

# Contest (1)

## template.cpp

47 lines

```
#include <bits/stdc++.h>
using namespace std;

template<typename A, typename B> ostream& operator<<(ostream &os, const pair<A, B> &p) { return os << '(' << p.first << ", " << p.second << ')'; }
template<typename T_container, typename T = typename enable_if<!is_same<T_container, string>::value, typename T_container::value_type>::type> ostream& operator<<(ostream &os, const T_container &v) { os << '{'; string sep; for (const T &x : v) os << sep << x, sep = ", "; return os << '}'; }
void dbg_out() { cerr << endl; }
template<typename Head, typename... Tail> void dbg_out(Head H, Tail... T) { cerr << ' ' << H; dbg_out(T...); }
#ifndef LOCAL
#define dbg(...) cerr << __LINE__ << ":" << #__VA_ARGS__, \
    dbg_out(__VA_ARGS__)
#else
#define dbg(...)
#endif

#define rep(i,s,e) for(ll i=(s); i<(e); i++)
#define repr(i,s,e) for(ll i=(s); i>(e); i--)
#define all(a) (a).begin(), (a).end()
#define sz(x) ((int)x.size())
#define pb push_back
#define ff first
#define ss second

using ll = long long;
using dl = double;
using vl = vector<ll>;
using vb = vector<bool>;
template<int SZ> using al = array<ll, SZ>;
using l2 = al<2>;
using v12 = vector<l2>;

const int MAXN = 1e5 + 5;
const ll MOD = 1e9 + 7;
const ll INF = 1e9;
const dl EPS = 1e-7;

int main() {
    ios::sync_with_stdio(0); cin.tie(0); cin.exceptions(cin.failbit);

    ll tcs = 1;
    cin >> tcs;
    for (ll tc = 1; tc <= tcs; tc++) {
        dbg(tc);

        // cout << "Case " << tc << ":" << ans;
        cout << '\n';
    }
    return 0;
}
```

## .bashrc

3 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
-fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps = <
```

## template .bashrc troubleshoot stress gen CodeBook

### troubleshoot.txt

54 lines

Pre-submit:  
 Write a few simple test cases if sample is not enough.  
 Are time limits close? If so, generate max cases.  
 Is the memory usage fine?  
 Could anything overflow?  
 Make sure to submit the right file.  
 Limits: int  $+2 \times 10^9$ , ll  $+9 \times 10^{18}$ , vector  $10^7$  usually

Wrong answer:  
 Print your solution! Print debug output, as well.  
 Are you clearing all data structures between test cases?  
 Can your algorithm handle the whole range of input?  
 Read the full problem statement again.  
 Do you handle all corner cases correctly?  
 Have you understood the problem correctly?  
 Any uninitialized variables?  
 Any overflows?  
 Confusing N and M, i and j, etc.?  
 Are you sure your algorithm works?  
 What special cases have you not thought of?  
 Are you sure the STL functions you use work as you think?  
 Add some assertions, maybe resubmit.  
 Create some testcases to run your algorithm on.  
 Go through the algorithm for a simple case.  
 Go through this list again.  
 Explain your algorithm to a teammate.  
 Ask the teammate to look at your code.  
 Go for a small walk, e.g. to the toilet.  
 Is your output format correct? (including whitespace)  
 Rewrite your solution from the start or let a teammate do it.

Runtime error:  
 Have you tested all corner cases locally?  
 Any uninitialized variables?  
 Are you reading or writing outside the range of any vector?  
 Any assertions that might fail?  
 Any possible division by 0? ( $\text{mod } 0$  for example)  
 Any possible infinite recursion?  
 Invalidated pointers or iterators?  
 Are you using too much memory?  
 Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:  
 Do you have any possible infinite loops?  
 What is the complexity of your algorithm?  
 Are you copying a lot of unnecessary data? (References)  
 How big is the input and output? (consider `scanf`)  
 Avoid vector, map. (use arrays/unordered\_map)  
 What do your teammates think about your algorithm?  
 $n < 10 O(n!)$ ,  $< 20 O(2^n)$ ,  $< 400 O(n^3)$ ,  $< 7500 O(n^2)$ ,  $< 7e4 O(nv/n)$ ,  $< 5e5 O(n\log n)$ ,  $< 5e6 O(n)$ ,  $< 1e18 O(\log n)$

Memory limit exceeded:  
 What is the max amount of memory your algorithm should need?  
 Are you clearing all data structures between test cases?

### stress.sh

18 lines

```
set -e
g++ code.cpp -o code
g++ gen.cpp -o gen
g++ brute.cpp -o brute
for((i = 1; ; ++i)); do
    ./gen $i > in
    ./code < in > out
    ./brute < in > ans
    diff -Z out ans > /dev/null || break
    # diff -w <./A < int> <./B < int> || break
```

```
echo "Passed test: " $i
done
echo "WA on the following test:"
cat in
echo "Your answer is:"
cat out
echo "Correct answer is:"
cat ans
```

### gen.h

**Description:** na

d3879c, 24 lines

```
using ud = uniform_int_distribution<ll>;
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch());
count());
```

```
void gen testcase() {
    ll tcs = ud(1,10)(rng); cout << tcs << "\n";
    rep(tc,0,tcs) {
        ll n = ud(1,100)(rng);
        cout << n << "\n";
        rep(i,0,n) cout << ud(1,1000)(rng) << " ";
        cout << "\n";
    }
    cout << "\n";
}

void gen tree() {
    ll n = ud(2,20)(rng);
    cout << n << "\n";
    vl perm(n); rep(i,0,n) perm[i] = i+1;
    shuffle(all(perm), rng);
    for (ll v=1; v<n; v++) {
        ll u = ud(0,v-1)(rng); // u=0 -> star, u=v-1 -> line, u = v/2 -> binary tree
        cout << perm[u] << " " << perm[v] << "\n";
    }
}
```

### CodeBook.h

**Description:** na

f8c3c8, 1126 lines

```
// DSU
vector<ll> parent;
vector<ll> sizes;
void make_set(ll v) {
    parent[v] = v;
    sizes[v] = 1;
}

ll find_set(ll v) {
    if (v == parent[v])
        return v;
    return parent[v] = find_set(parent[v]);
}

void union_sets(ll a, ll b) {
    a = find_set(a);
    b = find_set(b);
    if (a != b) {
        if (sizes[a] < sizes[b])
            swap(a, b);
        parent[b] = a;
        sizes[a] += sizes[b];
    }
}

// Linked List with Union
struct node{
```

```

node *nxt = nullptr;
ll val;
node(ll _v) {
    val = _v;
}
};

struct LI{
    node *head = nullptr, *tail = nullptr;
};

vector<LI*> p;
// b gonna own
void unionp(ll a, ll b){
    if (p[b]->head == nullptr)
        p[b]->head = p[a]->head;
    else
        p[b]->tail->nxt = p[a]->head;

    p[b]->tail = p[a]->tail;
    p[a]->head = nullptr;
    p[a]->tail = nullptr;
}

/*
for (ll i = 0; i < n; i++){
    node *nd = new node(i);
    LI *l = new LI;
    l->head = nd;
    l->tail = nd;
    p.push_back(l);
}
*/

// INVERSION COUNT
11 merge(vector<ll> &arr, vector<ll> &temp, 11 left, 11 mid, 11
right) {
    11 i = left, j = mid, k = left;
    11 inv_count = 0;

    while (i <= mid - 1 && j <= right) {
        if (arr[i] <= arr[j]) {
            temp[k++] = arr[i++];
        } else {
            temp[k++] = arr[j++];
            inv_count += (mid - i); // all remaining in left
            half are > arr[j]
        }
    }

    while (i <= mid - 1)
        temp[k++] = arr[i++];
    while (j <= right)
        temp[k++] = arr[j++];

    for (11 idx = left; idx <= right; idx++)
        arr[idx] = temp[idx];
}

return inv_count;
}

11 _mergeSort(vector<ll> &arr, vector<ll> &temp, 11 left, 11
right) {
    11 inv_count = 0;
    if (right > left) {
        11 mid = (left + right) / 2;

        inv_count += _mergeSort(arr, temp, left, mid);
        inv_count += _mergeSort(arr, temp, mid + 1, right);
        inv_count += merge(arr, temp, left, mid + 1, right);
    }
}

```

```

    }
    return inv_count;
}

11 mergeSort(vector<ll> &arr) {
    vector<ll> temp(arr.size());
    return _mergeSort(arr, temp, 0, arr.size() - 1);
}

// MOS ALGO
void remove_el(11 idx){
}

void add_el(11 idx){

}
11 get_answer(){

}
11 block_size{

    struct Query {
        11 l, r, idx;
        Query(11 _l, 11 _r, 11 _i){
            l = _l;
            r = _r;
            idx = _i;
        }

        bool operator<(Query other) const
        {
            return make_pair(l / block_size, r) <
                   make_pair(other.l / block_size, other.r);
        }
    };

    void mo_s_algorithm(vector<Query> &queries, vector<bool> &
answers) {
        answers = vector<bool> (queries.size());
        sort(queries.begin(), queries.end());

        odds = 0; // how many with odd occurrences

        11 cur_l = 0;
        11 cur_r = -1;

        for (Query q : queries) {
            if ((q.r - q.l + 1) % 2){
                answers[q.idx] = false;
                continue;
            }

            while (cur_l > q.l) {
                cur_l--;
                add_el(cur_l); // add
            }
            while (cur_r < q.r) {
                cur_r++;
                add_el(cur_r); // add
            }
            while (cur_l < q.l) {
                remove_el(cur_l); // remove
                cur_l++;
            }
            while (cur_r > q.r) {
                remove_el(cur_r); // remove
                cur_r--;
            }
        }
    }
}

answers[q.idx] = get_answer();
}

// SEGTREE
11 T[1000000]; // at least 2*n size (good to go 4*n, because
// we use 2*n and 2*n+1 stuffs) for segment trees
11 S[1000000]; // lazy

// p is the node/segment
void build(vector<ll> &a, 11 p, 11 L, 11 R){
    //base case
    if (L == R)
        T[p] = a[L];
    else{
        //let us now divide... i.e: for [1, 4] -> [1,2] + [3,4]
        11 mid = (L+R)/2;
        build(a, p*2, L, mid);
        build(a, p*2+1, mid+1, R);

        T[p] = min(T[p*2], T[p*2 + 1]);
    }
}

// Point Query
//update node u -> val
void update(11 p, 11 u, 11 val, 11 L, 11 R){
    if (L == R){
        T[p] = val;
    }
    else{
        11 mid = (L+R)/2;
        //we are only updating 1 index... so just go the right
        //direction
        if (u <= mid)
            update(p*2, u, val, L, mid);
        else
            update(p*2+1, u, val, mid +1, R);
        T[p] = min(T[p*2], T[p*2 + 1]);
    }
}

11 query(11 p, 11 ql, 11 qr, 11 L, 11 R){
    // [ql...qr] is outside [L...R]
    if (ql > qr)
        return INF; //returns INF so, doesn't hamper with other
        part values

    if (ql == L && qr == R)
        return T[p];

    // push(p); // uncomment IF lazy propagation (range updates
    )

    11 mid = (L+R)/2;
    11 left = query(p*2, ql, min(qr, mid), L, mid);
    11 right = query(p*2+1, max(ql, mid+1), qr, mid+1, R);
    return min(left, right);
}

// lazy push
void push(11 p) {
    11 val = S[p];

    //update the children
    T[p*2] += val;
    T[p*2 + 1] += val;
}

```

BUET\_TheEmptySet

```

//pass the lazys to children
S[p*2] += val;
S[p*2 + 1] += val;

}

//remove the current node lazy
S[p] = 0;

}

//update [ul,ur] by val
void update(ll p, ll ul, ll ur, ll val, ll L, ll R) {
    if (ul > ur)
        return;

    if (ul == L && ur == R) {
        S[p] += val;
        T[p] += val;
        return;
    }

    push(p);

    ll mid = (L+R)/2;

    update(p*2, ul, min(ur, mid), val, L, mid);      //update
    left segment
    update(p*2+1, max(ul, mid+1), ur, val, mid+1, R); //right
    segment

    T[p] = max(T[p*2], T[p*2 + 1]); //propagate up
}

// GRAPHS
void djikstra(vector<ll> &dist, ll s, vector<vector<Edge>> adj=,
front_adj){

    priority_queue<PII, vector<PII>, greater<PII>> PQ;
    dist[s] = 0;
    PQ.push({0, s}); //0 distance, 1 is the city

    while (!PQ.empty()) {
        PII t = PQ.top();
        PQ.pop();

        if (dist[t.ss] != t.ff) //if not equal, that means this
        t node is old
            continue;

        for (Edge e : adj[t.ss]) {
            if (e.w + t.ff < dist[e.v])
            {
                dist[e.v] = e.w + t.ff;
                PQ.push({dist[e.v], e.v});
            }
        }
    }
}

// BELLMAN FORD
dist = vector<PII>(n+1);

dist[1] = {0, 0};
for (ll i = 2; i <= n; i++)
    dist[i] = {INF, 0};

for (ll i = 1; i <= n; i++) {
    for (Edge e: edges) {
        if (e.w + dist[e.u].ff < dist[e.v].ff)
            dist[e.v] = {e.w + dist[e.u].ff, e.u}; // u -> v
            edge
    }
}

```

```

    }

bool neg_cyc = false;
ll neg_node = 0;
//check if a negative cycle exists
for (Edge e: edges){
    if (e.w + dist[e.u].ff < dist[e.v].ff) {
        dist[e.v] = {e.w + dist[e.u].ff, e.u};
        neg_cyc = true;
        neg_node = e.v;
        //break;
    }
}

// Negative Cycle Path
if (neg_cyc){
    for (ll i = 0; i < n; i++)
        neg_node = dist[neg_node].ss;

    stack<ll> path;
    path.push(neg_node);
    ll tmp_node = dist[neg_node].ss;
    while (tmp_node != neg_node){
        path.push(tmp_node);
        tmp_node = dist[tmp_node].ss;
    }
    path.push(neg_node);
}

// FLOYD-WARSHALL
for (ll k = 1; k <= n; k++){
    for (ll i = 1; i <= n; i++){
        for (ll j = 1; j <= n; j++){
            ll alt_dist = dist[i][k] + dist[k][j];
            if (alt_dist < dist[i][j]){
                dist[i][j] = alt_dist;
                dist[j][i] = alt_dist;
            }
        }
    }
}

// MAXFLOW DINIC
struct FlowEdge {
    int u, v;
    ll cap, flow = 0;
    FlowEdge(int u, int v, ll cap) : u(u), v(v), cap(cap)
};

struct Dinic {
    const ll flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;

    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }

    void add_edge(int u, int v, long long cap) {
        edges.emplace_back(u, v, cap);
        edges.emplace_back(v, u, 0);
        adj[u].push_back(m);
        adj[v].push_back(m+1);
        m += 2;
    }

    void bfs() {
        queue<int> q;
        q.push(s);
        level[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int v : adj[u]) {
                if (dist[u][v] == -1 && edges[u][v].cap > edges[u][v].flow) {
                    dist[u][v] = dist[u][u] + 1;
                    q.push(v);
                }
            }
        }
    }

    ll dfs(int u, int t, ll f) {
        if (u == t) return f;
        for (int v : adj[u]) {
            if (dist[u][v] == dist[u][u] + 1 && edges[u][v].cap > edges[u][v].flow) {
                ll res = dfs(v, t, min(f, edges[u][v].cap - edges[u][v].flow));
                if (res != -1) {
                    edges[u][v].flow += res;
                    edges[v][u].flow -= res;
                    return res;
                }
            }
        }
        return -1;
    }

    ll max_flow() {
        ll ans = 0;
        while (true) {
            bfs();
            if (dist[s][t] == -1) break;
            for (int u = 0; u < n; u++) {
                for (int v : adj[u]) {
                    if (dist[u][v] == dist[u][u] + 1 && edges[u][v].cap > edges[u][v].flow) {
                        edges[u][v].flow += 1;
                        edges[v][u].flow -= 1;
                    }
                }
            }
            if (dfs(s, t, flow_inf) == -1) break;
            ans += dfs(s, t, flow_inf);
        }
        return ans;
    }
}

```

```

        adj[v].push_back(m + 1);
        m += 2;
    }

    bool bfs() {
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int id : adj[u]) {
                // no remaining capacity
                if (edges[id].cap == edges[id].flow)
                    continue;
                // already visited
                if (level[edges[id].v] != -1)
                    continue;
                // node level assigned
                level[edges[id].v] = level[u] + 1;
                q.push(edges[id].v);
            }
        }
        // could sink be reached
        return level[t] != -1;
    }

    // find augmenting paths
    ll dfs(int u, ll pushed) {
        if (pushed == 0)
            return 0;
        if (u == t)
            return pushed;
        for (int& cid = ptr[u]; cid < (int)adj[u].size(); cid++)
        {
            int id = adj[u][cid];
            int v = edges[id].v;
            // not the forward path (level wise)
            if (level[u] + 1 != level[v])
                continue;

