TRANSPOSE OF LORENTZ TRANSFORMATION*

Júlio C. V. Barczyszyn
Federal University of Santa Catarina, Brazil
jucazyn@gmail.com – jucazyn.github.io

March 29, 2025

A good notation often plays a central role in making theoretical advances in the physical sciences, where calculations can easily become very lengthy. However, certain notations seem to create more confusion than advantages for the novice.

A quick search shows that there is a lot of confusion regarding index and Einstein's notation in the context of Relativity among students. In particular, the property $(\Lambda^t)^\mu{}_\nu = \Lambda_\nu{}^\mu$ of matrix of Lorentz transformations appears to be widely misunderstood (see, for example, Stack Exchange). I had the same doubt and was not satisfied with any of the answers I found, so I decided to write this and share it.

Definition. For any square matrix M, we define the **transpose** of M as

$$(M^t)_{\alpha\beta} = M_{\beta\alpha}. \quad (*)$$

Proposition. *Let Λ be the matrix of a Lorentz transformation. Then,*

$$(\Lambda^t)^\mu{}_\nu = \Lambda_\nu{}^\mu.$$

Proof. Let η denote the Minkowski metric. We simply compute

$$\begin{aligned} (\Lambda^t)^\mu{}_\nu &= \eta^{\mu\alpha}(\Lambda^t)_{\alpha\nu} \\ &= \eta^{\mu\alpha}\Lambda_{\nu\alpha} \quad (*) \\ &= \Lambda_\nu{}^\mu. \end{aligned}$$

□

*This is a first draft. In future versions, the text could evolve into lecture notes on tensors and index notation, especially if I come across more confusing things out there.