APPLICATION OF TWO DIFFERENT CONSTITUTIVE MODELS FOR NUMERICAL SIMULATION OF LIQUEFACTION

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ABSTRACT

Liquefaction phenomenon in saturated loose sandy layers occurs as an undrained response of soil when it is subjected to static or dynamic loads. A fully coupled dynamic numerical program is developed to predict the liquefaction potential of saturated sandy layers. Coupled dynamic field equations of extended Biot's theory with u-p formulation are used to determine pore fluid and soil skeleton responses. Generalized Newmark method is employed for integration in time. The soil behavior is characterized with densification model as well as a critical state two-surface plasticity model for sands. A class B analysis of experiment No.1 of the VELACS project is performed to simulate the dynamic response of level sand sites. In this analysis a horizontal liquefiable layer was excited horizontally. Computed results are compared to those recorded in centrifuge tests. The obtained numerical results demonstrate the capability of the model to simulate the liquefaction phenomenon within the critical state soil mechanics framework.

Keywords: Fully coupled, Plasticity, Critical state two-surface plasticity model, Densification, Liquefaction.

INTRODUCTION

Saturated soil and other two-phase media have been the subject of many investigations, both experimentally and numerically, over a number of years. The interaction of soil and pore fluid can be strong enough, due to the build up of pore pressure, to lead a catastrophic material softening, a phenomenon known as 'liquefaction'. Liquefaction occurs frequently in saturated granular materials under earthquake and other dynamic loadings such as blasting.

Granular particles such as sands are liable to compact during shaking. However, this reduction in volume is often prevented by the lack of drainage potential during the period of shaking. Hence, a nearly undrained condition results and pore pressure is forced to rise to counter such contractive behavior. This causes reduction of the effective stress and eventually, for loose sand, may lead to a phenomenon called 'initial liquefaction', which is defined as zero effective stress state in soil. On the other hand, for a denser material, the state of zero effective stress state may never occur, and cycles of alternative contraction and dilation may take place. This is termed 'cyclic mobility' [1].

Numerical modeling of soil liquefaction using advanced constitutive models and numerical techniques has been the subject of many investigations in recent years. Quantitative analysis of liquefaction can only be accomplished by considering the coupled interaction of the soil skeleton and of the pore fluid. For this purpose, a suitable formulation of the two-phase continuum behavior and a proper constitutive model is required. In this paper, Biot's modified theory is employed for modeling saturated soil behavior and two plasticity models namely densification model and a critical state two-surface plasticity model are used as the constitutive model for sand.

GENERAL CLASS OF FORMULATION

There are several different approaches to determine the behavior of two-phase medium. Generally, they can be classified as uncoupled and coupled analysis. In the uncoupled analysis, the responses of saturated soil are computed without considering the effect of soil-water interaction, then pore water pressure are calculated separately by means of a pore pressure generation model. In the coupled analysis, on the other hand, all unknowns are computed simultaneously at each time step. These are far more realistic representations of the physical phenomena than those provided by uncoupled.

In order to study the dynamic response of saturated soil system as an initial boundary value problem, the geotechnical finite element program PISA© which is a fully-coupled nonlinear effective stress finite element code for dynamic analysis of saturated porous media is modified and the mentioned constitutive model is implemented on it. In this code, saturated soil is modeled as a two-phase material based on the Biot's theory for porous media [2]. A simplified numerical framework of this theory, known as u-p formulation (in which displacement of the soil skeleton u, and pore pressure p, are the primary unknowns), was used in the above code.

The u-p formulation is defined by the equation of motion for the solid-fluid mixture, and the equation of mass conservation for the fluid phase, incorporating equation of motion for the fluid phase and Darcy's law. Using the finite element method of spatial discretization, the u-p formulation is as follows:

$$M\ddot{\mathbf{u}} + \int_V B^T \boldsymbol{\sigma}'' dV - Q\overline{P} - \mathbf{f}^{(1)} = 0$$

$$Q^T\dot{\mathbf{u}} + H\overline{P} + S\dot{\overline{P}} - \mathbf{f}^{(2)} = 0$$

where M is the mass matrix, $\bar{\mathbf{u}}$ the displacement vector, B the strain-displacement matrix, $\boldsymbol{\sigma}''$ the effective stress vector (determined by soil constitutive model discussed earlier), Q the discrete gradient operator coupling the solid and fluid phases, $\bar{P}$ the pore pressure vector, S the permeability matrix, H the compressibility matrix. The vectors $\mathbf{f}^{(1)}$ and $\mathbf{f}^{(2)}$ include the effect of body forces and prescribed boundary conditions for the solid-fluid mixture and the fluid phase respectively. In the equation of motion, the first term represents inertia force of the mixture, followed by internal force due to soil skeleton deformation, and internal force by pore-fluid pressure. In the equation of mass conservation, the first two terms represent the rate of volume change for the soil skeleton and the fluid phase, respectively, followed by seepage rate of the pore fluid. To complete the numerical solution, it is necessary to integrate the above equations in time. Here, the generalized Newmark method is used [3].

