A Study on Sensor Selection for Efficient Jet Engine Health Monitoring

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A STUDY ON SENSOR SELECTION
FOR EFFICIENT JET ENGINE HEALTH MONITORING

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ABSTRACT

Engine health monitoring has been an area of intensive research for many years. Numerous methods have been developed with the goal of determining a faithful picture of the engine condition. On the other hand, the issue of sensor selection allowing an efficient diagnosis has received less attention from the community.

The present contribution revisits the problem of sensor selection for engine performance monitoring within the scope of information theory. To this end, a metric that integrates the essential elements of the sensor selection problem is defined from the Fisher Information Matrix. An example application consisting in a commercial turbofan engine illustrates the enhancement that can be expected from a wise selection of the sensor set.

Keywords: sensor selection, Fisher Information Matrix, gas-path analysis

NOMENCLATURE

$\hat{\cdot}$	estimated value
$\tilde{\cdot}$	scaled value
A8IMP	nozzle exit area (baseline value: 1.4147 m^2)
bl	baseline conditions
FIM	Fisher Information Matrix
hpc	high pressure compressor
hpt	high pressure turbine
lpc	low pressure compressor
lpt	low pressure turbine
m	number of gas path measurements
n	number of health parameters
N	rotational speed
Pi	total pressure at station i
SEi	efficiency scaler at station i (baseline value: 1.0)
SWiR	flow capacity scaler at station i (baseline value: 1.0)
Ti	total temperature at station i
u	operating point parameters
w	health parameters
y	observed measurements
ϵ	measurement noise vector
σ	singular value
$\mathcal{N}(\mathbf{m}, \mathbf{C})$	a Gaussian probability distribution with mean **m** and covariance matrix **C**

INTRODUCTION

In the last years, condition-based maintenance has been widely promoted in the jet engine community. A maintenance schedule adapted to the level of deterioration of the engine leads to many advantages such as improved operability and safety or reduced life cycle costs. In this framework, generating a reliable information about the health condition of the engine is a requisite.

In this contribution, Module Performance Analysis, aka. **Gas Path Analysis**, is considered. Its purpose is to assess the changes in engine module performance on the basis of measurements collected along the gas path of the engine [1]. This approach allows to track the evolution of a particular engine relative to some baseline performance which may be engine-specific or fleet-averaged. The Gas Path Analysis approach is illustrated in figure 1 and is governed by the following principles:

- the condition of the components (e.g. fan, lpc, hpc, hpt, lpt, nozzle) can be represented by a set of performance indicators, so-called **health parameters w**, which are typically correcting factors on their efficiency and flow capacity,

- a degradation (progressive or accidental) affecting the engine induces a modification of its performance, which results in a drift of some **measurements y** collected along the engine flowpath (e.g. temperatures, pressures, mass flows, shaft speeds),

- a **physical or mathematical model** of the engine that relates the variation of the health parameters **w** to the drift of the gas-path measurements **y** can be derived.
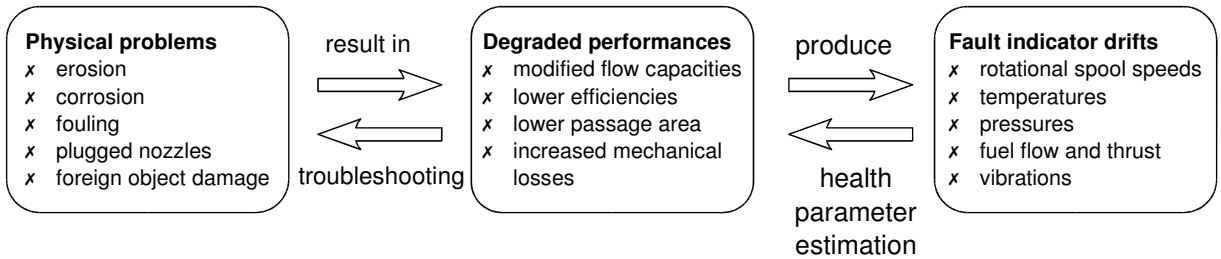

Figure 1: The Gas Path Analysis approach to jet engine diagnostics

As depicted in figure 1, the diagnosis problem (or health parameter estimation problem) can be considered as the inverse problem of performance simulation. Mathematically speaking, the health parameters are estimated by minimising a cost function $\rho(\cdot)$ of the residuals defined as the difference between the measured value **y** and the model prediction $\widehat{\mathbf{y}}_{\mathbf{w}}$:

$$\widehat{\mathbf{w}} = \arg \min_{\mathbf{w}} \{\rho(\mathbf{y} - \widehat{\mathbf{y}}_{\mathbf{w}})\} \tag{1}$$

Next to the definition of the cost function and the choice of an appropriate method to solve the inverse problem, which has been an area of intensive research (see [2] for a detailed review), the selection of a relevant configuration of the sensors is an important issue. Current sensors remain intrusive, their integration is limited by many constraints such as cost, weight or harsh environment. As a consequence, the number of sensors is usually kept to a minimum and hence outweighed by the number of health parameters. This makes the estimation problem ill-posed. Moreover, the measurements are corrupted with noise. The efficiency of the diagnosis algorithm depends on the sensor configuration which should therefore be selected so as to optimise the amount of information that is carried about the engine condition.

