APPM 4600
Homework 6

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1 Non-linear System

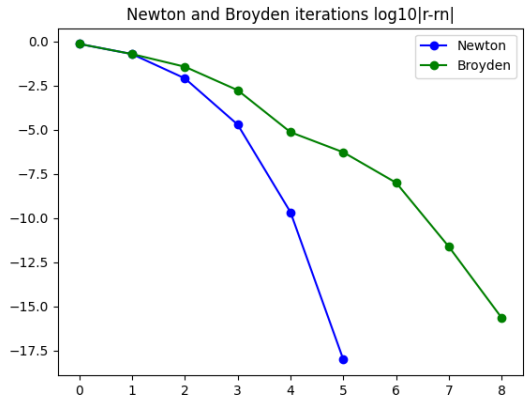
Consider the system below, which has two real solutions and the following initial guesses:

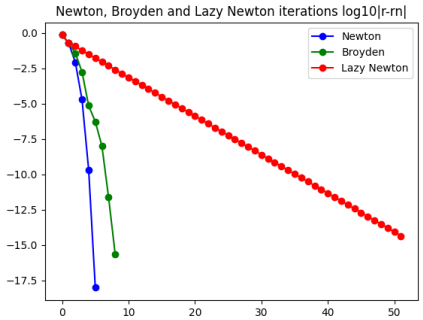
$$F(x,y) = \begin{Bmatrix} f(x,y) = x^2 + y^2 - 4 = 0 \\ g(x,y) = e^x + y - 1 = 0 \end{Bmatrix} \tag{1}$$

$$J_F(x,y) = \begin{bmatrix} 2x & 2y \\ e^x & 1 \end{bmatrix} \tag{2}$$

$$(x_0, y_0) = (1,1), (1,-1), (0,0) \tag{3}$$

We want to know how the performance of other Quasi-Newton Methods compares to Newton's method on this system. When applying the Newton method, Lazy-Newton, and Broydens method to the following system with the given initial guesses, we are left with a couple of situations; for the initial guess of $(1,1)$ and $(0,0)$, the system did not converge due to the Jacobian of these initial guesses being singular. However, when applying the initial guess of $(1,-1)$, we find results that match what we would expect for this system.





2 Non-Linear System

Consider the $3, 3$ system below:

$$\left\{ \begin{array}{c} x + cos(xyz) - 1 \\ (1-x)^{\frac{1}{4}} + y + 0.05z^2 - 0.15z - 1 \\ -x^2 - 0.1y^2 + 0.01y + z - 1 \end{array} \right\} = \underline{0} \tag{4}$$

We will apply Newtons Method, Steepest Descent, and a hybrid method which first uses steepest descent and then switches to newton once the values of our iteration has reached a difference less than $5 * 10^{-2}$. When using the same initial guess for all three methods of $(0, 0, 1)$ we can see the performance from this plot of the error. This Plot depicts that as usual newtons method converged the quickest, steepest descent converged the slowest and our hybrid method converged a slight bit slower than netwons method. While Newtons method does converge much faster it is far more unstable and computationally expense from this w can conclude that ther Hybrid method is the best for this situation as it is more stable at the begining than newtons method but still converges quickly while holding onto some stablilty as newtons method is generally stable within the basin of convergence.