OUTLINE OF THE CONSTITUTIVE MODELS

Development of plasticity models that accurately simulate the behavior of engineering materials has been of great interest to engineers in recent years. In the case of geo-materials such as granular soils, the mathematical formulation of appropriate plasticity models is rather complex. It requires pressure sensitivity of the elastic bulk and shear moduli, third stress invariant dependence and in some cases two or multi-surface plasticity formulations. Moreover, to be useful in engineering calculations these complex models require efficient and robust numerical implementation.

In this research two different classes of constitutive models for granular soils have been employed in the numerical analyses and their results are compared. The first one is a simple classical elasto-plastic model called "densification model" originally developed by Zienkiewicz et al. [4], the second constitutive model is an advanced critical-state plasticity model developed by Manzari and Dafalias [5].

The densification model contains a simple constitutive law, where elastoplastic behavior of the soil and the accumulation of pore water pressure are taken into account by two different mechanisms. The constitutive equation is written as:

$$d\sigma' = D_{ep}(d\varepsilon - d\varepsilon.)$$

where D_{ep} represents the elastoplastic constitutive matrix. In general a non-associative Mohr-Coulomb model with zero dilatancy is assumed for the elastoplastic behavior. $d\varepsilon_0$

represents the densification that occurs in the soil by cyclic loading and is called autogeneous strain. The autogeneous strain is determined based on the stress and strain values in each step.

The critical-state two-surface plasticity model used in this study has been developed by Manzari and Dafalias [5]. This model is proposed based on the following characteristics of the stress-strain behavior of sand [6]:

1. The strength and the volume change behavior of granular soils are governed by the combined effect of density (void ratio) and confining stress. This combined effect is represented by state parameter $\psi = e - e_c$ where e_c is the critical void ratio corresponding to the existing confining stress on the soil element.

2. When sheared in monotonic loading, granular soil that is denser than critical void ratio will exhibit a peak strength and upon further shearing a softening regime will appear in the F-E relationship. Granular soils with a void ratio less than their critical void ratio show a prevalent contractive response upon shearing toward critical state.

The formulation of the model is based on the general two-surface plasticity and the bounding surface plasticity theory [7]. The state parameter ψ is used as the key ingredient to accurately model the effect of critical state for sands. The numerical efficiency of the model is good because only the yield surface must be updated owing to kinematic and isotropic hardening. The other surfaces are fully determined by the value of state parameter ψ. A schematic representation of the two- surface model in the π-plane is shown in Fig.1.

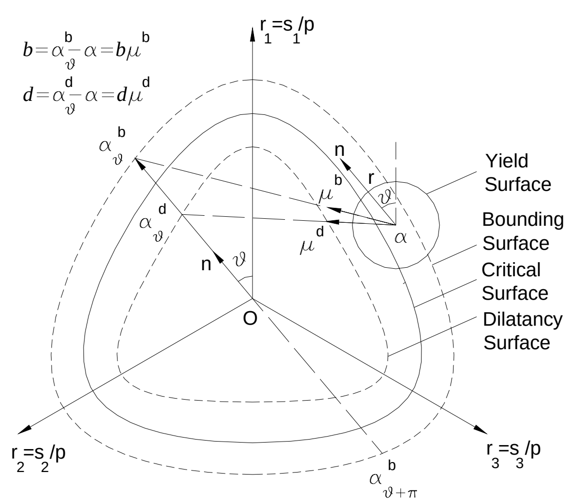


Figure.1 Schematic representation of the two-surface model in π-plane

MODIFIED PROGRAM PISA®

In order to study the dynamic response of the saturated soil system as an initial-boundary-value problem, the geotechnical finite element program PISA® has been used. The first version of this program PISA® was developed at the University of Alberta with the early name of SAGE [8]. This program was originally developed using FORTRAN. Later a commercial version of this program was released under the name of PISA® [9]. This geotechnical software analyzes stress and deformation fields in plane strain, plane stress and axisymmetric problems. Drained and undrained analysis can be performed using a wide range of elastic, elastoplastic and critical state models.