Optimal sensor selection has been investigated only sparsely in the engine health monitoring community, few contributions indeed address this issue. Most of the work so far is based on linear approaches. In [3], a metric based on the sensitivity of the sensors with respect to the parameters is defined. The use of the condition number of the influence coefficient matrix that relates measurements and parameters is also investigated in order to perform the sensor selection in the case of a square problem (i.e. as many sensors as parameters). In [4], a figure of merit based on the steady-state error covariance matrix of a Kalman filter is used to perform the optimisation of the sensor set, again in the case of a square problem. This study also takes into account the cost of the different sensors. In [5] the strong link between sensor selection and observability is underlined. A sensor configuration is considered as informative if it avoids both redundancy between measurements and correlation between fault directions. The solution of the sensor selection problem relies on a method for observability analysis developed in [6].

The same topic has received much more attention in other fields such as chemical or structural engineering. As a general trend (see for instance [7, 8, 9, 10]), the sensor selection problem is addressed in the framework of information theory. Metrics based on the Observability Grammian or on the Fisher Information Matrix are defined in order to optimise the configuration of the experiment.

In the light of these considerations, the present contribution revisits the problem of sensor selection for turbine engine performance monitoring within the scope of information theory. The application consists in a generic turbofan model developed in the frame of the European OBIDICOTE project. A metric that handles the aforementioned particularities is proposed. The research being focused on the metric definition itself, the optimal configuration is obtained through a brute-force technique. The quality of the resulting sensor set is assessed on the basis of fault cases that can be expected on a turbofan engine and an observability analysis is performed to get more understanding about the sensor configuration.

STATEMENT OF THE PROBLEM

The scope of this section is to present the theoretical background of the methodology developed for sensor selection. First, the system model relating the observations to the parameters is described. Elements of information theory are then introduced in the scope of sensor selection, with a particular focus on the Fisher Information Matrix (FIM). Metrics based on the FIM are proposed so as to optimise the measurement configuration for a given set of health parameters. Finally, directions to perform an observability analysis of a given configuration are drawn.

Simulation Model

One of the master pieces of the Gas Path Analysis approach is a simulation model of the engine. Considering a steady-state operation of the engine, these simulation tools are generally non-linear aero-thermodynamic models based on mass, energy and momentum conservation laws applied to the engine flow-path. Equation (2) represents such an engine model where $\mathbf{u}$ are the variables defining the operating point of the engine (e.g. fuel flow, altitude, Mach number), $\mathbf{w}$ are the aforementioned health parameters and $\mathbf{y}$ are the gas path measurements. A random variable $\epsilon \in \mathcal{N}(\mathbf{0}, \mathbf{R_y})$ which accounts for sensor inaccuracies and modelling errors is added to the deterministic part $\mathcal{G}(\cdot)$ of the model in order to reconcile the observed measurements and the model predictions. Equation (2) is therefore termed the statistical model.

$$\mathbf{y} = \mathcal{G}(\mathbf{u}, \mathbf{w}) + \epsilon \tag{2}$$

In the frame of turbine engine diagnosis, the statistical model is rarely used in the previous form stated by equation 2. Indeed, the quantity of interest is the difference between the actual engine health condition and a reference one represented by baseline values $\mathbf{w}^{bl}$. Assuming a linear relationship between the measurements and the health parameters, as well as fixed operating conditions, the previous statistical model is re-formulated according to equation (3).

$$
\begin{aligned}
\mathbf{y} &= \mathbf{y}^{bl} + \mathbf{G}\,(\mathbf{w} - \mathbf{w}^{bl}) + \boldsymbol{\epsilon} \\
\Leftrightarrow \delta\mathbf{y} &= \mathbf{G}\,\delta\mathbf{w} + \boldsymbol{\epsilon}
\end{aligned}
\tag{3}
$$

where

$$
\mathbf{y}^{bl} = \mathcal{G}(\mathbf{u}, \mathbf{w}^{bl}) \quad \text{and} \quad \mathbf{G} = \left.\frac{\partial}{\partial\mathbf{w}}\mathcal{G}(\mathbf{u}, \mathbf{w})\right|_{\mathbf{w}=\mathbf{w}^{bl}}
\tag{4}
$$

are respectively the prediction of the measurements and the Jacobian matrix of the engine model around the baseline $\mathbf{w}^{bl}$.

