

## Expt-4: Period of Oscillation for a Simple Pendulum

### OBJECTIVE:

1. To determine whether the period of oscillation is dependent on the mass, the angle of displacement or the length of the pendulum.
2. To measure the acceleration due to gravity.

### APPARATUS:

Digital stop watch, sample masses with hooks, meter stick, stand with clamp, digital weight balance, masses. The students need to bring a protractor for angle measurements.<sup>1</sup>

### THEORY:

A simple pendulum consists of a mass suspended by a light string of length  $L$ . By observation, one notices the very regular motion it takes, which makes one curious as to what affects this regular motion or period. This regular motion can be thought of, ideally, as being simple harmonic in nature. The period,  $T$ , can be defined as the point where the mass is released to the time where it returns to its original position.

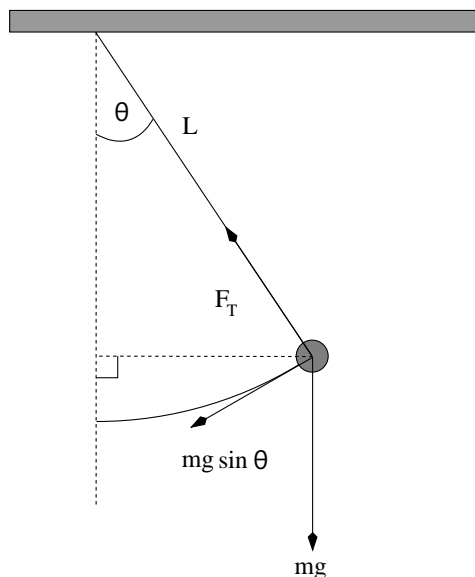


Figure 1: Free-Body Diagram of the Forces.

Outside of its equilibrium position (when it is at rest) the following diagram (Figure 1) with the forces labelled will apply. Consider the forces acting on the mass, we see that  $mg$  is the weight due to the force of gravity and that  $F_T$  is the tension in the string. The tangential force of the weight acting on the mass is the following:

$$F_{\text{tangential}} = mg \sin \theta. \quad (1)$$

The direction of this force is always toward the equilibrium and can be thought of as the restoring force. Now let us make an approximation that says that the period of oscillation for the pendulum is small and for small angles the following statement is true:  $\sin \theta \approx \theta$ .

Since the angles are small, determining the arc length that is swept can be defined as  $s = L\theta$  or rearranging it we get  $\theta = s/L$ . Applying both the approximation and the arc length to equation (1) we get the following:

$$F_{\text{tangential}} = mg\sin\theta \approx mg\theta = (mg/L)s \quad (2)$$

Hooke's Law ( $F = kx$ ) also works in a similar fashion, which is called Simple Harmonic Motion.

Comparing both equations, we see that  $k = mg/L$ , which makes  $s$  similar to the displacement,  $x$ , in Hooke's Law. Therefore, the period,  $T$ , of a simple pendulum can be described similarly to the period of a mass on a spring, which is already known. Taking that equation and the value of  $k$ , it can be transformed into this final result.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} \\ T &= 2\pi\sqrt{\frac{L}{g}} \end{aligned} \quad (3)$$

Now, the length  $L$  is the sum of the string length  $l$  and the radius of the metal block  $R$ . Substituting  $L = l + R$  in Eq. (3), squaring both sides and rearranging, we can write,

$$T^2 = \left(\frac{4\pi^2}{g}\right)l + \frac{4\pi^2 R}{g} \quad (4)$$

Look carefully at the transformed equation and take notice to what directly affects the period of a simple pendulum and remember what assumptions and approximations were made to get the final result.