In this research finite element program PISA® is modified for a fully-coupled dynamic nonlinear effective stress analysis of saturated porous media and appropriate constitutive models for liquefaction are implemented in it [10-12]. In this code, saturated soil is modeled as a two-phase material based on the Biot's theory [2] for consolidation of a saturated porous medium. A simplified numerical framework of this theory, known as u-p formulation is used in the above mentioned code. In this formulation displacement of the soil skeleton u, and pore pressure p, are the primary unknowns.

Two constitutive models described above i.e. densification model and two-surface critical state plasticity model have been implemented in PISA® for numerical simulation of liquefaction phenomenon. Performance evaluation of these models can be found in references [11, 12].

APPLICATION OF THE CODE FOR PREDICTION OF LIQUEFACTION

For application of the developed code towards liquefaction analysis, the class B prediction of the experiment No. 1 of VELACS (Verification of Liquefaction Analysis by Centrifuge Studies) project is considered. In the VELACS project, extensive research work has been conducted on experimental centrifuge simulation and numerical modeling of soil liquefaction under earthquake loading.

Description of the experiment

The case selected for this analysis is the centrifuge experiment No. 1 conducted during the course of VELACS project by Taboada and Dobry at RPI [13]. The experiment No. 1 consists of a 20cm high, horizontal, uniform Nevada sand layer, which is placed in a laminar box at a relative density of about 40%. The laminar box is constructed of rectangular aluminum rings stacked on top of each other with roller bearings in between. The purpose of using laminar box is to simulate the response of a semi-infinite sand layer during shaking. A sketch of the laminar box and the instrumentation used for this experiment is presented in figure 2. The sand layer is fully saturated with water, spun at a centrifuge acceleration of 50g, and excited horizontally at the base with the target prototype accelerogram reproduced in figure 3. A zero vertical acceleration was considered.

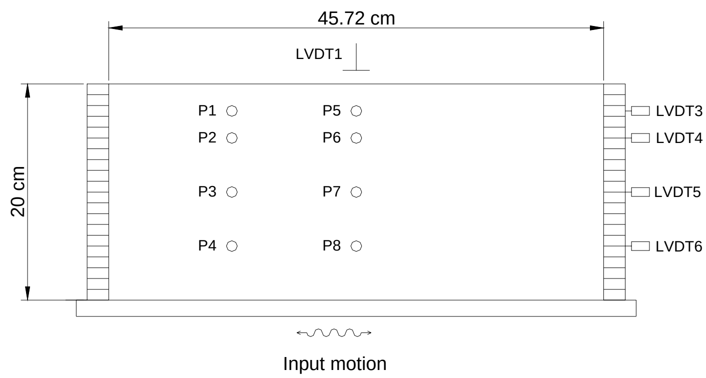


Figure 2. Cross-sectional view of the centrifuge experiment No.1

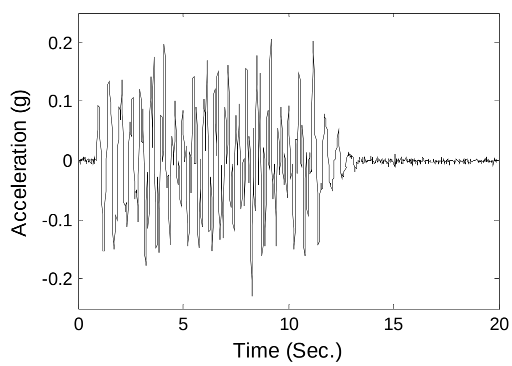


Figure 3. Horizontal input motion at the bottom

Numerical modeling

Numerical modeling is done in prototype scale using a two-dimensional formulation with plane-strain condition. 64 rectangular elements with 4-node for pore pressures and 8-node for displacements are used. The mesh is regular and uniform as shown in figure 8. The laminar box is modeled with the constraint of lateral tied nodes. The displacements of nodes located at the two ends of the soil at the same level are restrained to have the same value. The base nodes are fixed in both horizontal and vertical directions. Dissipation of pore pressure is allowed only through the top surface of the layer; the lateral boundaries and the base are kept impermeable. The effect of the lateral inertia of the rings is neglected. The model parameters used for simulations are listed below.