The linear model (3) can be scaled to a linear system with a noise distribution $\tilde{\boldsymbol{\epsilon}} \in \mathcal{N}(\mathbf{0}, \mathbf{I})$ provided that the covariance matrix $\mathbf{R_y}$ is positive definite. The scaled model is given by $\tilde{\mathbf{G}} = (\sqrt{\mathbf{R_y}})^{-1}\,\mathbf{G}$ where the scaling factor takes into account the relative accuracy of each sensor. The resulting linear model writes down:

$$
\delta\tilde{\mathbf{y}} = \tilde{\mathbf{G}}\,\delta\mathbf{w} + \tilde{\boldsymbol{\epsilon}}
\tag{5}
$$

Information Theory and Fisher Information Matrix

As mentioned in the introduction, several authors have turned to Information Theory to address the problem of sensor selection. Basically, the idea is to optimise the amount of information conveyed about the variables that are estimated. The concept of information is in fact useful for comparing different methods of observing a given random process such as described by equation (5). For this kind of static system, the Fisher Information Matrix (FIM) is the quantity of interest.

In statistics theory, the Fisher information is defined as the variance of the score function associated to the estimation problem [11]. The Fisher information quantifies the amount of information that an observation $\mathbf{y}$ carries about an unknown parameter $\mathbf{w}$.

Considering the joint probability distribution of the measurements and the parameters, $p(\mathbf{y}, \mathbf{w})$, that characterises the estimation problem the general term of the FIM is defined as:

$$
\mathbf{FIM}_{i,j} \overset{\text{def}}{=} E\left\{\frac{\partial}{\partial\mathbf{w}_i}\log p(\mathbf{y}, \mathbf{w}) \cdot \frac{\partial}{\partial\mathbf{w}_j}\log p(\mathbf{y}, \mathbf{w})\right\}
\tag{6}
$$

where $E\{\cdot\}$ is the mathematical expectation operator and log, the natural logarithm.

Applying this relation to the scaled statistical model (5) in a maximum likelihood (ML) framework, where only the measurements are viewed as random variables, it can be shown that the FIM writes down:

$$
\mathbf{FIM}_{\text{ML}} = \tilde{\mathbf{G}}^T\,\tilde{\mathbf{G}}
\tag{7}
$$

In the bayesian approach to estimation, where both the measurements and the health parameters are seen as random variables, the FIM takes the form:

$$
\mathbf{FIM}_{\text{Bayes}} = \tilde{\mathbf{G}}^T\,\tilde{\mathbf{G}} + \mathbf{Q}^{-1}
\tag{8}
$$

This latter expression is derived under the assumption of a prior distribution of the type $\delta \mathbf{w} = \mathcal{N}(\mathbf{0}, \mathbf{Q})$, where $\mathbf{Q}$ is the prior covariance matrix of the parameters, and (statistical) independence between the parameters and the measurements.

Interestingly the Cramer-Rao inequality [12] states that the diagonal terms of the inverse of the FIM are asymptotic lower bounds on the variance of any unbiased estimator of $\mathbf{w}$. This underlines the strong coupling between information, observability and estimation.

Engine performance monitoring is often characterised by a negative redundancy, which means that the number of parameters exceeds the number of sensors ($n > m$). In this case, the ML formulation of the estimation problem has no unique solution and bayesian estimation has to be applied. As already mentioned, sensor selection relies here on the FIM. Comparing equations (7) and (8), it can be seen that the FIM in the bayesian case is equal to the one of the ML case, that effectively depends on the sensor configuration, plus the contribution of the a priori knowledge. This latter term can be interpreted as an additional information that makes the mathematical problem well-posed. As a conclusion, the sensor selection problem will be solved by using the FIM of the ML framework, even in the case $m < n$.

Figures of Merit for Sensor Selection
In the previous section, it has been shown that the FIM is a relevant mathematical entity on which the sensor selection can rely. Practically, two main objectives govern the design of a diagnosis tool. On the one hand, high sensitivity is desirable in order to provide an early detection of a fault. On the other hand, the user's confidence in the system is conditioned by a minimum false alarm rate. These goals can be achieved by determining the configuration that maximises both the orthogonality between the sensors and the orthogonality between the parameters. The former means that two sensors should not react in the same way to an engine deterioration. The latter means that two parameters should have a distinct signature on the observations. Various scalar figures of merit based on the FIM can describe these objectives:

1. The **condition number** CN of the FIM is defined as the ratio of the largest to the smallest singular value. The condition number is linked to the rank of a matrix and to the difficulty in performing its inversion. In the case $m < n$, the rank of the FIM is at best equal to m which can be ensured by minimising the condition number. In that way, no sensor is redundant with another, which tends to decrease the false alarm rate.

