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Assessment of 94quad_UP finite element in OpenSees platform using pressure dependent multi yield model to simulate cyclic direct simple shear test

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I. Introduction to Elastic Plastic

An engineer processes the ability to form a scientific understanding and approach to solve real problems. According to Muir Wood, a scientific understanding proceeds by way of constructing and analyzing models of the segments of reality under study. However, it is rarely possible to perform an analysis in which full knowledge of the object being analyzed is known. Often time, it is up to the engineer to make up his or her own judgment in order to bridge the observed physical behaviors and the predicted behaviors. This is particularly true for geotechnical engineering. The practices of geotechnical engineering require us to work with very limited data about a complex environment where conditions can change radically over a short distance and with time. For example, most of the soil strength parameters are derived from just a few measurements such as: blow count, tip resistance, gradation, and shear wave velocity. This is the major difference between geotechnical engineering and structural engineering or mechanical engineering, in which it is feasible to specify and control the properties of structural or mechanical materials. The basic properties of the geologic materials – strength, stiffness, and permeability – are not constant, nor unique. These properties depend on the structure of the materials; the past, present, and future values of stresses including pore pressure acting on the materials; and time. (Lambe 1967; Lambe and Marr 1979; Ladd and DeGroot 2001) No device has yet been developed which can measures the properties of these materials for the exact conditions that exist during construction and operation of a constructed facility. No soil model exists to capture all of these effects within our analytical method. We apply judgment to transform the known information about the geometries and materials' behavior into parameters our analytical methods will accept. The objective of using conceptual models is to focus our attention on the important features of a problem and to leave aside features which are irrelevant. The choice of the model depends on the application. Simple models are easier to describe and comprehend. Extra complexities can always be incorporated subsequently if it is determined that they cannot be dispensed with.

Geotechnical engineering materials are classified as elastic plastic materials. These materials have two separate responses to the imposed loading which are elastic deformation and plastic deformation after yielding. Elastic response is much easier to describe and comprehend than plastic response. A familiar introduction to the elastic properties of materials is obtained by the simple experiment procedure of hanging weights on a wire and measuring the displacements with each loading increment

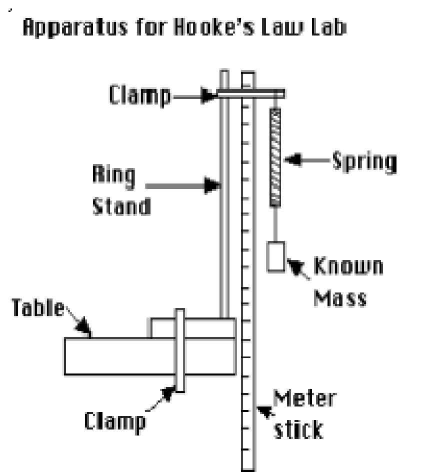


Figure 1: Set up for Hooke's Law Experiment

For some materials, there is a range of loading for which the displacement varies linearly with the applied load and is recovered when the load is removed. There is no permanent deformation in the material. The stress and strain are related by the following equation