Table 1. Material parameter for densification model [11]

Shear constant, G0	73000
Shear exponent, n	0.5
Poisson's ratio, ν	0.31
Cohesion, c (kPa)	0.0
Friction angle, Φ (deg)	34
Dilation angle, Ψ (deg)	0.0
Densification parameter, Γ	0.5
Densification parameter, A	0.05
Densification parameter, B	12.0

Table 2. Material parameter for the critical state two-surface plasticity model [12]

Elastic	G_0 (kPa)	**31400**	**Hardening**	h_0	**800**
	K_0 (kPa)	31400		m	0.05
	a	0.6		c_m	0.
Critical state	M_c	1.14	State parameter	k_c^b	3.975
	M_e	1.14		k_e^b	2.0
	λ	0.025		k_c^d	4.2
	$(e_c)_{ref}$	0.8		k_e^d	0.07
Dilatancy	A_0	0.6			
	C_f	100			
	$F_{\max}$	100			

Computations are performed in two steps: first a static analysis due to application of gravity (sand's own weight) is performed before seismic excitation. The resulting fluid hydrostatic pressures and stress-states along the soil column are used as initial conditions for the subsequent dynamic analysis.

Before discussing the results, it should be noted that a separate analysis with a single column of saturated sand comprised of 7 rectangular elements only has also been conducted to examine that the flow direction is vertical only. The analysis with 8 elements (single column of sand) has yielded exactly the same results comparing to the results obtained from the analysis of mesh shown at figure 4. This proves the one-dimensional pore pressure generation and consolidation procedure that take place in the laminar box filled with saturated sand.

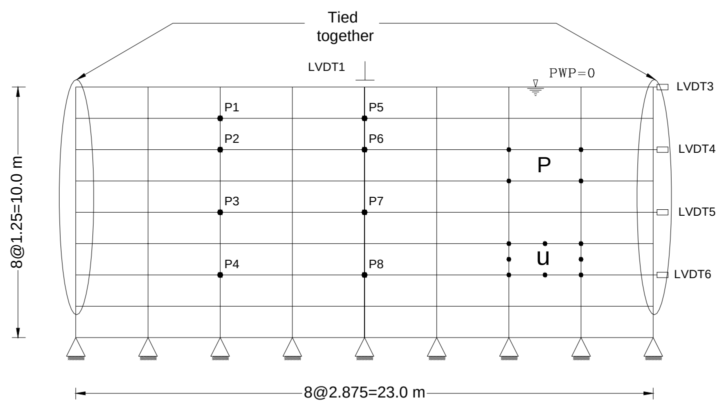


Figure 4. Finite element mesh and boundary conditions

Numerical modeling results and discussion

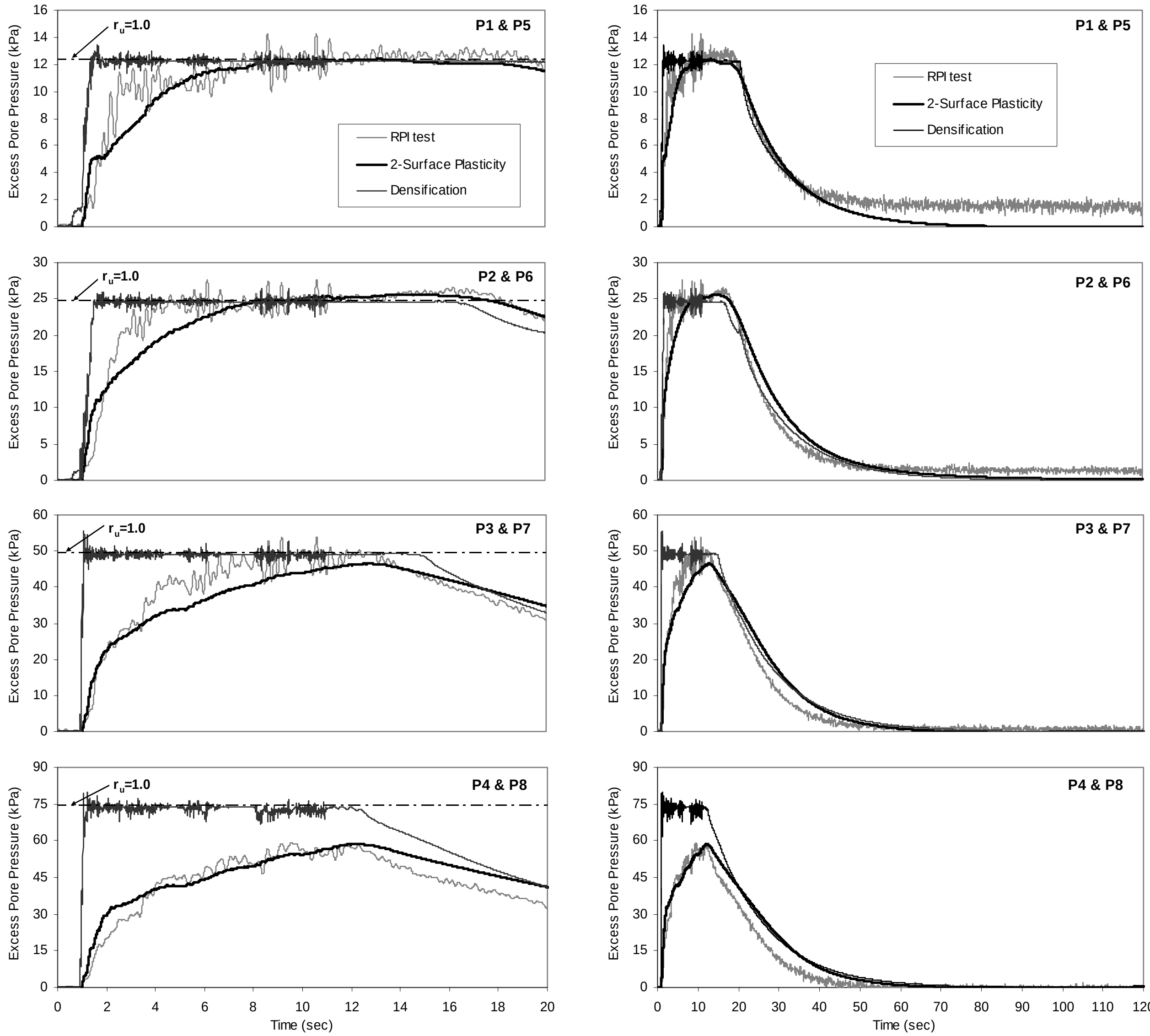
As mentioned above the liquefaction phenomenon has been numerically simulated in a fully coupled manner using two different constitutive models exclusively implemented in the code for modeling the liquefaction. Discussion on the results is divided into the following sections.

