Data Structures in C

Lecture Notes for CIS*2520

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Foreward

This document was originally written as supplement to the slides and lectures for CIS*2520. Recently(early 2017) it has been revised to serve as the main reference document for a distance education version of CIS*2520. Students using this material are assumed to have a background in programming in C.

Special thanks to David McCaughan, Pascal Matsakis and Xining Li for sharing slides and presentations from their offerings CIS*2520. D. McCaughan's materials influenced the overall design of the course materials significantly. Dr. Matsakis and and Dr. Li's work helped greatly to improve the material on computational complexity, heaps, and priority queues.

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Chapter 1

Introduction

Welcome to CIS 2520!

A data structures course is often the first course in a series of courses on software design. During this class we will examine a variety of abstract data types and how they can be used to improve the quality of the software you develop. This e-text contains all the notes provided as part of the course website as well as more detailed material for in-depth study.

The examples will be given in pseudo code that could be used as the outline for a program in nearly any programming language, or in c. The pseudocode format is as follows:

- function and procedures are defined with a name that begins with a lower case
- parameters are included as types only
- if there is a return value, it appears at the end of the function definition after a colon
- no ending punctuation is used for lines
- indentation is used rather than parentheses

For example a pseudocode function could be defined as :

```
calculateComplicateValue(int, String): int
```

If the same function were actually written in a C header file it would look something like this:

```
int calculateComplicateValue(int intVal, char * stringVal);
```

1.1 Why Software Design

Most software is not developed by a single person and most of it takes months or even years to complete and test. Most software development involves multiple people who subdivide the problem into chunks and then collaborate to make the entire program work.

The process of identifying suitable chunks is the essence of software engineering. When the chunks are defined so that a single chunk can be reused in more than one software project, the company benefits because it only pays once for the development of something than can be used more than once. Good software engineering practices save time (on the part of the developer) and money (on the part of the employer or funding organization).

In future courses, you will spend time learning about a variety of formal approaches to software development. Each formal approach represents one way to try to meet the objectives of modularity, maintainability, reuse, low coupling, high cohesion when creating software with medium-large teams. The effectiveness of each formal approach depends on the type of software being developed, on the development environment and context, and on the personalities of the development team.

1.2 Software Development- CIS 2520 Style

Clean, crisp coding habits are developed over time, not learned all at once. And, like any good (or bad) habit, if you dont practice, youll lose the habit. You are expected to follow good coding practices for all of your homework

in this class. Consistency is the key to writing readable code. For all work for 2520, you are expected to follow the following minimum set of conventions.

- Use a consistent naming convention for all variable names. Variable names must be meaningful with single-character names used only for counters.
- The names for types (including abstract data types) must be obviously different from the names for variables. For example, you could capitalize the first letter of the names of a type.
- Use spaces for indentation- do not use tabs
- Use .h files to declare functions and types and put implementation in .c files. You may, or may not, need a .h file for the file containing main. Use include-guards in your .h files.
- Organize your .h files and .c files so that functions are in the same order in both files. Use blank lines to create groups of code.
- Doxygen style comments go in the .h file for every function (as a minimum)
- Every source code or .h file begins with a doxygen comment that indicates author, purpose of code, and last modification date.
- Compile your code with -Wall and -g flags. You may use -ansi, -std=c99 or -std=c11

You must submit modular code for your homework solutions. This means you must break programs into separate files.

- Use makefiles to automate the build process so that you can divide your code into as many modules as necessary.
- Each file in your project should contain a single module that goes together in some way (i.e. a module for reading in files, a module for calculating things, a module for list operations).
- Your programs must be organized in a file structure that contains src/ bin/ include/ assets/ and docs/ folders and your makefile must make use of that file structure. src/ is for source code, include/ is for .h files, docs/ is for doxygen documentation, and assets is for testing files, data files, etc. The README goes in the root folder of an assignment, not in the sub folders.
- The README file should be based on the template shown at the end of this chapter. Read assignment specifications carefully as there may be additional information expected in assignment-specific README files.

1.3 Test driven development

It is not enough to simply have a program that runs, it must run properly and produce correct results for all allowable inputs. One way to do that is to set up the testing program as the first thing you do, and then to test early and to test often.

Instead of writing the entire solution to your homework, create a skeleton that that is just the framework of your program but no functionality. The skeleton should have all the files, all the functions definitions, all the structures and types, but none of the actual functionality. The values returned by functions and any actual data processing is completely hard coded. This is called writing a **stub** for a function. When you have created a complete skeleton for your program, you can then add functionality to one function or procedure at a time and test just that function.

Use a driver or test harness to test your code modules. Write the test harness as a series of function calls in main, using hard coded data for which you know the return value. Add checks and comparisons to determine if value returned by your function is the expected value. Once your software can pass all of the tests in your test harness, then it is time to write a new 'main' that is the actual application.

Create test data that cover all the different types of possible data. Test for edge cases (zero or null values, really big values, too many values, etc). Test each function with values having different attributes so that you can show you have thoroughly tested the different parts of your software.

For example, below you can find the skeleton code for a singly linked list of string data. You have likely worked with linked lists in a previous course. The code given contains no functionality, but it will compile and run with a test harness. Note: This skeleton code should not be your starting point for the first lab as it does not meet the requirements for the first lab. The skeleton code also gives you an example of doxygen-styled comments, which are required for this course and of how to use include guards in your .h file.

The .h file contains the function prototypes and the struct definitions and the comments for each function.

```
/**
 * @file linkedList.h
 * @author Judi McCuaig
 * @date January 2017
 * @brief API for a singly linked list
 */
#ifndef _JRM_LLIST
#define _JRM_LLIST
struct listNode {
    char * nodeValue;
    struct listNode * next;
};
typedef struct listNode node;

/** Creates a list and returns a pointer
*@return pointer to the list head
**/
node * createList ();
/** Destroys the list and frees all the allocated memory
*@param pointer to the head of the list
**/
void destroyList(node * head);
/** Adds an element to the head of the list
*@param pointer to the head of the list
*@param pointer to the string that will be stored in the list
**/
void addFront (node * head, char * data);
/** Removes a specific element from the list and frees the memory for that element
*@return 0 if the removal is successful, -1 on error
*@param pointer to the head of the list
*@param pointer to the string that will be removed from the list
**/
int removeFromList(node * head, char * data);
/** Prints the list in head-to-tail order
*@param pointer to the head of the list
**/
void printList(node * head);
/** Frees the memory allocated for a list node
*@param pointer to the node to be freed
**/
void freeElement (node * element);
/** Creates a list node with the data provided in the parameter
*@return pointer to the new node
*@param pointer to the data to be stored in the node
**/
node * initElement(char * data);

#endif
```

The source code (.c) file contains the implementation for the functions.

```
/**
 * @file linkedList.c
 * @author Judi McCuaig
 * @date January 2017
 * @brief skeleton code for a singly linked list
 */
#include "linkedList.h"

node * createList ()
{
       return (NULL);
}

void destroyList(node * head)
{
}

void addFront (node * head, char * data)
{
}


int removeFromList(node * head, char * data)
{
   return(0);
}


void printList(node * head)
{
    printf("\n");
}


void freeElement(node * element)
{
}

node * initElement(char * data)
{
}
```

1.4 Extending Activities

- The internet has many high quality resources about software engineering and software design. Build yourself a list of resources for information. Look for podcasts, videos, websites, or newsgroups that can help you arrive at definitions for each of the following: Modularity, Reuse, Maintainability, Coupling, Cohesion. You may use Wikipedia, but try to find other resources as well.
- There are many different processes for developing software. A common (and old) process is called the Waterfall cycle and involves rigidly dividing the process into definition, design, development and testing segments. Each segment is completed before the next segment is started. Other processes are more fluid, moving between design and development as new requirements are noted. It is very difficult to use a formal software development process as part of coursework because most of the design process is done for you when the assignment specification is done. Still, it is possible to use portions of specific approaches. The lab activities for 2520 are based on a modified Test-Driven development cycle. Use internet resources to inform yourself about Test-Driven development. Make list of the changes you will need to make to your own development habits in order to be successful in a test-driven process.

Chapter 2

Abstraction

2.1 Introduction

While computer hardware has changed significantly over the past several decades, the way in which we communicate with computers has not changed very much at all. The basic instructions and building blocks used in a computer program are the same now as they were when computers were constructed from mechanical parts. At the machine level, a program is a set of instructions composed of arithmetic and logic, and the data used and the instructions of the program are all represented as binary numbers. Most programmers rarely, if ever, interact directly with the binary machine code when creating computer programs. Instead a programmer uses a higher level language, an abstraction, that provides single instructions that represent several machine code steps. For example, in machine code, the instruction to add two numbers might look something like this:

00001001 0000000001100011 000000000110010

where the first number represents the instruction for add, and the second number is the memory location of one value to be added and the third number is the memory location of the second value to be added. This example code does not include putting the result of the addition anywhere useful for the programmer- that is a second step. Abstracted programming languages greatly improved the productivity and reliability of the programs written. Such languages are often called 'high level' languages. C, Pascal, Cobol, LISP, and Fortran are among the first high level languages developed. Each of these languages was specialized somewhat to create usable abstractions for operations that were common to a particular class of computer programs. Cobol has specialized operations for business and was created so that even non-programmers could understand the code. Fortran is specialized for mathematics, and LISP is specialized for artificial intelligence programming. In more recent years, even higher level languages have been abstracted from the early languages

2.2 Real World Abstraction

Abstraction is a tool that we use to navigate the world around us. A map is an example of an abstraction. A map contains information about a real space, but it only contains the information that is relevant to moving around in that space. Maps do not contain ALL the details about a space because that would make the map unreadable.

An abstraction is a representation of something that shows only the details that are essential for the task at hand, and ignores the other details. Think again about the map example. A topographical map has different details than a street map. A map that outlines points of interest for a nature walk would have different details than a map constructed for a geological survey.

Abstraction is used to make devices usable in situations where it is unnecessary to understand all the details in order to use something. The modern car is usable by persons who have little or no mechanical aptitude. The details that the driver sees are relevant for safe driving, and all the details about how the car operates have been hidden from the driver.

The utility supply to your house is an abstracted interface. You can use electrical power in your house without needing any details about how that power is generated, or how your house is wired. You don't even need to

understand how electricity works. A good abstraction is one that simplifies the use of something, but that still permits use in many general situations. Sometimes abstraction is unnecessary.

Consider the kitchen can opener. Many different kinds of can openers can be purchased, with a widely varying level of abstraction. Each higher level abstraction hides a few more details about how the can opener works. All can openers eventually open the can, the difference is in the amount of effort and understanding required of the operator.

This example makes it very clear that the can is opened by wiggling a sharp knife up and down through the metal.

This version has a circular knife, which hides the sawing motion with a crank, but the pressure required to operate it makes it very clear that a knife is still cutting through metal.

2.2.1 Implementation

An abstraction is really just a design until it is implemented in some way. It is possible to design an abstraction, and then have several different implementations, all with different strengths and weaknesses.

Suppose that you had been tasked to create a device to cook an egg without requiring the cook to crack the egg. One possible implementation for such an egg-cooking device is this

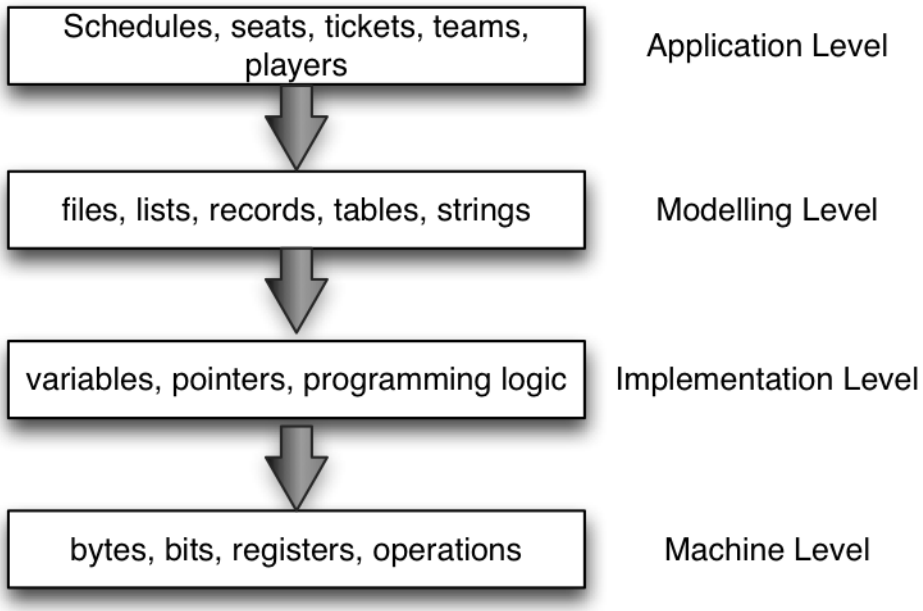
Another possible implementation could look like this

Another implementation can be found here: http://www.youtube.com/watch?v=XRb82E4_b38

While these implementations accomplish the task, they are not equal in terms of cost to construct, maintainability, likelihood of breakdown, etc.