            // recursively... the bottleneck
            ll tr = dfs(v, min(pushed, edges[id].cap - edges[id].flow));
            if (tr == 0)
                continue;

            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr; // reverse edge
            return tr;
        }
        return 0;
    }

    ll flow() {
        ll f = 0;
        while (true) {
            fill(level.begin(), level.end(), -1);
            level[s] = 0;
            q.push(s);
            if (!bfs())
                break;
            fill(ptr.begin(), ptr.end(), 0);
            while (ll pushed = dfs(s, flow_inf)) {
                f += pushed;
            }
        }
        return f;
    }
}
/*

```

## BUET\_TheEmptySet

```

Dinic D(n, 0, n-1);
for (ll i = 0; i < m; i++) {
    ll u, v, w;
    cin >> u >> v >> w;
    u--; v--;
    D.add_edge(u, v, w);
}
ll ans = D.flow();
*/
// PRIM MST
// MST returns minimum cost (return -1 if not connected)
ll prim(vector<vector<Edge>> adj) {
    ll ret = 0;

    priority_queue<PII, vector<PII>, greater<PII>> PQ;
    vector<bool> taken(adj.size());
    vector<ll> cost(adj.size(), INF);
    PQ.push({0, 1}); //0 distance, 1 node
    cost[1] = 0;

    while (!PQ.empty()) {
        PII t = PQ.top();
        PQ.pop();

        if (taken[t.ss])
            continue;
        taken[t.ss] = true;
        ret += t.ff;

        for (Edge e : adj[t.ss]) {
            // less weight edge to connect e.v
            if (e.w < cost[e.v])
            {
                PQ.push({e.w, e.v});
                cost[e.v] = e.w; // update the better cost
            }
        }
    }

    for (ll i = 1; i < adj.size(); i++) {
        if (cost[i] == INF){
            ret = -1;
            break;
        }
    }
    return ret;
}

// SCC
struct SCC {
    ll N;
    vvl adj, radj;
    vl topo, comp; // comp is component of node
    vector<bool> vis;
    vvl groups, condG;
    vector<bool> visSCC;
    vl topoSCC;
    void init(ll _N) {
        N = _N;
        adj.resize(N), radj.resize(N), comp = vl(N, -1), vis.resize(N);
    }
    void add_edge(ll u, ll v) { adj[u].pb(v), radj[v].pb(u); }
    void dfs(ll u) {
        vis[u] = 1;

```

```

        for (auto &v: adj[u]) if (!vis[v]) dfs(v);
        topo.pb(u);
    }
    void dfs2(ll u, ll p) {
        comp[u] = p;
        groups[p].pb(u);
        for (auto &v: radj[u]) if (comp[v] == -1) dfs2(v, p);
    }

    void dfssCC(ll u){
        visSCC[u] = 1;
        for (auto &v: condG[u]) if (!visSCC[v]) dfssCC(v);
        topoSCC.pb(u);
    }

    void gen(ll _N) { // fills allComp
        rep(i, 1, _N) if (!vis[i]) dfs(i);
        reverse(all(topo));

        ll cid = 0;
        for (auto &u: topo) if (comp[u] == -1){
            groups.pb({});
            dfs2(u, cid++);
        }

        // build cond. graph
        condG.resize(cid);
        visSCC.resize(cid);
        rep (i, 1, _N) {
            for (auto &j: adj[i]) {
                if (comp[i] == comp[j]) continue;
                condG[comp[i]].pb(comp[j]); //duplicate edges
            }
        }

        // topOrder of SCC
        rep(i, 1, _N) if (!visSCC[comp[i]]) dfssCC(comp[i]);
        reverse(all(topoSCC));
    };

    // MATHS

    ll gcd(ll a, ll b) {
        if (b == 0)
            return a;
        return gcd(b, a%b);
    }

    ll power(ll a, ll p, ll m = MOD) {
        if (p == 0)
            return 1;
        ll x = power(a, p/2, m);
        x = mul(x, x, m);
        if (p % 2)
            x = mul(a, x, m);
        return x;
    }

    ll inv_mod(ll a, ll m = MOD) {
        return power(a, m-2);
    }

    // NCR
    al<MAXN> fact, invfact;
    void initNCR(ll LIM = MAXN) {
        fact[0] = 1;
        for (i = 1; i < LIM; i++)
            fact[i] = mul(i, fact[i - 1]);
    }
    i--;
    invfact[i] = inv_mod(fact[i]);
    for (i--; i >= 0; i--) {
        invfact[i] = mul((i + 1), invfact[i + 1]);
    }
}

ll ncr(ll n, ll r) {
    if (r > n || n < 0 || r < 0)
        return 0;
    return mul(mul(fact[n], invfact[r]), invfact[n - r]);
}

// PRIMES & DIVS
vl primes;
map<ll, ll> primeMap;

void sieve(ll LIM = MAXN) {
    vector<bool> is_prime(LIM+1, true);

    is_prime[0] = is_prime[1] = false;
    for (ll i = 2; i <= LIM; i++) {
        if (is_prime[i]) {
            primeMap[i] = primes.size();
            primes.push_back(i);

            for (ll j = i * i; j <= LIM; j += i)
                is_prime[j] = false;
        }
    }
}

vl2 to_primes(ll x) {
    ll p = primes[0];
    ll i = 1;
    vl2 V;
    while (x != 1 && p * p <= x) {
        ll z = 0;
        while (x % p == 0)
        {
            z++;
            x /= p;
        }
        if (z > 0)
            V.push_back({i, z}); //just push index
        p = primes[++i];
    }
    if (x != 1)
        V.push_back({primeMap[x], 1}); // the remaining number itself is prime
    return V;
}

// Need divisors of all numbers 1..n
vector<vector<ll>> divs;
void find_divs(ll LIM = MAXN) {
    divs.assign(LIM+1, 0);
    for (ll i = 1; i <= LIM; i++)
        for (ll j = i; j <= LIM; j += i)
            divs[j].push_back(i);
}

// Need divisors of arbitrary numbers repeatedly
// vector<ll> spf;

```

```

// void precomputeSPF(ll n = 100000) {
//     spf.assign(n+1, 0);
//     for (ll i = 2; i <= n; i++) spf[i] = i;
//     for (ll i = 2; i*i <= n; i++)
//         if (spf[i] == i)
//             for (ll j = i*i; j <= n; j += i)
//                 if (spf[j] == j) spf[j] = i;
// }

// vector<ll> get_divisors(ll x) {
//     vector<pair<ll, ll>> fac;
//     while (x > 1) {
//         ll p = spf[x], cnt = 0;
//         while (x % p == 0) x /= p, cnt++;
//         fac.push_back({p, cnt});
//     }
//     vector<ll> res = {1};
//     for (auto &pr : fac) {
//         ll p = pr.first, c = pr.second;
//         ll sz = res.size();
//         long long mult = 1;
//         for (ll i = 1; i <= c; i++) {
//             mult *= p;
//             for (ll j = 0; j < sz; j++)
//                 res.push_back(res[j] * mult);
//         }
//     }
//     return res;
// }

// TOTIENT

// vector<ll> phi;
// void phi_1_to_n(ll n) {
//     phi.assign(n + 1);
//     for (ll i = 0; i <= n; i++)
//         phi[i] = i;
//
//     for (ll i = 2; i <= n; i++) {
//         if (phi[i] == i) {
//             for (ll j = i; j <= n; j += i)
//                 phi[j] = phi[j] / i;
//         }
//     }
// }

// MOBIUS
vector<ll> mu;
void mobius (int lim) {
    mu = vector<ll>(lim);
    vector<int> primes;
    vector<int> lp(lim, 0); // lowest prime factor
    mu[1] = 1;

    for (int i = 2; i < lim; i++) {
        if (lp[i] == 0) {
            lp[i] = i;
            primes.push_back(i);
            mu[i] = -1; // prime      (p) = -1
        }

        for (int p : primes) {
            if (p > lp[i] || 1LL * p * i > lim) break;
            lp[p * i] = p;
            if (p == lp[i]) {

```

```

                // p divides i      p^2 divides p*i      = 0
                mu[p * i] = 0;
                break;
            } else {
                // square-free, one more prime factor
                // multiply by -1
                mu[p * i] = -mu[i];
            }
        }
    }
}

// STRING
// KMP
void prefix_function(string s, vector<ll> &pi) {
    ll n = s.length();
    pi = vector<ll>(n);
    for (ll i = 1; i < n; i++) {
        ll j = pi[i-1];

        while (j > 0 && s[i] != s[j])
            j = pi[j-1];

        // abcabc... at index 6('a').. pi[6] = pi[5] + 1 =
        // ... as, pi[5] = 2.. and [2] == [6] == c
        // maximum value prefix function can reach is array size
        // (index) = (n-1) (example: 'aaaaaa')
        if (s[i] == s[j])
            j++;

        pi[i] = j;
    }
}

// p = pattern, t = text
void sub_search(string p, string t, vector<ll> &occurs) {
    vector<ll> pre;
    prefix_function(p+"#"+t, pre);

    ll k = p.length();
    ll n = k + t.length() + 1;
    for (ll i = k+1; i < n; i++) {
        //cout << pre[i] << "a ";

        // i is the end index of occurrence (*in the merged text
        // *)
        if (pre[i] == k)
            occurs.push_back(i-2*k); // pushes start index of
                                      // occurrences
    }
}

void count_prefixes(string s, vector<ll> &cnt) {
    vector<ll> pi;
    // calculate the prefix array
    prefix_function(s, pi);

    ll n = s.length();

    cnt = vector<ll>(n + 1); // cnt[i] = prefix of i length
    // at position i, the prefix of length pi[i] ends, so count
    // that
    for (ll i = 0; i < n; i++) // start from n+1 (if T), i < L
        (L = cat length)
        cnt[pi[i]]++;

    // for all prefixes of length i,
    // the smaller prefix within that prefix needs to be
    // counted as well

```

```

    // the smaller prefix = pi[i-1] ==> i = length and i-1 =
    // index
    for (ll i = n-1; i > 0; i--)
        cnt[pi[i-1]] += cnt[i];
    // original prefixes (if within T, then no originals)
    for (ll i = 0; i <= n; i++)
        cnt[i]++;
}

// TRIE
class Vertex{
public:
    vector<ll> string_ends; // we will store which strings end
    vector<ll> next;
    ll p;
    char pch; // edge we took to get here
    ll suffix_link, exit_link;
    vector<ll> go;
    Vertex(ll k, ll pp, char c){
        p = pp;
        pch = c;
        suffix_link = -1;
        exit_link = -1;
        next.resize(k, -1);
        go.resize(k, -1);
    }
};

class Trie{
public:
    ll K; // K=26 characters in Alphabet
    vector<Vertex> nodes;
    Trie (ll k){
        K = k;
        nodes.push_back(Vertex(K, -1, '#'));
        nodes[0].suffix_link = 0;
    }

    void add(string s, ll idx){
        ll cur = 0;
        for (char ch : s) {
            ll c = ch-'a';

            // no path => make path
            if (nodes[cur].next[c] == -1) {
                nodes[cur].next[c] = nodes.size();
                nodes.push_back(Vertex(K, cur, ch));
            }
            // travel to next path (whether by making or
            // existing)
            cur = nodes[cur].next[c];
        }
        // end of string (if we only wanted to know if any
        // string ends here
        // we would have kept a boolean like nodes[cur].isWord
        // = true
        nodes[cur].string_ends.push_back(idx);
    }

    // look how many times a string exist
    ll look(string s){
        ll cur = 0;
        for (char ch : s) {
            ll c = ch-'a';

            // no path => exit
            if (nodes[cur].next[c] == -1)
                return 0;

```

## BUET\_TheEmptySet

```

    // travel to next path (existing)
    cur = nodes[cur].next[c];
}
// end of string
return nodes[cur].string_ends.size();
}

// run full automation that will build up suffix
// suffix_links and failure paths (go)
void build_automation(){
    for (ll i = 0; i < nodes.size(); i++){
        //cout << i << endl;
        for (ll j = 0; j < K; j++) {
            go(i, j+'a');
        }
    }
}

// find all instances of patterns in the string
void run_automation(string s, vector<ll> &occurs) {
    ll cur = 0;
    //cout << cur << " ";
    for (char ch : s){
        ll c = ch-'a';
        cur = nodes[cur].go[c];

        for (ll v : nodes[cur].string_ends)
            occurs[v]++;
    }

    ll e = nodes[cur].exit_link;
    while (e != -1){
        for (ll v : nodes[e].string_ends)
            occurs[v]++;
        e = nodes[e].exit_link;
    }

    //cout << cur << " ";
    //cout << endl;
}

ll get_suffix_link(ll v){
    if (nodes[v].suffix_link == -1){

        //root node or parent is root
        if (v == 0 || nodes[v].p == 0){
            nodes[v].suffix_link = 0;
            nodes[v].exit_link = -1;
        }
        else{
            nodes[v].suffix_link = go(get_suffix_link(nodes[v].p), nodes[v].pch);
        }

        //now the exit link
        ll s = nodes[v].suffix_link;
        while (s != 0)
        {
            if (nodes[s].string_ends.size() > 0){
                nodes[v].exit_link = s;
                break;
            }
            s = get_suffix_link(s);
        }
    }
    return nodes[v].suffix_link;
}

```

```

ll go(ll v, char ch){
    ll c = ch-'a';

    if (nodes[v].go[c] == -1){
        //original edge exist
        if (nodes[v].next[c] != -1)
            nodes[v].go[c] = nodes[v].next[c];
        else{
            // if root then itself or thru failure
            //suffix_link (suffix)
            nodes[v].go[c] = (v == 0) ? 0 : go(
                get_suffix_link(v), ch);
        }
    }
    return nodes[v].go[c];
}

void print_trie(ll u, ll t) {
    //spaces
    for (ll s = 0; s < t; s++)
        cout << " ";

    // with suffix link & exit link
    cout << u << "->" << nodes[u].pch << " s=" << nodes[u].suffix_link << " e=" << nodes[u].exit_link << "(";
    //cout << u << "->" << nodes[u].pch << " ";
    for (ll z : nodes[u].string_ends){
        cout << z << " ";
    }
    cout << ")" << endl;
    for (ll i = 0; i < K; i++){
        if (nodes[u].next[i] != -1)
            print_trie(nodes[u].next[i], t+1);
    }
};

/*
Trie t(26);

string text;
cin >> text;

ll n;
cin >> n;

vector<string> patterns(n);

for (ll i = 0; i < n; i++){
    cin >> patterns[i];
    t.add(patterns[i], i);
}

t.build_automation();
t.print_trie(0, 0);

vector<ll> occurs(n, 0); // which pattern appears how many
times
t.run_automation(text, occurs);

for (ll i = 0; i < n; i++)
    cout << occurs[i] << endl;
*/
// DP
// 1. Order matters + unlimited repetition
for (int x = 0; x <= target; x++) // sum first

```

```

    for (int c : coins) // then coins
        if (x >= c)
            dp[x] += dp[x - c];

// 2. Order matters + no repetition
for (int x = target; x >= 0; x--) // sum backward
    for (int c : coins) // coins inner
        if (x >= c)
            dp[x] += dp[x - c];

// 3. Order doesn't matter + unlimited repetition
// STANDARD COIN CHANGE
for (int c : coins) // coins first
    for (int x = c; x <= target; x++) // then sum
        dp[x] += dp[x - c];

// 4. Order doesn't matter + no repetition
// 0-1 KNAPSACK style
for (int c : coins) // coins first
    for (int x = target; x >= c; x--) // sum backward
        dp[x] += dp[x - c];

// Binary Splitting (large k)
for (int i = 0; i < m; i++) {
    int c = coins[i];
    int k = cnt[i];

    // decompose k into sums of powers of 2
    int t = 1;
    while (k > 0) {
        int use = min(t, k);
        int weight = use * c;

        // 0/1 knapsack update
        for (int x = target; x >= weight; x--) {
            dp[x] += dp[x - weight];
        }

        k -= use;
        t <= 1;
    }
}

string a, b;
cin >> a >> b;
ll A = a.length();
ll B = b.length();
vector<vector<pair<ll, PII>> dp(A+1, vector<pair<ll, PII>>(B+1, {{0, {0, 0}}})) // first lcs, second is last common
point (for rebuild)
for (ll i = 1; i <= A; i++){
    for (ll j = 1; j <= B; j++){
        if (a[i-1] == b[j-1]){
            dp[i][j] = {dp[i-1][j-1].ff + 1, {i, j}};
        }
        else{
            if (dp[i-1][j].ff > dp[i][j-1].ff)
                dp[i][j] = dp[i-1][j];
            else
                dp[i][j] = dp[i][j-1];
        }
    }
}
string lcs = "";
PII pos = dp[A][B].ss;

while (pos.ff != 0){
    lcs += (char) a[pos.ff-1]; // 0-based
    pos.ff--;
}

```

# BUET\_TheEmptySet