$$\sigma = E\varepsilon \tag{1}$$

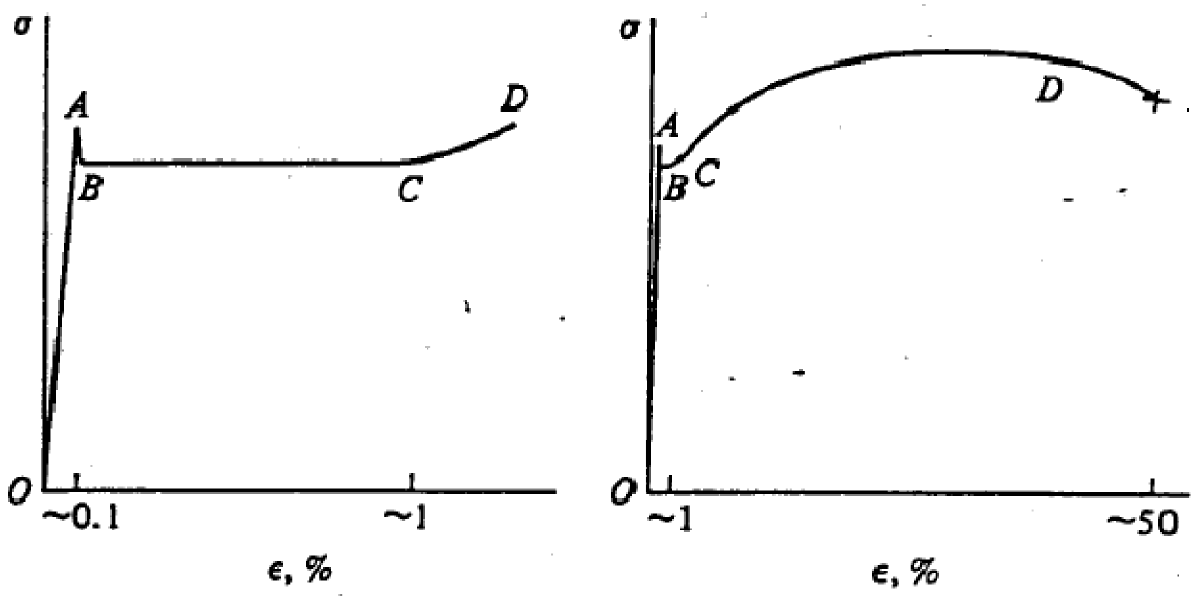


Figure 2: Observed behavior of mild steel in pure tension

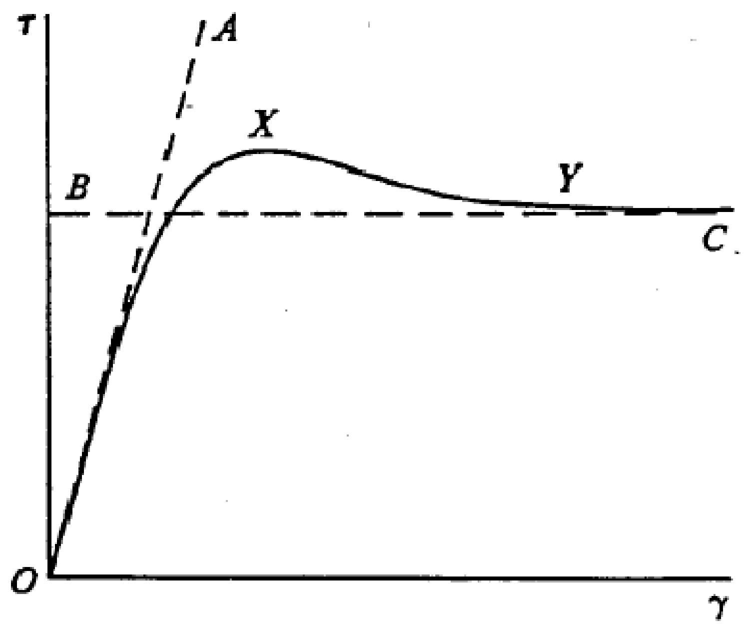


Figure 3: Observed shearing behavior of soil

σ is the applied stress, ε is the strain, and E is the Young's modulus. Working in term of Young's modulus and Poisson's ration (the ratio of horizontal strain to vertical strain), one can describe the response of a soil specimen to a general triaxial change of effective stress by the following equations:

$$\begin{bmatrix} \delta\varepsilon_a \\ \delta\varepsilon_r \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -2\nu \\ -\nu & 1-\nu \end{bmatrix}\begin{bmatrix} \delta\sigma_a \\ \delta\sigma_r \end{bmatrix} \tag{2}$$

or in p-q space:

$$\begin{bmatrix} \delta\varepsilon_p \\ \delta\varepsilon_q \end{bmatrix} = \begin{bmatrix} \frac{1}{K} & 0 \\ 0 & \frac{1}{3G} \end{bmatrix}\begin{bmatrix} \delta p \\ \delta q \end{bmatrix} \tag{3}$$

Whereas K is the stiffness and G is shear modulus.

In elasticity there is a one to one relationship between stress and strain. Such relationship can either be linear or nonlinear but an essential feature is that no net energy is dissipated under the application and removal of a stress to the soil specimen under pristine condition. However the overall stress-strain response of many materials can not be considered in to such a unique relationship. The irrecoverable, permanent extensions that remain under zero load are plastic deformations.