2.3 Abstraction in Software Design

The image below illustrates the different levels of abstraction required to create a computer program for managing a sports complex. The highest level of abstraction, the application level, maps the ideas and concepts of the real world to the specification of the program operation.



The second level, the modeling level, maps the objects and entities discovered in the first level to models that can be used to represent the entities in the software. The third level of abstraction creates the actual implementation of those models and the last level is the responsibility of the compiler.

In computing, an abstraction lets you understand 'what' can be accomplished without requiring that you know the details of 'how' the thing is accomplished.

The reason we use abstractions in programming is really for economics. It is more time-efficient for programmers to reuse previously developed (and debugged) libraries than it is for them to program everything in binary. It is also more cost-efficient because the reusable library only needs to be debugged once, and then used many times. This is in contrast to building from scratch each time, which incurs the same debugging requirements for every development effort.

Programming languages, file system libraries, input-output libraries, and string libraries are all examples of abstractions that programmers use every day. Abstract data types are another kind of abstraction that programmers use.

Learning how to design a program in layers, and how to work at an appropriate level of abstraction for each layer is a crucial skill for software developers. Most developers are only concerned with the first three levels of abstraction (application, modeling, and implementation), and must learn how to design applications within those layers.

When you are writing software, you must first ensure that you understand the problem. Make sure you know what the software is supposed to do, and that you understand all the conditions surrounding its operation.

Your first task as a software developer is to decompose the problem into application-level constructs. Identify the main ideas in the software problem (like teams and schedules). These are the core 'things' that the software must create, but they are not things that the computer programming language knows anything about.

The second task is to identify a suitable model for each of the things you find in the first step. For example, you might decide that the best way to represent the teams is as a list, but that the best way to represent the ticket line is by using a queue.

In many cases, the developer can use third party libraries as the implementation layer for the models. Thoroughly debugged modules of abstract data types are available for nearly every programming language. However, in this class you will be asked to create those modules . By the end of the course, you should have your own, thoroughly debugged, library of Abstract Data Types that you can reuse whenever you wish.

2.3.1 Abstract Data Types

Remember that specification (i.e. "what") and implementation (i.e. "how") are two separate things. For example, if you are writing functions in your code, the specification is the function header + pre/post conditions and the implementation is the local variables + body of subroutine.

When you are writing abstract data types, the specification is the definition of data type + operations defined on that type. The implementation is the program needed to effect those operations. In C the specification is usually a header file containing type and function declarations together with a .c file in which they are implemented.

Abstract data types are organized into modules. Each module contains the variable definitions (structures) and operations required to meet the specification of the ADT.

The module exports a 'type', such as list or tree or stack that a programmer can use to declare variables. The variables can then be manipulated using the operations defined within the ADT.

The most important part of this process is that the user of the variable types only needs to know WHAT is possible, but has no need to know HOW it is implemented. The type is abstract from the point of view of the user.

An abstract data type is usually provided to the developer in the form of a library or module.

Some operations are common to most ADT specifications. In particular, pretty much all ADT modules need to provide mechanisms for the following operations:

- Creating the ADT (initialization of internal variables/allocation of dynamic structures)
- Destroying the ADT (management of de-allocation of dynamic resources)
- Adding data to the ADT
- Removing data from the ADT
- Getting the value of data in the ADT

The details of how those operations behave vary in the different kinds of ADTs, but the core purpose of the operation remains constant.

2.3.2 Worked Example: The Fraction ADT

Suppose you were writing a program that required a representation of fractions. It is not always sufficient to convert fractions to decimals, sometimes fractions need to stay as fractions. A fraction ADT that allowed the programmer to declare a variable of type fraction would be extremely useful.

First, we need a definition of what a fraction is. Fraction: A fraction is a number a/b where a,b are integers, b is non-zero.

Operations

The first step in designing an ADT is to imagine what operations are required. In addition to the core operations of create, insert, read, destroy the fraction ADT will need:

- add (subtraction is just adding with negative numbers)
- multiply (really just repeated adding, but would be nice to have it as a separate operation)

```
ADT specification for Fraction

create_fraction(numerator, denominator): Fraction
    preconditions: none
    postconditions: a fraction is created with the appropriate numerator
                    and denominator
get_numerator(Fraction): number
    preconditions: an initialized Fraction is given as the parameter
    postconditions: none
get_denominator(Fraction): number
    preconditions: an initialized Fraction is given as the parameter
    postconditions: none
destroy_fraction(Fraction)
    preconditons: the parameter Fraction is initialized
    postconditions: the fraction is destroyed and memory released if necessary
add(Fraction, Fraction): Fraction
    preconditions: two initialized fractions are passed in as parameters
```

```
    postconditions: the two fractions are added together and the result is
                    placed in a new Fraction variable that is returned to the
                    calling procedure
```

The ADT could have many other operations as well. For example, it could have a function to display/print a fraction or the ability to create a fraction type from a string representation of the fraction (i.e. "one half" or "three fifths").

Example Code

Here's an example of what some of the fraction code might look like in C. First, the .h file contains the definition of the struct as well as the prototypes for the functions that operate on that struct.

```
/**
 * @file fraction.h
 * @author Judi McCuaig
 * @date January 2017
 * @brief API for a fraction ADT
 */
#ifndef _JRM_FRACTION
#define _JRM_FRACTION

fraction.h

typedef struct {
  int integer;
  int numerator;
  int denominator;
} Fraction;

/** Creates a fraction from an integer numerator and denominator
*@return NULL if the creation is unsuccessful
*@param integer value for the numerator
*@param integer value for the denominator
**/
Fraction * create_fraction(int numer, int denom);

/** Adds two fractions and returns a new fraction that is the result of the addition
*@return NULL if the addition is unsuccessful
*@param Pointer the first Fraction operand
*@param Pointer to the second Fraction operand
**/
Fraction * add ( Fraction *fractOne, Fraction *fractTwo );

#endif
```

The .c file doesn't need to redefine the struct because the .h file is included. The .c file is used to flesh out the implementation of the functions.

```
/**
 * @file fraction.c
 * @author Judi McCuaig
 * @date January 2017
 * @brief implementation of fraction ADT
 */
#include "fraction.h"

Fraction * create_fraction(int numer, int denom){
  Fraction * temp = malloc(sizeof(Fraction)*1);
  temp->numerator = numer;
  temp->denominator = denom;
  return(temp)
}

Fraction * add ( Fraction *fractOne, Fraction *fractTwo ){
  Fraction result= malloc(sizeof(Fraction)*1);
```

```
  int largeCommonDenominator;
  int part1, part2, numeratorResult;
  hcd = fractOne->denominator * fractTwo->denominator;
  part1 = fractTwo->denominator * fractOne->numerator;
  part2 = fractOne->denominator * fractTwo->numerator;
  numeratorResult = part1 + part2;
  result->numerator = numeratorResult;
  result->denominator = largeCommonDenominator;
  return result;
}
```

The fraction ADT can be used by simply including the .h file in the c program that needs the ADT and then compiling the fraction.c program in with the application.c program.

```
/**
 * @file application.c
 * @author Judi McCuaig
 * @date January 2017
 * @brief use of Fraction ADT
 */
#include "fraction.h"

int main(void){
  Fraction * myFraction = create_fraction(1,2);
  Fraction * myOtherFraction = create_fraction(3,4);
  Fraction * theAnswer = add(myFraction, myOtherFraction);
}
```

2.4 Information Hiding (Encapsulation)

Information Hiding is one way to achieve abstraction. The details of the library implementation are hidden by providing functions to perform operations on the data structure instead of allowing programmers to work directly with the attributes of the data structure. The user of the ADT must use ONLY the interface (the available operations) of the ADT and must resist the temptation to go 'under the hood' and use the component parts.

Information Hiding is also called encapsulation. Encapsulation helps to prevent errors when a library of functions is updated. For example, suppose you were using a String ADT that provides a stringLength(String) operation to return the length of the string. Suppose further that you knew that the length of the string was stored as an integer in the ADT because you had looked at the source code. If, in your code, you write int size = stringLength(myString); you are guaranteed (because of preconditions and postconditions) that you will get the length of the string returned.

However, if you chose to ignore the encapsulation and bypassed the interface, writing the following instead int size = myString-¿length you could end up with code that gave errors. In this situation what would happen if the author of the String library gives you an update that changes the declaration of length in the struct from int to double? Your previously working code will break, because you didn't use the interface.

2.5 Extending Activities

- Compile a list of 10 different programming languages. Categorize the languages on your list based on their level of abstraction (Moderate, High, Very High).
- **Sidequest: Rube-Goldberg Machines**
  Another example of implementation differences can be found in the notion of Rub-Goldberg machines. A Rube-Goldberg machine is something that performs a simple task in as complex a way as possible. There are several examples of Rube Goldberg machines for making toast on YouTube. Find at least three, watch the video, and count the number of steps in each machine. As you can see, even though the task specified for a Rube Goldberg machine is simple, the implementations of that specification are endless. For a silly example of a Rube-Goldberg machine, have a look at: http://www.youtube.com/watch?v=lCYg_gz4fDo The point of this diversion is that it is really easy, especially in software construction, to meet the specifications

for a program, but to do it with the software equivalent of a rube-goldberg machine and end up with something that works, but is not easily understood and is difficult to maintain.

- Write pseudocode for a multiply and a subtract operation for the fraction ADT What does the integer portion of the Fraction struct represent? List any additional operation(s) would be required in order to make use of that part of the struct.

Chapter 3

List ADT

3.1 Introduction

Usually when we write programs we dont know how many records or data items will be available to the program. Frequently that number isnt known even when data is entered into the program. Data storage structures like arrays are sometimes inconvenient because the length of an array must be known in order to allocate memory for the structure before data can be entered.

Linked Data structures solve this problem by allocating exactly enough memory for a single data item, filling the item, then allocating enough memory for the next item and connecting the two together so that they form a collection of data items. This process is repeated for however many items there are.

These types of constructs are called **dynamic data structures**. Dont confuse this term with dynamic memory allocation- the word dynamic simply means that the action happens as the program is executing. So dynamic memory allocation (malloc) happens during program execution and a dynamic data structure is created during program execution, but the two things are separate.

3.2 Linked Lists

A linked list can be constructed in several different ways.The differences between the construction is in the number and purpose of the pointers in the node structure. Sometimes all that is needed is a simple list, where the first item in the list leads to the second item which leads to the third item and so on. This is called a **singly linked list**. A singly linked list provides no mechanism to return to the previous item. Imagine a collaborative story-writing task where each person writes a sentence on paper and then passes the paper to someone else who writes another sentence on the paper. At the end of the task, there is a story on each piece of paper and the last author is holding the list of sentences, but there is no record of who the previous authors were. That is how a singly linked list works.

A **double linked list** provides a mechanism to identify both the next item on the list as well as the previous item on the list from any position in the list. A double linked list is similar to a group of people standing in line. Any individual person can identify both the person ahead of them in the line and the person behind them.

A **circular linked list** is a list of a fixed size. The last element of the circular list is connected to the first element of the list so that from the last element the program can easily return to the first element. A circular linked list doesn't really have an end point.

The next three images show a singly linked list, a double linked list and a circular linked list

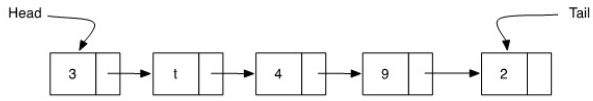


Figure 3.1: Single Linked List

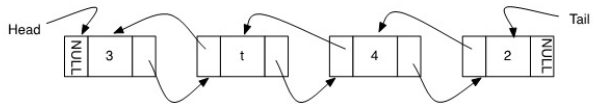


Figure 3.2: Double Linked List

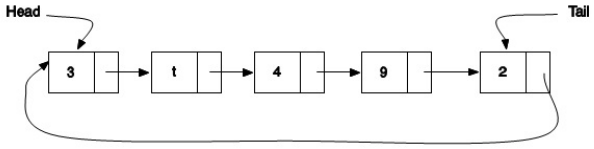


Figure 3.3: Circular Linked List

3.2.1 Implementation Details

When you write the operations for a linked list, the most challenging aspect is to keep track of the pointers that give access to the adjacent nodes in the list.

The pseudocode below illustrates the process for adding an element to the beginning of the list. This pseudocode assumes that currentElement is the beginning of the list and that newElement has already been allocated and is pointing to the list data. The order of these two lines of code is very important. Can you explain why?

```
newElement->$next = currentElement->$next
currentElement->$next = newElement
```

The following steps are required to add a node to the end of the list:

1. Initialize a new element with the desired data. This should be a separate function call.

2. Walk through until end of list is reached

3. Set the *next* attribute of the element at the end of the list to the new element.

The most challenging part of this algorithm is Step 2, walking through the list. This is an operation that you will find yourself doing frequently with linked data structures. The pseudocode for that step is shown below. Draw a list on some paper and work through the algorithm until you understand how to walk through a list to get to the end of the list.

```
currentElement = firstElementOfList //usually called the head of the list
while(currentElement->next is not NULL)
    currentElement = currentElement->next
lastElement = currentElement
```

3.2.2 Nodes

A node, or element, is the fundamental structure within a list. Regardless of your choice of implementation for a list, you will need some kind of a node structure to contain the data for the list.