```

pos.ss--;
pos = dp[pos.ff][pos.ss].ss;
}
reverse(lcs.begin(), lcs.end());

vector<PII> dp(1<<n); // first is min. rides, second is min.
weight
dp[0] = {1, 0}; // First elevator
for (ll i = 1; i < (1<<n); i++) {
    dp[i] = {INF, INF};

    // j is person (the last person to be included in last ride)
    // try to find without j and consider including j in subset
    [j belongs to i]
    for (ll j = 0; j < n; j++) {
        if (i & (1 << j)){
            // without j
            PII p = dp[i ^ (1 << j)];
            ll s = p.ff;
            ll W = p.ss;

            // needs new lift for "j"
            if (W + w[j] > x){
                s++;
                W = min(W, w[j]);
            }
            else
                W += w[j];

            // less steps OR equal steps but less weight in
            // last lift
            if ((s < dp[i].ff) || (s == dp[i].ff && W < dp[i].
            ss))
                dp[i] = {s, W};
        }
    }
}

cout << dp[(1<<n)-1].ff << endl;
}

```

```

ll n, m;
cin >> n >> m;
vector<vector<ll>> dp(2, vector<ll> (1<<n, 0));
vector<bool> validmasks((1<<n)-1, false);

ll lim = (1<<n)-1;
// base case & valid masks
for (ll i = 0; i < n; i++) {
    for (ll s = 0; s <= lim; s++) {
        // if 1s are in even segments then valid
        bool valid = true;
        ll c = 0;
        for (ll x = 0; x < n; x++) {
            if (s & (1 << x))
                c++;
            else{
                if (c % 2){
                    valid = false;
                    break;
                }
                c = 0;
            }
        }
        if (c % 2)
            valid = false;
        if (valid){
            validmasks[s] = true;
        }
    }
}

```

```

dp[0][~s & lim] = 1; // horizontal tiles in those
                       // empty places, leaking into next column
}

for (ll j = 1; j < m; j++){
    for (ll s = 0; s <= lim; s++) {
        // Column transition: dp[0][s] => dp[1][s]
        // for mask s, the previous column must have 0 where
        // there's 1 (horizontal block used that leaks into
        // next)
        // and in places where 0, it can be 1 or 0
        // So, subsets of ~s
        dp[1][s] = 0;
        ll x = ~s & lim;
        for (ll t = x; ; t=(t-1)&x){
            // t is covered from previous cell (horizontal
            // block)
            // s is covered from this cell (horizontal block)
            if (validmasks[~(t|s) & lim]) // remaining cells
                // can be covered with vertical blocks
                dp[1][s] = (dp[1][s]+dp[0][t])%M;

            if (t == 0)
                break;
        }
        for (ll s = 0; s <= lim; s++)
            dp[0][s] = dp[1][s];
    }
}

cout << dp[0][0] << endl;
}

```

## Mathematics (2)

### 2.1 Trigonometry

$$\begin{aligned}\sin(v+w) &= \sin v \cos w + \cos v \sin w \\ \cos(v+w) &= \cos v \cos w - \sin v \sin w\end{aligned}$$

$$\begin{aligned}\tan(v+w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}\end{aligned}$$

$$(V+W) \tan(v-w)/2 = (V-W) \tan(v+w)/2$$

where  $V, W$  are lengths of sides opposite angles  $v, w$ .

$$\begin{aligned}a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi)\end{aligned}$$

where  $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$ .

## 2.2 Geometry

### 2.2.1 Triangles

Side lengths:  $a, b, c$

$$\text{Semiperimeter: } p = \frac{a+b+c}{2}$$

$$\text{Area: } A = \sqrt{p(p-a)(p-b)(p-c)}$$

$$\text{Circumradius: } R = \frac{abc}{4A}$$

$$\text{Inradius: } r = \frac{A}{p}$$

Length of median (divides triangle into two equal-area triangles):

$$m_a = \frac{1}{2} \sqrt{2b^2 + 2c^2 - a^2}$$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[ 1 - \left( \frac{a}{b+c} \right)^2 \right]}$$

$$\text{Law of sines: } \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$$

$$\text{Law of cosines: } a^2 = b^2 + c^2 - 2bc \cos \alpha$$

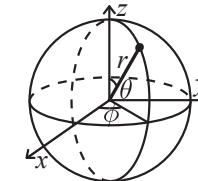
### 2.2.2 Quadrilaterals

With side lengths  $a, b, c, d$ , diagonals  $e, f$ , diagonals angle  $\theta$ , area  $A$  and magic flux  $F = b^2 + d^2 - \frac{a^2 + c^2}{4}$ :

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

### 2.2.3 Spherical coordinates

For cyclic quadrilaterals the sum of opposite angles is  $180^\circ$ ,  $ef = ac + bd$ , and  $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$ .



$$\begin{aligned}x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \arctan(y/x)\end{aligned}$$

## 2.3 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1+x^2} \\ \int \tan ax \, dx &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax \, dx &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} \, dx &= \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) & \int xe^{ax} \, dx &= \frac{e^{ax}}{a^2} (ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x) \, dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x) \, dx$$

## 2.4 Probability theory

Let  $X$  be a discrete random variable with probability  $p_X(x)$  of assuming the value  $x$ . It will then have an expected value (mean)  $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$  and variance  $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$  where  $\sigma$  is the standard deviation. If  $X$  is instead continuous it will have a probability density function  $f_X(x)$  and the sums above will instead be integrals with  $p_X(x)$  replaced by  $f_X(x)$ .

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent  $X$  and  $Y$ ,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

### 2.4.1 Discrete distributions

#### Binomial distribution

The number of successes in  $n$  independent yes/no experiments, each which yields success with probability  $p$  is

$\text{Bin}(n, p)$ ,  $n = 1, 2, \dots, 0 \leq p \leq 1$ .

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$  is approximately  $\text{Po}(np)$  for small  $p$ .

#### First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability  $p$  is  $\text{Fs}(p)$ ,  $0 \leq p \leq 1$ .

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

## OrderStatisticTree HashMap

### Poisson distribution

The number of events occurring in a fixed period of time  $t$  if these events occur with a known average rate  $\kappa$  and independently of the time since the last event is  $\text{Po}(\lambda)$ ,  $\lambda = t\kappa$ .

$$\begin{aligned} p(k) &= e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots \\ \mu &= \lambda, \sigma^2 = \lambda \end{aligned}$$

### 2.4.2 Continuous distributions

#### Uniform distribution

If the probability density function is constant between  $a$  and  $b$  and 0 elsewhere it is  $\text{U}(a, b)$ ,  $a < b$ .

$$\begin{aligned} f(x) &= \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases} \\ \mu &= \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12} \end{aligned}$$

#### Exponential distribution

The time between events in a Poisson process is  $\text{Exp}(\lambda)$ ,  $\lambda > 0$ .

$$\begin{aligned} f(x) &= \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \\ \mu &= \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2} \end{aligned}$$

#### Normal distribution

Most real random values with mean  $\mu$  and variance  $\sigma^2$  are well described by  $\mathcal{N}(\mu, \sigma^2)$ ,  $\sigma > 0$ .

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If  $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$  and  $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$  then

$$aX_1 + bX_2 + c \sim \mathcal{N}(a\mu_1 + b\mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

## 2.5 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let  $X_1, X_2, \dots$  be a sequence of random variables generated by the Markov process. Then there is a transition matrix  $\mathbf{P} = (p_{ij})$ , with  $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$ , and  $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$  is the probability distribution for  $X_n$  (i.e.,  $p_i^{(n)} = \Pr(X_n = i)$ ), where  $\mathbf{p}^{(0)}$  is the initial distribution.

$\pi$  is a stationary distribution if  $\pi = \pi \mathbf{P}$ . If the Markov chain is *irreducible* (it is possible to get to any state from any state), then  $\pi_i = \frac{1}{\mathbb{E}(T_i)}$  where  $\mathbb{E}(T_i)$  is the expected time between two visits in state  $i$ .  $\pi_j/\pi_i$  is the expected number of visits in state  $j$  between two visits in state  $i$ .

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors,  $\pi_i$  is proportional to node  $i$ 's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1).  $\lim_{k \rightarrow \infty} \mathbf{P}^k = \mathbf{1}\pi$ .

A Markov chain is an *A-chain* if the states can be partitioned into two sets  $\mathbf{A}$  and  $\mathbf{G}$ , such that all states in  $\mathbf{A}$  are absorbing ( $p_{ii} = 1$ ), and all states in  $\mathbf{G}$  leads to an absorbing state in  $\mathbf{A}$ . The probability for absorption in state  $i \in \mathbf{A}$ , when the initial state is  $j$ , is  $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$ . The expected time until absorption, when the initial state is  $i$ , is  $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ki} t_k$ .

## Data structures (3)

### OrderStatisticTree.h

**Description:** A set (not multiset!) with support for finding the  $n$ 'th element, and finding the index of an element. To get a map, change `null_type`.

**Time:**  $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
            tree_order_statistics_node_update>;

void example() {
    Tree<int> t, t2; t.insert(8);
    auto it = t.insert(10).first;
    assert(it == t.lower_bound(9));
    assert(t.order_of_key(10) == 1);
    assert(t.order_of_key(11) == 2);
    assert(*t.find_by_order(0) == 8);
    t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

### HashMap.h

**Description:** Hash map with mostly the same API as `unordered_map`, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
    const uint64_t C = 11(4e18 * acos(0)) | 71;
    ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};

__gnu_pbds::gp_hash_table<ll, int, chash> h({}, {}, {}, {}, {1<<16});
```

## BUET\_TheEmptySet

## UnionFindRollback Matrix LineContainer Treap FenwickTree FenwickTree2d

## UnionFindRollback.h

**Description:** Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().

**Usage:** int t = uf.time(); ...; uf.rollback(t);

**Time:**  $\mathcal{O}(\log N)$

de4ad0, 21 lines

```
struct RollbackUF {
    vi e; vector<pii> st;
    RollbackUF(int n) : e(n, {-1}) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return sz(st); }
    void rollback(int t) {
        for (int i = time(); i --> t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    }
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a]});
        st.push_back({b, e[b]});
        e[a] += e[b]; e[b] = a;
        return true;
    }
};
```

## Matrix.h

**Description:** Basic operations on square matrices.

**Usage:** Matrix<int, 3> A;

A.d = {{{1,2,3}}, {{4,5,6}}, {{7,8,9}}};

array<int, 3> vec = {1,2,3};

vec = (A^N) \* vec;

4da5a2, 26 lines

```
template<class T, int N> struct Matrix {
    typedef Matrix M;
    array<array<T, N>, N> d{};
    M operator*(const M& m) const {
        M a;
        rep(i, 0, N) rep(j, 0, N)
            rep(k, 0, N) a.d[i][k] += d[i][j] * m.d[j][k];
        return a;
    }
    array<T, N> operator*(const array<T, N>& vec) const {
        array<T, N> ret{};
        rep(i, 0, N) rep(j, 0, N) ret[i] += d[i][j] * vec[j];
        return ret;
    }
    M operator^(ll p) const {
        assert(p >= 0);
        M a, b(*this);
        rep(i, 0, N) a.d[i][i] = 1;
        while (p) {
            if (p&1) a = a*b;
            b = b*b;
            p >>= 1;
        }
        return a;
    }
};
```

## LineContainer.h

**Description:** Container where you can add lines of the form  $kx+m$ , and query maximum values at points  $x$ . Useful for dynamic programming (“convex hull trick”).

**Time:**  $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```
struct Line {
    mutable ll k, m, p;
```

```
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};
```

## Treap.h

**Description:** A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.

**Time:**  $\mathcal{O}(\log N)$

1754b4, 53 lines

```
struct Node {
    Node *l = 0, *r = 0;
    int val, y, c = 1;
    Node(int val) : val(val), y(rand()) {}
    void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
    if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
    if (!n) return {};
    if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
        auto [L,R] = split(n->l, k);
        n->l = R;
        n->recalc();
        return {L, n};
    } else {
        auto [L,R] = split(n->r, k - cnt(n->l) - 1); // and just "k"
        n->r = L;
        n->recalc();
        return {n, R};
    }
}

Node* merge(Node* l, Node* r) {
    if (!l) return r;
    if (!r) return l;
    if (l->y > r->y) {
        l->r = merge(l->r, r);
        return l->recalc(), l;
    } else {
        r->l = merge(l, r->l);
        return r->recalc(), r;
    }
}
```

```
    } else {
        r->l = merge(l, r->l);
        return r->recalc(), r;
    }
}

Node* ins(Node* t, Node* n, int pos) {
    auto [l,r] = split(t, pos);
    return merge(merge(l, n), r);
}

// Example application: move the range [l, r) to index k
void move(Node*& t, int l, int r, int k) {
    Node *a, *b, *c;
    tie(a,b) = split(t, l); tie(b,c) = split(b, r - 1);
    if (k <= l) t = merge(ins(a, b, k), c);
    else t = merge(a, ins(c, b, k - r));
}
```

## FenwickTree.h

**Description:** Computes partial sums  $a[0] + a[1] + \dots + a[pos - 1]$ , and updates single elements  $a[i]$ , taking the difference between the old and new value.

**Time:** Both operations are  $\mathcal{O}(\log N)$ .

e62fac, 22 lines

```
struct FT {
    vector<ll> s;
    FT(int n) : s(n) {}
    void update(int pos, ll dif) { // a[pos] += dif
        for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
    }
    ll query(int pos) { // sum of values in [0, pos)
        ll res = 0;
        for (; pos > 0; pos &= pos - 1) res += s[pos-1];
        return res;
    }
    int lower_bound(ll sum) { // min pos st sum of [0, pos] >= sum
        // Returns n if no sum is >= sum, or -1 if empty sum is.
        if (sum <= 0) return -1;
        int pos = 0;
        for (int pw = 1 << 25; pw; pw >>= 1) {
            if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
                pos += pw, sum -= s[pos-1];
        }
        return pos;
    }
};
```

## FenwickTree2d.h

**Description:** Computes sums  $a[i,j]$  for all  $i < I, j < J$ , and increases single elements  $a[i,j]$ . Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).

**Time:**  $\mathcal{O}(\log^2 N)$ . (Use persistent segment trees for  $\mathcal{O}(\log N)$ .)

```
"FenwickTree.h" 157f07, 22 lines
struct FT2 {
    vector<vi> ys; vector<FT> ft;
    FT2(int limx) : ys(limx) {}
    void fakeUpdate(int x, int y) {
        for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
    }
    void init() {
        for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
    }
    int ind(int x, int y) {
        return (int)(lower_bound(all(ys[x]), y) - ys[x].begin());
    }
    void update(int x, int y, ll dif) {
        for (; x < sz(ys); x |= x + 1)
            ft[x].update(ind(x, y), dif);
    }
};
```

```
11 query(int x, int y) {
11 sum = 0;
for (; x; x &= x - 1)
sum += ft[x-1].query(ind(x-1, y));
return sum;
}
```

**RMQ.h**

**Description:** Range Minimum Queries on an array. Returns  $\min(V[a], V[a+1], \dots, V[b-1])$  in constant time.  
**Usage:** RMQ rmq(values);  
rmq.query(inclusive, exclusive);  
**Time:**  $\mathcal{O}(|V| \log |V| + Q)$

510c32, 16 lines

```
template<class T>
struct RMQ {
    vector<vector<T>> jmp;
    RMQ(const vector<T>& V) : jmp(1, V) {
        for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k) {
            jmp.emplace_back(sz(V) - pw * 2 + 1);
            rep(j, 0, sz(jmp[k]))
                jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
        }
    }
    T query(int a, int b) {
        assert(a < b); // or return inf if a == b
        int dep = 31 - __builtin_clz(b - a);
        return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
    }
};
```

**MoQueries.h**

**Description:** Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge  $(a, c)$  and remove the initial add call (but keep in).  
**Time:**  $\mathcal{O}(N\sqrt{Q})$

a12ef4, 49 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vi mo(vector<pii> Q) {
    int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
    for (int qi : s) {
        pii q = Q[qi];
        while (L > q.first) add(--L, 0);
        while (R < q.second) add(R++, 1);
        while (L < q.first) del(L++, 0);
        while (R > q.second) del(--R, 1);
        res[qi] = calc();
    }
    return res;
}

vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0) {
    int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
    add(0, 0), in[0] = 1;
    auto dfs = [&](int x, int p, int dep, auto& f) -> void {
        par[x] = p;
        L[x] = N++;
        if (dep) I[x] = N++;
        for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    };
}
```

```
if (!dep) I[x] = N++;
R[x] = N;
}
dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
iota(all(s), 0);
sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
for (int qi : s) rep(end, 0, 2) {
    int &a = pos[qi], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
    else { add(c, end); in[c] = 1; } a = c; }
    while (!!(L[b] <= L[a] && R[a] <= R[b])) {
        I[i++] = b, b = par[b];
        while (a != b) step(par[a]);
        while (i--) step(I[i]);
        if (end) res[qi] = calc();
    }
}
return res;
}
```

## Numerical (4)

### 4.1 Polynomials and recurrences

**Polynomial.h**

c9b7b0, 17 lines

```
struct Poly {
    vector<double> a;
    double operator()(double x) const {
        double val = 0;
        for (int i = sz(a); i--;) (val *= x) += a[i];
        return val;
    }
    void diff() {
        rep(i, 1, sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    }
    void divroot(double x0) {
        double b = a.back(), c; a.back() = 0;
        for (int i = sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
        a.pop_back();
    }
};
```

**PolyRoots.h**

**Description:** Finds the real roots to a polynomial.  
**Usage:** polyRoots({{2,-3,1}}, -1e9, 1e9) // solve  $x^2-3x+2 = 0$   
**Time:**  $\mathcal{O}(n^2 \log(1/\epsilon))$

```
*polynomial.h*                                         b00bfe, 23 lines
vector<double> polyRoots(Poly p, double xmin, double xmax) {
    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
    vector<double> ret;
    Poly der = p;
    der.diff();
    auto dr = polyRoots(der, xmin, xmax);
    dr.push_back(xmin-1);
    dr.push_back(xmax+1);
    sort(all(dr));
    rep(i, 0, sz(dr)-1) {
        double l = dr[i], h = dr[i+1];
        bool sign = p(l) > 0;
        if (sign ^ (p(h) > 0)) {
            rep(it, 0, 60) { // while (h - l > 1e-8)
                double m = (l + h) / 2, f = p(m);
                if ((f <= 0) ^ sign) l = m;
                else h = m;
            }
            ret.push_back((l + h) / 2);
        }
    }
}
```

```
}
}
return ret;
}
```

## 4.2 Optimization

**GoldenSectionSearch.h**

**Description:** Finds the argument minimizing the function  $f$  in the interval  $[a, b]$  assuming  $f$  is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is  $\epsilon ps$ . Works equally well for maximization with a small change in the code. See *Ternary-Search.h* in the Various chapter for a discrete version.