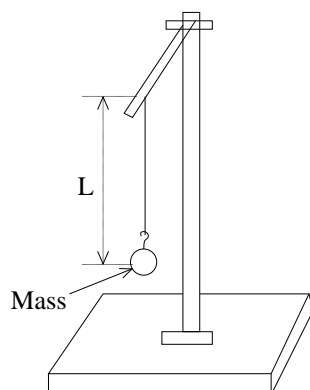


Figure 2: The Experimental Setup

## PROCEDURE:

### Part I: Mass Dependence

1. First, the apparatus should look similar to Figure 2.
2. Next, we want to vary the mass, but keep the length and angle of oscillation constant. Use a length greater than 50cm and record the value. Note that the length of the pendulum is the sum of the length of the string from the hanging point up to the hook attached with the mass block and the distance from the top of the hook up to the center of the mass of the block. Note: Use the digital balance to weigh your masses. The effective length of the pendulum is the sum of the length of the string and the distance of the center of mass of the mass. Use a protractor to keep the angle of oscillation less than or equal to 15 degrees.
3. After the first period is recorded press, repeat for two more times. Fill in Table 1 accordingly, where  $T_{\text{avg}}$  is the average period of the three trials.
4. Repeat steps 2 and 3 for two more masses, and complete Table-1.

5. Construct a graph of  $T_{\text{avg}}^2$  vs.  $m$ . This should look like a horizontal line.

### Part II: Angle Dependence

1. This time hold the mass constant and record this value. Keep the same length from Part I. Use a protractor to vary the angle of oscillation to take data. Fill in Table 2 accordingly.
2. Construct a graph of  $T_{\text{avg}}^2$  vs.  $\theta$ . This should look like a nonlinear line.

### Part III: Length Dependence

1. Keep the same mass from Part II as well as keep the angle less than or equal to 15 degrees, but vary the length ( $l$ ) of the pendulum by increasing or decreasing the string length. Fill in Table 3.
2. Construct a graph of  $T_{\text{avg}}^2$  vs.  $l$ . Notice that the equation of a simple pendulum can be used to determine the acceleration due to gravity,  $g$ , by manipulating it from Eq. (4), where we see that it is similar to  $y = mx + b$ . The slope,  $m$ , equals  $4\pi^2/g$ . Use this expression to calculate the experimental value of acceleration due to gravity,  $g_{\text{exp}}$ .

### CALCULATIONS:

1. Calculate  $T_{\text{avg}}$  by summing the total number of trials and dividing by three.
2. Calculate the slope using the data from your best-fit line.
3. Calculate  $g_{\text{exp}}$  using the expression at the end of Part III.
4. Calculate the percent error of your gravitational acceleration,  $g_{\text{exp}}$ , by comparing it to the accepted value,  $g = 9.80\text{m/s}^2$ .

## Lab Report

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Name of the Experiment :

Your Name :

Your ID # :

Name of the 1st Partner :

Date :

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Instructor's comments:

Table 1. Mass Dependence of the Period

Length of Pendulum,  $L =$  \_\_\_\_\_ m

Mass (grams)	A Single Period (sec)			$T_{avg}$ (sec)	$T_{avg}^2$ (sec <sup>2</sup> )

Table 2. Angle Dependence of the Period

Mass of Pendulum = \_\_\_\_\_ grams

Angle (degrees)	A Single Period (sec)			$T_{avg}$ (sec)	$T_{avg}^2$ (sec <sup>2</sup> )
10					
15					
20					
30					
40					

Table 3. Length Dependence of the Period

Length $l$ (m)	A Single Period (sec)			$T_{avg}$ (sec)	$T_{avg}^2$ (sec <sup>2</sup> )
0.40					
0.45					
0.50					
0.55					
0.60					

Slope of the best fit line = \_\_\_\_\_ s<sup>2</sup>/m.

$g_{exp}$  = \_\_\_\_\_ m/s<sup>2</sup>.

Percent error = \_\_\_\_\_

**QUESTIONS:**

1. Does the period of a simple pendulum depend on the mass?
2. Is the period constant over small angles? Does it vary when one reaches larger angles?
3. Does the period depend on the length of the pendulum?
4. Of the three parameters explored in this experiment, which has the strongest influence?
5. Is your best-fit line in form Table-3 goes through the origin? Explain why or explain not?

## Prelab-4

Your name:

ID#:

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1. What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?
2. An engineer builds two simple pendulums. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10 kg. Pendulum 2 has a bob with a mass of 100 kg. Describe how the motion of the pendulums will differ if the bobs are both displaced by  $12^\circ$ .
3. Suppose that a grandfather clock (a simple pendulum) is running slowly; that is, the time it takes to complete each cycle is longer than it should be. Should you (a) shorten or (b) lengthen the pendulum to make the clock keep attain the preferred time?