When we take the time to create an ADT, we want it to be truly abstract. That means that the ADT should not be tied to a particular kind of data, since the operations on a list are identical whether it is storing integers, strings, or structs. It is a waste of time and testing to create separate ADT libraries for every possible type of data.

Instead we abstract the data by creating a data structure called a **node**. As a minimum, the node contains a pointer that points to the data being stored, and a pointer to the next node in the list.

```
typedef struct Node {
    void * data;
    struct Node * next;
}Node;
```

The type of the data doesn't matter because it is cast to a void pointer (void *). The data stored in the list might be a string; it might be an integer; or it might be a complex struct that represents a larger data record about some entity.

A linked list ADT should have a function that creates a new node, sets the next pointer to NULL, and assigns the data to the data pointer. Algorithms for working with linked lists assume that the *next pointer* for the tail element is set to NULL so that the end of the list is easily identified. It is very important to ensure that all new nodes have their next pointer initialized to NULL.

The data stored in a linked list is separate from the node definition. A node simply points to the data element. Suppose you were storing addresses in the list. The data representation would then contain elements for name, phone number, mailing address and possibly email address. A struct to represent the data might be given as follows.

```
typedef struct Address {
    char * name;
    char * telephone;
    char * mailingAddress;
    char * email;
} Address;
```

3.2.3 Operations

A list ADT typically provides functions that add elements to the list, remove elements from the list, report how long the list is, and possibly sort the list. It also must provide functions to create and destroy the list.

A minimum set of operations is shown below. The names of the functions can vary, but an operation with comparable functionality is necessary. The parameters given in the pseudcode are also a minimum set of parameters. Implemented functions may need additional parameters.

```
create(...): List
destroy(List)
insertFront(List,  DataElement):theHead
getFront(List):DataElement
deleteFront(List)
```

Once a list is created, it is manipulated through operations on the ADT functions. These functions and procedures can do whatever the programmer desires, as long as they are written carefully to **encapsulate** the implementation details of the list. While a list ADT is functional with just the minimum set of functions, usually more functions are provided with an ADT. Some common additional operations on lists include

- Finding the length of a list
  - returns an int and takes the root of the list as a parameter
  - getLength(List):int
- Finding an element of a list
  - returns a pointer to the element in the list, without removing the element from the list

  - needs a search criteria and the list as parameters, and returns a pointer to the data element, not the node
  - find(Element, List)
- Printing a list.
  - might print the entire list to stdout
  - a more elegant version returns a string (or pointer to a string) that represents a nicely formatted printout of the list elements.
  - print(List):char*
- Adding/removing Nodes at the end of the list
- Adding/removing nodes after or before a specific element
- Adding/removing nodes in a specific position in the list
- getting the length of the list
- **Iterator** operations (current, next, previous, head, tail)
  An iterator is a function that allows the user to step through the elements in a data structure. Iterators typically remember the current position so that the user can move backwards and forwards within the data structure.

3.2.4 Adding Elements

Elements can be added into a list at the beginning, in the middle, at some arbitrary location, at the end, etc. Each insert algorithm is slightly different than the others but the basic idea is the same in all cases.

1. Construct a new list element
2. Put the desired data into the new element
3. Find the location where the element is supposed to be inserted
4. Adjust the other list elements so that the new one is in the right location and so all existing elements are still part of the list

Algorithms for inserting an element in the first position and last position are shown below. The pseudocode or algorithm for inserting in a specific location is left as an exercise for you to do.

```
insertFirst(List, Element):theHead
Also Known as:addFront, insertFront, addHead, etc
Purpose: To add an element to the list at the front of the list
Preconditions: An initialized list is available.
PostConditions: The node containing the desired data is added to the front of the list, the
    length of the list is increased by one, the head of the list is set to point at the
    newly added element.

insertFirst(List, Element):theHead
     initialize a new node with the desired data
     set the next pointer of the new node to point at the first nod of the list
     set theHead  of the list to point at  the new node
```

```
insertLast(List, DataElement):void
Also Known as: addBack, insertBack, addTail, etc
Purpose: To add an element to the list at the tail of the list
Preconditions: An initialized list is available.  The new node has the next pointer
    initialized to NULL
PostConditions: The node containing the desired data is added to the end of the list, the
    length of the list is increased by one.

insertLast(List, DataElement):void
        initialize a new element with the desired data.
        walk through until end of list is reached
        set the next attribute of the element at the end of the list to the new element.
```

3.2.5 Deleting Nodes

Deleting nodes in a list uses algorithms that are the reverse of adding nodes. Nodes can be deleted from the front, the back, or any location in the list. The simplest algorithms delete nodes from the front or the back of the list.

```
deleteFront(List):Element  //often a delete method returns the value it has deleted
Also Known As: deleteFirst, removeFront, removeFirst
Purpose: To remove the first element from the list and return it to the calling procedure
Preconditions: A non-empty list is available
PostConditions: The first element of the list is removed, the length of the list is
    decreased by one,  the removed element is returned.

deleteFirst(List):Element
    set a temporary pointer(temp) to point at the first node in the list
   set the head pointer of the list to point at the second node in the list
   set the next pointer of the temporary node (the former first node)  to be NULL
    return(temp->data)
```

```
deleteFromBack(List):Element  //often a delete method returns the value it has deleted
Also Known As: deleteLast, removeBack,  etc
Purpose: To remove the last element from the list and return it to the calling procedure
Preconditions: A non-empty list is available
PostConditions: The last element of the list is removed, the length of the list is decreased
     by one,  the removed element is returned.

deleteLast(List):Element
        walk the list to find the second last element //node->next->next == NULL
    set a temporary pointer(temp) to point at the last node in the list
    set the next pointer of the second last element to NULL
   set the next pointer of the temporary node (the former last node)  to be NULL
    return(temp->data)
```

3.2.6 Other Core List Operations

```
isEmpty(List):Boolean
Purpose: To determine if the list has any elements stored in it
Preconditions: An initialized list is available
PostConditions: None

isEmpty(List):Boolean
    if theHead == NULL and theTail == NULL
    then return (true)
    else return (false)
```

isFull() is a function that is only useful in situations where a list could be full. This might happen in situations where a specific amount of memory has been allocated to the list. In that case when the memory is full, a reallocation must be effected before the list size can be increased by adding another element.

```
isFull():Boolean
Purpose:To determine if the list is filled to capacity
Preconditions: An initialized, non-empty list is available
PostConditions: None

isFull():Boolean
    if length == maxSize
    then return (true)
    else return (false)
```

```
create(...): List
Purpose: Create a new List initialized to be empty
Inputs: the function pointers for functions to manage the data stored in the list
Preconditions: None
PostConditions: A new list is created and is empty

create(...):List
```

```
    create the struct to hold the list metadata (head, tail, function pointers, etc)
    assign NULL to head (and tail if necessary)
    assign function pointers
    return(List)
\end {lstlisting}


\begin{lstlisting}
destroy(List)
Purpose: To destroy a list, freeing memory if required
Preconditions: A list exists
PostConditions: The list is destroyed

destroy(List)
   for each node in the list
       delete the data in the node using function pointer
       delete the node
   delete the struct holding the list metadata
```

3.3 Array implementation for a list

The previous sections of this document have focussed on the list composed of linked data structures (the linked list). A list ADT does not have to be a constructed using linked nodes. An array can be used to construct a List ADT. At its simplest, the array holds the data in the list and each location in the array is one element of the list. The head of the list is at the first position in the array and the tail of the list is wherever the last element is.

The array-based list stores void * data in the array in the same way that the linked node stored void * data for the linked data structure.

The choice of implementation does not change the operations required for the list ADT but it does change some of the implementation details. For example, if the user wished to insert data into the list maintaining a sorted order, the insert operation would require that room is made for the new element by shifting any subsequent elements one position in the array. If a data value was to be deleted from the list, the delete operation must ensure that the space left from the deletion is filled by shifting the tail-end of the array one space towards the head.

Because arrays must be allocated to be a fixed size in C, a new longer array must be constructed and the data copied into it if the list grows longer than the size of the array.

Fortunately, because of information hiding and encapsulation, the user of the the List ADT should never need to know whether the List is implemented as a linked data structure or an array-based list.

3.4 Array Lists vs Linked Lists

Linked Lists

- Advantages
  - Linked lists can be an arbitrary size because the list grows and shrinks as elements are added.
  - Insertion and deletion of data do not require moving other data elements, so the operations are more efficient than the comparable operation on an array structure.
- Disadvantages
  - Linked lists are less efficient in situations where the program must be able to access any element of the data at any point in the program. This type of accessibility is called **random access** and is more efficiently implemented with an array.

Array Lists

- Advantages
  - Many operations are very fast because the array indices provide direct access
  - functions are simple and easy to debug, making ADT development simpler
- Disadvantages
  - The resizing operations can be processor intensive if the list is large. There are many different strategies for deciding how much bigger to make the new array when resizing.

– To keep an array sorted, you must shuffle the elements each time you add another element.

3.5 List Iterator

An iterator is a mechanism that allows navigation of a data structure. An iterator is usually a different structure (or class in the case of object oriented programs) that is fairly tightly coupled with the implementation of the data structure being iterated.

When creating an ADT library in C, iterator functions can be included easily either with, or without the use of additional structures.

List iterator methods permit forwards and backwards navigation of the list. They are extremely useful for accomplishing insertions and deletions because the navigation code is encapsulated within the iterator operation.

Iterator methods for a list might include:

- next()
- previous()
- first()
- last()
- moveToNth()
- getCurrentElement()
- setCurrentElement()

While some of the iterator methods might seem to be duplicates of the basic list functions, an iterator has an important role to play in encapsulation. An iterator can hide the implementation of the list from the user of the library and can provide only the navigational functions to the user.

For example, an iterator for a list could be set up as follows:

```
typedef struct Iterator
{
    List * theList;
    Node * currentListPos;

}ListIterator;
```

Given this structure, the functions shown above could take a ListIterator as a parameter and provide the user with data that is stored in the list. Of course, the ListIterator would need to be initialized with enough parameters that it could, in turn, initialize the underlying list.

An algorithm for the *next()* function is shown below.

```
next(ListIterator):DataElement
Purpose: To move to the next element in a list and return the value of that element
Preconditions: The List member of the ListIterator is non-empty
PostConditions: The currentListPos of the iterator is increased by one and the data stored
    at that node is returned

next(ListIterator):DataElement
        if currentListPos is not the end of the list
           dataToReturn = currentListPos->data
           currentListPos = currentListPos->next
        return (dataToReturn)
```

Some list iterator operations, such as previous() are easier using a double linked list, but all operations are possible regardless of list implementation. The algorithm for other list iterator operations are left as a practice exercise.

3.6 Extending Activities

- Write an algorithm for the addToLocation() operation. This operation should add an element to the list at a specific location in the list (identified by a number). Use the same specification format as has been used to describe operations in this lesson. Include all necessary parameters and return values in the signature of your specification. Be sure to include preconditions and postconditions

- Write the algorithm to delete a node from the nth position of a linked list. The operation should return the deleted data and ensure that the remaining elements of the list are properly connected.
- Write the algorithm for inserting a node in sorted order given an array implementation of a list. The algorithm should take the data as a parameter.
- Write the algorithm for the list iterator operation *previous()*. It should take a ListIterator as a parameter and return the data associated with the previous node. It should also adjust the current position pointer. Write the algorithm for a double linked list and then write it again for a single linked list.

Chapter 4

Stack ADT

A full understanding of the List ADT is needed in order to fully understand stacks. If you do not understand how lists work and the associated operations of lists, please review that material before attempting to understand the Stack ADT.

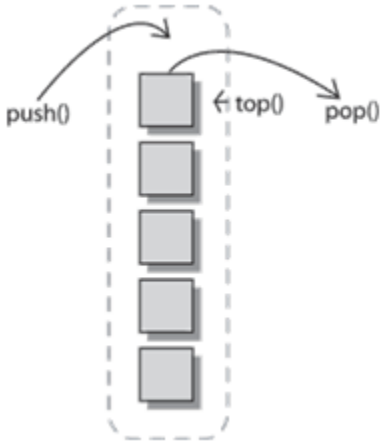


Figure 4.1: A stack data structure

A stack is a linear data structure in which all insertions and deletions occur at the head, or top of the stack. A stack ADT can only be interacted with from the front, or top, of the stack. Items can be placed on the top, and then taken off the top, but never shuffled or sorted through. A stack of building blocks on a base is a good visual metaphor for a stack. Imagine that you want to add height to the stack of blocks- you add blocks to the top of the stack. To reduce the height of the stack of blocks, you remove blocks from the stack.

