

Jude Kenwell 3. Indigohead

(6825) - NL

| WEEK | Avg Bet per Visit (W) | Visits per month |
|------|-----------------------|------------------|
| A | 1 | 2 |
| B | 2 | 3 |
| C | 3 | 4 |
| D | 4 | 5 |



$$x = \frac{1+2+3+4}{4}$$

$$y = 2+3+4+5$$

$$x = 2.5$$

$$y = 3.5$$

$$\begin{array}{c} \text{4. } A^{\text{centered}} \\ \left[\begin{array}{cccc} 1-2.5 & 2-3.5 & 3-3.5 & 4-3.5 \\ 2-2.5 & 3-3.5 & 4-3.5 & 5-3.5 \\ 3-2.5 & 4-3.5 & 5-3.5 & 6-3.5 \\ 4-2.5 & 5-3.5 & 6-3.5 & 7-3.5 \end{array} \right] \end{array} = \left[\begin{array}{cc} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{array} \right]$$

$$5. \sum_{n=1}^4 \frac{1}{n-1} A^{\text{centered}} \cdot t^{\text{centered}}$$

$$At^{\text{centered}} = \left[\begin{array}{cccc} -1.5 & -0.5 & 0.5 & 1.5 \\ -1.5 & -0.5 & 0.5 & 1.5 \end{array} \right]$$

$$\text{At}^{\text{centered}} = \left[\begin{array}{cc} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{array} \right]$$

$$\begin{aligned}
 & \sum \frac{1}{3} \left[(-1.5)^2 + (0.5)^2 + (0.5)^2 + (1.5)^2 \quad (-1.5)^2 + (-1.5)^2 + (0.5)^2 + (1.5)^2 \right. \\
 & \quad \left. (-1.5)^2 + (0.5)^2 + (0.5)^2 + (1.5)^2 \quad (-1.5)^2 + (-0.5)^2 + (0.5)^2 + (1.5)^2 \right] \\
 & \sum -\frac{1}{3} \left[(1.25) + (0.25) + (0.25) + (2.25) \quad (1.25)(0.25)(0.25) + (1.25) \right. \\
 & \quad \left. (1.25) + (0.25) + (1.25) + (1.25) - (1.25)(1.25) + (0.25)(0.25) + (0.25)(2.25) \right] \\
 & \sum = \frac{1}{3} \left[\begin{matrix} 5 & 5 \\ 5 & 5 \end{matrix} \right] = \left[\begin{matrix} \frac{5}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{5}{3} \end{matrix} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \text{det}(I - \lambda I) = 0 \\
 & \text{det} \left| \begin{matrix} \frac{2}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{2}{3} \end{matrix} - \lambda \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right| = 0 \quad = \left(\frac{2}{3} - \lambda \right)^2 - \left(\frac{5}{3} \right)^2 \\
 & \quad = \left(\frac{2}{3} - \lambda \right) \left(\frac{2}{3} - \lambda \right) - \left(\frac{5}{3} \right)^2 \\
 & \quad = \left(\frac{2}{3} \right)^2 + 2 \left(\frac{2}{3} \lambda \right) + \lambda^2 - \left(\frac{5}{3} \right)^2 \\
 & \quad = \frac{4}{9} + \frac{10}{3} \lambda + \lambda^2 - \frac{25}{9} = 0 \\
 & \text{det} \left| \begin{matrix} \frac{2}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{2}{3} \end{matrix} - \begin{matrix} \lambda & 0 \\ 0 & \lambda \end{matrix} \right| = 0 \quad \lambda + \frac{10}{3} \lambda = 0 \\
 & \quad = \left(\lambda + 0 \right) \left(\lambda + \frac{10}{3} \right) \\
 & \text{det} \left| \begin{matrix} \frac{2}{3} - \lambda & \frac{5}{3} \\ \frac{5}{3} & \frac{2}{3} - \lambda \end{matrix} \right| \quad \lambda_1 = \frac{10}{3}, \quad \lambda_2 = 0 \\
 & \quad = 0
 \end{aligned}$$

