The Riemann Zeta Function

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References

This document serves as a repository for sources that might be used for our final project. We need to provide historical background, how the mathematics were developed over the years, the current developments, and any other information

1. Derbyshire, John. *Prime Obsession - Bernhard Riemann and the Greatest Unsolved Problem in Mathematics,* 2003. Reading through the opening chapters of this book, it seems to provide biographical information about Riemann and his origin as a mathematician, among many other things. It will be useful when providing background information about Riemann, his Hypothesis, and the zeta function. Another useful feature of this source is that it contains the notation Riemann himself used, which could be helpful when reading the material written by him. Overall, I feel like this source will be very helpful for providing background information about his life, and less so about the zeta function and its applicability to number theory.

2. Mazur, Barry and Stein, William A. *Prime Numbers and Riemann Hypothesis,* 2016. This relatively short text serves as an excellent introduction to the Riemann Zeta Function. Starting with elementary number theory it swiftly segues into discussing the complexities of prime numbers, including the apparent vagaries of their distribution. As the range of numbers increases, the step-like function of the primes smoothes out, allowing us to take a probabilistic approach. After discussing Fourier transforms and delta functions, the authors lead us to the Riemann Spectrum. This book should serve as an excellent path from the number theory we are familiar with to the intricacies of the Riemann Zeta function.

3. Riffer-Reinert. *The Zeta Function and its Relation to the Prime Number Theorem.* University of Chicago, 2011. As the title suggests, this research paper explores the zeta function and how it relates to a specific topic in number theory, namely, the Prime Number Theorem. This source will be useful to understand exactly how the zeta function is useful to number theory and whether it applies to other contexts in mathematics, such as complex analysis. It contains many definitions, lemmas, theorems, and

proofs that are related to the topic we are researching and will help to understand the mechanics of its use. Also, as far as mathematical papers go, it is relatively easy to read and not very long, while at the same time containing useful information.

4. Xu, Mingrui. *Riemann Zeta Function and Prime Number Distribution.* University of Washington, 2014. This paper, again, contains information related to the zeta function and how it relates to prime number distribution, as opposed to how it relates to the Prime Number Theorem in the previous source. This paper builds on top of the last one, in that it attempts to tackle the distribution of primes using the Prime Number Theorem (which it includes but isn't the sole focus of the paper). It has many of the same equations and definitions, which will help when trying to understand the function and its mathematical relevance (based off of previous experience, some authors write in a way that is more accessible than others, even when writing about similar material).

5. Borwein, Jonathan. Bradley, David. Crandall, Richard. *Computational Strategies for the Riemann Zeta Function.* CECM, 1999. As in the case with Derbyshire, this source provides background information about the zeta function and possible motivations for efficient evaluation. In other words, it provides real-world applications where the zeta function might be useful. It is noticeably focused on the computational and optimization aspect of the zeta function. It contains algorithms that can be used to compute the zeta function, along with pseudo-code that can be translated into a scientific computing problem for computer scientists. From what I have read, it is very thorough (and dense at times).

6. Luo Qiang and Wang, Zhidan. *Numerical Calculation of the Riemann Zeta Function at Odd-Integer Arguments: A Direct Formula Method.* Mathematical Sciences 9.1, 2015. This paper approaches the Riemann Zeta function connection to Number Theory from the opposite end, providing a way to calculate values of the function at odd integers, given the results from the bracketing even integers. While the result cannot be precise, it is accurate to within 10^{-n} for odd integer n. We hope this result will provide a relatively simple computational way to understand the behavior of the complex valued Riemann Zeta function.

7. Ramos, R.V. and Mendes, F.V. *Riemann Quantum Circuit.* arXiv, 2014. Adopting a cross-disciplinary approach, the authors of draw a connection from the Riemann Zeta function to quantum systems and computation. The operator of the quantum system has eigenvalues related to the zeroes of the Riemann Zeta function. This work suggests that for any finite set of zeros to the function it is possible to build a quantum circuit with eigenvalues correlated to those zeros. While this paper is both information-dense and includes a discussion of Quantum Mechanics which diverges from our field of study, it also offers practical applications of the Reimann Zeta function.