2. The **trace** Tr of the FIM is defined as the sum of the singular values. In the case $m < n$, the sum is limited to the first m singular values. The trace is a measure of the global sensitivity of the sensors with respect to the parameters and hence has to be maximised. It is linked to some extent to the sensitivity measure defined in [3].

3. The **determinant** Det of the FIM is defined as the product of the singular values. Again, the product is restricted to the first m singular values. Indeed, as stated in point 1, at most m singular values are non-zero. This quantity has to be maximised too as the inverse of the determinant is a measure of the overall uncertainty on the estimated parameters. Furthermore, this criterion tends to standardise the singular values and hence the contribution of each sensor. This criterion is similar to the one used in [5].

Equation (9) expresses the figures of merit in a mathematical form, where $\sigma_i, i = 1, \ldots, m$ are the singular values[1] of the FIM sorted in descending order.

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[1] see next section for a definition of singular value

$$CN = \frac{\sigma_1}{\sigma_m} \quad Tr = \sum_{i=1}^{m} \sigma_i \quad Det = \prod_{i=1}^{m} \sigma_i \tag{9}$$

Each of those metrics puts emphasis on a particular aspect of the problem at hand. To perform the sensor selection, it is thus proposed to combine them in an aggregated figure of merit:

$$FOM = -W_1 \log(CN) + W_2 \log(Tr) + W_3 \log(Det) \tag{10}$$

where W_i are factors that allow both normalisation of the magnitude of the various components and variable weighting between the objectives. For the problem considered in this study, FOM has to be maximised with respect to the sensor configuration under the constraint of a fixed number of sensors.

Metrics for Observability Analysis

As pointed by [13], the classical test for observability, i.e. checking the invertibility (or full-rank) of the system matrix, gives merely a boolean answer. In the case of negative reduncancy, the system is always classified as non-observable. Practically, it is however **partially** observable and it is worth assessing the observability level of the different parameters as well as the unobservable fault directions. Inspired by the work of [6], a method for generating this information is proposed in the following.

Singular value decomposition (see [14]) applied to the scaled Jacobian matrix $\tilde{G}$ serves as the mathematical tool to carry out the observability analysis:

$$\tilde{G} = U \, \Sigma \, V^T \tag{11}$$

where

- $U \in \mathcal{R}^{m \times m}$ is an orthogonal matrix whose columns define an orthonormal basis for the output (measurement) space,
- $V \in \mathcal{R}^{n \times n}$ is an orthogonal matrix whose columns define an orthonormal basis for the input (parameter) space,
- $\Sigma \in \mathcal{R}^{m \times n}$ is a rectangular matrix whose main diagonal contains the singular values σ, which can be seen as stretching factors along the directions defined in U and V.

Given the meaning of U and Σ, a sensitivity index SI is defined for each sensor as a weighted norm of each row of U:

$$SI_i = U_i \, \sqrt{\Sigma^T \Sigma} \, U_i^T \qquad i = 1, \ldots, m \tag{12}$$

This sensitivity index is an image of the overall contribution of a given sensor to the estimation process. The higher the SI, the more important the associated sensor is to estimate the health parameters.

An observability index for the parameters, and to a larger extent for any fault (i.e. combination of several parameters) could be defined in a similar way as the sensitivity index is. Nonetheless, in the case of negative redundancy, a loss of information is introduced due to the fact that the number of parameters outweighs the number of sensors. From linear algebra theory, it can be stated that the rank of the Jacobian matrix $\tilde{G}$ is at best equal to m. Hence, $n - m$ singular values are equal to zero. The columns of V associated to these nought singular values define the null space of the input basis and do not carry any information about the state of the system.

The definition of the observability index proposed in the present study quantifies this loss of information. For some fault $\mathbf{f}$, the observability index OI is computed according to:

$$\text{OI}_{\mathbf{f}} = \frac{\left\| \text{pinv}(\mathbf{\Sigma}) \, \mathbf{\Sigma} \, \mathbf{V}^T \, \mathbf{f} \right\|^2}{\left\| \mathbf{V}^T \, \mathbf{f} \right\|^2} \tag{13}$$

where the numerator is a picture of the intensity of fault $\mathbf{f}$ measured in the observable sub-space (vectors of $\mathbf{V}$ associated to non-zero singular values), and the denominator represents the total intensity of fault $\mathbf{f}$. pinv is the pseudo-inverse operator (see [14]).

An observability index equal to unity means that the considered fault has no component located in the null space and can then be estimated with a great accuracy. On the contrary, a small observabillity index characterises a large loss of information. The estimation of the associated fault is hence less accurate.