Having established that yield function exist for soils, it is follows that, for a stress path inside a current yield surface, the response is elastic. As soon as, a stress change engages a current yield surface, a combination of elastic and plastic responses would occur. It is the necessary to decide on the nature of the plastic deformations, the magnitudes and relative magnitudes of various components of plastic deformation and the link between these magnitudes and the changing size of the yield surface.

The basic structure of a typical elastic plastic constitutive model is followed:

1) Constitutive Equation

2) Yield Function which specifies the combination of stresses for which plastic deformation may occur $F(\sigma_{ij})=0$

3) Flow Rule which determines the direction of plastic strains and relative sizes of the strain increments $(\delta\varepsilon_p, \delta\varepsilon_q)$

4) Hardening law which specifies how yield criteria changes with plastic strains

$$\sigma_o = \sigma_o(\varepsilon_{ij}^p)$$

2. Elastic Plastic Model (Pressure Dependent Multi Yield Surface)

A number of constitutive models have been developed to simulate cyclic mobility and/or flow liquefaction of soil response. To name a few, there are UBCSAND (Byrne et al. 2004), SANISAND (Dafalias & Taiebat et al. 2008), and Pressure dependent multi yield surfaces (Elgama, Yang, and Parra et al. 2002). Conventional stress-space constitutive models, in which inelastic material behavior is controlled by the stress path, may not be easily calibrated to reproduce the observed magnitudes of perfectly plastic shear deformation. In this regard, direct control of permanent strain accumulation is more practical approach. This is accomplished by using strain-space parameters within an existing multi yield surface stress space model (Parra et al. 1996, Yang et al. 2000).

Following the usual cyclic soil plasticity concepts, a strain increment $\dot{\varepsilon}$ is assumed to be the sum of elastic ($\dot{\varepsilon}^{e}$) and plastic ($\dot{\varepsilon}^{p}$) strain increments. The material elasticity is linear and isotropic, and nonlinearity and anisotropic result from plasticity.

a) The constitutive equations:

$$\dot{\sigma} = E : (\dot{\varepsilon} - \frac{Q : E : \dot{\varepsilon}}{H' + Q : E : P} P) \tag{4}$$

$\dot{\sigma}$: The effective Cauchy stress increment.

$\dot{\varepsilon}$: Strain increment.

P : Symmetric second-order tensor that defines direction plastic deformation in stress-space.

E : Isotropic elastic coefficient tensor.

Q : Symmetric second-order tensor that defines direction of normal to the yield surface

H : The plastic modulus

b) Yield function:

The yield surfaces (Figure 4) can be defined by the following equation (Prevost et al. 1985 and Lacy et al. 1986)

$$f = \frac{3}{2}(s-(p-p_o)\alpha):(s-(p-p_o)\alpha) - M^2(p-p_o)^2 = 0 \tag{5}$$

$s = \sigma - p\delta$: Deviatoric stress tensor (σ effective Cauchy stress tensor and δ is the identity tensor

p : Effective hydrostatic stress

p_o : Modeling parameter used to define a residual strength for cohesionless soils at p=0 *(see Figure 4a)*