Excess pore pressure

Figure 5 displays the computed and recorded pore pressures at nodes P1 to P8. For a better illustration of pore pressure variation, the first 20 seconds of the time history is shown separately on Fig. 5(a). The complete time history of pore pressure variation is depicted on Fig 5(b). Along with three pore pressure variations, dotted lines representing $r_u = u_e / \sigma_v$ at each elevation are plotted in the figures (where u_e is excess pore-pressure, and σ_v initial effective vertical stress). The r_u lines indicate whether or not the generated pore pressure due to bottom excitation reaches to the condition of zero effective stress or primary liquefaction state. The recorded pore pressure time histories indicate that soil at the P1 and P2 levels is liquefied. At P3 level it is liquefied just for a short period of time and at the P4 level, liquefaction has not taken place. Numerical modeling using densification model shows a state of liquefaction at all levels in the soil. Although the sharp increase of pore pressure in this model at the onset of shaking does not match with the experimental results but the pore pressure dissipation (consolidation phase) matches well with the experimental results. The results with densification model have been obtained by calibrating the value of the coefficient of permeability k=3.3×10^{-4} m/sec for a best fit to the experimental results. On the other hand numerical results using the 2-surface plasticity model reveals very good agreement with the experimental values. Pattern of the initial increasing, leveling off, and then decreasing of the pore pressure watches well with lab results. The value of k for this model has found to be k=0.00135 m/sec for obtaining the results.

Selection of the coefficients of permeability for these models has been performed in order to achieve the best match with the experimental values at the consolidation phase, at the end of shaking. This procedure shows k values 5 and 20 times greater than the value measured from the laboratory constant head test (which was 6.6×10^{-5} m/sec) for densification model and 2-surface plasticity model, respectively. It has been observed by previous investigators that the soil permeability obtained from a standard laboratory test is different from the in-flight permeability of the soil in a liquefying state. Although numerical simulations confirm that the coefficient of permeability during liquefaction becomes greater than their static values, yet the large difference between the calibrated k values for two different constitutive models indicate that the calibrated values may not necessarily represent the actual soil permeability. Comparing the numerical results with the recorded pore pressure responses over time in Figure 5(b) show that the predicted drainage process at P1 and P2 is faster and at P4 is slower than the experimental one. At P3 the drainage process of numerical and experimental results show a good agreement. These observations indicate that the coefficient of permeability is not a stationary parameter during shaking as well as drainage processes, therefore using a constant value for permeability in the numerical analysis possesses an inherent pitfall by which the drainage can not be simulated in a desirable manner. It seems that the change of the coefficient of permeability at the end of shaking takes places in such a way that it decreases at lower depths and increases at higher depths. More research is required on this matter.

The drainage sequence starting from bottom to top is evident from Figure 5. This is confirmed by the experimental results. The magnitude of the excess pore pressure would be higher at the bottom, and lower at the top, thus causing a pore pressure gradient resulting upward flow. Due to this gradient, the excess pore pressure at the bottom of the sand layer would dissipate first as the sand particles at the bottom come into contact with each other. Hence, the excess pore pressure at locations P8 and P7 would dissipate before the excess pore pressure at locations P5 and P6.