Moreover, any fault direction that is a linear combination of the vectors of the null space is unobservable in the sense that it produces no shifts in the measurements. The examination of the unobservable fault directions gives also clues about possible confusions between weakly observable faults.

APPLICATION OF THE METHOD
Engine Layout
The application considered as a test-case is a large bypass ratio, mixed-flow, turbofan. The engine performance model was developed in the frame of the OBIDICOTE[2] project and is detailed in [15]. A schematic of the engine is sketched in figure 2 where the location of the eleven health parameters and the station numbering are also indicated. One control variable is considered in the following, namely the fuel flow rate fed in the combustor.
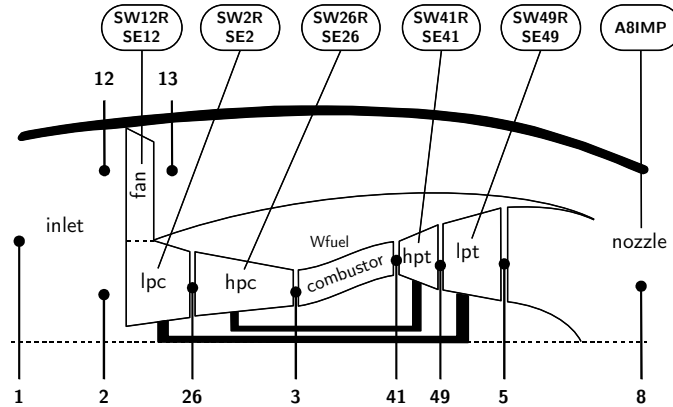

Figure 2: Turbofan layout with health parameters location

The sensor selection problem is formulated for the case of an on-board engine performance monitoring tool. As steady-state operation of the engine is achieved nearly exclusively during the cruise phase, the corresponding operating point is selected for this study. Cruise conditions are defined in table 1.

Table 1: Cruise point definition

Label	W_{fuel}	Altitude	Mach	ΔT_{ISA}
Value	0.350 kg/s	10668 m	0.80	0.0 K

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[2]A Brite-Euram project for On-board Identification, Diagnosis and Control of Turbofan Engine

Sensors that may be fitted on the engine are listed with their associated uncertainty (noise level) in table 2. They mostly consist in total temperature and pressure at various component faces as well as both spool speeds.

Table 2: Available sensors (uncertainty is three times the standard deviation)

Label	P13	P26	NLP	P3	P49	P5
Uncertainty	±100 Pa	±500 Pa	±6 rpm	±5000 Pa	±500 Pa	±300 Pa
Label	T13	T26	NHP	T3	T49	T5
Uncertainty	±2 K	±2 K	±12 rpm	±2 K	±2 K	±2 K

Optimisation of the Sensor Configuration

The optimisation problem is to determine the m (among twelve) sensors that maximise the figure of merit of equation (10) for the set of $n = 11$ health parameters detailed in figure 2 at cruise conditions. The only parameter that still needs to be set is the number of sensors that are installed on the engine. According to the information reported in [16], a number of $m = 7$ sensors seems a reasonable choice for contemporary commercial turbofans.

This optimisation problem is typically combinatorial. Although algorithms are especially dedicated to this kind of problem, the optimal configuration is found here by means of a brute force technique: the figure of merit is computed for every of the 792 (12 choose 7 binomial coefficient) possible combinations.

As highlighted in [17], it becomes harder and harder to install an inter-turbine pressure sensor on modern engines. The search has then been carried out a second time by removing P49 from the list of available sensors, leaving a mere 330 configurations to be examined.

Table 3 reports three sensor sets. Set **A** is the standard instrumentation defined in the frame of the OBIDICOTE project and serves as the baseline for comparison, set **B** is the optimal one when considering P49 as available and set **C** is the optimal one when discarding P49 from the list. Note that the notion of optimality is relative to the previously defined metric.

Table 3: Selected sensors sets

set **A**	P13	T13	NLP	NHP	P3	T3	T5
set **B**	P13	P26	NLP	NHP	P3	P49	T49
set **C**	P13	P26	NLP	NHP	P3	T3	T49

Examining table 3, one first notices that P13, both spool speeds and P3 are common to the three sensor sets. P13 is indeed required to correctly assess fan performance. Concerning the spool speeds and P3, these sensors are mandatory for control purpose. It is thus a good point that they appear in the optimal sets. Furthermore, spool speeds are generally very accurately measured, which is one of the reasons of their presence in the optimal configurations. Set **B** is characterised by the presence of P49. This sensor is in fact the only one in the list that allows full differentiation of lpt and hpt faults.