M : Modeling parameter related to friction angle

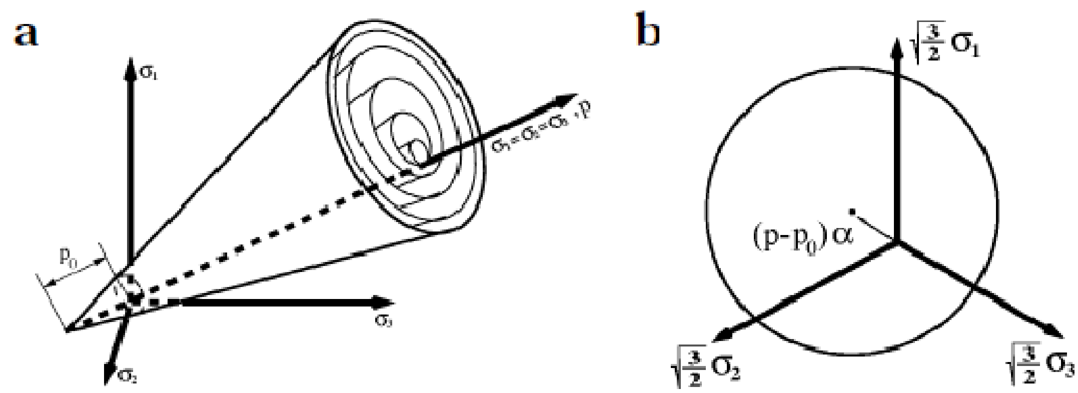


Figure 4: Multi-yield surfaces in the new constitutive model (Prevost 1985, Lacy 1986, Parra 1996)

As shown in Figure 4a this yield function forms a conical surface in stress-space with its apex at p_o along the hydrostatic axis. A number of similar yield surface with common apex p_o and different sizes form a hardening zone. The outer most surface is designated as the failure surface. This pressure dependent multi-yield surface model is used effectively to predict the yielding of cohesionless soils under cyclic loading.

c) Flow rule:

Experimentally observed soil contractive/dilative behaviors are handled in this model by employing a non-associative flow rule so as to achieve appropriate interaction between shear and volumetric response. The nonassociativity is restricted to the volumetric component of plastic flow P.

$$3P'' = \chi(\eta, \eta_{pt}, p, \gamma^p) \quad (6)$$

$\eta = (\frac{3}{2} s : s)^{1/2} / (p - p_o)$: The effective stress ratio

η_{pt} : Material parameter defining effective stress ration along the phase transformation line

γ^p

: Cumulative plastic shear deformation

The function χ is defined according to the η value relative to η_{pt}

d) Strain-space formulation

For the special case when $\eta=\eta_{pt}$ (at low confining pressure), a significant amount of permanent shear strain may be accumulated rapidly with minimal change of shear stress as shown in Figure 5.

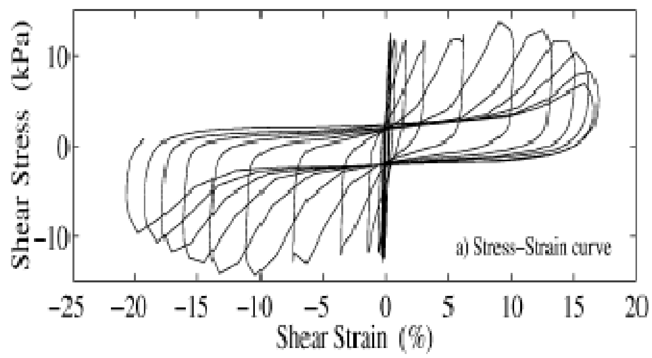


Figure 5: Stress-strain responses in undrained cyclic simple shear laboratory test with symmetric loading cycles

This is accomplished in the model by including an additional perfectly plastic zone (PPZ) before the initiation of dilation (Figure 6, phase 1-2). The PPZ is defined in deviatoric strain space, and has a circular and initially isotropic shape. Depending on the current strain state and plastic loading history, the PPZ may enlarge and/or translate in deviatoric strain space.

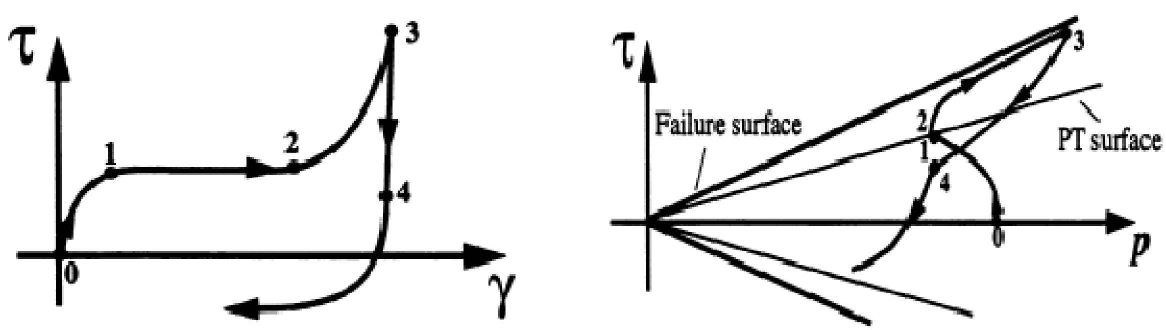


Figure 6: Response mechanism of the developed constitutive model (Parra 1996, Yang 2000)

e) Hardening law:

All surfaces, but the outer most surface may be translated in stress space without changing its form and with no intersection. The translation of the active yield surface f_m

is defined by a new relationship such that no overlap or intersection is allowed between the yield surfaces. The outermost surface remains stationary at all time.