**Usage:** double func(double x) { return 4+x+.3\*x\*x; }  
**Time:**  $\mathcal{O}(\log((b-a)/\epsilon))$

31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5)-1)/2, eps = 1e-7;
    double x1 = b - r*(b-a), x2 = a + r*(b-a);
    double f1 = f(x1), f2 = f(x2);
    while (b-a > eps)
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r*(b-a); f2 = f(x2);
        }
    return a;
}
```

**HillClimbing.h**

**Description:** Poor man's optimization for unimodal functions

seeef, 14 lines

```
typedef array<double, 2> P;
template<class F> pair<double, P> hillClimb(P start, F f) {
    pair<double, P> cur(f(start), start);
    for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
        rep(j, 0, 100) rep(dx, -1, 2) rep(dy, -1, 2) {
            P p = cur.second;
            p[0] += dx*jmp;
            p[1] += dy*jmp;
            cur = min(cur, make_pair(f(p), p));
        }
    }
    return cur;
}
```

**Integrate.h**

**Description:** Simple integration of a function over an interval using Simpson's rule. The error should be proportional to  $h^4$ , although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

4756fc, 7 lines

```
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
    double h = (b - a) / 2 / n, v = f(a) + f(b);
    rep(i, 1, n*2)
        v += f(a + i*h) * (i&1 ? 4 : 2);
    return v * h / 3;
}
```

**IntegrateAdaptive.h**

**Description:** Fast integration using an adaptive Simpson's rule.

## BUET\_TheEmptySet

```

Usage: double sphereVolume = quad(-1, 1, [] (double x) {
    return quad(-1, 1, [&] (double y) {
        return quad(-1, 1, [&] (double z) {
            return x*x + y*y + z*z < 1; });
    });
}
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b)/2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
    d c = (a + b) / 2;
    d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
    if (abs(T - S) <= 15 * eps || b - a < 1e-10)
        return T + (T - S) / 15;
    return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
    return rec(f, a, b, eps, S(a, b));
}

```

## Simplex.h

**Description:** Solves a general linear maximization problem: maximize  $c^T x$  subject to  $Ax \leq b$ ,  $x \geq 0$ . Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of  $c^T x$  otherwise. The input vector is set to an optimal  $x$  (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that  $x = 0$  is viable.

**Usage:** vvd A = {{1,-1}, {-1,1}, {-1,-2}};

vd b = {1,1,-4}, c = {-1,-1}, x;

T val = LPSolver(A, b, c).solve(x);

**Time:**  $\mathcal{O}(NM * \#pivots)$ , where a pivot may be e.g. an edge relaxation.  $\mathcal{O}(2^n)$  in the general case.

## Simplex Determinant IntDeterminant SolveLinear SolveLinear2

```

typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
    int m, n;
    vi N, B;
    vvd D;
}

LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
    rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
    rep(i,0,m) { B[i] = +i; D[i][n] = -1; D[i][n+1] = b[i]; }
    rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m+1][n] = 1;
}

void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
        T *b = D[i].data(), inv2 = b[s] * inv;
        rep(j,0,n+2) b[j] -= a[j] * inv2;
        b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
}

bool simplex(int phase) {

```

```

    int x = m + phase - 1;
    for (;;) {
        int s = -1;
        rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
        if (D[x][s] >= -eps) return true;
        int r = -1;
        rep(i,0,m) {
            if (D[i][s] <= eps) continue;
            if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                < MP(D[r][n+1] / D[r][s], B[r])) r = i;
        }
        if (r == -1) return false;
        pivot(r, s);
    }

    T solve(vd &x) {
        int r = 0;
        rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
        if (D[r][n+1] < -eps) {
            pivot(r, n);
            if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
            rep(i,0,m) if (B[i] == -1) {
                int s = 0;
                rep(j,1,n+1) ltj(D[i]);
                pivot(i, s);
            }
        }
        bool ok = simplex(1); x = vd(n);
        rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
        return ok ? D[m][n+1] : inf;
    }
}

```

## 4.3 Matrices

## Determinant.h

**Description:** Calculates determinant of a matrix. Destroys the matrix.

**Time:**  $\mathcal{O}(N^3)$

```

double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1;
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
        }
    }
    return res;
}

```

## IntDeterminant.h

**Description:** Calculates determinant using modular arithmetics. Modulus can also be removed to get a pure-integer version.

**Time:**  $\mathcal{O}(N^3)$

```

const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
    int n = sz(a); ll ans = 1;
    rep(i,0,n) {
        rep(j,i+1,n) {
            while (a[j][i] != 0) { // gcd step
                ll t = a[i][i] / a[j][i];
                if (t) rep(k,i,n)
                    a[i][k] = (a[i][k] - a[j][k] * t) % mod;
            }
        }
    }
    swap(a[0], a[n-1]);
    ans *= a[0];
}

```

```

    swap(a[i], a[j]);
    ans *= -1;
}
ans = ans * a[i][i] % mod;
if (!ans) return 0;
return (ans + mod) % mod;
}

```

## SolveLinear.h

**Description:** Solves  $A * x = b$ . If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in  $A$  and  $b$  is lost.

**Time:**  $\mathcal{O}(n^2m)$

```

typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m);
    vi col(m); iota(all(col), 0);

    rep(i,0,n) {
        double v, bv = 0;
        rep(r,i,n) rep(c,i,m)
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            rep(j,i,n) if (fabs(b[j]) > eps) return -1;
            break;
        }
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i];
        rep(j,i+1,n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k,i+1,m) A[j][k] -= fac * A[i][k];
        }
        rank++;
    }

    x.assign(m, 0);
    for (int i = rank; i--;) {
        b[i] /= A[i][i];
        x[col[i]] = b[i];
        rep(j,0,i) b[j] -= A[j][i] * b[i];
    }
    return rank; // (multiple solutions if rank < m)
}

```

## SolveLinear2.h

**Description:** To get all uniquely determined values of  $x$  back from SolveLinear, make the following changes:

```

"SolveLinear.h" 08e495, 7 lines
rep(j,0,n) if (j != i) // instead of rep(j, i+1, n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i];
    fail:;
}

```

## SolveLinearBinary.h

**Description:** Solves  $Ax = b$  over  $\mathbb{F}_2$ . If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys  $A$  and  $b$ .

**Time:**  $\mathcal{O}(n^2m)$

fa2d7a, 34 lines

typedef bitset<1000> bs;

```
int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
    int n = sz(A), rank = 0, br;
    assert(m <= sz(x));
    vi col(m); iota(all(col), 0);
    rep(i, 0, n) {
        for (br=i; br<n; ++br) if (A[br].any()) break;
        if (br == n) {
            rep(j, i, n) if (b[j]) return -1;
            break;
        }
        int bc = (int)A[br]._Find_next(i-1);
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j, 0, n) if (A[j][i] != A[j][bc]) {
            A[j].flip(i); A[j].flip(bc);
        }
        rep(j, i+1, n) if (A[j][i]) {
            b[j] ^= b[i];
            A[j] ^= A[i];
        }
        rank++;
    }

    x = bs();
    for (int i = rank; i--;) {
        if (!b[i]) continue;
        x[col[i]] = 1;
        rep(j, 0, i) b[j] ^= A[j][i];
    }
    return rank; // (multiple solutions if rank < m)
}
```

## MatrixInverse.h

**Description:** Invert matrix  $A$ . Returns rank; result is stored in  $A$  unless singular ( $\text{rank} < n$ ). Can easily be extended to prime moduli; for prime powers, repeatedly set  $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$  where  $A^{-1}$  starts as the inverse of  $A$  mod  $p$ , and  $k$  is doubled in each step.

**Time:**  $\mathcal{O}(N^3)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i, 0, n) tmp[i][i] = 1, col[i] = i;

    rep(i, 0, n) {
        int r = i, c = i;
        rep(j, i, n) rep(k, i, n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i;
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j, 0, n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i];
        rep(j, i+1, n)
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k, i+1, n) A[j][k] -= f * A[i][k];
            rep(k, 0, n) tmp[j][k] -= f * tmp[i][k];
    }
}
```

```
rep(j, i+1, n) A[i][j] /= v;
rep(j, 0, n) tmp[i][j] /= v;
A[i][i] = 1;
}

for (int i = n-1; i > 0; --i) rep(j, 0, i) {
    double v = A[j][i];
    rep(k, 0, n) tmp[j][k] -= v * tmp[i][k];
}

rep(i, 0, n) rep(j, 0, n) A[col[i]][col[j]] = tmp[i][j];
return n;
}
```

## Tridiagonal.h

**Description:**  $x = \text{tridiagonal}(d, p, q, b)$  solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, \quad 1 \leq i \leq n,$$

where  $a_0, a_{n+1}, b_i, c_i$  and  $d_i$  are known.  $a$  can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, \dots, -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.

If  $|d_i| > |p_i| + |q_{i-1}|$  for all  $i$ , or  $|d_i| > |p_{i-1}| + |q_i|$ , or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

**Time:**  $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
                       const vector<T>& sub, vector<T> b) {
    int n = sz(b); vi tr(n);
    rep(i, 0, n-1) {
        if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
            b[i+1] -= b[i] * diag[i+1] / super[i];
            if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
            diag[i+1] = sub[i]; tr[++i] = 1;
        } else {
            diag[i+1] -= super[i] * sub[i] / diag[i];
            b[i+1] -= b[i] * sub[i] / diag[i];
        }
    }
    for (int i = n; i--;) {
        if (tr[i]) {
            swap(b[i], b[i-1]);
            diag[i-1] = diag[i];
            b[i] /= super[i-1];
        } else {
            b[i] /= diag[i];
            if (i) b[i-1] -= b[i] * super[i-1];
        }
    }
    return b;
}
```

## 4.4 Fourier transforms

FastFourierTransform.h

**Description:**  $\text{fft}(a)$  computes  $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$  for all  $k$ .  $N$  must be a power of 2. Useful for convolution:  $\text{conv}(a, b) = c$ , where  $c[x] = \sum a[i]b[x-i]$ . For convolution of complex numbers or more than two vectors: `FFT`, multiply pointwise, divide by  $n$ , reverse(`start+1, end`), `FFT` back. Rounding is safe if  $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$  (in practice  $10^{16}$ ; higher for random inputs). Otherwise, use NTT/FFTMod.

**Time:**  $\mathcal{O}(N \log N)$  with  $N = |A| + |B|$  ( $\sim 1s$  for  $N = 2^{22}$ )

00ced6, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1); // (^ 10% faster if double)
    for (static int k = 2; k < n; k *= 2) {
        R.resize(n); rt.resize(n);
        auto x = polar(1.0L, acos(-1.0L) / k);
        rep(i, k, 2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
    }
    vi rev(n);
    rep(i, 0, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i, 0, n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j, 0, k) {
            C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
            a[i + j + k] = a[i + j] - z;
            a[i + j] += z;
        }
    }
    vd conv(const vd& a, const vd& b) {
        if (a.empty() || b.empty()) return {};
        vd res(sz(a) + sz(b) - 1);
        int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
        vector<C> in(n), out(n);
        copy(all(a), begin(in));
        rep(i, 0, sz(b)) in[i].imag(b[i]);
        fft(in);
        for (C &x : in) x *= x;
        rep(i, 0, n) out[i] = in[-i & (n - 1)] - conj(in[i]);
        fft(out);
        rep(i, 0, sz(res)) res[i] = imag(out[i]) / (4 * n);
        return res;
    }
}
```

## FastFourierTransformMod.h

**Description:** Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as  $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$  (in practice  $10^{16}$  or higher). Inputs must be in  $[0, \text{mod}]$ .

**Time:**  $\mathcal{O}(N \log N)$ , where  $N = |A| + |B|$  (twice as slow as NTT or FFT)

"FastFourierTransform.h"

```
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
    if (a.empty() || b.empty()) return {};
    vl res(sz(a) + sz(b) - 1);
    int B=32-__builtin_clz(sz(res)), n=1<<B, cut=__sqrt(M);
    vector<C> L(n), R(n), outs(n), out1(n);
    rep(i, 0, sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
    rep(i, 0, sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
    fft(L), fft(R);
    rep(i, 0, n) {
        int j = -i & (n - 1);
        out1[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
        outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / li;
    }
    fft(out1), fft(outs);
    rep(i, 0, sz(res)) {
        ll av = ll(real(out1[i])+.5), cv = ll(imag(outs[i])+.5);
        ll bv = ll(imag(out1[i])+.5) + ll(real(outs[i])+.5);
        res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
    }
}
```

```

    }
    return res;
}

```

## NumberTheoreticTransform.h

**Description:** nt(a) computes  $\hat{f}(k) = \sum_x a[x]g^{xk}$  for all  $k$ , where  $g = \text{root}^{(mod-1)/N}$ .  $N$  must be a power of 2. Useful for convolution modulo specific nice primes of the form  $2^a b + 1$ , where the convolution result has size at most  $2^a$ . For arbitrary modulo, see FFTMod. conv(a, b) = c, where  $c[x] = \sum a[i]b[x - i]$ . For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).

**Time:**  $\mathcal{O}(N \log N)$

```

.../number-theory/ModPow.h"                                     ced03d, 35 lines
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void nttr(vl &a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vl rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, s++) {
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s)};
        rep(i, k, 2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
    }
    vi rev(n);
    rep(i, 0, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i, 0, n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j, 0, k) {
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        }
    }
vl conv(const vl &a, const vl &b) {
    if (a.empty() || b.empty()) return {};
    int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
        n = 1 << B;
    vl L(a), R(b), out(n);
    L.resize(n), R.resize(n);
    nttr(L), nttr(R);
    rep(i, 0, n)
        out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
    nttr(out);
    return {out.begin(), out.begin() + s};
}

```

## FastSubsetTransform.h

**Description:** Transform to a basis with fast convolutions of the form  $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$ , where  $\oplus$  is one of AND, OR, XOR. The size of  $a$  must be a power of two.

**Time:**  $\mathcal{O}(N \log N)$

```

464cf3, 16 lines
void FST(vi& a, bool inv) {
    for (int n = sz(a), step = 1; step < n; step *= 2) {
        for (int i = 0; i < n; i += 2 * step) rep(j, i, i+step) {
            int &u = a[j], &v = a[j + step]; tie(u, v) =
                inv ? pii(v - u, u) : pii(v, u + v); // AND
                inv ? pii(v, u - v) : pii(u + v, u); // OR
                pii(u + v, u - v); // XOR
        }
        if (inv) for (int& x : a) x /= sz(a); // XOR only
    }
    vi conv(vi a, vi b) {

```

```

        FST(a, 0); FST(b, 0);
        rep(i, 0, sz(a)) a[i] *= b[i];
        FST(a, 1); return a;
    }

```

## Number theory (5)

## 5.1 Primality

## FastEratosthenes.h

**Description:** Prime sieve for generating all primes smaller than LIM.

**Time:**  $\text{LIM}=1e9 \approx 1.5s$

```

6b2912, 20 lines
const int LIM = le6;
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int)round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1));
    vector<pii> cp;
    for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
        cp.push_back({i, i * i / 2});
        for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
    }
    for (int L = 1; L <= R; L += S) {
        array<bool, S> block{};
        for (auto &[p, idx] : cp)
            for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
        rep(i, 0, min(S, R - L))
            if (!block[i]) pr.push_back((L + i) * 2 + 1);
    }
    for (int i : pr) isPrime[i] = 1;
    return pr;
}

```

## MillerRabin.h

**Description:** Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to  $7 \cdot 10^{18}$ ; for larger numbers, use Python and extend A randomly.