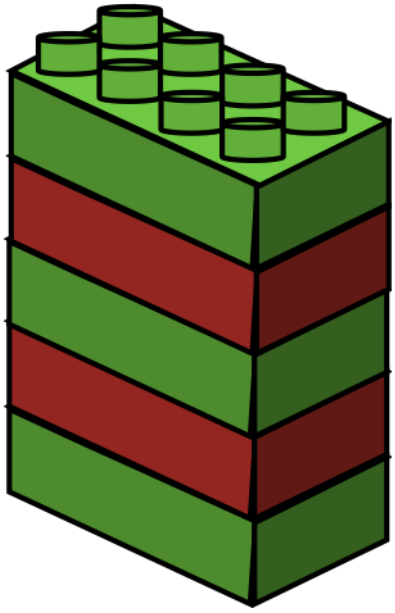


Figure 4.2: A stack of bricks

Another good metaphor for a stack is a stack of pennies. If you want a penny of a specific year on top you could remove pennies from the stack until you reach it. If the penny you want is on the bottom of the stack you would need to remove every penny above it in order to get to it. You couldnt just pull it out from beneath all the other pennies. And no, a stack implementation wont let you knock over the stack and grab the penny you want.

Figure 4.3: A stack of pennies

4.1 Why use stacks?

Often programmers feel that if they have one usable data structure that is enough. The list ADT is powerful and flexible and many programmers use it for all purposes. However, there are many times when a special purpose ADT is more appropriate. Some applications do not require the flexibility of a list, but need to meet speed or data storage requirements. A specialized ADT can often reduce overhead and speed up operations. Below are some additional reasons to be familiar with and understand multiple different types of ADTs.

- A program becomes more readable and errors become easier to find therein if explicit use of specialized data structures is made.
- Simpler interfaces (like in the case of the class stack) make specialized implementations possible which are particularly time or space efficient. For example, the Stack ADT could be implemented with the help of a particularly space-efficient list ADT, thus reducing space requirements overall.
- The use of a specialized data structure often leads to more insight into the problem definition from which further improvements in the implementation can arise then.
- To give an example: That a given stack will store only a certain number of elements known in advance during the overall runtime could be noticed by the programmer only by explicit use of a stack. A b_stack then would be the structure of choice and its use would imply an improvement of the space efficiency.

4.2 Operations on a Stack ADT

A Stack is a **LIFO data structure**. LIFO stands for Last In, First Out. The last element to be placed in the stack will be the first to be removed from it.

The most modular way to create a Stack ADT is to encapsulate a List ADT. To create a stack using an encapsulated list, you simply make use of the operations provided by the LIST ADT. For example, a list ADT might have operations such as addHead, removeHead. Those operations can be used inside the Stack ADT to provide the operations needed by the Stack. Often a Stack ADT is implemented as a **wrapper** around a List ADT. In the same way that the ListIterator used the List ADT and provided iterator operations to the user while hiding the details of the list from the user, the Stack ADT uses the List ADT and provides stack operations.

A Stack ADT structure might look as follows:

```
typedef struct Stack
{
    List * theList;
    Node * top;  //top of the stack
    int stackSize;
}Stack;
```

The operations provided with any Stack ADT must allow for creation and destruction of the stack as well as insertion and removal of elements as a minimum. Because the goal is an **abstract** data type, the stack must be initialized with pointers to functions for managing the stored data.

Since the Stack ADT presented here is a wrapper around a List ADT, most of the stack operations simply call the operations of the underlying list. The Stack ADT hides the list operations from the user of the stack.

```
create(): Stack
Purpose: To create and initialize a stack
PreConditions: None
PostConditions: A stack is created and initialized to empty

create(): Stack
      create the List sending in the appropriate function pointer parameters
     set the top  pointer to the first position in list
     return (the stack we just created)
```

```
destroy(stack)
Purpose: To destroy a stack
PreConditions: An initialized stack exists
PostConditions: The stack is destroyed and all associated memory is freed.

destroy(Stack)
```

```
     destroy theList using the list ADT destroy function
     destroy the top pointer
     destroy the stack struct
```

The push() function of a stack is equivalent to the addFront() function for a list. One of the main purposes for creating the specialized ADT is program readability and maintainability so it is important to use function names that are widely associated with the specific ADT.

```
push(Stack, Element)
Also Known As: add, insert
Purpose: Places an element in the stack
PreConditions: The stack is not full
PostConditions: An element is added to the stack, the length is increased by one, the top of
     the stack points to the newly added element

push(Stack, DataElement)
     insertFront(theList, DataElement)
     top is set to the position of the data just added
     stackSize is increased by one
```

```
pop(Stack): Element
Also Known As: remove, delete
Purpose: Removes the first element in the stack
PreConditions: The stack is not empty
PostConditions: The first (top) element of the stack is removed and returned to the caller.
    The top of the stack is set to the successor of the removed element, the length of the
    stack is decremented by one.

pop(Stack):Element
     theRemovedElement  is set to the return value of getFront(theList)
     the first element of the list is removed  by calling removeFront(theList)
     top  is set to  the new first element of theList
     return(theRemovedElement)
```

```
peek(Stack): Element
Also Known As: top
Purpose: To examine the element at the top of the stack without removing it from the stack.
PreConditions: The stack is not empty
PostConditions: Returns the element that is at the top of the stack but does not remove that
     element from the stack.

peek(Stack):Element
     theValue  is set to point at the data element at position (top) in the list. Sometimes
         a copy of the data element is made rather than providing a pointer to the actual
         data element.   Use the list operation getFront(List)
     return (theValue)
```

```
isEmpty(Stack): Boolean
Purpose: To determine if the stack is empty
Preconditions: An initialized stack exists
PostConditions: evaluates to true if the stack has no elements, false otherwise

isEmpty(Stack):Boolean
     if(theList is empty)  //use the List isEmpty() function
     then return(true)
     else return(false)
```

```
isFull(Stack): Boolean
Purpose: Relevant only in situations where the size of the stack is limited. Is used to
    determine whether the stack is full.
PreConditions: An initialized stack is available
PostConditions: Evaluates to true if the stack has reached its maximum size, false otherwise
    .

isFull(Stack):Boolean
     if(theList is full)  or the MAX_SIZE of the stack has been reached
     then return(true)
     else return(false)
```

The size of a stack is often an input that is used in algorithms involving a Stack ADT. As such it is an important piece of data to keep, even if the underlying list implementation does not keep a length. Stack operations may need to have logic added to update the stackSize struct member.

```
length(Stack): int
Purpose: To obtain a count of the number of elements currently in the stack
PreConditions: An initialized stack is available
PostConditions: returns the count of the number of elements

length (Stack): Int
     return  stackSize  (or theList->length if the list ADT keeps track of length)
```

4.3 Array Implementation of a Stack

Arrays can also be used to create a stack ADT. To use an array as a stack you simply need to ensure that the elements are added to the array sequentially and that your stack ADT keeps track of where the 'top' of the stack is in the array.

For example, suppose we allocated an array of size 8 to store a stack of integers, and a separate variable called top to keep track of the position of the top of the stack. The figure below shows the array after the integer 7 has been pushed onto the stack. The variable top has the value 0 because the top of the stack is at position 0 in the array.

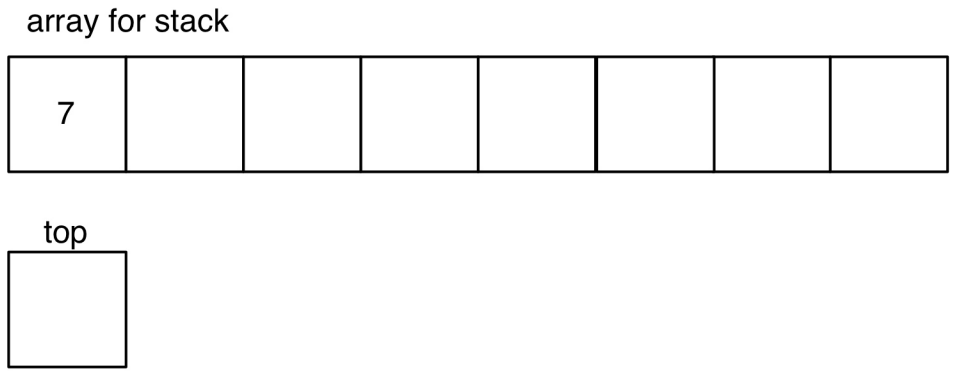


Figure 4.4: 7 at position 0

After three more elements are pushed onto the stack, the array contains four items and the top variable holds the value of 3 because the top of the stack is at position 3 in the array.

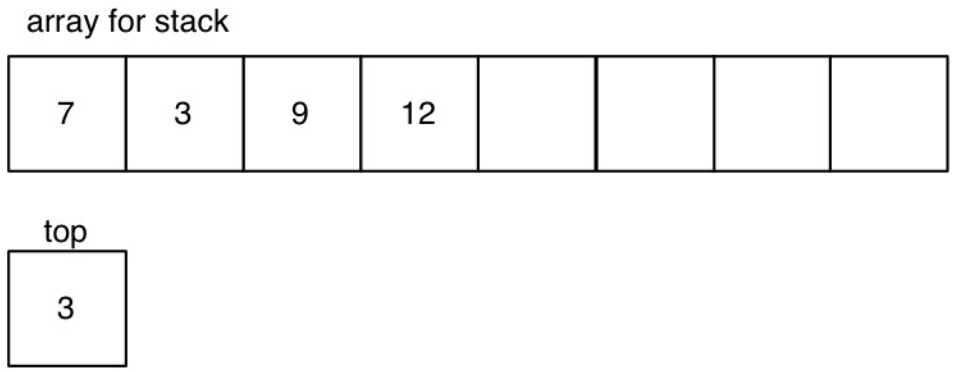


Figure 4.5: 7 at position 3

If an element is popped off the stack, the array then holds three elements and the top variable would hold the value 2 because the top of the stack moves to position 2 when an element is taken off the stack.

An array implementation of a Stack ADT needs to provide the same set of stack operations as the encapsulated list implementation. However, the operations will not call list ADT operations, they will directly manipulate the array. Because an array is a fixed size, your ADT will need to provide an isFull operation so that programmers using your ADT can avoid overflowing your stack.

4.4 Applications

A stack reverses the order of the elements stored. This property makes a stack very useful in certain programming applications including memory management and many mathematical applications

It is always a good idea to use the correct data structure for the problem. While you could use a list for everything, the use of the more specialized data structures makes your code more readable and makes it much easier to see errors of logic. A specialized data structure can be created with efficiencies designed to make the program faster, or use less memory. Also, specialized data structures make it easier for the programmer using the ADT to conceptualize the problem. If a programmer is using a stack ADT for a program that requires the properties of a stack, then the programmer cannot make errors such as taking a data element out of the middle.

Reverse Polish Notation (Postfix notation)

The most common mathematical notation is *infix* notation. The operators appear between the operands for the operation (e.g. 1 + 2, 5 * 4, a/b). Infix notation is ambiguous by itself and must be annotated with parenthesis and a set of rules for the order in which operations are carried out.

For example, consider the arithmetic problem 7 * 5 - 4 * 2. The answer could be 23 or 61 or ??. The 'right' answer is clear to those who understand the order of operations rules.

A polish logician, Jan Lukasiewicz, developed a notation that does not require parenthesis and that embeds the required order of operations in the notation. Statements are read from left to right and the previous two operands are evaluated when an operator is encountered. The table below shows several examples of Infix and the equivalent Postfix notation.

Infix	Postfix
a + b	ab+
(a-b)*c	ab-c*
(a+b)/(b*a)	ab+ba*/
(a*(b+c))/d	abc+*d/
a*((b-c)/(d+e))	bc-de+/a*

Consider the second example. Reading left to right, the first two operands are a and b, the next character is a minus sign, so b is subtracted from a giving an answer that is stored as a single operand. The next item read is a c which is an operand. The next item read indicates multiplication so c is multiplied by the next most recent operand, which is the result of subtracting b from a.

Work through each of the examples in the table so that you are confident in working with postfix notation.

A stack is the data structure of choice for creating a reverse polish calculator. Each time you read a character from the input stream you push it onto the stack. When an operation is encountered, you pop two operands off the stack, perform the operation, and push the result back onto the stack. You then continue reading the input stream.

Consider, once more, the second example in the table. As the input stream is read, a is pushed onto the stack, followed by b. The next character is an operand so two pop operations are executed and a-b is calculated. The result is pushed onto the stack. At this point the stack contains one value. The next symbol is read and is pushed onto the stack because it is not an operand. There are now two elements on the stack. The final symbol is read, two pop operations are executed and the multiplication operation is executed.

4.5 Resources

There are many good resources about stacks on the internet. A few are listed below, but this is only a small sample. It is worth spending some time to find resources that work well for your personal style of learning.