Figure 7 presented the result of direct simple shear on a liquefiable cohesionless soil element

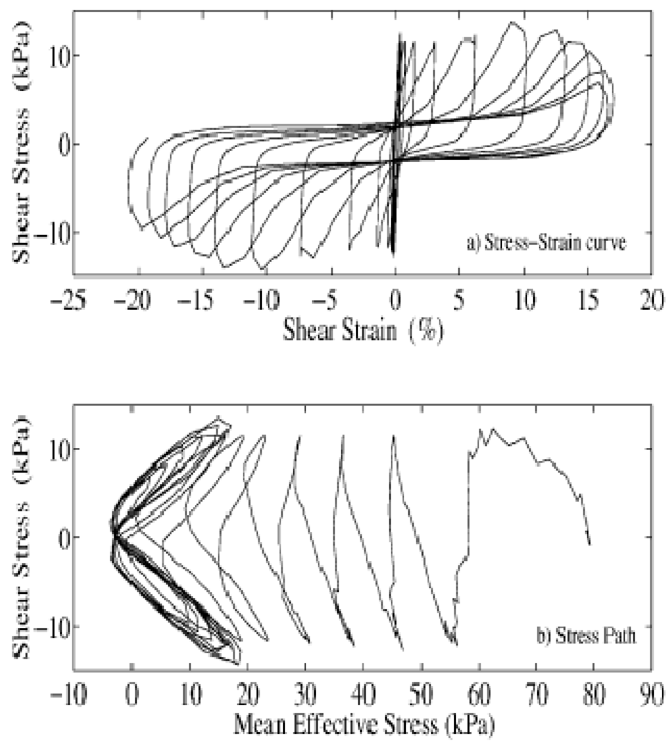


Figure 7: Stress-strain and stress path response for Nevada sand (Dr=60%) in a stress-controlled, undrained cyclic simple shear test

3. Numerical results on the Finite Element Method (Global) level for Pressure Dependent Multi-Yield surface soil model

3.1 94quad_UP Finite Element

In order to study the dynamic response of saturated soil system as an initial-boundary-value problem, a two dimensional plain-strain Finite Element (FE) is considered. The saturated soil element is modeled as two-phase material based on Biot's theory for porous material. A simplified numerical frame work of this theory, known as u-p formulation (in which displacement of the soil skeleton u, and pore pressure, are the primary unknowns) was implemented. A 94quad-UP FE is selected to best represented the response of a cohesionless soil element under cyclic simple shear test. Figure 8 presents the configuration of this element.

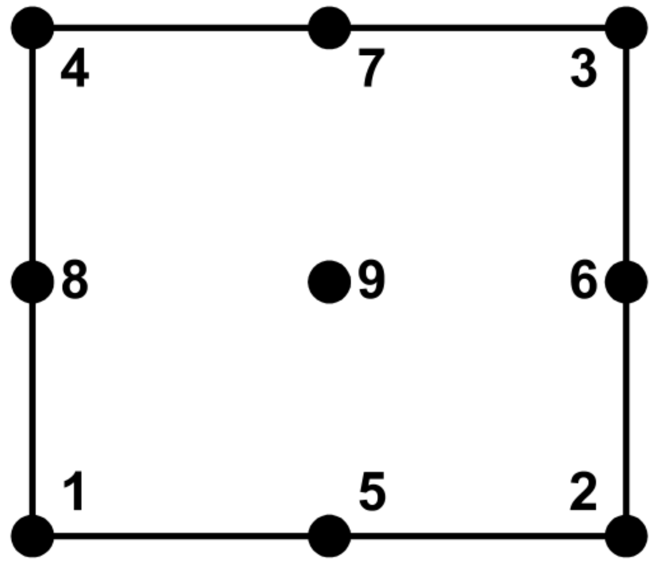


Figure 8: 94quad_UP element

The four corner nodes have 3 degrees of freedom (DOF); DOF 1 and DOF 2 for displacement and DOF 3 for fluid pressure. The other nodes only have 2 degree of freedoms. Node 1, node 2, and node 5 are fully supported. Node 3, node 4, and node 7 are at the free surface. The analysis was carried out in two phase:

- Phase one: The third degree of freedom of node 1 and node 2 was released, and the soil element is under drain isotropic consolidation.
- Phase two: The third degree of freedom of node 1 and node 2 was reverted, and the soil element is under cyclic simple shear loading applied at node 3, node 4, and node 7.

Krylov Newton algorithm was used in the analysis. A simple Mathlab program was used for post processing.

3.2 Effect of constraint type:

Four types of constraint are available in OpenSees environment: Plain constraint, Penalty Method, Lagrange Multipliers, and Transformation Method.

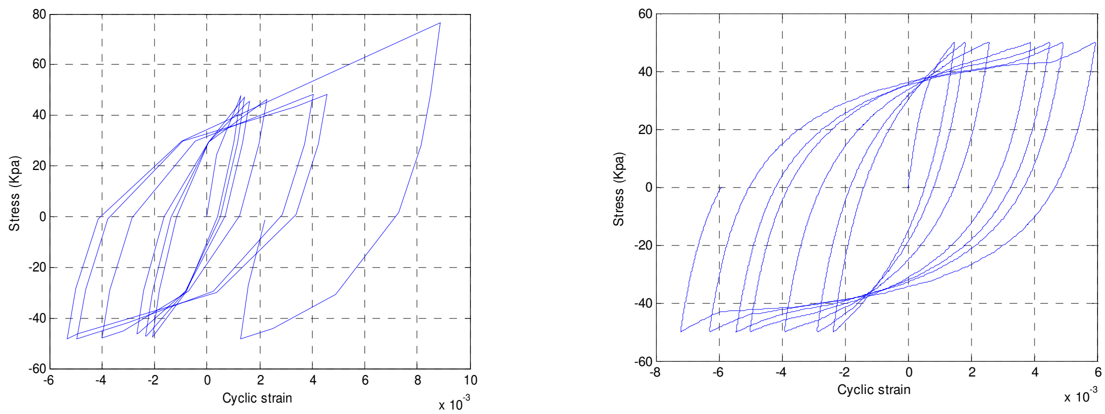
+ Plain constraint: All boundary conditions are fixity using a single point constraint.

+ Penalty Method: This method applies very stiff element (numerically) at the boundary conditions.

+ Lagrange Multipliers: Once the Lagrange Multipliers have been applied, the resulting stiffness matrix is no longer positive definite.

+ Transformation Method: This method reduces the size of the system for multi point constraints.

Of the four constraint types presented above, Plain constraint and Penalty method were selected to investigate the effect of constraint type to numerical simulated results. Figure 9 shows the results of analyzing the above 94quad_UP soil element under cyclic simple shear test and it was done implicitly. The soil element was loaded under 7 loading cycles with step size of 0.001



a b

Figure 9: Numerical simulated results of 94quad_UP plain strain element under 7 cycles of simple shear loading. (a): Plain constraint, (b) Constraint penalty.

As it is shown, the result obtained from Plain constraint yield excessive shear deformation for the 7th loading cycle while the Penalty method shows a more expected result.