**Time:** 7 times the complexity of  $a^b \bmod c$ .

## "ModMuLL.h"

```

60dc01, 12 lines
bool isPrime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
    for (ull a : A) { // ^ count trailing zeroes
        ull p = modpow(a % n, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    }
    return 1;
}

```

## Factor.h

**Description:** Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

**Time:**  $\mathcal{O}(n^{1/4})$ , less for numbers with small factors.

## "ModMuLL.h", "MillerRabin.h"

```

d8d98d, 18 lines
ull pollard(ull n) {
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    auto f = [&](ull x) { return modmul(x, x, n) + i; };
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x += i, y = f(x);
        if ((q = modmul(prd, max(x, y) - min(x, y), n))) prd = q;
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}

```

```

}
vector<ull> factor(ull n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), all(r));
    return l;
}

```

## 5.2 Divisibility

## euclid.h

**Description:** Finds two integers  $x$  and  $y$ , such that  $ax + by = \gcd(a, b)$ . If you just need gcd, use the built in `_gcd` instead. If  $a$  and  $b$  are coprime, then  $x$  is the inverse of  $a$  (mod  $b$ ).

33ba8f, 5 lines

```

11 euclid(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
}

```

## CRT.h

**Description:** Chinese Remainder Theorem.

crt(a, m, b, n) computes  $x$  such that  $x \equiv a \pmod{m}$ ,  $x \equiv b \pmod{n}$ . If  $|a| < m$  and  $|b| < n$ ,  $x$  will obey  $0 \leq x < \text{lcm}(m, n)$ . Assumes  $mn < 2^{62}$ .

**Time:**  $\log(n)$

```

"euclid.h"
04d93a, 7 lines
11 crt(ll a, ll m, ll b, ll n) {
    if (n > m) swap(a, b), swap(m, n);
    ll x, y, g = euclid(m, n, x, y);
    assert((a - b) % g == 0); // else no solution
    x = (b - a) % n * x % n / g * m + a;
    return x < 0 ? x + m*n/g : x;
}

```

## 5.2.1 Bézout's identity

For  $a \neq b \neq 0$ , then  $d = \gcd(a, b)$  is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If  $(x, y)$  is one solution, then all solutions are given by

$$\left( x + \frac{kb}{\gcd(a, b)}, y - \frac{ka}{\gcd(a, b)} \right), \quad k \in \mathbb{Z}$$

## phiFunction.h

**Description:** Euler's  $\phi$  function is defined as  $\phi(n) := \#$  of positive integers  $\leq n$  that are coprime with  $n$ .  $\phi(1) = 1$ ,  $p$  prime  $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$ ,  $m, n$  coprime  $\Rightarrow \phi(mn) = \phi(m)\phi(n)$ . If  $n = p_1^{k_1}p_2^{k_2}\dots p_r^{k_r}$  then  $\phi(n) = (p_1 - 1)p_1^{k_1 - 1}\dots(p_r - 1)p_r^{k_r - 1}$ .  $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$ .  $\sum_{d|n} \phi(d) = n$ ,  $\sum_{1 \leq k \leq n, \gcd(k, n)=1} k = n\phi(n)/2$ ,  $n > 1$

**Euler's thm:**  $a, n$  coprime  $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$ .

**Fermat's little thm:**  $p$  prime  $\Rightarrow a^{p-1} \equiv 1 \pmod{p} \forall a$ .

cf7d6d, 8 lines

```

const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
    rep(i, 0, LIM) phi[i] = i & 1 ? i : i/2;
    for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
        for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}

```

### 5.3 Fractions

ContinuedFractions.h

**Description:** Given  $N$  and a real number  $x \geq 0$ , finds the closest rational approximation  $p/q$  with  $p, q \leq N$ . It will obey  $|p/q - x| \leq 1/qN$ .

For consecutive convergents,  $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$ . ( $p_k/q_k$  alternates between  $> x$  and  $< x$ .) If  $x$  is rational,  $y$  eventually becomes  $\infty$ ; if  $x$  is the root of a degree 2 polynomial the  $a$ 's eventually become cyclic.

**Time:**  $\mathcal{O}(\log N)$

dd6c5e, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
    ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
    for (;;) {
        ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
            a = (ll)floor(y), b = min(a, lim),
            NP = b*P + LP, NQ = b*Q + LQ;
        if (a > b) {
            // If b > a/2, we have a semi-convergent that gives us a
            // better approximation; if b = a/2, we *may* have one.
            // Return {P, Q} here for a more canonical approximation.
            return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
                make_pair(NP, NQ) : make_pair(P, Q);
        }
        if (abs(y = 1/(y - (d)a)) > 3*N) {
            return {NP, NQ};
        }
        LP = P; P = NP;
        LQ = Q; Q = NQ;
    }
}
```

FracBinarySearch.h

**Description:** Given  $f$  and  $N$ , finds the smallest fraction  $p/q \in [0, 1]$  such that  $f(p/q)$  is true, and  $p, q \leq N$ . You may want to throw an exception from  $f$  if it finds an exact solution, in which case  $N$  can be removed.

**Usage:** `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1, 3}`

**Time:**  $\mathcal{O}(\log(N))$

27ab3e, 25 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
    bool dir = 1, A = 1, B = 1;
    Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
    if (f(lo)) return lo;
    assert(f(hi));
    while (A || B) {
        ll adv = 0, step = 1; // move hi if dir, else lo
        for (int si = 0; step; (step *= 2) >>= si) {
            adv += step;
            Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
            if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
                adv -= step; si = 2;
            }
        }
        hi.p += lo.p * adv;
        hi.q += lo.q * adv;
        dir = !dir;
        swap(lo, hi);
        A = B; B = !adv;
    }
    return dir ? hi : lo;
}
```

### 5.4 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

ContinuedFractions FracBinarySearch IntPerm multinomial

with  $m > n > 0$ ,  $k > 0$ ,  $m \perp n$ , and either  $m$  or  $n$  even.

### 5.5 Primes

$p = 962592769$  is such that  $2^{21} \mid p - 1$ , which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power  $p^a$ , except for  $p = 2, a > 2$ , and there are  $\phi(\phi(p^a))$  many. For  $p = 2, a > 2$ , the group  $\mathbb{Z}_{2^a}^\times$  is instead isomorphic to  $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$ .

### 5.6 Estimates

$$\sum_{d \mid n} d = O(n \log \log n).$$

The number of divisors of  $n$  is at most around 100 for  $n < 5 \cdot 10^4$ , 500 for  $n < 10^7$ , 2000 for  $n < 10^{10}$ , 200 000 for  $n < 10^{19}$ .

### 5.7 Möbius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Möbius Inversion:

$$g(n) = \sum_{d \mid n} f(d) \Leftrightarrow f(n) = \sum_{d \mid n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$$\sum_{d \mid n} \mu(d) = [n = 1] \text{ (very useful)}$$

$$g(n) = \sum_{n \mid d} f(d) \Leftrightarrow f(n) = \sum_{n \mid d} \mu(d/n) g(d)$$

$$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$$

## Combinatorial (6)

### 6.1 Permutations

#### 6.1.1 Factorial

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$n$	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$n$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

**Description:** Permutation  $\rightarrow$  integer conversion. (Not order preserving.) Integer  $\rightarrow$  permutation can use a lookup table.

**Time:**  $\mathcal{O}(n)$

044568, 6 lines

```
int permToInt(vi& v) {
    int use = 0, i = 0, r = 0;
    for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
        use |= 1 << x; // (note: minus, not ~!)
    return r;
}
```

044568, 6 lines

### 6.1.2 Cycles

Let  $gs(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} gs(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

### 6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

### 6.1.4 Burnside's lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g \cdot x = x$ ).

If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k \mid n} f(k) \phi(n/k).$$

## 6.2 Partitions and subsets

### 6.2.1 Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2e5$	$\sim 2e8$

### 6.2.2 Lucas' Theorem

Let  $n, m$  be non-negative integers and  $p$  a prime. Write  $n = n_k p^k + \dots + n_1 p + n_0$  and  $m = m_k p^k + \dots + m_1 p + m_0$ . Then  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$ .

### 6.2.3 Binomials

multinomial.h

**Description:** Computes  $\binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \dots k_n!}$

```
ll multinomial(vi& v) {
    ll c = 1, m = v.empty() ? 1 : v[0];
    rep(i, 1, sz(v)) rep(j, 0, v[i]) c = c * ++m / (j+1);
    return c;
}
```

a0a312, 5 lines

## 6.3 General purpose numbers

### 6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-able).  
 $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^{\infty} f(i) &= \int_m^{\infty} f(x) dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^{\infty} f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

### 6.3.2 Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$$\begin{aligned} c(n, k) &= c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1 \\ \sum_{k=0}^n c(n, k)x^k &= x(x+1)\dots(x+n-1) \end{aligned}$$

$$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$$

$$c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$$

### 6.3.3 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$  j:s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$  j:s s.t.  $\pi(j) \geq j$ ,  $k$  j:s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

### 6.3.4 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

### 6.3.5 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ . For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

## TopoSort MinCostMaxFlow MinCut

### 6.3.6 Labeled unrooted trees

# on  $n$  vertices:  $n^{n-2}$   
# on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \cdots n_k n^{k-2}$   
# with degrees  $d_i$ :  $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

### 6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

- $C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$
- sub-diagonal monotone paths in an  $n \times n$  grid.
  - strings with  $n$  pairs of parenthesis, correctly nested.
  - binary trees with  $n+1$  leaves (0 or 2 children).
  - ordered trees with  $n+1$  vertices.
  - ways a convex polygon with  $n+2$  sides can be cut into triangles by connecting vertices with straight lines.
  - permutations of  $[n]$  with no 3-term increasing subseq.

## Graph (7)

### 7.1 Fundamentals

#### TopoSort.h

**Description:** Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than  $n$  – nodes reachable from cycles will not be returned.

**Time:**  $\mathcal{O}(|V| + |E|)$

```
d678d8, 8 lines
vi topoSort(const vector<vi>& gr) {
    vi indeg(sz(gr)), q;
    for (auto& li : gr) for (int x : li) indeg[x]++;
    rep(i, 0, sz(gr)) if (indeg[i] == 0) q.push_back(i);
    rep(j, 0, sz(q)) for (int x : gr[q[j]]) if (--indeg[x] == 0) q.push_back(x);
    return q;
```

### 7.2 Network flow

#### MinCostMaxFlow.h

**Description:** Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.

**Time:**  $\mathcal{O}(FE \log(V))$  where  $F$  is max flow.  $\mathcal{O}(VE)$  for setpi.

```
58385b, 79 lines
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;

struct MCMF {
    struct edge {
        int from, to, rev;
        ll cap, cost, flow;
    };
    int N;
    vector<vector<edge>> ed;
    vi seen;
    vector<ll> dist, pi;
```

vector<edge\*> par;

```
MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N), par(N) {}
void addEdge(int from, int to, ll cap, ll cost) {
    if (from == to) return;
    ed[from].push_back(edge{from, to, sz(ed[to]), cap, cost, 0});
    ed[to].push_back(edge{to, from, sz(ed[from]) - 1, 0, -cost, 0});}
```

```
} void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;
```

```
_gnu_pbds::priority_queue<pair<ll, int>> q;
vector<decltype(q)::point_iterator> its(N);
q.push({0, s});
```

```
while (!q.empty()) {
    s = q.top().second; q.pop();
    seen[s] = 1; di = dist[s] + pi[s];
    for (edge& e : ed[s]) if (!seen[e.to]) {
        ll val = di - pi[e.to] + e.cost;
        if (e.cap - e.flow > 0 && val < dist[e.to]) {
            dist[e.to] = val;
            par[e.to] = &e;
            if (its[e.to] == q.end())
                its[e.to] = q.push({-dist[e.to], e.to});
            else
                q.modify(its[e.to], { -dist[e.to], e.to });
        }
    }
}
rep(i, 0, N) pi[i] = min(pi[i] + dist[i], INF);
```

```
pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
        ll fl = INF;
        for (edge* x = par[t]; x; x = par[x->from])
            fl = min(fl, x->cap - x->flow);

        totflow += fl;
        for (edge* x = par[t]; x; x = par[x->from]) {
            x->flow += fl;
            ed[x->to][x->rev].flow -= fl;
        }
    }
    rep(i, 0, N) for (edge& e : ed[i]) totcost += e.cost * e.flow;
    return {totflow, totcost / 2};
```

// If some costs can be negative, call this before maxflow:

```
void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1, ll v;
    while (ch-- && it--) {
        rep(i, 0, N) if (pi[i] != INF)
            for (edge& e : ed[i]) if (e.cap)
                if ((v = pi[i] + e.cost) < pi[e.to])
                    pi[e.to] = v, ch = 1;
        assert(it >= 0); // negative cost cycle
    }
};
```

## MinCut.h

**Description:** After running max-flow, the left side of a min-cut from  $s$  to  $t$  is given by all vertices reachable from  $s$ , only traversing edges with positive residual capacity.

## GlobalMinCut.h

**Description:** Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

**Time:**  $\mathcal{O}(V^3)$

8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
    pair<int, vi> best = {INT_MAX, {}};
    int n = sz(mat);
    vector<vi> co(n);
    rep(i, 0, n) co[i] = {i};
    rep(ph, 1, n) {
        vi w = mat[0];
        size_t s = 0, t = 0;
        rep(it, 0, n-ph) { //  $O(V^2)$   $\rightarrow O(E \log V)$  with prio. queue
            w[t] = INT_MIN;
            s = t, t = max_element(all(w)) - w.begin();
            rep(i, 0, n) w[i] += mat[t][i];
        }
        best = min(best, {w[t] - mat[t][t], co[t]} );
        co[s].insert(co[s].end(), all(co[t]));
        rep(i, 0, n) mat[s][i] += mat[t][i];
        rep(i, 0, n) mat[i][s] = mat[s][i];
        mat[0][t] = INT_MIN;
    }
    return best;
}
```

## GomoryHu.h

**Description:** Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.

**Time:**  $\mathcal{O}(V)$  Flow Computations

PushRelabel.h<sup>\*</sup> 0418b3, 13 lines

```
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
    vector<Edge> tree;
    vi par(N);
    rep(i, 1, N) {
        PushRelabel D(N); // Dinic also works
        for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
        tree.push_back({i, par[i], D.calc(i, par[i])});
        rep(j, i+1, N)
            if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
    }
    return tree;
}
```

## 7.3 Matching

## hopcroftKarp.h

**Description:** Fast bipartite matching algorithm. Graph  $g$  should be a list of neighbors of the left partition, and  $btoa$  should be a vector full of -1's of the same size as the right partition. Returns the size of the matching,  $btoa[i]$  will be the match for vertex  $i$  on the right side, or -1 if it's not matched.

**Usage:**  $vi btoa(m, -1);$  hopcroftKarp(g, btoa);

**Time:**  $\mathcal{O}(\sqrt{VE})$

f612e4, 42 lines

```
bool dfs(int a, int L, vector<vi> &g, vi &btoa, vi &A, vi &B) {
    if (A[a] != L) return 0;
    A[a] = -1;
    for (int b : g[a]) if (B[b] == L + 1) {
        B[b] = 0;
        if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
            return 1;
    }
}
return sz(btoa) - (int)count(all(btoa), -1);
}
```

```
    return btoa[b] = a, 1;
}
return 0;
}

int hopcroftKarp(vector<vi> &g, vi &btoa) {
    int res = 0;
    vi A(g.size()), B(btoa.size()), cur, next;
    for (;;) {
        fill(all(A), 0);
        fill(all(B), 0);
        cur.clear();
        for (int a : btoa) if (a != -1) A[a] = -1;
        rep(a, 0, sz(g)) if (A[a] == 0) cur.push_back(a);
        for (int lay = 1; lay++) {
            bool islast = 0;
            next.clear();
            for (int a : cur) for (int b : g[a]) {
                if (btoa[b] == -1) {
                    B[b] = lay;
                    islast = 1;
                }
                else if (btoa[b] != a && !B[b]) {
                    B[b] = lay;
                    next.push_back(btoa[b]);
                }
            }
            if (islast) break;
            if (next.empty()) return res;
            for (int a : next) A[a] = lay;
            cur.swap(next);
        }
        rep(a, 0, sz(g))
            res += dfs(a, 0, g, btoa, A, B);
    }
}
```

## DFSMatching.h

**Description:** Simple bipartite matching algorithm. Graph  $g$  should be a list of neighbors of the left partition, and  $btoa$  should be a vector full of -1's of the same size as the right partition. Returns the size of the matching.  $btoa[i]$  will be the match for vertex  $i$  on the right side, or -1 if it's not matched.

**Usage:**  $vi btoa(m, -1);$  dfsMatching(g, btoa);

**Time:**  $\mathcal{O}(VE)$

522b98, 22 lines

