\nabla*(\epsilon^2\nabla\mathbf{N}\,)\mathrm{dV})*\mathbf{X}=\int_V \mathbf{N}^{\mathrm{t}}\eta^3\,dV\;+\int_V \mathbf{N}^{\mathrm{t}}\eta\,dV$$

$$(-[\mathbf{N}^{\mathrm{t}}\eta^3\epsilon^2\nabla\mathbf{N}\,]v\;+\int_v \nabla\mathbf{N}^{\mathrm{t}}\eta^3\epsilon^2\nabla\mathbf{N}\,dV)*\mathbf{X}\;=\int_V \mathbf{N}^{\mathrm{t}}\eta^3\,dV\;+\int_V \mathbf{N}^{\mathrm{t}}\eta\,dV$$

$$(-[\mathbf{N}^{\mathrm{t}}\eta^3\epsilon^2\nabla\mathbf{N}\ ]v\ +\ \frac{1}{J}\boldsymbol{B^tA^tAB} * \frac{1}{3360}) * \mathbf{X}\ = \frac{J}{105} * \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$(\frac{1}{J}\boldsymbol{B^tA^tAB} * \frac{1}{3360}) * \mathbf{X}\ = \frac{J}{105} * \begin{bmatrix}1\\1\\1\\1\end{bmatrix} +\ [\mathbf{N}^{\mathrm{t}}\eta^3\epsilon^2\nabla\mathbf{N}\ ]v$$

$$(\frac{1}{J}\boldsymbol{B^tA^tAB} * \frac{1}{3360}) \cdot \mathbf{X}\ = \frac{J}{105} * \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$\mathbf{KX} =\ \mathbf{b}$$