(a) (b)

Figure 5. Calculated and measured pore pressure time histories

For a better illustration of this phenomenon contours of r_u in different time steps are shown in Figure 6. The values of r_u at different depths up to the time of t=1.0 sec are more than those on the sand surface. This causes an upward dissipation process in the soil. But at time t=1.2 sec, r_u at the surface becomes greater. This indicate that the excess pore water pressure depends upon the contraction characteristics of sand and inflow/out flow of the pore water. Change in the pattern of variation of r_u along the sand column reveals the extremely complicated behavior of the pore fluid during the liquefaction process. The stress paths depicted on Figure 7 show the typical mechanism of cyclic decrease in effective stress due to pore pressure build-up. For saturated medium-dense cohesionless soils, the presented dynamic excitation response demonstrates minor cyclic mobility effect under level ground conditions.

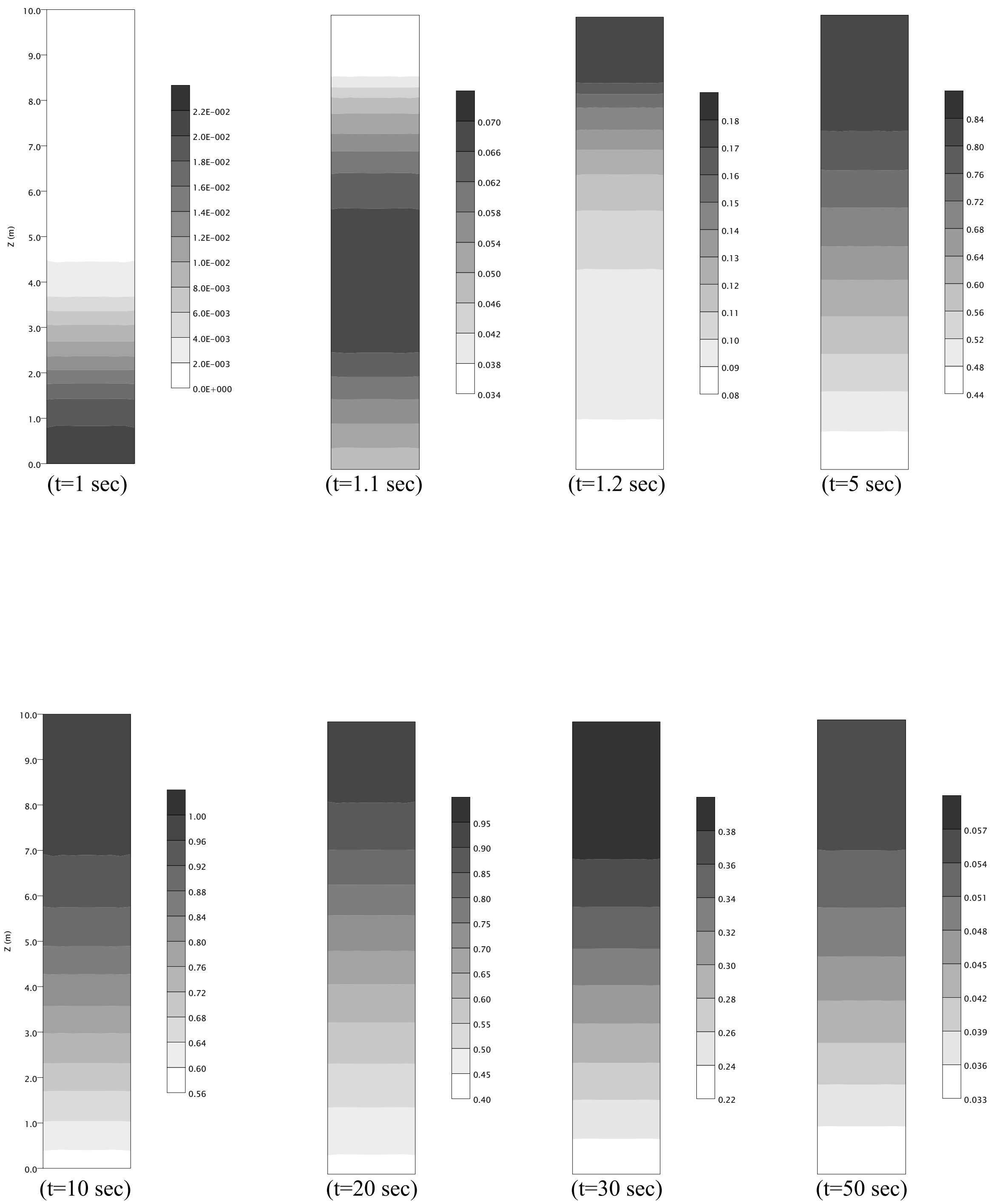


Figure 6. Contours of r_u in different time steps

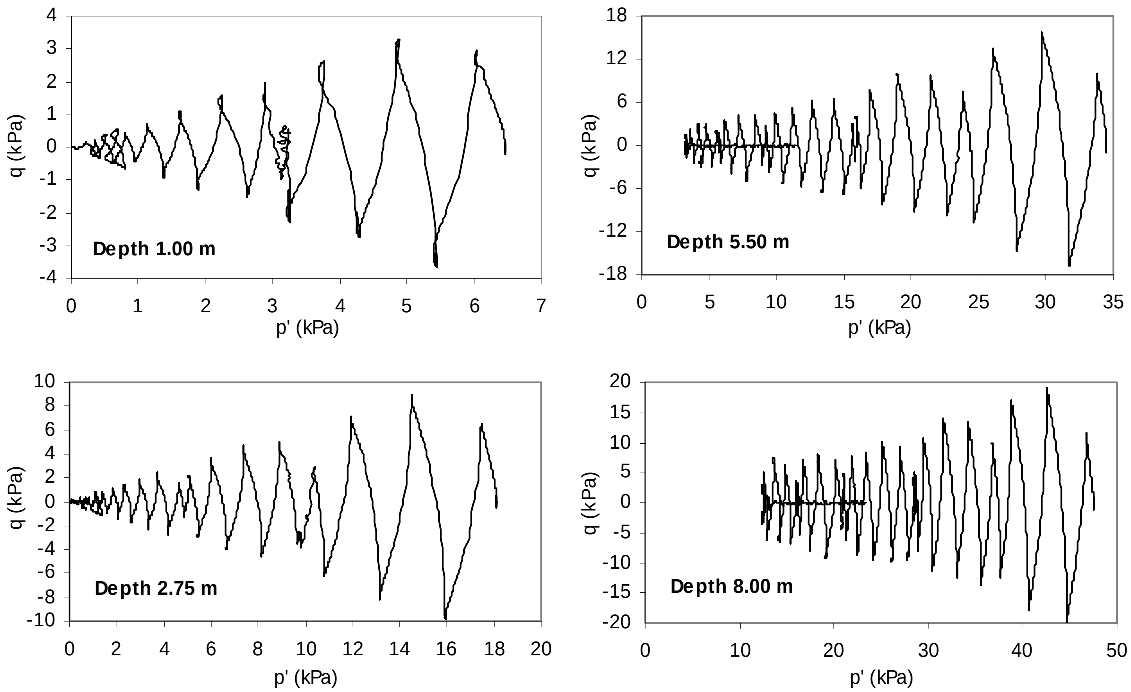


Figure 7. Computed effective stress paths at different depths

Horizontal displacements

Horizontal displacements variation over time is shown in Figure 8. On these figures only the calculated displacements using 2-surface plasticity model is compared with the laboratory results because the densification model predictions are poor. Generally speaking, the amplitude of the predicted horizontal displacement oscillations are smaller than the measured values in the lab. Their peak values are smaller than the recordings as well. The final value of the horizontal displacement at LVDT3 & 4 are simulated very well. This shows that the 2-surface plasticity model is capable of modeling the behavior of saturated sand under low confining stresses.

