TRANSPOSE OF LORENTZ TRANSFORMATION*

Júlio C. V. Barczyszyn
Federal University of Santa Catarina
jucazyn@gmail.com

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Good notations often play central roles in making theoretical advances in the physical sciences, where calculations can easily become lengthy. However, certain notations create more confusion than clarity for beginners.

A quick search reveals widespread confusion regarding index notation or Einstein summation convention among students. In particular, the property

$$(\Lambda^t)^\mu{}_\nu = \Lambda_\nu{}^\mu \tag{1}$$

of matrices of Lorentz transformations is often misunderstood (see, for example, Stack Exchange). Of course, (1) is not the usual definition for the transpose of a matrix. To my surprise, I couldn't find any simple proof for the equality. Even worse, in some places the discussion ended up complicating more. Unsatisfied with the answers I found, I decided to write and share this.

Definition. For any square matrix M, the **transpose** of M is defined as

$$(M^t)_{\alpha\beta} = M_{\beta\alpha}. \quad (*)$$

Proposition. *Let Λ be the matrix of a Lorentz transformation. Then,*

$$(\Lambda^t)^\mu{}_\nu = \Lambda_\nu{}^\mu.$$

Proof. Let η denote the Minkowski metric. We compute:

$$\begin{aligned}(\Lambda^t)^\mu{}_\nu &= \eta^{\mu\alpha}(\Lambda^t)_{\alpha\nu} \\ &= \eta^{\mu\alpha}\Lambda_{\nu\alpha} \quad (*) \\ &= \Lambda_\nu{}^\mu.\end{aligned}$$

□

Remark. It's worth mentioning that the proof of

$$(\Lambda^t)_\nu{}^\mu = \Lambda^\mu{}_\nu$$

is analogous.

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*This is a first draft. In future versions, the text could evolve into lecture notes on tensors and index notation, especially if I come across more confusing things out there.