```
bool find(int j, vector<vi> &g, vi &btoa, vi &vis) {
    if (btoa[j] == -1) return 1;
    vis[j] = 1; int di = btoa[j];
    for (int e : g[di])
        if (!vis[e] && find(e, g, btoa, vis)) {
            btoa[e] = di;
            return 1;
        }
    return 0;
}
int dfsMatching(vector<vi> &g, vi &btoa) {
    vi vis;
    rep(i, 0, sz(g)) {
        vis.assign(sz(btoa), 0);
        for (int j : g[i])
            if (find(j, g, btoa, vis)) {
                btoa[j] = i;
                break;
            }
    }
    return sz(btoa) - (int)count(all(btoa), -1);
}
```

## MinimumVertexCover.h

**Description:** Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h" da4196, 20 lines

```
vi cover(vector<vi> &g, int n, int m) {
    vi match(m, -1);
    int res = dfsMatching(g, match);
    vector<bool> lfound(n, true), seen(m);
    for (int it : match) if (it != -1) lfound[it] = false;
    vi q, cover;
    rep(i, 0, n) if (!lfound[i]) q.push_back(i);
    while (!q.empty()) {
        int i = q.back(); q.pop_back();
        lfound[i] = 1;
        for (int e : g[i]) if (!seen[e] && match[e] != -1) {
            seen[e] = true;
            q.push_back(match[e]);
        }
    }
    rep(i, 0, n) if (!lfound[i]) cover.push_back(i);
    rep(i, 0, m) if (seen[i]) cover.push_back(n+i);
    assert(sz(cover) == res);
    return cover;
}
```

## WeightedMatching.h

**Description:** Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for  $L[i]$  to be matched with  $R[j]$  and returns (min cost, match), where  $L[i]$  is matched with  $R[\text{match}[i]]$ . Negate costs for max cost. Requires  $N \leq M$ .

**Time:**  $\mathcal{O}(N^2 M)$

1e0fe9, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
    if (a.empty()) return {0, {}};
    int n = sz(a) + 1, m = sz(a[0]) + 1;
    vi u(n), v(m), p(m), ans(n - 1);
    rep(i, 1, n) {
        p[0] = i;
        int j0 = 0; // add "dummy" worker 0
        vi dist(m, INT_MAX), pre(m, -1);
        vector<bool> done(m + 1);
        do { // dijkstra
            done[j0] = true;
            int i0 = p[j0], j1, delta = INT_MAX;
            rep(j, 1, m) if (!done[j]) {
                auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
                if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
                if (dist[j] < delta) delta = dist[j], j1 = j;
            }
        } while (p[j0]);
        while (j0) { // update alternating path
            int j1 = pre[j0];
            p[j0] = p[j1], j0 = j1;
        }
        rep(j, 0, m) {
            if (done[j]) u[p[j]] += delta, v[j] -= delta;
            else dist[j] -= delta;
        }
        j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
        int j1 = pre[j0];
        p[j0] = p[j1], j0 = j1;
    }
    rep(j, 1, m) if (p[j]) ans[p[j] - 1] = j - 1;
    return {-v[0], ans}; // min cost
}
```

## BUET\_TheEmptySet

```
GeneralMatching.h
Description: Matching for general graphs. Fails with probability  $N/\text{mod}$ .
Time:  $\mathcal{O}(N^3)$ 


---


...numerical/MatrixInverse-mod.h" cb1912, 40 lines
vector<pii> generalMatching(int N, vector<pii>& ed) {
    vector<vector<ll>> mat(N, vector<ll>(N), A);
    for (pii pa : ed) {
        int a = pa.first, b = pa.second, r = rand() % mod;
        mat[a][b] = r, mat[b][a] = (mod - r) % mod;
    }

    int r = matInv(A = mat), M = 2*N - r, fi, fj;
    assert(r % 2 == 0);

    if (M != N) do {
        mat.resize(M, vector<ll>(M));
        rep(i, 0, N) {
            mat[i].resize(M);
            rep(j, N, M) {
                int r = rand() % mod;
                mat[i][j] = r, mat[j][i] = (mod - r) % mod;
            }
        }
    } while (matInv(A = mat) != M);

    vi has(M, 1); vector<pii> ret;
    rep(it, 0, M/2) {
        rep(i, 0, M) if (has[i])
            rep(j, i+1, M) if (A[i][j] && mat[i][j]) {
                fi = i; fj = j; goto done;
            }
        assert(0); done:
        if (fj < N) ret.emplace_back(fi, fj);
        has[fi] = has[fj] = 0;
        rep(sw, 0, 2) {
            ll a = modpow(A[fi][fj], mod-2);
            rep(i, 0, M) if (has[i] && A[i][fj]) {
                ll b = A[i][fj] * a % mod;
                rep(j, 0, M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
            }
            swap(fi, fj);
        }
    }
    return ret;
}
```

## 7.4 DFS algorithms

### BiconnectedComponents.h

**Description:** Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two internally disjoint paths between any two nodes (a cycle exists through them). Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

**Usage:** int eid = 0; ed.resize(N);  
for each edge (a,b) {  
ed[a].emplace\_back(b, eid);  
ed[b].emplace\_back(a, eid++);}  
bicomps([&](const vi& edgelist) {...});  
**Time:**  $\mathcal{O}(E + V)$

## GeneralMatching BiconnectedComponents 2sat EulerWalk EdgeColoring

```
st.push_back(e);
} else {
    int si = sz(st);
    int up = dfs(y, e, f);
    top = min(top, up);
    if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
    }
    else if (up < me) st.push_back(e);
    else /* e is a bridge */
}
return top;
}

template<class F>
void bicomps(F f) {
    num.assign(sz(ed), 0);
    rep(i, 0, sz(ed)) if (!num[i]) dfs(i, -1, f);
}
```

### 2sat.h

**Description:** Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type  $(a||b)\&\&(\neg a||c)\&\&(\neg d||\neg b)\&\&...$  becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions ( $\sim x$ ).

**Usage:** TwoSat ts(number of boolean variables);  
ts.either(0, ~3); // Var 0 is true or var 3 is false  
ts.setValue(2); // Var 2 is true  
ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true  
ts.solve(); // Returns true iff it is solvable  
ts.values[0..N-1] holds the assigned values to the vars  
**Time:**  $\mathcal{O}(N + E)$ , where N is the number of boolean variables, and E is the number of clauses.

```
5f9706, 56 lines
struct TwoSat {
    int N;
    vector<vi> gr;
    vi values; // 0 = false, 1 = true

    TwoSat(int n = 0) : N(n), gr(2*n) {}

    int addVar() { // (optional)
        gr.emplace_back();
        gr.emplace_back();
        return N++;
    }

    void either(int f, int j) {
        f = max(2*f, -1-2*f);
        j = max(2*j, -1-2*j);
        gr[f].push_back(j^1);
        gr[j].push_back(f^1);
    }

    void setValue(int x) { either(x, x); }

    void atMostOne(const vi& li) { // (optional)
        if (sz(li) <= 1) return;
        int cur = ~li[0];
        rep(i, 1, sz(li)) {
            int next = addVar();
            either(cur, ~li[i]);
            either(cur, next);
            either(~li[i], next);
            cur = ~next;
        }
        either(cur, ~li[1]);
    }
}
```

```
} // val, comp, z; int time = 0;
int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
        low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
        x = z.back(); z.pop_back();
        comp[x] = low;
        if (values[x>>1] == -1)
            values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
}

bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i, 0, 2*N) if (!comp[i]) dfs(i);
    rep(i, 0, N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
}
```

### EulerWalk.h

**Description:** Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

**Time:**  $\mathcal{O}(V + E)$  780b64, 15 lines

---

vi eulerWalk(vector<vector<pii>> &gr, int nedges, int src=0) {
 int n = sz(gr);
 vi D(n), its(n), eu(nedges), ret, s = {src};
 D[src]++;
 while (!s.empty()) {
 int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
 if (it == end) ret.push\_back(x); s.pop\_back(); continue;
 tie(y, e) = gr[x][it++];
 if (!eu[e]) {
 D[x]--, D[y]++;
 eu[e] = 1; s.push\_back(y);
 }
 }
 for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
 return {ret.rbegin(), ret rend()};
}

## 7.5 Coloring

### EdgeColoring.h

**Description:** Given a simple, undirected graph with max degree  $D$ , computes a  $(D+1)$ -coloring of the edges such that no neighboring edges share a color.  $(D$ -coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

**Time:**  $\mathcal{O}(NM)$  e210e2, 31 lines

---

vi edgeColoring(int N, vector<pii> eds) {
 vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
 for (pii e : eds) ++cc[e.first], ++cc[e.second];
 int u, v, ncols = \*max\_element(all(cc)) + 1;
 vector<vi> adj(N, vi(ncols, -1));
 for (pii e : eds) {
 tie(u, v) = e;
 fan[0] = v;
 loc.assign(ncols, 0);
 int at = u, end = u, d, c = free[u], ind = 0, i = 0;
 while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
 loc[d] = ++ind, cc[ind] = d, fan[ind] = v;

## BUET\_TheEmptySet

```
cc[loc[d]] = c;
for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
    swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
while (adj[fan[i][d] != -1) {
    int left = fan[i], right = fan[++i], e = cc[i];
    adj[u][e] = left;
    adj[left][e] = u;
    adj[right][e] = -1;
    free[right] = e;
}
adj[u][d] = fan[i];
adj[fan[i][d]] = u;
for (int y : {fan[0], u, end})
    for (int& z = free[y] = 0; adj[y][z] != -1; z++);
}
rep(i, 0, sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
return ret;
}
```

## 7.6 Trees

### BinaryLifting.h

**Description:** Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.

**Time:** construction  $\mathcal{O}(N \log N)$ , queries  $\mathcal{O}(\log N)$

bffce85, 25 lines

```
vector<vi> treeJump(vi& P) {
    int on = 1, d = 1;
    while(on < sz(P)) on *= 2, d++;
    vector<vi> jmp(d, P);
    rep(i, 1, d) rep(j, 0, sz(P))
        jmp[i][j] = jmp[i-1][jmp[i-1][j]];
    return jmp;
}

int jmp(vector<vi>& tbl, int nod, int steps) {
    rep(i, 0, sz(tbl))
        if(steps & (1<<i)) nod = tbl[i][nod];
    return nod;
}

int lca(vector<vi>& tbl, vi& depth, int a, int b) {
    if (depth[a] < depth[b]) swap(a, b);
    a = jmp(tbl, a, depth[a] - depth[b]);
    if (a == b) return a;
    for (int i = sz(tbl); i--;) {
        int c = tbl[i][a], d = tbl[i][b];
        if (c != d) a = c, b = d;
    }
    return tbl[0][a];
}
```

### LCA.h

**Description:** Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.

**Time:**  $\mathcal{O}(N \log N + Q)$

0f62fb, 21 lines

```
struct LCA {
    int T = 0;
    vi time, path, ret;
    RMQ<int> rmq;
    LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C, 0, -1), ret)) {}
    void dfs(vector<vi>& C, int v, int par) {
        time[v] = T++;
        for (int y : C[v]) if (y != par) {
            path.push_back(v), ret.push_back(time[v]);
            dfs(C, y, v);
        }
    }
}
```

## BinaryLifting LCA CompressTree HLD LinkCutTree

```
dfs(C, y, v);
}

int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)];
}
//dist(a, b){return depth[a] + depth[b] - 2*depth[lca(a, b)];}
};
```

### CompressTree.h

**Description:** Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most  $|S| - 1$ ) pairwise LCAs and compressing edges. Returns a list of (par, orig\_index) representing a tree rooted at 0. The root points to itself.

**Time:**  $\mathcal{O}(|S| \log |S|)$

"LCA.h"

9775a0, 21 lines

```
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
    static vi rev; rev.resize(sz(lca.time));
    vi li = subset, &T = lca.time;
    auto cmp = [&](int a, int b) { return T[a] < T[b]; };
    sort(all(li), cmp);
    int m = sz(li)-1;
    rep(i, 0, m) {
        int a = li[i], b = li[i+1];
        li.push_back(lca.lca(a, b));
    }
    sort(all(li), cmp);
    li.erase(unique(all(li)), li.end());
    rep(i, 0, sz(li)) rev[li[i]] = i;
    vpi ret = {pii(0, li[0])};
    rep(i, 0, sz(li)-1) {
        int a = li[i], b = li[i+1];
        ret.emplace_back(rev[lca.lca(a, b)], b);
    }
    return ret;
}
```

### HLD.h

**Description:** Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most  $\log(n)$  light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS\_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.

**Time:**  $\mathcal{O}((\log N)^2)$

"../data\_structures/LazySegmentTree.h"

9547af, 46 lines

```
template <bool VALS_EDGES> struct HLD {
    int N, tim = 0;
    vector<vi> adj;
    vi par, siz, rt, pos;
    Node *tree;
    HLD(vector<vi> adj_) {
        : N(sz(adj_)), adj(adj_), par(N, -1), siz(N, 1),
          rt(N), pos(N), tree(new Node(0, N)) { dfsSz(0); dfsHld(0); }
    }
    void dfsSz(int v) {
        for (int& u : adj[v]) {
            adj[u].erase(find(all(adj[u]), v));
            par[u] = v;
            dfsSz(u);
            siz[v] += siz[u];
            if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
        }
    }
}
```

```
void dfsHld(int v) {
    pos[v] = tim++;
    for (int u : adj[v]) {
        rt[u] = (u == adj[v][0] ? rt[v] : u);
        dfsHld(u);
    }
}
template <class B> void process(int u, int v, B op) {
    for (;;) v = par[rt[v]]);
        if (pos[u] > pos[v]) swap(u, v);
        if (rt[u] == rt[v]) break;
        op(pos[rt[v]], pos[v] + 1);
    }
    op(pos[u] + VALS_EDGES, pos[v] + 1);
}
void modifyPath(int u, int v, int val) {
    process(u, v, [&](int l, int r) { tree->add(l, r, val); });
}
int queryPath(int u, int v) { // Modify depending on problem
    int res = -1e9;
    process(u, v, [&](int l, int r) {
        res = max(res, tree->query(l, r));
    });
    return res;
}
int querySubtree(int v) { // modifySubtree is similar
    return tree->query(pos[v] + VALS_EDGES, pos[v] + siz[v]);
}
```

### LinkCutTree.h

**Description:** Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.

**Time:** All operations take amortized  $\mathcal{O}(\log N)$ .

0fb462, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
    Node *p = 0, *pp = 0, *c[2];
    bool flip = 0;
    Node() { c[0] = c[1] = 0; fix(); }
    void fix() {
        if (c[0]) c[0]->p = this;
        if (c[1]) c[1]->p = this;
        // (+ update sum of subtree elements etc. if wanted)
    }
    void pushFlip() {
        if (!flip) return;
        flip = 0; swap(c[0], c[1]);
        if (c[0]) c[0]->flip ^= 1;
        if (c[1]) c[1]->flip ^= 1;
    }
    int up() { return p ? p->c[1] == this : -1; }
    void rot(int i, int b) {
        int h = i ^ b;
        Node *x = c[i], *y = b == 2 ? x->c[h], *z = b ? y : x;
        if ((y->p) == p) p->c[up()] = y;
        c[i] = z->c[i ^ 1];
        if (b < 2) {
            x->c[h] = y->c[h ^ 1];
            y->c[h ^ 1] = x;
        }
        z->c[i ^ 1] = this;
        fix(); x->fix(); y->fix();
        if (p) p->fix();
        swap(pp, y->pp);
    }
    void splay() {
        for (pushFlip(); p; ) {
            if (p->p) p->p->pushFlip();
        }
    }
}
```

## BUET\_TheEmptySet

```

    p->pushFlip(); pushFlip();
    int c1 = up(), c2 = p->up();
    if (c2 == -1) p->rot(c1, 2);
    else p->p->rot(c2, c1 != c2);
}

Node* first() {
    pushFlip();
    return c[0] ? c[0]->first() : (splay(), this);
}

struct LinkCut {
    vector<Node> node;
    LinkCut(int N) : node(N) {}

    void link(int u, int v) { // add an edge (u, v)
        assert(!connected(u, v));
        makeRoot(&node[u]);
        node[u].pp = &node[v];
    }

    void cut(int u, int v) { // remove an edge (u, v)
        Node *x = &node[u], *top = &node[v];
        makeRoot(top); x->splay();
        assert(top == (x->pp ?: x->c[0]));
        if (x->pp) x->pp = 0;
        else {
            x->c[0] = top->p = 0;
            x->fix();
        }
    }

    bool connected(int u, int v) { // are u, v in the same tree?
        Node* nu = access(&node[u])->first();
        return nu == access(&node[v])->first();
    }

    void makeRoot(Node* u) {
        access(u);
        u->splay();
        if(u->c[0]) {
            u->c[0]->p = 0;
            u->c[0]->flip ^= 1;
            u->c[0]->pp = u;
            u->c[0] = 0;
            u->fix();
        }
    }

