

Jude Kennell 8. Indigalood

10/15/11-M

OWNER	Avg Bet per Visit (USD)	Visits per month
A	1	2
B	2	3
C	3	4
D	4	5



2.

$$x = \frac{1 + 2 + 3 + 4}{4}$$

$$x = 2.5$$

$$y = 2 + 3 + 4 + 5$$

$$y = 3.5$$

3. A_{center}

$$\begin{bmatrix} 1 - 2.5 & 2 - 3.5 \\ 2 - 1.5 & 3 - 2.5 \\ 3 - 2.5 & 4 - 1.5 \\ 4 - 2.5 & 5 - 0.5 \end{bmatrix} = \begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix}$$

5. $\sum \frac{1}{n-1} A_{center}^T A_{center}$

$$A_{center} = \begin{bmatrix} -1.5 & -0.5 & 0.5 & 1.5 \\ -1.5 & -0.5 & 0.5 & 1.5 \end{bmatrix}$$

$$A_{center}^T = \begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix}$$

3.

$$\sum \frac{1}{3} \begin{bmatrix} (-1.5)^6 + (-0.5)^6 + (0.5)^6 + (1.5)^6 & (-1.5)^6 + (-0.5)^6 + (0.5)^6 + (1.5)^6 \\ (-1.5)^6 + (-0.5)^6 + (0.5)^6 + (1.5)^6 & (-1.5)^6 + (-0.5)^6 + (0.5)^6 + (1.5)^6 \end{bmatrix}$$

$$\sum = \frac{1}{3} \begin{bmatrix} (2.25) + (0.25) + (0.25) + (2.25) & (2.25)(0.25) + (0.25) + (2.25) \\ (2.25) + (0.25) + (0.25) + (2.25) & (2.25) + (0.25) + (0.25) + (2.25) \end{bmatrix}$$

$$\sum = \frac{1}{3} \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{5}{3} \end{bmatrix}$$

6.

$$\det (I - \lambda I) = 0$$

$$\det \begin{bmatrix} \frac{1}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{1}{3} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{aligned} &= \left(\frac{1}{3} - \lambda\right)^2 - \left(\frac{5}{3}\right)^2 \\ &= \left(\frac{1}{3} - \lambda\right)\left(\frac{1}{3} - \lambda\right) - \left(\frac{5}{3}\right)^2 \\ &= \left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\lambda\right) + \lambda^2 - \left(\frac{5}{3}\right)^2 \\ &= \frac{1}{9} + \frac{10}{3}\lambda + \lambda^2 - \frac{25}{9} = 0 \\ &\lambda^2 + \frac{10}{3}\lambda - \frac{24}{9} = 0 \\ &(\lambda + 0)(\lambda + \frac{10}{3}) \\ &\lambda_1 = -\frac{10}{3} \quad \lambda_2 = 0 \end{aligned}$$

$$\det \begin{bmatrix} \frac{1}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{1}{3} \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\det \begin{bmatrix} \frac{1}{3} - \lambda & \frac{5}{3} \\ \frac{5}{3} & \frac{1}{3} - \lambda \end{bmatrix}$$

$$2. \quad A = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \quad V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \lambda = \frac{10}{3}$$

$$\left(\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} - \begin{bmatrix} \frac{10}{3} & 0 \\ 0 & \frac{10}{3} \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{5}{3} \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\begin{cases} -\frac{5}{3}v_1 + \frac{5}{3}v_2 = 0 \\ \frac{5}{3}v_1 - \frac{5}{3}v_2 = 0 \end{cases}$$

$$\begin{cases} \frac{5}{3}v_2 = \frac{5}{3}v_1 \\ \frac{5}{3}v_2 = \frac{5}{3}v_1 \end{cases}$$

$$V = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \quad V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \lambda_2 = 0$$

$$\left(\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\lambda_1 = \frac{10}{3} \quad V = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\|V\| = \sqrt{1^2 + 1^2} = \sqrt{2} \quad V_{\text{normal}} = \frac{V}{\|V\|} = \frac{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 0 \quad V = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\|V\| = \sqrt{(-1)^2 + 1^2} = \sqrt{2} \quad V_{\text{normal}} = \frac{V}{\|V\|} = \frac{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$9. \quad AV = \lambda V$$

$$\lambda_1 \begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{5}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{10}{3} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{5}{3} + \frac{5}{3} \\ \frac{5}{3} + \frac{5}{3} \end{bmatrix} = \begin{bmatrix} \frac{10}{3} \\ \frac{10}{3} \end{bmatrix} = \begin{bmatrix} \frac{10}{3} \\ \frac{10}{3} \end{bmatrix} //$$

$$\lambda_2 \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} V = \lambda V$$

$$\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 5-5 \\ 5-5 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 //$$

$$\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$$

10. the first principal component is ~~$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$~~
 - because it has the largest eigenvalue.

11. - 12.

new projection = X centered $\times V_{\text{normal}}$

$$\begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix} \times \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{-1.5-1.5}{\sqrt{2}} = \frac{-3}{\sqrt{2}} \\ \frac{-0.5-0.5}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \\ \frac{0.5+0.5}{\sqrt{2}} = \frac{1}{\sqrt{2}} \\ \frac{1.5+1.5}{\sqrt{2}} = \frac{3}{\sqrt{2}} \end{bmatrix}$$