Derivative

1 Last Time

1.1 Polynomial Function

† $y = x^n \rightarrow \frac{dy}{dx} = nx^{n-1}$
† $y = x^2 \rightarrow \frac{dy}{dx} = 2x$

1.2 Chain Rule

† $y = (3x+4)^2 \rightarrow \frac{dy}{dx} = 2(3x+4) \cdot \frac{d(3x+4)}{dx} = 2(3x+4) \cdot 3$

1.3 Partial Derivative

† $y = (3x+4w)^2 \rightarrow \frac{\partial y}{\partial x} = 2(3x+4w) \cdot \frac{\partial(3x+4w)}{\partial x} = 2(3x+4) \cdot 3$
† $y = (3x+4w)^2 \rightarrow \frac{\partial y}{\partial w} = 2(3x+4w) \cdot \frac{\partial(3x+4w)}{\partial w} = 2(3x+4) \cdot 4$

1.4 Squared Error Function

† $E(w,b) = \frac{1}{2}\sum_{i=1}^{n}(w \cdot x_i + b - y_i)^2 \rightarrow$
$\frac{\partial E(w,b)}{\partial w} = \frac{1}{2}\sum_{i=1}^{n} 2(w \cdot x_i + b - y_i) \cdot \frac{\partial(w \cdot x_i + b - y_i)}{\partial w} = \sum_{i=1}^{n}(w \cdot x_i + b - y_i) \cdot x_i$
$\frac{\partial E(w,b)}{\partial b} = \frac{1}{2}\sum_{i=1}^{n} 2(w \cdot x_i + b - y_i) \cdot \frac{\partial(w \cdot x_i + b - y_i)}{\partial b} = \sum_{i=1}^{n}(w \cdot x_i + b - y_i) \cdot 1$

2 New Function

2.1 Exponential Function

† $y = e^x \rightarrow \frac{dy}{dx} = e^x$

2.2 Chain Rule

† $y = e^{wx} \rightarrow \frac{dy}{dx} = e^{wx} \cdot \frac{d(wx)}{dx} = e^{wx} \cdot w$

2.3 Partial Derivative

† $y = e^{wx} \rightarrow \frac{\partial y}{\partial w} = e^{wx} \cdot \frac{\partial (wx)}{\partial w} = e^{wx} \cdot x$

3 Logistic Function

3.1 Basic

† $y = \sigma(x) = \frac{1}{1+e^{-x}} = (1+e^{-x})^{-1} \rightarrow$

$$\frac{dy}{dx} = (-1)\cdot(1+e^{-x})^{-2}\cdot\frac{d(1+e^{-x})}{dx} = (-1)\cdot(1+e^{-x})^{-2}\cdot e^{-x}\cdot\frac{d(-x)}{dx} = (-1)\cdot(1+e^{-x})^{-2}\cdot e^{-x}\cdot(-1)$$

$$= (1+e^{-x})^{-2}\cdot e^{-x} = \frac{1}{1+e^{-x}}\cdot\frac{e^{-x}}{1+e^{-x}} = \frac{1}{1+e^{-x}}\cdot\left(1-\frac{1}{1+e^{-x}}\right) = \sigma(x)(1-\sigma(x))$$

3.2 Partial Derivative

† $y = \sigma(w) = \frac{1}{1+e^{-wx}} = (1+e^{-wx})^{-1} \rightarrow$

$$\frac{\partial y}{\partial w} = (-1)\cdot(1+e^{-wx})^{-2}\cdot\frac{\partial(1+e^{-wx})}{\partial w} = (-1)\cdot(1+e^{-wx})^{-2}\cdot e^{-wx}\cdot\frac{\partial(-wx)}{\partial w} = (-1)\cdot(1+e^{-wx})^{-2}\cdot e^{-wx}\cdot(-x)$$

$$= (1+e^{-wx})^{-2}\cdot e^{-wx}\cdot x = \frac{1}{1+e^{-wx}}\cdot\frac{e^{-wx}}{1+e^{-wx}}\cdot x = \frac{1}{1+e^{-wx}}\cdot\left(1-\frac{1}{1+e^{-wx}}\right)\cdot x = \sigma(w)(1-\sigma(w))\cdot x$$

3.3 Error Function

† $E(w) = \frac{1}{2}\sum_{i=1}^{n}\left(\frac{1}{1+e^{-wx_i}} - y_i\right)^2 \rightarrow$

$$\frac{\partial E(w)}{\partial w} = \sum_{i=1}^{n}\left(\frac{1}{1+e^{-wx_i}} - y_i\right)\cdot\left(\frac{1}{1+e^{-wx_i}}\right)\cdot\left(1-\frac{1}{1+e^{-wx_i}}\right)\cdot x_i$$