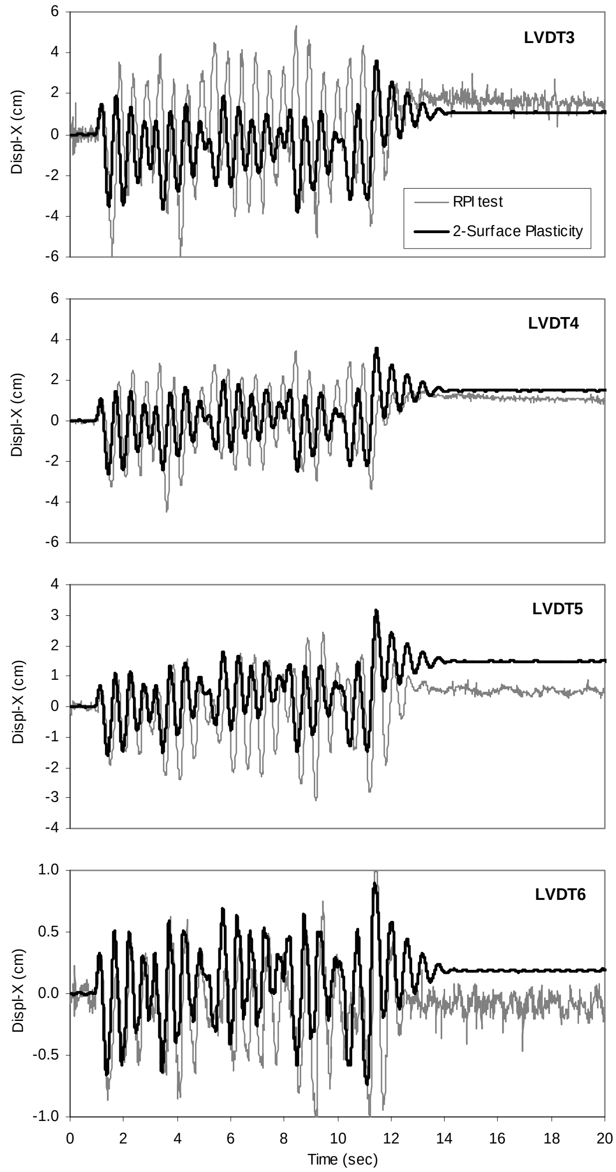


Figure 8. Calculated and measured time histories of horizontal displacements

Vertical displacements

Calculated and recorded vertical displacement time histories at the free surface of sand are presented in Figure 9. Predicted settlement is about 17 cm at LVDT1 using 2-surface plasticity model. The predicated settlement using densification model is vitally zero. Same as the experimental results, most of the calculated settlements occurs during the shaking period. Maximum predicted settlement has found to be 17 cm which is in good agreement with the experimental observations.

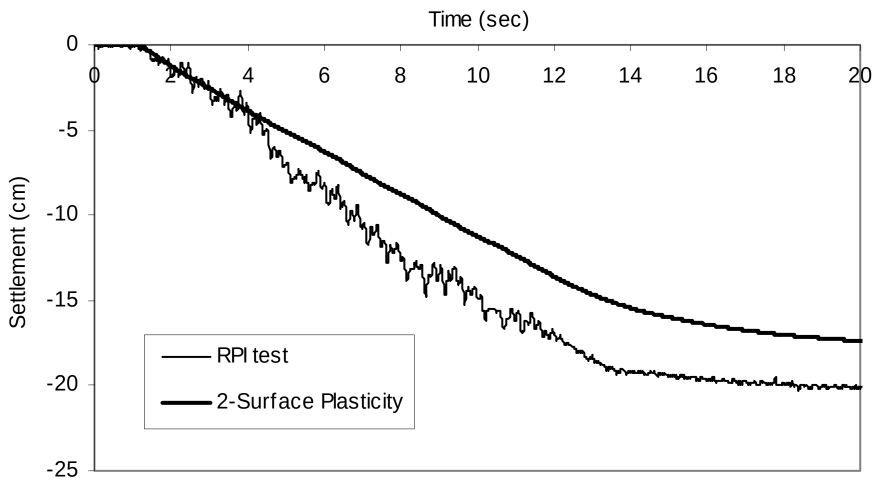


Figure 9. Calculated and measured settlement at the free surface of sand column

CONCLUSIONS

Liquefaction phenomenon of loose saturated sand layer in a laminar box in centrifuge experiment is simulated using a fully coupled dynamic program with a u-p formulation and a critical state two-surface plasticity model for sands. The results indicate that:

1. Densification model based on a classical elastoplastic framework can predict the liquefaction phenomenon at low elevations below ground (low confining stresses) but is unable to accurately predict this phenomenon at high depths.
2. The capability of densification model in predicting the horizontal and vertical displacement during liquefaction process is poor.
3. The critical state 2-surface plasticity model captures most of the important features of the complex interaction of pore fluid and sand particles subjected to cyclic loads.
4. Pore pressure variation and vertical displacements during liquefaction can be simulated with very good accuracy, which shows the superiority of using advanced plasticity models for simulation of liquefaction.
5. Behavior of saturated sand in a laminar box subjected to lateral excitation is basically one-dimensional and the pore-fluid drainage path is in the vertical direction only.
6. Liquefaction usually causes a significant increase of the coefficient of permeability, but rapid change in the pattern of r_u in the soil column during shaking demonstrates that permeability is not a stationary parameter in the liquefaction process and it may either

increase or decrease at different depths. Hence assuming a constant value of k can not be regarded a suitable approach. However, finding a realistic assumption for variation of permeability during liquefaction requires much more investigations.

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