- http://www.cs.bu.edu/teaching/c/stack/array/
- http://www.algolist.net/Data_structures/Stack

4.6 Extending Activities

- Would a double linked list or a single linked list be a better choice for encapsulation in a Stack ADT? Justify your opinion.
- Write the algorithms for push() and pop() given an array implementation of a stack. Show the stack struct definition as well.
- Create a design or prototype of a reverse polish calculator program. You should limit operations to + - * and /. Use a stack ADT in your design.

Chapter 5

Queues

5.1 Introduction

A queue is an example of a **FIFO data structure**. FIFO stands for First In First Out. The defining characteristic of a FIFO data structure is that the data that has been stored the longest is the next piece of data that will be returned by a 'get data' operation. FIFO data structures do not allow the user to retrieve specific pieces of data.

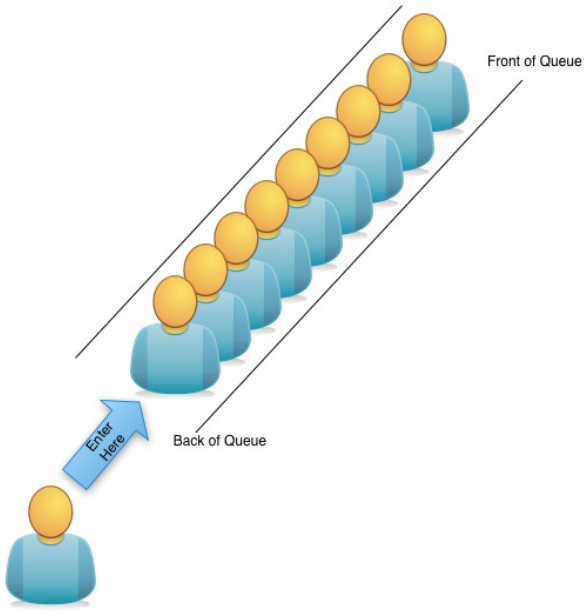


Figure 5.1: Queue

A Queue ADT usually has no size restrictions and can grow or shrink unrestricted. In some applications it is possible to constrain the size of the queue to some predetermined maximum, which allows the software developer to select extremely efficient representations for the queue.

Similar to the Stack ADT, one of the most common implementations for a Queue is the encapsulation of a List ADT. Because a Queue typically does not have a size, it is most common to use a linked list as the encapsulated ADT but an array implementation of a list could be used where max size is known and the characteristics of arrays give some needed performance advantage.

5.2 Implementation: Encapsulate List ADT

The most modular way to create a Queue ADT is to encapsulate a List ADT. To encapsulate an ADT you write your new ADT using the encapsulated one as a variable or data structure.

The Queue ADT must retrieve data elements from the front of the list (getFront(List)) and add elements at the back of the list (addToBack(List)). If the chosen List ADT does not provide an addToBack function, it cannot easily be used as the underlying ADT for a queue.

Encapsulation makes it possible to write the Queue ADT operations without worrying about the actual representation or memory management. By using a good List ADT we ensure that the representation and memory management are handled properly, and we make use of that to create our Queue.

The minimum set of operations on a Queue are:

- create()
- enqueue() //AKA add()
- dequeue() //AKA remove()
- destroy()

Other optional but useful operations allow the programmer to examine the front of the queue, determine the length of the queue, discover whether the queue is full or empty, etc.

The encapsulated Queue ADT is usually represented by a struct that is accompanied by several functions. One possible representation of the struct is shown below.

```
typedef struct Queue{
    List * theList;
    Node * front;
    Node * back;  //back is optional
    int * length;
}Queue;
```

```
create(): Queue
Purpose: To create and initialize a queue
PreConditions: MAX\_LEN has been defined previously (it could be passed in as a parameter if
     desired)
PostConditions: A queue is created and initialized to empty


create(): Queue
     theList  is set to point at the return value from create a list- sending in the
         appropriate function pointers
     front  is set to point at the first position in list
     back  is set to point at the last position in list  //back is an optional pointer
     return (the queue  just created)
```

```
destroy(Queue)
Purpose: To destroy a queue
PreConditions: An initialized queue exists
PostConditions: The queue is destroyed and all associated memory is
       freed.

destroy(Queue)
     destroy theList using the list ADT destroy procedure
     destroy the front and back pointers
     destroy the Queue struct
```

```
add(Queue, DataElement)
Also Known As: insert(), enqueue()
Purpose: adds an element to the end of the queue
PreConditions: The queue is not full
PostConditions: The new element is added as the last element in queue

add(Queue, Element)
      insertBack(theList, DataElement)
      back pointer is set last position of theList
      length of queue is updated
```

```
remove(Queue):DataElement
Also Known As: delete(),  dequeue()
Purpose: removes the first element in the queue
PreConditions: The queue is not empty
PostConditions: The first (front) element of the queue is removed and
       returned to the caller. The front of the queue is set to the successor
       of the removed element.


remove (Queue): Element
     theRemovedElement <- getFront(theList)
     remove the first element from the list
     front is set to point at the new first element of theList
     length of the queue is updated
     return(theRemovedElement)
```

```
peek(Queue):Element
Also Known As: front()
Purpose: To examine the element at the front of the queue without
       removing it from the queue.
PreConditions: The queue is not empty
PostConditions: Returns the element that is at the front of the queue
       but does not remove that element from the queue.

peek(Queue):Element
     return(getFront(theList)
```

```
isFull(Queue):Boolean
Purpose: Relevant only in situations where the size of the queue is
       limited. Is used to determine whether the queue is full.
PreConditions: An initialized queue is available
PostConditions: evaluates to true if the queue has reached its maximum
       size, false otherwise.

isFull():Boolean
     if(theList is full)  OR MAX_SIZE has been reached
     then return(true)
     else return(false)
```

```
isFull(Queue):Boolean
Purpose: To obtain a count of the number of elements currently in the
       queue
PreConditions: An initialized queue is available
PostConditions: returns the count of the number of elements

isEmpty():Boolean
     if(theList is empty)
     then return(true)
     else return(false)
```

```
length(Queue):int
Purpose: To obtain a count of the number of elements currently in the
       queue
PreConditions: An initialized queue is available
PostConditions: returns the count of the number of elements

length(Queue):int
     return length OR length(List) if the list ADT provides a length function
```

5.3 Array Implementation of a Queue

Queues can be implemented using an array in much the same way as a Stack ADT can be implemented with arrays.

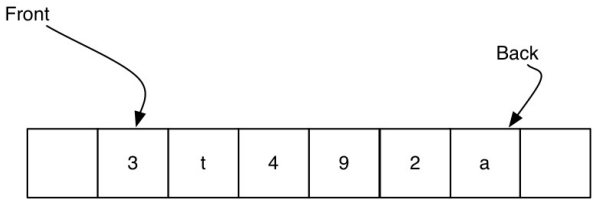


Figure 5.2: Array Queue

The programmer must keep track of the front and the back positions of the queue and, if a linear array is used, items must be shuffled up periodically in order to avoid using up all the memory in the computer.

In the example above, the array allocated to the queue has two empty spaces, one at the front and one at the back. One more element can be added to the back of the queue, and then all of the elements will need to be shuffled forwards in the queue in order to add a second additional element.

Removing the element 3 from this queue will free up one more space, but the entire set of elements will have to be shuffled forwards in order to use that space.

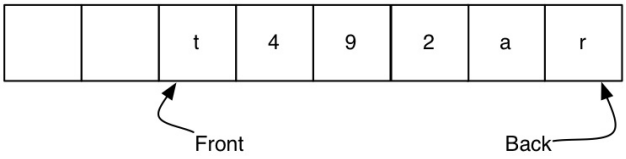


Figure 5.3: Array Queue Before Shuffling

Shuffling the elements forward is a simple algorithm, but requires that every element is moved, which can take some time if the queue is several thousand elements long. The diagram below shows that there are two spaces at the end of the queue after elements are shuffled forwards.

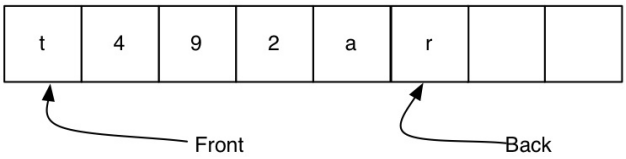


Figure 5.4: Array Queue After Shuffling

While the array implementation is conceptually simple to describe to collaborators, it comes with several complications as well. The array implementation can use extra memory if many items are added to the queue and then many are removed. The queue must either be firmly limited in size or memory must be reallocated as the queue grows and memory reallocation affects the running time of the algorithm.

A common model for an array implementation of a queue is to use a circular buffer as the representation for the queue.

A circular buffer is really just an array, but the method of dealing with the start and endpoints of the queue is slightly different. Imagine an array, but one that is arranged in a circle. Consider the picture below. The queue contains eight elements, beginning with an 'a'. After removing one element, and adding two more elements the queue will look like the second picture and has two spaces available for additional elements.

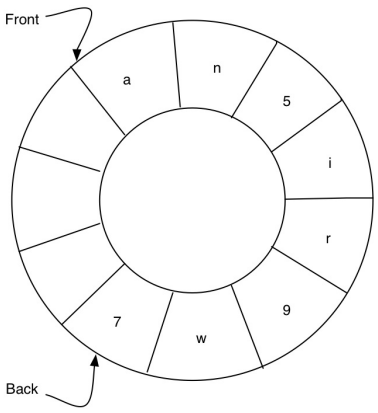


Figure 5.5: Circular Buffer Queue Before

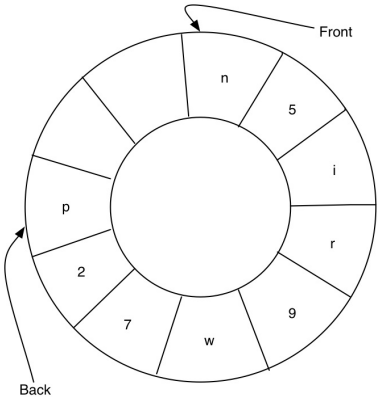


Figure 5.6: Circular Buffer Queue After

One of the advantages of a circular buffer is that there is no need to shuffle elements around as there is with an array. One of the disadvantages is that the queue must be a fixed length, else the circular buffer suffers from the same memory reallocation requirements as a linear array implementation.

Of course, computer memory isn't circular, so a 'circular' buffer is really implemented using a linear array.

5.4 Additional Resources

5.5 Extending Activities

- Read the first few sections of the Computational Complexity chapter of this work. Be sure you understand how to represent the complexity of an algorithm using **Big O** notation. What is the complexity of the enqueue() and dequeue() operations?
- What additional information must be kept track of in order to use a conventional array as a circular buffer? Give the Queue struct that you would use if you were writing a circular buffer queue implementation.
- Write the algorithm for dequeuing an item from a circular buffer queue.
- Which of the following statements about queues is untrue?
  - a) Queues can have elements inserted at any position in the data structure.
  - b) The first element inserted into a queue will be the first element taken out of the queue.
  - c) Queues can be found in the real world.
  - d) The size of a queue data structure is bounded only by the size of the computer memory.

  - e)A queue is somewhat similar to a stack.
- Which List operation would be most likely to be the one encapsulated if you were writing a remove operation for a queue?
  - a) addHead(elementToBeAdded)
  - b)length()
  - c)removeBack()
  - d)removeHead()
  - e)insert(position, elementToBeAdded)
- Given the following queue: A B b E r S T, where A is the front of the queue, what will the queue content be after two remove operations?
  - a) A B b E r S T
  - b) b E r S T
  - c) A B b E r S
  - d) T A B b E r S
  - e) The queue will be empty

  Given the following queue: t y 5 8 i 2 d t e, what should a call to length() return?
  - a)9
  - b) 8
  - c) 10
  - d) it will generate an error because of the mixed data types
  - e) 3

Chapter 6

Associative Arrays

6.1 Introduction

An associative array is a data structure that lets you manage pairs of data. Each data pair consists of a key and a value, where the key is a unique identifier and the value might repeat. An associative array does not permit the collection of data to have duplicate key values. For example, if we were storing a collection of data that consisted of student numbers and student names an associative array might be suitable. Student id numbers are unique within the university so we could guarantee that there would not be any duplicate numbers. The student number would be the key in this example and the student name would be the value. It is perfectly permissible for values to repeat. For example, it is very likely that there would be many students with names like Erin or Lynne.

Student records are a good example of data that is easily stored in an associative array. The student ID (your student number) is the key, and the other data about you (your name, your program, your GPA, etc) forms the value. It is common to have a complex data type (a struct in C) as the value in an associative array.

6.2 Implementation

Associative Arrays can be implemented using a simple array, lists, hash tables and a variety of trees. In this section we look briefly at the array and list implementations.

6.2.1 Array Implementation

An array implementation of an associative array is useful when the keys for the key/value pairs can be restricted to a finite size and the key of the data pair is an integer because the keys are used as the indices into the array.

The picture below shows an array based associative array. The left hand column is just to illustrate the position of the data and would not be stored separately. The 'key' would simply be the index into the array.