It is also of interest to point out that sets **B** and **C** contain respectively 4 and 3 pressure sensors out of 7. This is explained by the fact that pressure sensors are more informative about the engine condition than temperature sensors. It may appear strange that lpt exhaust pressure P5 does not appear in the optimal selection. For a mixed-flow engine, this quantity is nonetheless highly correlated with fan

outlet pressure P13. So selecting P13 discards P5 and vice-versa. It can also be seen that Exhaust Gas Temperature (defined as T49 or T5, depending on the manufacturer) is part of the optimal sensor sets. An EGT sensor is already fitted on every engine as this quantity serves as a global indicator of engine health for the pilot (so-called EGT margin).

Evaluation of the Sensor Sets

In order to evaluate the benefits brought by optimal instrumentation, a series of fault cases, taken from [18] and summarised in table 4, are considered. They are intended to represent degradations that can be expected on modern turbofans for all individual components. Considered faults involve single as well as multiple health parameters. In the present application, faults have a constant magnitude. Snapshot data are simulated by means of the engine performance model. Gaussian noise, whose magnitude is specified in table 2 is added to the generated measurements to make them closer to typical test data.

Table 4: Considered fault cases

a	-1% on SW12R	-0.5% on SE12	fan, lpc
	-0.7% on SW2R	-0.4% on SE2	
b	-1% on SE12		
c	-1% on SW26R	-0.7 % on SE26	hpc
d	-1% on SE26		
e	-1% on SW26R		
f	+1% on SW42R		hpt
g	-1% on SW42R	-1 % on SE42	
h	-1% on SE42		
i	-1% on SE49		lpt
j	-1% on SW49R	-0.4 % on SE49	
k	-1% on SW49R		
l	+1% on SW49R	-0.6 % on SE49	
m	+1% on A8IMP		nozzle
n	-1% on A8IMP		

A diagnosis tool based on a Kalman filter (see [19] for further details) processes the data. Given its stochastic nature, each test-case is run twenty times with different realisation of measurement noise. Table 5 reports the averaged results in a very synthetic way. Identification of the fault is declared successful if the estimation error lies within its bounds (see [20] for more details) and is indicated by a checkmark ($\checkmark$) in the table.

Table 5: Fault identification results with the 3 sensor sets

Fault case	a	b	c	d	e	f	g	h	i	j	k	l	m	n
set **A**	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$	$\checkmark$	$\checkmark$
set **B**	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$
set **C**	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$

The standard instrumentation, set **A**, performs already quite well. It can however be seen that it fails in diagnosing correctly most of the lpt faults (cases **j**, **k** and **l**). Sensor set **B**, featuring the inter-turbine pressure sensor, resolves this issue, as previously mentioned. However, fault case **m** is now misidentified. The increase in exhaust nozzle area is confounded with a fan fault. Finally, set **C**

achieves some improvement with respect to set **A** too. Most of the lpt faults but case **j** are correctly assessed. As for set **B**, the diagnosis tool is misled by fault case **m**.

Observability Analysis

The study of the optimal sensor sets is concluded by an observabilty analysis as per the concepts defined in a previous section. Figure 3 depicts the observability indices of the health parameters for the three sensors sets. Concerning set **A**, the observability of most of the parameters is quite good, except for the booster flow factor, SW2R, and the lpt flow factor SW49R. The optimal set **B** improves the observability of those parameters, on the other hand, it degrades the observability of SE12 and SE26 due to the loss respectively of T13 and T3. The nozzle area is also affected. Finally, optimal set **C** offers an intermediate solution. With respect to set **A**, the observability of the lpc is improved, as well as the one of the core components. As for set **B**, one can notice a decrease in observability for SE12 and A8IMP.
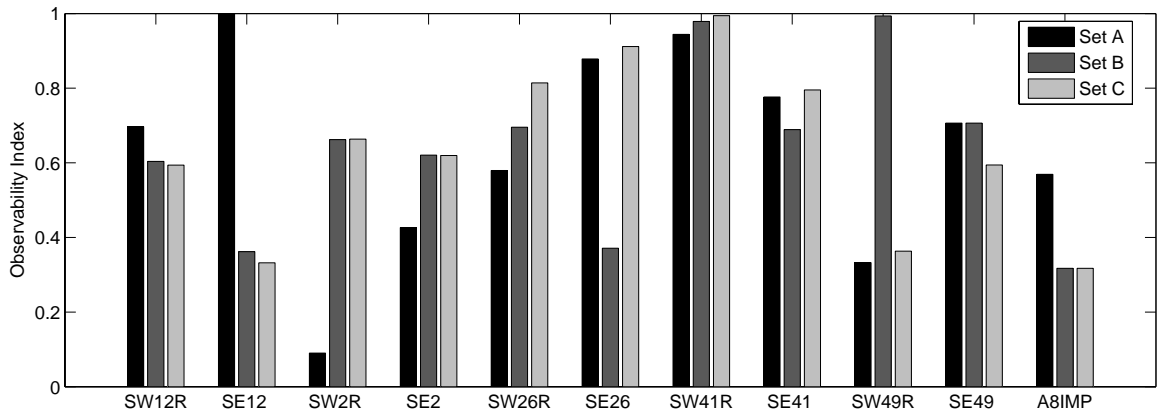