$$A = \begin{bmatrix} \frac{2}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{2}{3} \end{bmatrix} \quad V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \lambda_1 = \frac{10}{3}$$

$$\begin{aligned}
 & \left(\begin{bmatrix} \frac{2}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{2}{3} \end{bmatrix} - \lambda_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \quad \left[\begin{bmatrix} -\frac{5}{3}v_1 + \frac{2}{3}v_2 = 0 \\ \frac{2}{3}v_1 - \frac{5}{3}v_2 = 0 \end{bmatrix} \right] \\
 & \left(\begin{bmatrix} \frac{2}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{2}{3} \end{bmatrix} - \lambda_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \quad \left[\begin{bmatrix} \frac{5}{3}v_1 = \frac{2}{3}v_2 \\ \frac{2}{3}v_1 = \frac{5}{3}v_2 \end{bmatrix} \right] \\
 & \left(\begin{bmatrix} \frac{2}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{2}{3} \end{bmatrix} - \frac{10}{3} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \\
 & \left(\begin{bmatrix} -\frac{2}{3} & \frac{5}{3} \\ \frac{5}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right) = 0
 \end{aligned}$$

$$V = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A \begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix} \quad V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad k_2 = 0$$

$$\left(\begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix} - k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \quad \begin{bmatrix} 5v_1 + 5v_2 = 0 \\ 5v_1 + 5v_2 = 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \quad \begin{bmatrix} 5v_2 = -5v_1 \\ 5v_1 = -5v_2 \end{bmatrix}$$

$$\left(\begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \quad V = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$k_1 = \frac{10}{3} \quad V = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\|V\| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2} \quad V_{\text{normal}} = \frac{V}{\|V\|} = \frac{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$k_2 = 0 \quad V = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\|V\| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \quad V_{\text{normal}} = \frac{V}{\|V\|} = \frac{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$9. \quad A V = k V$$

$$k_1 \begin{bmatrix} \frac{5}{3} & 5 \\ \frac{5}{3} & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{10}{3} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{5}{3} + \frac{5}{3} & 5 \\ \frac{5}{3} + \frac{5}{3} & 3 \end{bmatrix} = \begin{bmatrix} \frac{10}{3} & 5 \\ \frac{10}{3} & 3 \end{bmatrix} = \begin{bmatrix} \frac{10}{3} \\ \frac{10}{3} \end{bmatrix},$$

$$k_2 \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} V = k V \quad \begin{bmatrix} 5-5 & 5 \\ 5-5 & 5 \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0 \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

$$\boxed{\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \boxed{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}}$$

10. the first principal component is ~~$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$~~ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
because it has the largest eigenvalue.

11. - 12.

new projection = A centered \times Unnormalized

$$\begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix} \times \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{-1.5 - 1.5}{\sqrt{2}} = \frac{-3}{\sqrt{2}} \\ \frac{-0.5 - 0.5}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \\ \frac{0.5 + 0.5}{\sqrt{2}} = \frac{1}{\sqrt{2}} \\ \frac{1.5 + 1.5}{\sqrt{2}} = \frac{3}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} -1.5 \\ -0.5 \\ 0.5 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1.5 \\ 1.5 \\ 1.5 \end{bmatrix} = \frac{V}{\|V\|} = \frac{\text{original } V}{\|V\|} = \frac{V}{\|V\|} = \frac{V}{\|V\|}$$

$$\begin{bmatrix} 1.5 \\ 1.5 \\ 1.5 \\ 1.5 \end{bmatrix} = \frac{V}{\|V\|} = \frac{V}{\|V\|} = \frac{\text{original } V}{\|V\|} = \frac{V}{\|V\|} = \frac{V}{\|V\|}$$

$$\begin{bmatrix} 1.5 \\ 1.5 \\ 1.5 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1.5 \\ 1.5 \\ 1.5 \end{bmatrix} = \frac{V}{\|V\|} = \frac{\text{original } V}{\|V\|} = \frac{V}{\|V\|} = \frac{V}{\|V\|}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$