0	Sleepy
1	Doc
2	Grumpy
3	Happy
4	EMPTY
5	Bashful
6	Dopey
7	EMPTY

Figure 6.1: Array Implemented Associative Array

A sentinel value is used when there is no value for the key (The sentinel values is EMPTY in the example picture).

- Advantages:
  - An array based implementation gives very fast operations because most require only a simple index into the array (O(1)).
  - The array based implementation is also simple and easy to understand.
- Disadvantages:
  - The entire storage space must be created and initialized to EMPTY, meaning that the storage requirements are quite large compared to other implementations, especially if the set of possible keys is large.

6.2.2 Linked List Implementation

A linked list implementation of an associative array can use a specialized list node that contains the key value as an integer as well as pointers to the one to the next item in the list and the associated value.

Note that there is no **requirement** to keep a linked list table sorted in any particular order (in the picture below the list is not sorted by the key) but it often improves performance if the list is kept sorted.

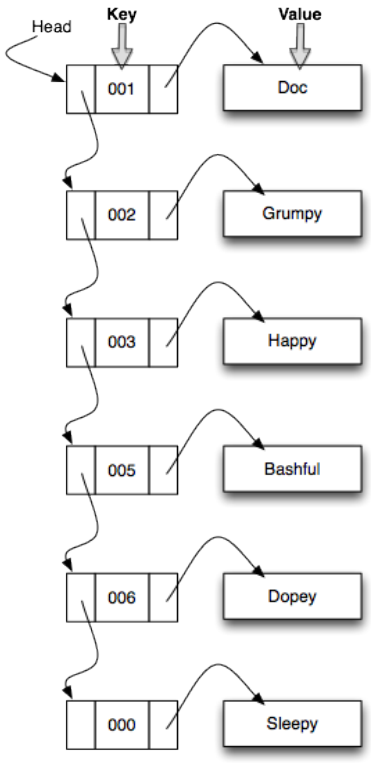


Figure 6.2: Linked List Implemented Associative Array

The picture shows a associative array that is a single linked list where the key is stored in the list node along with a pointer to the value. What would be the challenge if you tried to simply use the position of the node in the list as the key?
The struct for a linked version of an associative array might look like this:

```
struct aarray{
    int * key;
    node * next
    void * data
    };
```

The reason for adding the key explicitly into the struct rather than leaving it as part of the void*data is that the operations on the array need to compare the key value and would not be able to do that if the key was hidden in the abstracted data.

- Advantages:
  - The size of the associative array can grow arbitrarily
  - The keys do not need to be restricted to a simple integer- for example the key could be a unique alphanumeric
- Disadvantages:
  - The cost of each operation is dependent on the size of the associative array

6.3 Operations on Associative Arrays

Operations List (Minimum Set):

- create(): AArray
- destroy(AArray)
- insert (AArray, key, value)

- remove(AArray, key)
- lookup(AArray, key):value
- update(AArray, key, newValue)

```
create():AArray
Purpose: to create an empty, initialized associative array
Preconditions: none
Postconditions: an empty, initialized associative array is created

create():AArray
      1. create the storage ADT
      2. create the key ADT
      3. return(the AArray we just created)
```

```
destroy(AArray)
Purpose: to destroy an associative array and free the  memory if required
Preconditions: an initialized associative array
Postconditions: the array  is destroyed along with all references to data. (Important note:
    the data itself may not be destroyed by this operation and should be removed prior to
    destroying the AArray).

destroy(AArray)
      1. the appropriate destroy procedure
      2. release any other pointers
```

```
insert(AArray, key, value)
Purpose: To add a key/value pair to the associative array
Preconditions: no value exists for the key being added
Postconditions: the size of the AArray has increased by one, the key and value are stored
    with a reference leading from key to value.

insert(AArray,key,value)
      1. key into key ADT
      2. connect key and value with reference (pointer)
      3. increase length counter of AArray by one
```

```
remove (AArray,key): value
Purpose: to remove a key/value pair from the associative array
Preconditions: the  key/value pair is stored in the AArray
Postconditions: the key/value pair is no longer in the AArray. The length of the AArray has
    decreased by one.

remove (AArray,key): value
      1. look up key in key ADT
      2. follow reference to value in value ADT
      3. store value in temporary variable
      4. remove value from value ADT
      5. remove key from key ADT
      6. return(temporary variable)
```

```
lookup (AArray, key): value
Purpose: to retrieve the value stored for a particular key
Preconditions: the key/value pair is in the AArray
Postconditions: the value is returned. The AArray is unchanged.

lookup (AArray,key): value
      1. look up key in key ADT
      2. follow reference to value in value ADT
      3. store value in temporary variable
      4. return(temporary variable)
```

```
update (AArray, key, newValue)
Purpose: the change the value associated with a key that already exists in the associative
    array
Preconditions: a key/value pair exists for the given key
Postconditions: the new value is associated with the given key
```

```
update (AArray, key, newValue)
      1. look up key in key ADT
      2. follow reference to value in value ADT
      3. set value to newValue
```

```
exists(key):Boolean
Purpose: to determine if a key is represented in the associative array
Preconditions: an initialized AArray is available
Postconditions: n/a

exists(key):Boolean
      1. look up key in key ADT
      2. if key is found return(true)
      3. else return(false)
```

```
isEmpty():Boolean
Purpose: to determine if any key/value pairs are represented in the associative array
Preconditions: an initialized AArray is available
Postconditions: n/a

isEmpty():Boolean
      1. if the key ADT is empty; return(true)
      2. else return (false)
```

```
isFull():Boolean
Purpose: to determine if the AArray is full
Preconditions: an initialized AArray is available
Postconditions: n/a

isFull():Boolean
      1. if the key ADT is empty; return(true)
      2. else return (false)
```

6.4 Applications

Associative arrays are useful in any situation where you need to maintain a connection between two pieces of data, but where the data might be changed frequently during the running of the program. As an abstract example, AArays are less appropriate for connections between a person and their phone number- which typically stays the same, and more appropriate for representing the connections between a parking lot ticket number on a vehicle and a parking spot.

Associative arrays are really only useful if the key portion of the key-data pair maps nicely to the positions in an array. This is most likely to occur in situations where data is generated sequentially, such as ticket numbers for a coat check, or order numbers for an online store.

6.4.1 Lending Library

Scenario: Suppose you wanted to implement a lending library for your collection of computer games so that each game could only be lent out to one person at a time (no copies allowed). The core data structure used by this application could be an associative array.

Requirements:

- Each computer game is assigned a unique ID number (0-N)
- Borrowers can have more than one game checked out at a time
- A game can only be checked out by a single borrower at any one time

Context

The application would obviously need some user interface constructs to enable data entry and some file storage utilities to permit persistent storage. For the purposes of this example, we will assume those details have been worked out already.

Programming Constructs we would need to write this program

- A way of creating borrowers
- A way of identifying borrowers
  - The set of borrowers will also need to be in an ADT. If you give each borrower an ID, then an AArray might be a good choice for this as well.
- A way of creating and populating the list of games that can be borrowed
- The AArray representation for the set of borrowed games

Worked example

Lending Library pseudocode

- Create and populate borrowers data structure
- Create and populate the games data structure
- Initialize the AArray of borrowed games to empty
- If a game is borrowed add the game/borrower pair to the AArray representing the borrowed games
- if a game is returned remove the game/borrower pair from the AArray representing the borrowed games

6.5 Extending Activities

- If you were implementing a student record system for a small music school with fewer than 200 students, and were using associative arrays as the data structure, how would you implement your associative array? Would you make a different choice if your student record system was for a university?
- Write a 'todo list' of the things you would need to do (including research and learning) in order to implement the borrowers library example.Try to make the list very specific. Use the todo list to estimate the amount of time it would take you to implement and test the application (round up to the nearest hour). subsubsection

Chapter 7

Hash Tables

7.1 Introduction

Definition: A Hash Table is an array (aka Table) that uses a hash function to map keys to a specific location in the array. The position of the entry in the table is determined by the value of the key.

A hash table (also knowns as a Hash Map) is a more flexible version of an associative array that does not require that the key of the the key-data pair be something that easily maps to a position in a table. Instead the hash table calculates the table position by algorithmically manipulating (hashing) the key to determine the correct position in the table. A hash table is useful in situations where the data key-value pairs might not have key values that are nicely distributed to allow them to be mapped to table positions between 0 and the end of the table.

The **Hash Function** a function that, when given a key as a parameter returns a an integer within a fixed range. The hash function ALWAYS returns the same value for a given input, but may return the same value for more than one input. For example, a hash function could take in the province and plate number from a car licence plate (something like ONBWCC123) and manipulate the digits and letters to produce an integer that maps to a position on the table. If the hash table were of size 357, the integer would be between 0 and 356.

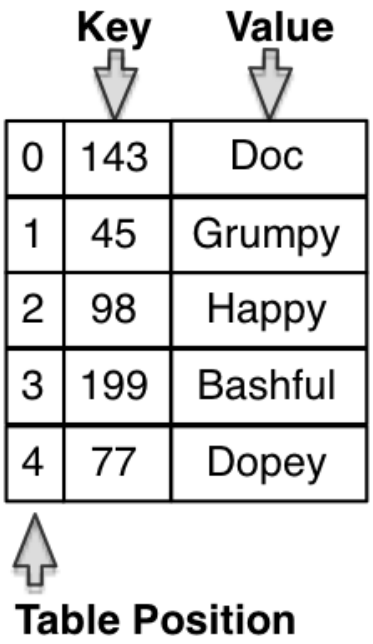


Figure 7.1: Hash Table

Image 7.1 shows a 3x5 table. The first column is labeled as Table Position, the second shows the key, the

third shows the value. There are five sets of data in the table.

The hash table shown is a simplified table that has the data stored directly in the table. A finished implementation of a hash table ADT would store void * data in the same way that a linked list does. A hash table is nearly always implemented using an array, since the primary purpose of using a hash table to facilitate speedy lookups for data.

Real World Example: Spelling checkers are an application that can employ a hash table. Because the dictionary of properly spelled words can be quite large, a search through that dictionary for each word in the document being checked is the most time consuming part of the operation. The dictionary can be read once into a hash table, where each word in the dictionary is put through a hash function and stored in the table at the appropriate position. In this example the word is both the key AND the data. After the table is loaded each word of the document to be spell checked is hashed

7.2 Hash Table Operations

The minimum set of operations for a hash table are exactly the same as the operations for an associative array.

The only change is the process for inserting and finding data because the hash table must use the hash function to determine where the new data goes and where the data can be found.

The implementation of the operations changes depending on the collision resolution strategy chosen.

Operations List (Minimum Set): This is exactly the same operations list is for an associative array. The algorithms for the operations are also the same and are not repeated here.

- create(): HTable
- destroy(HTable)
- insert (HTable, key, value)
- remove(HTable, key)
- lookup(HTable, key):value
- update(HTable, key, newValue)

7.3 Hash Table Characteristics

- Hash tables can provide an improvement over the search time for binary trees (binary tree search time is O(log 2 n). The cost of search in a hash table is the time to execute the hash function + any time to resolve collisions which is usually O(1).
- Hash tables are efficient for large sets of data, but sometimes the hash function is computationally expensive which makes other data structures more effective for small sets of data.
- The most often used operations on a hash table are the operations for finding and inserting. These operations are dependent on the hash function. The selection of hash function is key to the efficient operation of the hash table.
- The set of keys for the data is called the key space. The key space is often larger than the size of the table in memory (the address space), which
- The hash function maps the key space into the address space. More specifically the hash function takes a key and uses it to produce an address that is in the defined address space of the allocated table. The same address must be produced every time given the same key.
- Because the key space is typically larger than the address space, more than one key will be mapped to the same address. This is called a collision and must be handled through collision resolution. The selection of collision resolution method is an important part of hash table design.

7.4 Hash Functions

The hash function must map the keys in the data to a position in the table. A hash function needs to know the size of the table (usually sent in as a parameter) in order to determine the appropriate position.

The characteristics of a good hash function are:

- speed: the function cannot be slow to run
- space: the hash function should not take a lot of storage space
- generation of positions: the function should spread the keys out evenly across the whole hash table (no clustering in one part of the table)

All hash functions take in a key of a particular type as a parameter and return a value between 0 and the size of the table. Some hash functions are written with a hard coded table size, some take the table size as a parameter. Some hash functions work on numeric keys, some work on alphanumerics. There are many different possible hash functions. The subsections below detail some possible approaches to writing hash functions.

7.4.1 Before Hashing: Preconditioning

Keys in the key space frequently contain alphanumeric characters, but hash functions are often more effective on keys that can be manipulated numerically. One common approach to hashing is to convert alphanumeric keys to a numeric value through a process called preconditioning. Preconditioning can be as simple as replacing each letter with the ascii code for the letter, or it can do more complex manipulations such as normalizing the length of the keys in addition to converting them.