Figure 3: Comparison of the observability indices OI

In figure 4, the sensitivity indices are compared for each sensor set. From the left picture, it can be seen that both temperature sensors in the cold section, namely T13 and T3, contribute to a lesser extent in set **A**. The center picture, associated to set **B**, illustrates the effect of the optimisation. The distribution of sensitivity indices is more uniform and values are higher. Finally, the right graph shows the sensitivity indices for set **C**. Again, the general trend reveals an increase in the average level. T3 has the lowest contribution among all sensors.
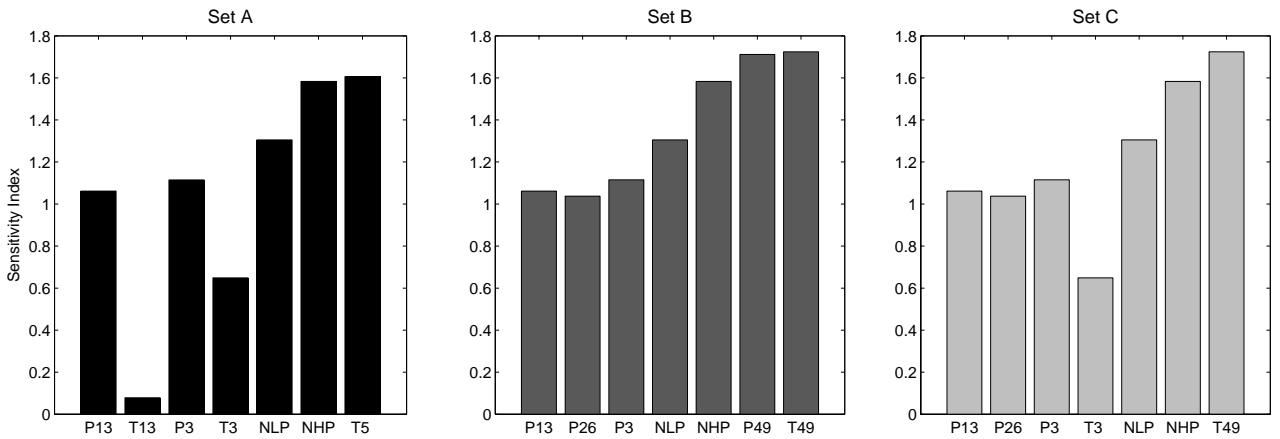

Figure 4: Comparison of the sensitivity indices SI

10

DISCUSSION

To complete the analysis of the results, some issues that may lead to further developments of the proposed methodology are discussed in the following.

The first question is related to the assumptions on which the proposed approach relies. The metric defined for sensor selection has indeed been derived for a linearised system, around a given operating point and reference health conditions. Yet, the behaviour of the engine is essentially non-linear. It would be of interest to evaluate the effects of the non-linearities on the selection process. On the other hand, the method should be extended to multi-point estimation. A solution could consist in computing the metric from a weighted sum of Fisher Information Matrices derived for various conditions (both operating and health). Another concern is that the current methodology does not take into account preferential directions for some faults (e.g. the efficiencies are not expected to improve over time). This might impact the sensor selection and should hence be integrated within the figure of merit.

The present contribution is dedicated to the selection of the optimal sensor configuration for a given set of health parameters. Variations of this problem could make up another interesting field of research. A first variant consists in the selection of the best sensors to add to an existing set (for instance imposed by control system requirements). A second one is the selection of the subset of the most observable health parameters given a sensor configuration in order to have a square problem ($m = n$). This problem has already been investigated within the scope of thrust estimation in [21]. The reduction is however carried out in a transformed space, so that the health parameters become actually tuners which turn out to be of little interest for diagnostics purpose. With a redefined metric, problems such as the minimisation of the number of sensors that allow a satisfactory estimation of a given set of parameters could be investigated. A valuable complementary study could consist in the assessment of the modification in the observability properties of the system in case a sensor is removed either to simplify the configuration or because it is faulty.

Finally, the search for the optimal sensor configuration is achieved here by means of a brute force technique. The primary focus of the present paper is indeed the definition of relevant metrics that describe the problem under consideration. An optimisation algorithm fitted to the combinatorial nature of the problem could supersede the brute force technique which rapidly turns prohibitive as far as computational burden is concerned. In this framework, multi-objective optimisation could be considered. This would leave the designer to select the optimal configuration by trading-off the metrics a posteriori rather than a priori through the definition of an aggregated figure of merit. Additional features such as cost and reliability of the sensors could be taken into account in the definition of the metric.