    Node* access(Node* u) {
        u->splay();
        while ((Node* pp = u->pp)) {
            pp->splay(); u->pp = 0;
            if (pp->c[1]) {
                pp->c[1]->p = 0; pp->c[1]->pp = pp; }
            pp->c[1] = u; pp->fix(); u = pp;
        }
        return u;
    }
}

```

## DirectedMST.h

**Description:** Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

**Time:**  $\mathcal{O}(E \log V)$

.../data-structures/UnionFindRollback.h"

39e620, 60 lines

```

struct Edge { int a, b; ll w; };
struct Node {
    Edge key;
    Node *l, *r;
    ll delta;
    void prop() {

```

## DirectedMST Point lineDistance SegmentDistance

```

        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    Edge top() { prop(); return key; }
};

Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ?: b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}

void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n);
    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
    ll res = 0;
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n), in(n, {-1, -1}), comp;
    deque<tuple<int, int, vector<Edge>>> cycs;
    rep(s, 0, n) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return {-1, {}};
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            Q[qi] = e, path[qi++] = u, seen[u] = s;
            res += e.w, u = uf.find(e.a);
            if (seen[u] == s) {
                Node* cyc = 0;
                int end = qi, time = uf.time();
                do cyc = merge(cyc, heap[w = path[--qi]]);
                while (uf.join(u, w));
                u = uf.find(u), heap[u] = cyc, seen[u] = -1;
                cycs.push_front({u, time, {&Q[qi], &Q[end]}});
            }
            rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
        }

        for (auto& [u, t, comp] : cycs) { // restore sol (optional)
            uf.rollback(t);
            Edge inEdge = in[u];
            for (auto& e : comp) in[uf.find(e.b)] = e;
            in[uf.find(inEdge.b)] = inEdge;
        }
        rep(i, 0, n) par[i] = in[i].a;
        return {res, par};
    }
}

```

## 7.7 Math

## 7.7.1 Number of Spanning Trees

Create an  $N \times N$  matrix mat, and for each edge  $a \rightarrow b \in G$ , do  $\text{mat}[a][b]--$ ,  $\text{mat}[b][b]++$  (and  $\text{mat}[b][a]--$ ,  $\text{mat}[a][a]++$  if  $G$  is undirected). Remove the  $i$ th row and column and take the determinant; this yields the number of directed spanning trees rooted at  $i$  (if  $G$  is undirected, remove any row/column).

## 7.7.2 Erdős–Gallai theorem

A simple graph with node degrees  $d_1 \geq \dots \geq d_n$  exists iff  $d_1 + \dots + d_n$  is even and for every  $k = 1 \dots n$ ,

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k).$$

## Geometry (8)

## 8.1 Geometric primitives

## Point.h

**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 28 lines

```

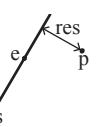
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template <class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {}
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
    P operator+(P p) const { return P(x+p.x, y+p.y); }
    P operator-(P p) const { return P(x-p.x, y-p.y); }
    P operator*(T d) const { return P(x*d, y*d); }
    P operator/(T d) const { return P(x/d, y/d); }
    T dot(P p) const { return x*p.x + y*p.y; }
    T cross(P p) const { return x*p.y - y*p.x; }
    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
    T dist2() const { return x*x + y*y; }
    double dist() const { return sqrt((double)dist2()); }
    // angle to x-axis in interval [-pi, pi]
    double angle() const { return atan2(y, x); }
    P unit() const { return *this/dist(); } // makes dist()==1
    P perp() const { return P(-y, x); } // rotates +90 degrees
    P normal() const { return perp().unit(); }
    // returns point rotated 'a' radians ccw around the origin
    P rotate(double a) const {
        return P(x*cos(a)-y*sin(a), x*sin(a)+y*cos(a)); }
    friend ostream& operator<<(ostream& os, P p) {
        return os << "(" << p.x << "," << p.y << ")";
    }
};

```

## lineDistance.h

**Description:**

Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.



Point.h\*

```

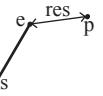
template <class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double)(b-a).cross(p-a) / (b-a).dist();
}

```

## SegmentDistance.h

**Description:**

Returns the shortest distance between point p and the line segment from point s to e.



**Usage:** Point<double> a, b(2,2), p(1,1);  
bool onSegment = segDist(a,b,p) < 1e-10;  
"Point.h"

5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
    return ((p-s)*d-(e-s)*t).dist()/d;
}
```

## SegmentIntersection.h

### Description:

If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

**Usage:** vector<P> inter = segInter(s1,e1,s2,e2);  
cout << "segments intersect at " << inter[0] << endl;  
"Point.h", "OnSegment.h"



```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
    auto oa = c.cross(d, a), ob = c.cross(d, b),
        oc = a.cross(b, c), od = a.cross(b, d);
    // Checks if intersection is single non-endpoint point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)};
    set<P> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d);
    return {all(s)};
}
```

## lineIntersection.h

### Description:

If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

**Usage:** auto res = lineInter(s1,e1,s2,e2);  
if (res.first == 1)  
cout << "intersection point at " << res.second << endl;  
"Point.h"



```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)};
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
    return {1, (s1 * p + e1 * q) / d};
}
```

## sideOf.h

**Description:** Returns where p is as seen from s towards e. 1/0/-1  $\leftrightarrow$  left/on/line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

**Usage:** bool left = sideOf(p1,p2,q)==1;  
"Point.h"

3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
    auto a = (e-s).cross(p-s);
    double l = (e-s).dist()*eps;
    return (a > l) - (a < -l);
}
```

## OnSegment.h

**Description:** Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

"Point.h"

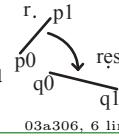
c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

## linearTransformation.h

### Description:

Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h"

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
                      const P& q0, const P& q1, const P& r) {
    P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dist2());
    return q0 + P((r-p0).cross(num), (r-p0).dot(num)) / dp.dist2();
}
```

## Angle.h

**Description:** A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

**Usage:** vector<Angle> v = {w[0], w[0].t360() ...}; // sorted  
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }  
// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i

0f0602, 35 lines

```
struct Angle {
    int x, y;
    int t;
    Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
    Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
    int half() const {
        assert(x || y);
        return y < 0 || (y == 0 && x < 0);
    }
    Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
    Angle t180() const { return {-x, -y, t + half()} };
    Angle t360() const { return {x, y, t + 1}; }
};

bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
           make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
    if (b < a) swap(a, b);
    return (b < a.t180() ?
            make_pair(a, b) : make_pair(b, a.t360()));
}
```

// Given two points, this calculates the smallest angle between  
// them, i.e., the angle that covers the defined line segment.  
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {  
 if (b < a) swap(a, b);  
 return (b < a.t180() ?  
 make\_pair(a, b) : make\_pair(b, a.t360()));
}

Angle operator+(Angle a, Angle b) { // point a + vector b  
 Angle r(a.x + b.x, a.y + b.y, a.t);
 if (a.t180() < r) r.t--;
 return r.t180() < a ? r.t360() : r;
}

Angle angleDiff(Angle a, Angle b) { // angle b - angle a  
 int tu = b.t - a.t; a.t = b.t;
 return {a.x\*b.y + a.y\*b.y, a.x\*b.x - a.y\*b.x, tu - (b < a)};
}

## 8.2 Circles

### CircleIntersection.h

**Description:** Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h"

84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P*> out) {
    if (a == b) { assert(r1 != r2); return false; }
    P vec = b - a;
    double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
        p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p/d2;
    if (sum*sum < d2 || dif*dif > d2) return false;
    P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
    *out = {mid + per, mid - per};
    return true;
}
```

### CircleTangents.h

**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h"

b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
    P d = c2 - c1;
    double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
    if (d2 == 0 || h2 < 0) return {};
    vector<pair<P, P>> out;
    for (double sign : {-1, 1}) {
        P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
        out.push_back({c1 + v * r1, c2 + v * r2});
    }
    if (h2 == 0) out.pop_back();
    return out;
}
```

### CirclePolygonIntersection.h

**Description:** Returns the area of the intersection of a circle with a ccw polygon.

**Time:**  $\mathcal{O}(n)$

".../content/geometry/Point.h"

19add1, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
    auto tri = [&](P p, P q) {
        auto r2 = r * r / 2;
        P d = q - p;
        auto a = d.dot(p)/d.dist2(), b = (p.dist2() - r*r)/d.dist2();
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
        if (t < 0 || 1 <= s) return arg(p, q) * r2;
        P u = p + d * s, v = q + d * (t-1);
    };
}
```

```

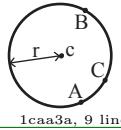
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
}
auto sum = 0.0;
rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i+1)%sz(ps)] - c);
return sum;
}

```

### circumcircle.h

**Description:**

The circumcircle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



1caa3a, 9 lines

```

"Point.h"
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()*(C-B).dist()*(A-C).dist()/
        abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}

```

### MinimumEnclosingCircle.h

**Description:** Computes the minimum circle that encloses a set of points.

**Time:** expected  $\mathcal{O}(n)$

```

"circumcircle.h"
09dd0a, 17 lines
pair<P, double> mec(vector<P> ps) {
    shuffle(all(ps), mt19937(time(0)));
    P o = ps[0];
    double r = 0, EPS = 1 + 1e-8;
    rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
        o = ps[i], r = 0;
        rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
            o = (ps[i] + ps[j]) / 2;
            r = (o - ps[i]).dist();
        }
        rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
            o = ccCenter(ps[i], ps[j], ps[k]);
            r = (o - ps[i]).dist();
        }
    }
    return {o, r};
}

```

## 8.3 Polygons

### InsidePolygon.h

**Description:** Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

**Usage:** vector<P> v = {P{4,4}, P{1,2}, P{2,1}};

bool in = inPolygon(v, P{3, 3}, false);

**Time:**  $\mathcal{O}(n)$

```

"Point.h", "OnSegment.h", "SegmentDistance.h"
2bf504, 11 lines
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
    int cnt = 0, n = sz(p);
    rep(i,0,n) {
        P q = p[(i+1)%n];
        if (onSegment(p[i], q, a)) return !strict;
        //or: if (segDist(p[i], q, a) <= eps) return !strict;
        cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
    }
    return cnt;
}

```

### PolygonArea.h

**Description:** Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 6 lines

```

template<class T>
T polygonArea2(vector<Point<T>>& v) {
    T a = v.back().cross(v[0]);
    rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
    return a;
}

```

### PolygonCenter.h

**Description:** Returns the center of mass for a polygon.

**Time:**  $\mathcal{O}(n)$

```

"Point.h" 9706dc, 9 lines
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
    P res(0, 0); double A = 0;
    for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
        res = res + (v[i] + v[j]) * v[j].cross(v[i]);
        A += v[j].cross(v[i]);
    }
    return res / A / 3;
}

```

### PolygonCut.h

**Description:**

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

**Usage:** vector<P> p = ...;

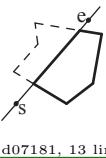
p = polygonCut(p, P(0,0), P(1,0));

"Point.h"

```

typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
    vector<P> res;
    rep(i,0,sz(poly)) {
        P cur = poly[i], prev = i ? poly[i-1] : poly.back();
        auto a = s.cross(e, cur), b = s.cross(e, prev);
        if ((a < 0) != (b < 0))
            res.push_back(cur + (prev - cur) * (a / (a - b)));
        if (a < 0)
            res.push_back(cur);
    }
    return res;
}

```



d07181, 13 lines

### ConvexHull.h

**Description:**

Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

**Time:**  $\mathcal{O}(n \log n)$

```

"Point.h" 310954, 13 lines
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
    if (sz(pts) <= 1) return pts;
    sort(all(pts));
    vector<P> h(sz(pts)+1);
    int s = 0, t = 0;
    for (int it = 2; it-->t; s = -t, reverse(all(pts)))
        for (P p : pts) {
            while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
            h[t++] = p;
        }
    return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}

```



### HullDiameter.h

**Description:** Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

**Time:**  $\mathcal{O}(n)$

```

"Point.h" c571b8, 12 lines
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
    int n = sz(S), j = n < 2 ? 0 : 1;
    pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
    rep(i,0,j)
        for (;;) j = (j + 1) % n
        res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
        if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
            break;
    }
    return res.second;
}

```

### PointInsideHull.h

**Description:** Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

**Time:**  $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

typedef Point<ll> P;

```

bool inHull(const vector<P>& l, P p, bool strict = true) {
    int a = 1, b = sz(l) - 1, r = !strict;
    if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
    if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
    if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        if (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
    }
    return sgn(l[a].cross(l[b], p)) < r;
}

```

### LineHullIntersection.h

**Description:** Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • (-1, -1) if no collision, • (i, -1) if touching the corner i, • (i, i) if along side (i, i+1), • (i, j) if crossing sides (i, i+1) and (j, j+1). In the last case, if a corner i is crossed, this is treated as happening on side (i, i+1). The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.

**Time:**  $\mathcal{O}(\log n)$

```

"Point.h" 7cf45b, 39 lines
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i+1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
    int n = sz(poly), lo = 0, hi = n;
    if (extr(0)) return 0;
    while (lo + 1 < hi) {
        int m = (lo + hi) / 2;
        if (extr(m)) return m;
        int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
        if (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
    }
    return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp());

```

## BUET\_TheEmptySet

```

int endB = extrVertex(poly, (b - a).perp());
if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1};
array<int, 2> res;
rep(i, 0, 2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
        int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
        (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
}
if (res[0] == res[1]) return {res[0], -1};
if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
        case 0: return {res[0], res[0]};
        case 2: return {res[1], res[1]};
    }
return res;
}

```

## 8.4 Misc. Point Set Problems

## ClosestPair.h

**Description:** Finds the closest pair of points.**Time:**  $\mathcal{O}(n \log n)$ 

`"Point.h"` ac41a6, 17 lines

```

typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
    int j = 0;
    for (P p : v) {
        P d{1 + (ll)sqrt(ret.first), 0};
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
            ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
        S.insert(p);
    }
    return ret.second;
}

```

## kdTree.h

**Description:** KD-tree (2d, can be extended to 3d)

`"Point.h"` bac5b0, 63 lines

```

typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;

    T distance(const P& p) { // min squared distance to a point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x, y) - p).dist2();
    }

    Node(vector<P>&& vp) : pt(vp[0]) {
        for (P p : vp) {

```

## ClosestPair kdTree FastDelaunay

```

        x0 = min(x0, p.x); x1 = max(x1, p.x);
        y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
        // split on x if width >= height (not ideal...)
        sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
        // divide by taking half the array for each child (not
        // best performance with many duplicates in the middle)
        int half = sz(vp)/2;
        first = new Node{vp.begin(), vp.begin() + half};
        second = new Node{vp.begin() + half, vp.end()};
    }
}

struct KDTree {
    Node* root;
    KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

    pair<T, P> search(Node *node, const P& p) {
        if (!node->first) {
            // uncomment if we should not find the point itself:
            // if (p == node->pt) return {INF, P()};
            return make_pair((p - node->pt).dist2(), node->pt);
        }

        Node *f = node->first, *s = node->second;
        T bfirst = f->distance(p), bsec = s->distance(p);
        if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

        // search closest side first, other side if needed
        auto best = search(f, p);
        if (bsec < best.first)
            best = min(best, search(s, p));
        return best;
    }

    // find nearest point to a point, and its squared distance
    // (requires an arbitrary operator< for Point)
    pair<T, P> nearest(const P& p) {
        return search(root, p);
    }
};

FastDelaunay.h

```

**Description:** Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.**Time:**  $\mathcal{O}(n \log n)$ 

`"Point.h"` eefdf5, 88 lines

```

typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point

struct Quad {
    Q rot, o; P p = arb; bool mark;
    P& F() { return r()->p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = p.dist2(), A = a.dist2() - p2,
        B = b.dist2() - p2, C = c.dist2() - p2;
    return p.cross(a, b)*C + p.cross(b, c)*A + p.cross(c, a)*B > 0;
}

```

```

}
Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
    H = r->o; r->r()->r() = r;
    rep(i, 0, 4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
    return r;
}
void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q;
}
pair<Q, Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (sz(s) == 2) return {a, a->r()};
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r()};
    }
}

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
Q A, B, ra, rb;
int half = sz(s) / 2;
tie(ra, A) = rec({all(s) - half});
tie(B, rb) = rec({sz(s) - half + all(s)});
while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
       (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
Q base = connect(B->r(), A);
if (A->p == ra->p) ra = base->r();
if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
    }
for (;;) {
    DEL(LC, base->r(), o); DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC)))) {
        base = connect(RC, base->r());
    } else
        base = connect(base->r(), LC->r());
}
return {ra, rb};
}

vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (e->o->F().cross(e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
    q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < sz(q)) if (!!(e = q[qi++])->mark) ADD;
}

```