7.4.2 Hashing by Truncation

The **truncation** method of hashing simply uses a portion of the key as the address space and ignores the rest of the key. This method is also known as **Digit Analysis**

The truncation method only works for keys that have consistent characteristics. For example, suppose you had a set of 8 digit keys such as 46 234 789 and your hash table size was 1000 (hash values between 0 and 999).

One approach to truncation for hashing this number could be to simply select the 1 ^{s}t , 3 ^{r}d and 5 ^{t}h digits of the key (424 in this case). Another approach to truncation for hashing this type of key could be to select the first, third and last digits and then reverse the order (924 in this case).

The approach to truncation must be consistently applied to every key and must always give the same results, but otherwise any sort of digit manipulation that results in a suitably sized address space can be considered as an approach to truncation.

The truncation method has the advantage that it is very fast, but it often results in many collisions because the hashed values often dont distribute well within the entire table.

7.4.3 Hashing by Folding

The **folding method** of hashing involves separating the key into parts that are the same length as the address space and performing some kind of math on those parts. Often one part is composed of the leftovers and is a different length. A common method of folding is to add the parts together once the parts are identified, ignoring the final carry. The result is used as the table address for the original key.

For example, suppose that the key value of 456 987 321 must be hashed into a table of size 1000 (hash values between 0 and 999). The original key is partitioned into three segments, each three digits long and those segments are added together. $456 + 987 + 321 = 1764$. The 1 thousand is ignored and the hash key becomes 764.

The folding method is a fast hashing algorithm and results in more evenly distributed addresses than truncation. Even though it is better than truncation, it can still result in poorly distributed addresses.

7.4.4 Hashing by Division

Integer division can be used to consistently reduce a large integer to a small one. Recall the modulo operator, which is used to find the remainder value after integer division. 15 modulo 4 (15%4) yields the value 3. The only possible values from an expression that is modulo 4 is 0, 1, 2 or 3. If we use larger numbers, say 10000 modulo 999, the remainder value will have to be a number between 0 and 998. We can use this characteristic of modulo to calculate a hash value of a large number.

The division method hash function calculates the hash value of a key, H(k) by finding the remainder that results from dividing k by some positive integer (modulo). More formally **H(k) = ABS(k)%m** where m is a large positive integer, % is the modulo operator and ABS is the absolute value.

The division method of hashing will return addresses that are between zero and one less than the divisor (m), so it is important that m is also the size of the table, otherwise this method will result in invalid addresses.

This method will map keys that are close in value to different addresses, but will map keys that are multiples of each other to the same address, so its effectiveness depends on the characteristics of the original key space. Consider the hash values calculated below, for an m value of 11.

Key	Hash
12	1
13	2
14	3
56	1
57	2
58	3
239	8

The division method can result in very good distribution of address locations depending on the value chosen for m. The table size (m) should not be set to an even number, or a power of 2 or 10. Prime numbers, and odd numbers whose factors are all over 20 are good choices for the table size (m).

7.5 Collision Resolution for Hashing

When two keys hash to the same table position we say that a **collision** has occured. Even if the keys for the data are unique, it is likely that sometimes the hash function will calculate the same table position for two keys. When this happens, the hash table must find a different location to store the second set of data, since two pieces of data cannot be stored in the same memory location. The process of locating the alternate storage location is called **collision resolution**.

Two common approaches to collision resolution are **open addressing** and **separate chaining**. Open addressing solutions use an algorithm to search in a repeatable fashion for an alternate location. Separate chaining solutions create additional table space by using a linked data structure.

7.5.1 Open Addressing

When a collision occurs in an open addressing hashing system, the collision is resolved by trying alternative cells until an empty cell in the hash table is found. In effect some number is added to the calculated cell position to arrive at a new cell to try, and the formula is set to wrap-around to the beginning of the table when the end is reached.

The number of filled cells in the hash table is used to calculate the *load factor* of the table (number filled/size of table). Open addressing is a collision resolution technique that works best with a table that has a load factor of .5 or less.

The offset is calculated by a collision resolution algorithm. A collision resolution algorithm must ensure that it always finds an empty cell if one is available. We will examine three open-addressing collision resolution algorithms that are used : linear probing, random probing, and double hashing. There are many more possible approaches and several variations on the three we will examine, but these three serve to illustrate the concepts behind open addressing to resolve collisions.

Suppose that the items in the table are names.

- Sleepy is hashed to position 0
- Doc and Grumpy are hashed to position 3
- Sneezy is hashed to position 4
- Happy is hashed to position 5
- Bashful and Dopey are hashed to position 7

The table state, after each addition is shown in the diagram below.

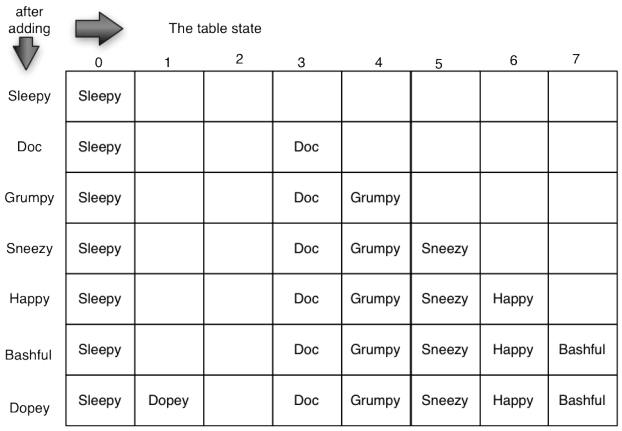


Figure 7.2: Linear Probing

While linear probing always finds a cell if one is available, it tends to cluster the data (as you can see in the diagram). This type of clustering is called **primary clustering**. Also, as the table gets full, the number of probes required increases which slows down insertions and searching.

Random Probing

One way to avoid primary clustering is to change the probing mechanism so that it does not return consecutive positions. The probing strategy cannot truly be random because must still generate every position in the table exactly once for any given collision, and it must generate exactly the same sequence again given the same collision. However, it does not need to generate new table positions in the order that they appear in the table.

One example of such a strategy adds some positive number to the location of the collision, and then finds the modulo value with respect to the table size.

```
{ newLocation = (mostRecentLocation + constant) mod tableSize}
```

This strategy works well when the constant and the tableSize share no common divisor other than 1. Random probing suffers from **secondary clustering**. Secondary clustering occurs when the same exact sequence of positions is generated when two keys are mapped to the same position more than once during a search for position.

Double Hashing or Rehashing

Double hashing is an effective collision resolution strategy that avoids clustering. When there is a collision of hashed values for keys, the colliding key is hashed again using a different hash function. The result of the second hash function is not used as a position, but is used as the constant value in the equation for random probing.

```
newLocation = (mostRecentLocation + valueFromSecondHashFunction) mod tableSize
```

Linear Probing

Linear probing is a strategy that places the collided data in the next available cell. When the end of the table is reached, the search wraps around to the beginning of the table.

Double hashing usually eliminates clustering in the hash table because the sequence of values generate for any two colliding keys will not be the same.

7.5.2 Separate Chaining

Separate chaining is another approach for handling hashing collisions. Rather than try to find an alternate table position, the separate chaining approach stores the colliding records for a single table position in a linked list. Instead of storing data directly in the hash table, the linked list is stored in the hash table at the hashed position.

The separate chaining method requires memory storage space for the hash table as well as for the linked lists, so it is more memory intensive than an open addressing approach to collision resolution.

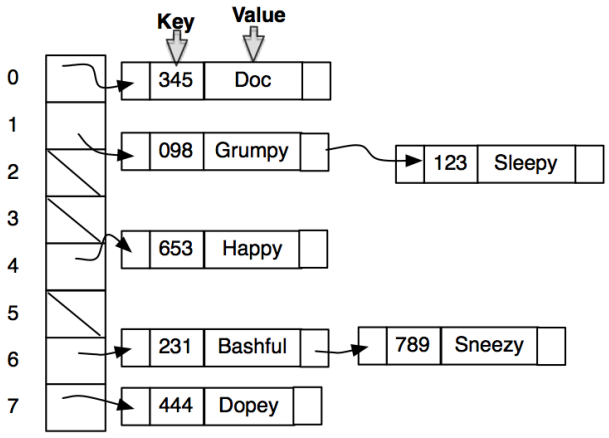


Figure 7.3: Separate Chaining

Separate chaining is often implemented so that none of the data items are stored in the actual hash table. Instead the hash table simply houses pointers to the linked lists of data that hash to that location.

Algorithms for separate chaining are easy to construct, as the first step is a simple lookup in an array and the rest of the data structure can be manipulated through linked list operations. If the hash function is good, there will be few collisions and the linked lists will be short.

Even though this technique requires more storage space than an open addressing approach, the simplicity and its performance make it a popular choice in hash table design.

7.6 Additional Resources

An excellent hashing tutorial with several animations can be found here:

http://research.cs.vt.edu/AVresearch/hashing/

This tutorial reviews most of what we have learned in this lesson, and expands the topics somewhat. The animations are excellent and worth exploring.

7.7 Extending Activities

- Using the division method, calculate hash values for the following set of keys. Calculate once using a table size of 11. Calculate a second time using a table size of 12. do you notice anything about the distributions of the calculated values?
  Keys: 54 77 82 13 991 308 68 45 1001 73
  x
- The expected search time for Linear probing can be calculated by the following formula:
  O(1 + 1/(1-loadFactor)) for a successful search and **O(1+1/(1-loadFactor)$\hat{}$2)** for an unsuccessful search.
  Create a chart showing the expected search times (successful and unsuccessful) for the following load factors: .10, .20., .30, .40, .50, .60, .70, .80, .85, .90, .95.
  What do these numbers tell you?
- Suppose you have two hash functions H1 and H2 where H1(87) = 10, H2(87) = 3 and H1(42)=10, H2(42)=7. Further suppose that the key 87 is inserted into the table first and then the key of 42. Show the sequence of table positions tried when using random probing with a constant of 37 and a table size of 11.

- The expected time for a search using Double hashing can be calculated as shown below:
  successful search: (1/loadfactor)(1+ ln(1/(1-loadfactor)))
  unsuccessful search: 1/(1-loadfactor)
  Calculate a chart of the predicted search times (successful and unsuccessful) for the following load factors: .10, .20., .30, .40, .50, .60, .70, .80, .85, .90, .95.
  What do these numbers tell you?
- The loadFactor of a hash table using separate chaining to resolve collisions is calculated as the **number of keys/ the number of chains** The expected length of search , successful or unsuccessful, of Separate Chaining is O(1+loadFactor).
  Calculate a chart of the predicted search times for the following load factors: .10, .20., .30, .40, .50, .60, .70, .80, .85, .90, .95.
  What do these numbers tell you?
- What factors would you consider when selecting a collision resolution approach for a hash table?

Chapter 8

Introduction to Trees

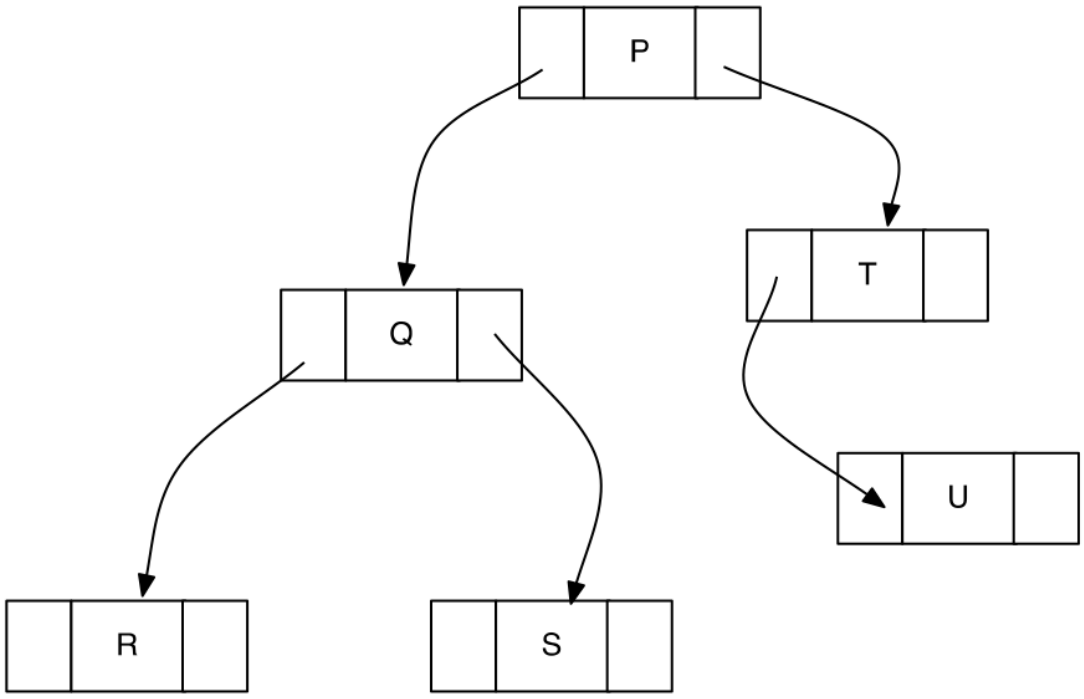
A tree is somewhat similar to a linked list, but, when used properly, can provide very fast access to specific data elements because data is stored in a sorted fashion. There are many new terms in this chapter.