CONCLUSION

In this contribution, the problem of optimal selection of the sensor configuration for diagnostics has been revisited from the viewpoint of information theory. From sound mathematical arguments, the Fisher Information Matrix has appeared to be a relevant quantity for the problem of sensor selection. A figure of merit addressing various issues such as sensor noise, negative redundancy or orthogonality has been defined based on the Fisher Information Matrix.

The selection of the optimal sensor configuration with respect to the defined metric has been performed by a naive, brute force technique. The enhancement brought by the use of an optimal instrumentation have been underlined with the estimation of a series of simulated, but still realistic fault cases that may occur on a contemporary turbofan engine.

REFERENCES

[1] A. J. Volponi. Foundation of gas path analysis (part i and ii). In *von Karman Institute Lecture Series*, number 01 in Gas Turbine Condition Monitoring and Fault Diagnosis, 2003.

[2] Y. G. Li. Performance-analysis-based gas turbine diagnostics: A review. *IMechE J. of Power and Energy*, 216(5):363–377, 2002.

[3] Ph. Kamboukos, P. Oikonomou, A. Stamatis, and K. Mathioudakis. Optimizing effectiveness of mixed turbofans by means of adaptive modelling and choice of appropriate monitoring parameters. In *AGARD RTO-MP-79: Ageing Mechanisms and Control*, 2001.

[4] R. Mushini and D. Simon. On optimization of sensor selection for aircraft gas turbine engine. In *18th International Conference on Systems Engineering*, pages 9–18, 2005.

[5] G. Bechini, G. Ameyugo, L. Marinai, and R. Singh. Gas path diagnostics: the importance of measurement selection in the monitoring process. In *17th International Symposium on Air Breathing Engines*, number ISABE-2005-1281, 2005.

[6] M. J. Provost. *The use of optimal estimation techniques in the analysis of gas turbines*. PhD thesis, Cranfield University, 1994.

[7] P. H. Kirkegaard and R. Brincker. On the optimal locations of sensors for parametric identification of linear structural systems. *Mechanical Systems and Signal Processing*, 8:639–647, 1994.

[8] F. van den Berg, H. Hoefsloot, H. Boelens, and A. Smilde. Selection of optimal sensor position in a tubular reactor using robust degree of observability criteria. *Chemical Engineering Science*, 55:827–837, 2000.

[9] A. Vande Wouwer, N. Point, S. Porteman, and M. Remy. An approach to the selection of optimal sensor locations in distributed parameters systems. *Journal of Process Control*, 10:291–300, 2000.

[10] C. Papadimitriou. Optimal sensor placement methodology for parametric identification of structural systems. *Journal of Sound and Vibration*, 278:923–947, 2004.

[11] M. J. Schervish. *Theory of Statistics*. Springer, 1995.

[12] C. R. Rao. *Linear Statistical Inference and Its Applications*. John Wiley & Sons, 1973.

[13] R. G. Brown. Not just observable, but how observable. In *National Electronics Conference*, volume 22, pages 709–714, 1966.

[14] G. Strang. *Linear Algebra and its Applications*. Brooks Cole, 3rd edition, 1988.

[15] A. Stamatis, K. Mathioudakis, J. Ruiz, and B. Curnock. Real-time engine model implementation for adaptive control and performance monitoring of large civil turbofans. In *ASME Turbo Expo*, number 2001-GT-0362, 2001.

[16] L. Jaw and S. Garg. Propulsion control technology development in the united states (a historical perspective). Technical Memorandum 2005-213978, NASA, 2005.

[17] B. A. Roth, D. L. Doel, and J. J. Cissell. Probabilistic matching of turbofan performance models to test data. In *ASME Turbo Expo*, number GT2005-68201, 2005.

[18] B. Curnock. Obidicote project - word package 4: Steady-state test cases. Technical Report DNS62433, Rolls-Royce PLc, 2000.

[19] P. Dewallef. *Application of the Kalman Filter to Health Monitoring of Gas Turbine Engines: A Sequential Approach to Robust Diagnosis*. PhD thesis, University of Liège, 2005.

[20] S. Borguet and O. Léonard. Coupling principal component analysis and kalman filter algorithms for on-line aircraft engine diagnostics. In *18th International Symposium on Air Breathing Engines*, number ISABE-2007-1275, 2007.

[21] J. S. Litt. An optimal orthogonal decomposition method for kalman filter-based turbofan engine thrust estimation. In *ASME Turbo Expo*, number GT2005-68808, 2005.