3.3 Effect of different solver

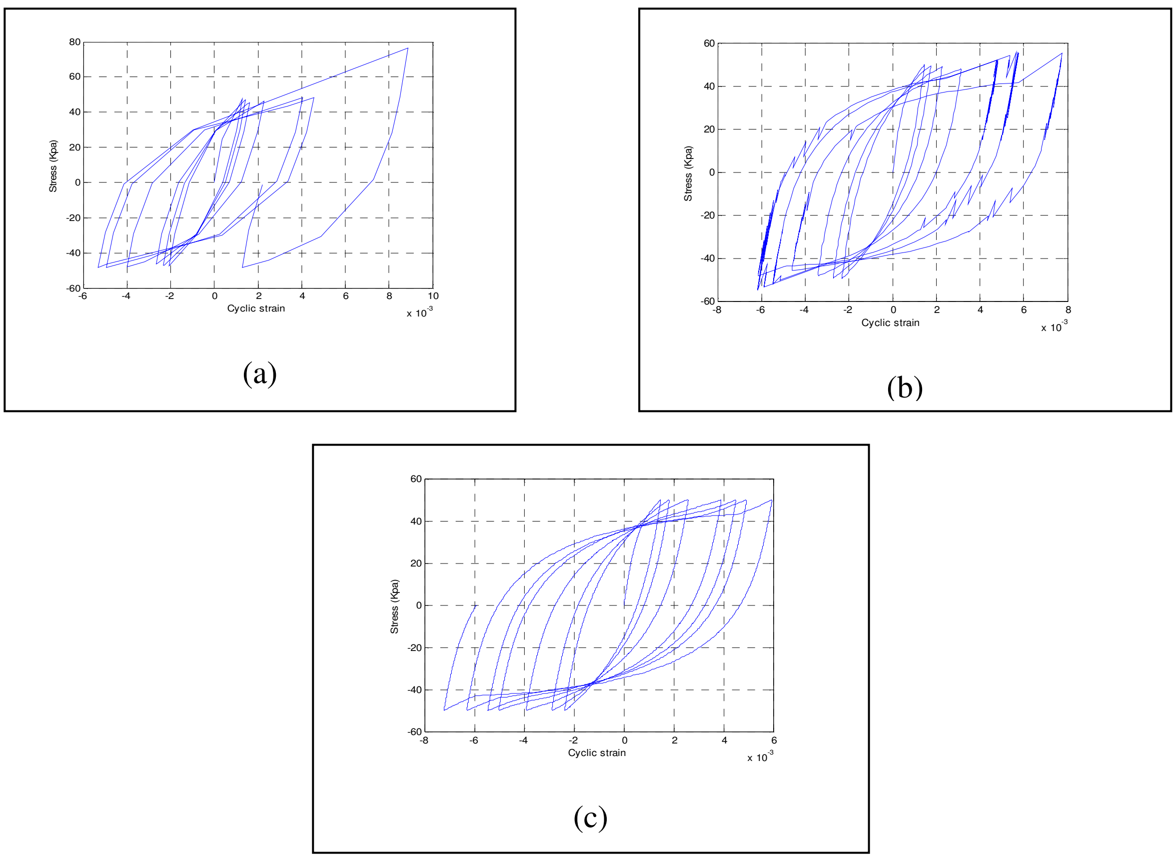
Presented here is the effect of different solvers on the run time of the simulation. As we can see in Table 1 below, the most appropriate solver for this particular problem is the BandGeneral solver. Both Umfpack and ProfileSPD solver yield the same runtime. SparseGeneral and SparseSPD solver take much more time to run while BandSPD solver failed to produce any result.

Solver	ProfileSPD	BandGeneral	SparseSPD	SparseGeneral	Umfpack	BandSPD
Run Time (sec)	18	16	24	21	18	NA

Table 1: Solver selection and run time

3.4 Effect of Step Size

Presented here are the results of finite element numerical analysis and the effect of choosing different step size. Figure 10 shows the stress-strain response of the 94quad_UP element under cyclic direct simple shear loading. The results shown below are obtained from simulating the subjected soil element under: Constraint Penalty and BandGeneral solver. Step size of 0.1, 0.01, and 0.001 were applied to the simulation respectively. As we can see, a coarse step size of 0.1 results in an excessive strain response for the 7th loading cycle. As we decrease the step size down to 0.01 and 0.001. The stress-strain responses start looking more and more similar to the responses obtained from laboratory test.



(a) (b) (c)

Figure 10: Numerical results for cyclic direct shear test: (a) 7 loading cycles with stepsize =0.1, (b) 7 loading cycles with stepsize=0.01, (c) 7 loading cycles with stepsize=0.001

4. Future works

For future work the other constitutive models like SANISAND and UBCSAND can also be use. Also, cyclic triaxial loading can also be simulated and the same procedure could be done.

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