```
    return pts;
}
```

## 8.5 3D

### PolyhedronVolume.h

**Description:** Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
    double v = 0;
    for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
}
```

### Point3D.h

**Description:** Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    }
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const { return sqrt((double)dist2()); }
    //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
    double phi() const { return atan2(y, x); }
    //Zenith angle (latitude) to the z-axis in interval [0, pi]
    double theta() const { return atan2(sqrt(x*x+y*y), z); }
    P unit() const { return *this/(T)dist(); } //makes dist()==1
    //returns unit vector normal to *this and p
    P normal(P p) const { return cross(p).unit(); }
    //returns point rotated 'angle' radians ccw around axis
    P rotate(double angle, P axis) const {
        double s = sin(angle), c = cos(angle); P u = axis.unit();
        return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
    }
};
```

### 3dHull.h

**Description:** Computes all faces of the 3-dimension hull of a point set. \*No four points must be coplanar\*, or else random results will be returned. All faces will point outwards.

**Time:**  $\mathcal{O}(n^2)$

\*Point3D.h\*

5b45fc, 49 lines

```
typedef Point3D<double> P3;

struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); }
    int a, b;
};

struct F { P3 q; int a, b, c; };
```

```
vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
    vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
    vector<F> FS;
    auto mf = [&](int i, int j, int k, int l) {
        P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
        if (q.dot(A[l]) > q.dot(A[i]))
            q = q * -1;
        F f{q, i, j, k};
        E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
        FS.push_back(f);
    };
    rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
        mf(i, j, k, 6 - i - j - k);
    rep(i,4,sz(A)) {
        rep(j,0,sz(FS)) {
            F f = FS[j];
            if (f.q.dot(A[i]) > f.q.dot(A[f.a])) {
                E(a,b).rem(f.c);
                E(a,c).rem(f.b);
                E(b,c).rem(f.a);
                swap(FS[j--], FS.back());
                FS.pop_back();
            }
        }
        int nw = sz(FS);
        rep(j,0,nw) {
            F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
            C(a, b, c); C(a, c, b); C(b, c, a);
        }
        for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
            A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
        return FS;
    };
}
```

### sphericalDistance.h

**Description:** Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude)  $f_1(\phi_1)$  and  $f_2(\phi_2)$  from x axis and zenith angles (latitude)  $t_1(\theta_1)$  and  $t_2(\theta_2)$  from z axis ( $0 =$  north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows.  $dx \cdot \text{radius}$  is then the difference between the two points in the x direction and  $d \cdot \text{radius}$  is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
                        double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);
    return radius*2*asin(d/2);
}
```

## Strings (9)

### Manacher.h

**Description:** For each position in a string, computes  $p[0][i]$  = half length of longest even palindrome around pos i,  $p[1][i]$  = longest odd (half rounded down).

**Time:**  $\mathcal{O}(N)$

e7ad79, 13 lines

```
array<vi, 2> manacher(const string& s) {
```

```
int n = sz(s);
array<vi, 2> p = {vi(n+1), vi(n)};
rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i < r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
        p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
}
return p;
}
```

### MinRotation.h

**Description:** Finds the lexicographically smallest rotation of a string.

**Usage:** `rotate(v.begin(), v.begin() + minRotation(v), v.end());`

**Time:**  $\mathcal{O}(N)$

d07a42, 8 lines

```
int minRotation(string s) {
    int a=0, N=sz(s); s += s;
    rep(b,0,N) rep(k,0,N) {
        if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
        if (s[a+k] > s[b+k]) {a = b; break; }
    }
    return a;
}
```

### SuffixArray.h

**Description:** Builds suffix array for a string.  $sa[i]$  is the starting index of the suffix which is  $i$ 'th in the sorted suffix array. The returned vector is of size  $n + 1$ , and  $sa[0] = n$ . The lcp array contains longest common prefixes for neighbouring strings in the suffix array:  $lcp[i] = lcp(sa[i], sa[i-1])$ ,  $lcp[0] = 0$ . The input string must not contain any null chars.

**Time:**  $\mathcal{O}(n \log n)$

635552, 22 lines

```
struct SuffixArray {
    vi sa, lcp;
    SuffixArray(string s, int lim=256) { // or vector<int>
        s.push_back(0); int n = sz(s), k = 0, a, b;
        vi x(all(s)), y(n), ws(max(n, lim));
        sa = lcp = y, iota(all(sa), 0);
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
            p = j, iota(all(y), n - j);
            rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
            fill(all(ws), 0);
            rep(i,0,n) ws[x[i]]++;
            rep(i,1,lim) ws[i] += ws[i - 1];
            for (int i = n; i-->0; sa[--ws[x[y[i]]]] = y[i]);
            swap(x, y), p = 1, x[sa[0]] = 0;
            rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
        }
        for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
            for (k && k--, j = sa[x[i] - 1];
                 s[i + k] == s[j + k]; k++);
    }
};
```

### SuffixTree.h

**Description:** Ukkonen's algorithm for online suffix tree construction. Each node contains indices  $[l, r]$  into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining  $[l, r]$  substrings. The root is 0 (has  $l = -1, r = 0$ ), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

**Time:**  $\mathcal{O}(26N)$

aae0b8, 50 lines

```
struct SuffixTree {
    enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
```

```

int toi(char c) { return c - 'a'; }
string a; // v = cur node, q = cur position
int t[N][ALPHA], l[N], r[N], p[N], s[N], v=0, q=0, m=2;

void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
        if (t[v][c]==-1) { t[v][c]=m; l[m]=i;
            p[m+1]=v; v=s[v]; q=r[v]; goto suff; }
        v=t[v][c]; q=l[v];
    }
    if (q===-1 || c==toi(a[q])) q++; else {
        l[m+1]=i; p[m+1]=m; l[m]=l[v]; r[m]=q;
        p[m]=p[v]; t[m][c]=m+1; t[m][toi(a[q])]=v;
        l[v]=q; p[v]=m; t[p[m]][toi(a[l[m]])]=m;
        v=s[p[m]]; q=l[m];
        while (q<r[m]) { v=t[v][toi(a[q])]; q+=r[v]-l[v]; }
        if (q==r[m]) s[m]=v; else s[m]=m+2;
        q=r[v]-(q-r[m]); m+=2; goto suff;
    }
}

SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1], t[1]+ALPHA, 0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
}

// example: find longest common substring (uses ALPHA = 28)
pii best;
int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
        mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
        best = max(best, {len, r[node] - len});
    return mask;
}

static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
}

```

## Hashing.h

**Description:** Self-explanatory methods for string hashing.

2d2a67, 44 lines

```

// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
typedef uint64_t ull;
struct H {
    ull x; H(ull x=0) : x(x) {}
    H operator+(H o) { return x + o.x + (x + o.x < x); }
    H operator-(H o) { return *this + ~o.x; }
    H operator*(H o) { auto m = (_uint128_t)x * o.x;
        return H((ull)m) + (ull)(m > 64); }
    ull get() const { return x + !~x; }
    bool operator==(H o) const { return get() == o.get(); }
    bool operator<(H o) const { return get() < o.get(); }
};

static const H C = (11)1e11+3; // (order ~ 3e9; random also ok)

```

## Hashing AhoCorasick IntervalContainer

```

struct HashInterval {
    vector<H> ha, pw;
    HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
        pw[0] = 1;
        rep(i,0,sz(str))
            ha[i+1] = ha[i] * C + str[i],
            pw[i+1] = pw[i] * C;
    }
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b-a];
    }
};

vector<H> getHashes(string& str, int length) {
    if (sz(str) < length) return {};
    H h = 0, pw = 1;
    rep(i,0,length)
        h = h * C + str[i], pw = pw * C;
    vector<H> ret = {h};
    rep(i,length,sz(str)) {
        ret.push_back(h = h * C + str[i] - pw * str[i-length]);
    }
    return ret;
}

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}

```

### AhoCorasick.h

**Description:** Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(-, word) finds all words (up to  $N\sqrt{N}$  many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.

**Time:** construction takes  $\mathcal{O}(26N)$ , where  $N$  = sum of length of patterns. find(x) is  $\mathcal{O}(N)$ , where  $N$  = length of x. findAll is  $\mathcal{O}(NM)$ .

```

struct AhoCorasick {
    enum {alpha = 26, first = 'A'}; // change this!
    struct Node {
        // (nmatches is optional)
        int back, next[alpha], start = -1, end = -1, nmatches = 0;
        Node(int v) { memset(next, v, sizeof(next)); }
    };
    vector<Node> N;
    vi backp;
    void insert(string& s, int j) {
        assert(!s.empty());
        int n = 0;
        for (char c : s) {
            int& m = N[n].next[c - first];
            if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
            else n = m;
        }
        if (N[n].end == -1) N[n].start = j;
        backp.push_back(N[n].end);
        N[n].end = j;
        N[n].nmatches++;
    }
    AhoCorasick(vector<string>& pat) : N(1, -1) {
        rep(i,0,sz(pat)) insert(pat[i], i);
        N[0].back = sz(N);
        N.emplace_back(0);
    }

    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {

```

```

        int n = q.front(), prev = N[n].back;
        rep(i,0,alpha) {
            int ed = N[n].next[i], y = N[prev].next[i];
            if (ed == -1) ed = y;
            else {
                N[ed].back = y;
                (N[ed].end == -1 ? N[ed].end : backp[N[ed].start])
                    = N[y].end;
                N[ed].nmatches += N[y].nmatches;
                q.push(ed);
            }
        }
    }
    vi find(string word) {
        int n = 0;
        vi res; // ll count = 0;
        for (char c : word) {
            n = N[n].next[c - first];
            res.push_back(N[n].end);
            // count += N[n].nmatches;
        }
        return res;
    }
    vector<vi> findAll(vector<string>& pat, string word) {
        vi r = find(word);
        vector<vi> res(sz(word));
        rep(i,0,sz(word)) {
            int ind = r[i];
            while (ind != -1) {
                res[i - sz(pat[ind]) + 1].push_back(ind);
                ind = backp[ind];
            }
        }
        return res;
    }
}

```

## Various (10)

### 10.1 Intervals

#### IntervalContainer.h

**Description:** Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).

**Time:**  $\mathcal{O}(\log N)$

edce47, 23 lines

```

set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
        R = max(R, it->second);
        before = it = is.erase(it);
    }
    if (it != is.begin() && (--it)->second >= L) {
        L = min(L, it->first);
        R = max(R, it->second);
        is.erase(it);
    }
    return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second;
    if (it->first == L) is.erase(it);
}

```

```
else (int&) it->second = L;
if (R != r2) is.emplace(R, r2);
}
```

## IntervalCover.h

**Description:** Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add || R.empty(). Returns empty set on failure (or if G is empty).

**Time:**  $\mathcal{O}(N \log N)$

9e9d8d, 19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
    vi S(sz(I)), R;
    iota(all(S), 0);
    sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
    T cur = G.first;
    int at = 0;
    while (cur < G.second) { // (A)
        pair<T, int> mx = make_pair(cur, -1);
        while (at < sz(I) && I[S[at]].first <= cur) {
            mx = max(mx, make_pair(I[S[at]].second, S[at]));
            at++;
        }
        if (mx.second == -1) return {};
        cur = mx.first;
        R.push_back(mx.second);
    }
    return R;
}
```

## ConstantIntervals.h

**Description:** Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.

**Usage:** constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});

**Time:**  $\mathcal{O}(k \log \frac{n}{k})$

753a4c, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
    if (p == q) return;
    if (from == to) {
        g(i, to, p);
        i = to; p = q;
    } else {
        int mid = (from + to) >> 1;
        rec(from, mid, f, g, i, p, f(mid));
        rec(mid+1, to, f, g, i, p, q);
    }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
    if (to <= from) return;
    int i = from; auto p = f(i), q = f(to-1);
    rec(from, to-1, f, g, i, p, q);
    g(i, to, q);
}
```

## 10.2 Misc. algorithms

## TernarySearch.h

**Description:** Find the smallest i in  $[a, b]$  that maximizes  $f(i)$ , assuming that  $f(a) < \dots < f(i) \geq \dots \geq f(b)$ . To reverse which of the sides allows non-strict inequalities, change the  $<$  marked with (A) to  $\leq$ , and reverse the loop at (B). To minimize  $f$ , change it to  $>$ , also at (B).

**Usage:** int ind = ternSearch(0, n-1, [&](int i){return a[i];});

**Time:**  $\mathcal{O}(\log(b-a))$

9155b4, 11 lines

```
int ternSearch(int a, int b, F f) {
    assert(a <= b);
    while (b - a >= 5) {
        int mid = (a + b) / 2;
        if (f(mid) < f(mid+1)) a = mid; // (A)
        else b = mid+1;
    }
    rep(i, a+1, b+1) if (f(a) < f(i)) a = i; // (B)
    return a;
}
```

## LIS.h

**Description:** Compute indices for the longest increasing subsequence.

**Time:**  $\mathcal{O}(N \log N)$

2932a0, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
    if (S.empty()) return {};
    vi prev(sz(S));
    typedef pair<I, int> p;
    vector<p> res;
    rep(i, 0, sz(S)) {
        // change 0 -> i for longest non-decreasing subsequence
        auto it = lower_bound(all(res), p{S[i], 0});
        if (it == res.end()) res.emplace_back(), it = res.end()-1;
        *it = {S[i], i};
        prev[i] = it == res.begin() ? 0 : (it-1)->second;
    }
    int L = sz(res), cur = res.back().second;
    vi ans(L);
    while (L--) ans[L] = cur, cur = prev[cur];
    return ans;
}
```

## FastKnapsack.h

**Description:** Given N non-negative integer weights w and a non-negative target t, computes the maximum S  $\leq t$  such that S is the sum of some subset of the weights.

**Time:**  $\mathcal{O}(N \max(w_i))$

b20ccc, 16 lines

```
int knapsack(vi w, int t) {
    int a = 0, b = 0, x;
    while (b < sz(w) && a + w[b] <= t) a += w[b++];
    if (b == sz(w)) return a;
    int m = *max_element(all(w));
    vi u, v(2*m, -1);
    v[a+m-t] = b;
    rep(i, b, sz(w)) {
        u = v;
        rep(x, 0, m) v[x+w[i]] = max(v[x+w[i]], u[x]);
        for (x = 2*m; --x > m; ) rep(j, max(0, u[x]), v[x])
            v[x-w[j]] = max(v[x-w[j]], j);
    }
    for (a = t; v[a+m-t] < 0; a--);
    return a;
}
```

## 10.3 Dynamic programming

## KnuthDP.h

**Description:** When doing DP on intervals:  $a[i][j] = \min_{i < k < j} (a[i][k] + a[k][j]) + f(i, j)$ , where the (minimal) optimal  $k$  increases with both  $i$  and  $j$ , one can solve intervals in increasing order of length, and search  $k = p[i][j]$  for  $a[i][j]$  only between  $p[i][j-1]$  and  $p[i+1][j]$ . This is known as Knuth DP. Sufficient criteria for this are if  $f(b, c) \leq f(a, d)$  and  $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$  for all  $a \leq b \leq c \leq d$ . Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

**Time:**  $\mathcal{O}(N^2)$

## DivideAndConquerDP.h

**Description:** Given  $a[i] = \min_{lo(i) \leq k \leq hi(i)} (f(i, k))$  where the (minimal) optimal  $k$  increases with  $i$ , computes  $a[i]$  for  $i = L..R-1$ .

**Time:**  $\mathcal{O}((N + (hi - lo)) \log N)$

d38d2b, 18 lines

```
struct DP { // Modify at will:
    int lo(int ind) { return 0; }
    int hi(int ind) { return ind; }
    ll f(int ind, int k) { return dp[ind][k]; }
    void store(int ind, int k, ll v) { res[ind] = pii(k, v); }
```

```
void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO, lo(mid)), min(HI, hi(mid)))
        best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
}
```

```
void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
```

## 10.4 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

## 10.4.1 XOR PROPERTIES

- $a|b = a^\wedge b + a\&b$
- $a^\wedge (a\&b) = (a|b)^\wedge b$  (b turned off but a on gives on)
- $b^\wedge (a\&b) = (a|b)^\wedge a \dots$
- $(a\&b) | (a|b) = a|b$
- $a+b = a|b + a\&b$
- $a+b = (a^\wedge b) + 2*(a\&b)$

## 10.4.2 Bit hacks

- $x \& -x$  is the least bit in x.
- for (int x = m; x; ) { --x &= m; ... } loops over all subset masks of m (except m itself).
- $c = x\&-x$ ,  $r = x+c$ ;  $((r^\wedge x) >> 2)/c$  | r is the next number after x with the same number of bits set.
- rep(b, 0, K) rep(i, 0, (1 << K))
 

```
        if (i & 1 << b) D[i] += D[i^\wedge (1 << b)];
```

 computes all sums of subsets.