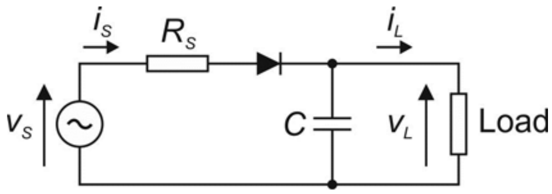
Half-wave and full-wave rectifier efficiencies
Note for Luke Morton, ASH 21.02.16

Assumptions:

- Sinusoidal voltage source v_S with amplitude v_0 and purely resistive source impedance R_S
- Ideal diodes, with zero forward voltage and zero reverse current
- Large reservoir capacitance, giving DC output voltage v_L with negligible ripple

Half-wave rectifier



The current drawn from the source is:

$$i_S = \begin{cases} (v_S - v_L)/R_S & v_S > v_L \\ 0 & otherwise \end{cases}$$

The steady state load current $i_L$is just the average value of i_S:

$$i_L = \overline{i_S} = \frac{1}{2\pi R_S} \int_{-\Delta\varphi/2}^{\Delta\varphi/2} (v_0 \cos\theta - v_L) d\theta$$

where $\Delta\varphi$ is the conduction angle of the diode, which is given by:

$$\Delta\varphi = 2\cos^{-1}(v_L/v_0)$$

Evaluating the above integral we find:

$$i_L = \overline{i_S} = \frac{v_0}{\pi R_S}\left[\sqrt{1-u^2} - u\cos^{-1} u\right]$$

where $u = v_L/v_0$.

The (constant) power delivered to the load is then:

$$P_{L\_hw} = v_L i_L = \frac{v_0^2}{8R_S}\frac{8u}{\pi}\left[\sqrt{1-u^2} - u\cos^{-1} u\right]$$

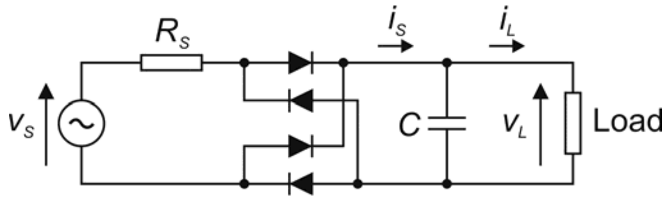
At the same time, the average power drawn from the source is:

$$P_{S\_hw} = \overline{i_S v_S} = \frac{1}{2\pi R_S} \int_{-\frac{\Delta\varphi}{2}}^{\frac{\Delta\varphi}{2}} v_0 \cos\theta\, (v_0 \cos\theta - v_L) d\theta$$

Evaluating this integral gives:

$$P_{S\_hw} = \overline{i_S v_S} = \frac{v_0^2}{8R_S}\frac{4}{\pi}\left[\cos^{-1} u - u\sqrt{1-u^2}\right]$$

Full-wave rectifier



For this case the source current i_S (NB measured on output side of rectifier) is:

$$i_S = \begin{cases} (|v_S| - v_L)/R_S & |v_S| > v_L \\ 0 & otherwise \end{cases}$$

For any given value of $u = v_L/v_0$, there will be two current pulses per cycle, each having the same form as the single current pulse that occurred in the half-wave case. It follows that, for a given value of u, the load current, load power and average source power will all be twice as large as in the half-wave case, i.e.:

$$P_{L-fw}(u) = 2P_{L-hw}(u) \quad ; \quad P_{S-fw}(u) = 2P_{S-hw}(u)$$

It also follows that, for a given value of u, the overall system efficiency P_L/P_S, taking into account the power loss in the source resistance, is the same for both configurations. (NB the rectifiers themselves are 100% efficient in this analysis because there are no diode losses.)

Performance comparison

The plot below shows the variations of the normalised output power and overall efficiency for the idealised half-wave and full-wave rectifiers, as a function of normalised output voltage $u = v_L/v_0$. The output powers are normalised to $v_0^2/(8R_S)$ which is the power that would be delivered to a matched load.