The first section will familiarize you with the terms and concepts you will need to understand the algorithms for working with trees in your programs.

Subsequent sections will help you understand the algorithms associated with constructing and maintaining a tree. It is to your advantage to ensure that you understand the concepts in this chapter before trying to learn about the specific algorithms for binary trees. Trees are a fundamental data structure in many programming languages and are the base data structure for several other data types including heaps, priority queues, and search trees.

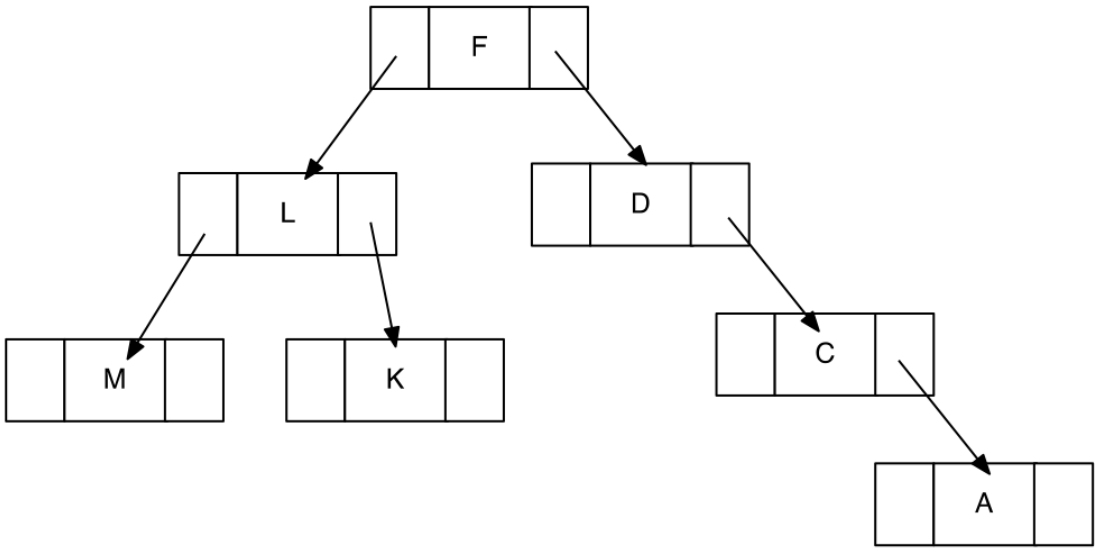
8.1 Simple Rooted Trees

A tree ADT is similar to a linked list in that it consists of a series of connected nodes. Unlike a linked list node, a tree node can be connected to more than two other nodes.



Consider the picture above. It shows a tree with six nodes. There are several terms that you must understand before you can really understand how a tree ADT works.

- Nodes in trees have relationships to other nodes. We label these relationships as *parent*, *child*, *ancestor*.
  - Node Q in the picture is a child of node P. Node P is the parent of nodes Q and T. Node P is the ancestor of node S.
  - Nodes can only have one parent
- A tree has a *root node*. In fact a tree can have only one root node. The root node has no parent. In the example tree, P is the root node.
- Trees have branches. The path between two nodes is called a branch. In the example, nodes P and Q have two branches, node T has one branch and nodes R, S and U have zero branches.
- Nodes with no branches are known as leaf nodes.
- A tree has a height. The height is the number of nodes in the longest path from the root to a leaf. The height of the example tree is 2 because the longest path is from P to Q to (R or S). Some algorithms for calculating height count the root node as height one. For this course we will not count the root node in the height by default. If the root node is counted it will be explicitly noted.
- Each node in the tree has a level. In the example, nodes Q and T are on level 1 because they are children of the root. R, S and U are on level 2.
- Trees are recursive: each branch of a tree (each subtree) is also a tree.



Can you answer the following questions about the tree image shown above?
How many nodes are in the tree? (answer 7)
What is the height of this tree? (answer 3)
Which node is the root node of the tree? (answer F)
Which nodes are leaf nodes of the tree? (M, K, A)
Which node is the parent of node K? (L)
What is the largest number of branches that any node in this tree has? (2)

8.2 Operations on Trees

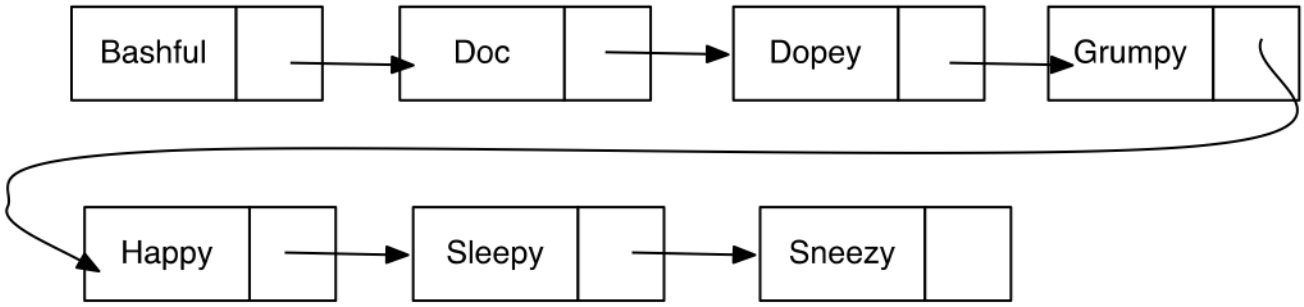
8.2.1 Tree Insertions

When nodes are inserted into a tree, the operation is performed such that the nodes are kept in sorted order in the tree. Sorted order can be defined any way that the programmer wishes. For example nodes could be sorted alphabetically, or they could be sorted by numeric value, length, or by some other attribute.

Data entry into a tree is a bit more complicated than it is for linked lists. Suppose we were creating a linked list from the following names and we wished to keep the list sorted:

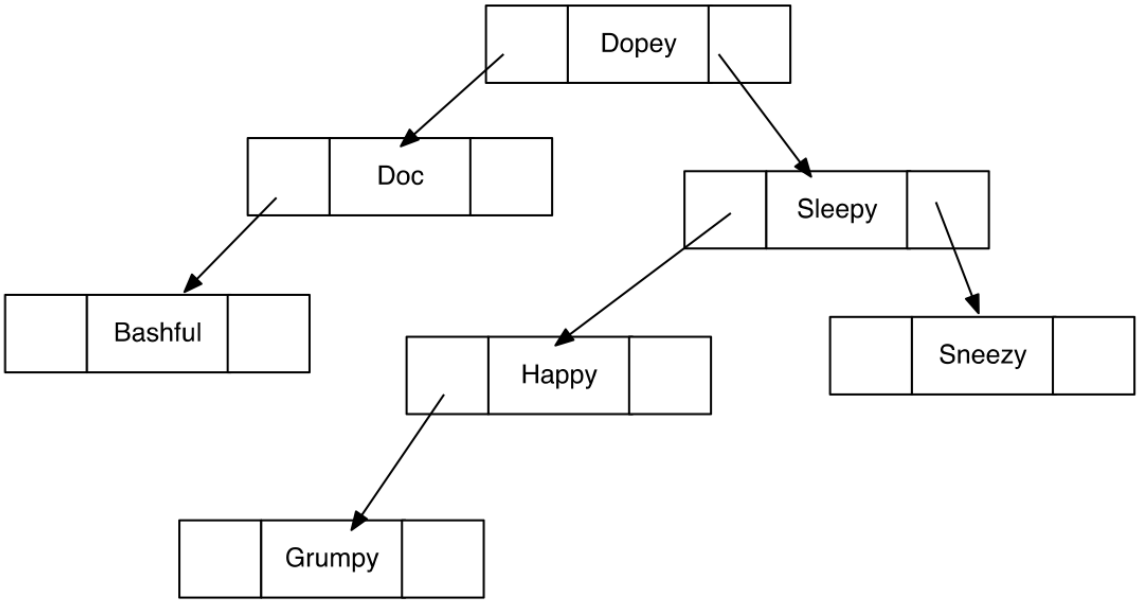
- Dopey
- Sleepy
- Happy
- Doc
- Sneezy
- Grumpy
- Bashful

We would create the list with the first entry of Dopey and then we'd insert Sleepy behind it. Happy would get inserted between Dopey and Sleepy, Doc would be inserted before Dopey, Grumpy would be added after Dopey and Bashful would be inserted at the head of the list. The completed list would look something like this:



However, if we were storing the same data in a tree ADT the insertion/addition process would be slightly different. Trees use a compare function to decide if the node to be inserted is larger, or smaller than the node being compared. Nodes that are larger than the current node are placed in the right branch, nodes that are smaller than the current node are placed in the left branch.

So if we started the tree by entering Dopey, the second node, Sleepy, would be placed in the right branch because Sleepy is 'larger' alphabetically than Dopey. Happy also goes to in the right hand branch of Dopey, but in the left hand branch of Sleepy because Happy is bigger than Dopey, but smaller than Sleepy. The finished tree can be seen below:



8.2.2 Removing nodes from a tree

Because tree nodes can have two children, the removal of nodes from a tree is more complicated than removing elements from a list. Consider the tree shown in the previous section. If "Doc" were removed from the tree, the tree could be reconfigured so that Dopey pointed to Bashful and the sorted nature of the tree would not be compromised.

However, if "Sleepy" were to be removed from the tree, the reconfiguration is much more challenging. Should Dopey point to Happy or to Sneezy? What happens to the other node? Many tree libraries do not actually delete an element from a tree on removal, instead the library marks the element "inactive". It holds its place in the tree, but cannot be returned on searches and cannot be updated.

We will discuss removing nodes from trees more in subsequent chapters.

8.2.3 Tree Traversal

Nodes in the trees are accessed by traversing the tree. Traversal is simply a process of walking through the tree by using the pointers that connect nodes to move from one node to the next. Tree traversal always starts with the root node of the tree. An **iterator** can be used to traverse a tree if the Tree ADT comes with an iterator function.

Imagine that your task was to print all of the values of the nodes in the tree. What order would they be printed in if you started at the root node? The answer to that question depends on when you printed each node and which branches of the tree you followed first. The different patterns of walking through the tree are traversal schemes. Two common schemes are in-order traversal and pre-order traversal. Both of these traversal algorithms are recursive.

Pre-order traversal

Another common traversal technique is Pre-order traversal. The general idea is the same but the algorithm changes slightly to process the root node first, before any subtrees are processed. The general algorithm is:

process current node
process left subtree
process right subtree

The pre-order traversal of the tree example shown in section YYYYY is Dopey, Doc, Bashful, Sleepy, Happy, Grumpy, Sneezy.

```
We start at the root (the current node) and print it - Dopey.
  Recurse on the left subtree - printing Doc
        Recurse on Doc's left subtree- printing Bashful
           Bashful has no left subtree
           Bashful has no right subtree
           Return
        Doc has no right subtree
     Recurse on the right subtree - printing Sleepy
        Recurse on Sleepy's left subtree - printing Happy
           Recurse on Happy's left subtree printing Grumpy
              Grumpy has no left subtree
              Grumpy has no right subtree
              return
            Happy has no right subtree
            Return
          Recurse on Sleepy's right subtree printing Sneezy
                Sneezy has no left subtree
                Sneezy has no right subtree
                Return
```

In-order traversal

An in-order traversal visits each node in sort order and processes the node as it is visited. For example, if the nodes in the example tree were traversed in order, and the processing action was to print the value, the result would be an alphabetized list of names.

To conduct an in-order traversal, the algorithm must move to the smallest value in the tree (which will be the left most leaf), process that leaf, move to its parent, process the parent, move to the right branch and repeat.

For each tree (and subtree) the algorithm is:

- process left subtree
- process root
- process right subtree

The key difference between pre-order and in-order traversal is when the root node is processed. The in-order traversal for the example tree is shown below:

```
Start at the root but do not process it
Recurse to process the left subtree.
        This means we must move to Doc but do not process it because it is the root
                Recurse to process the left subtree
                        This means we must move to Bashful but do not process it
                        Bashful has no left subtree
                        process the root and print Bashful
                        Bashful has no right subtree so we return
                Process the root and print Doc
                Doc has no right subtree so we return
        process the root and print Dopey
        Recurse to Process the right subtree
                This means we must move to Sleepy
                        Recurse to process the left subtree
                        This means we must move to Happy
                        Recurse to process the left subtree
                                This means we must move to Grumpy
                                Grumpy has no left subtree
                                process the root and print Grumpy
                                Grumpy has no right subtree so we return
                        process the root and print Happy
                        Happy has no right subtree so we return
                Process the root and print Sleepy
                Recurse to Process the right subtree
                        This means we move to Sneezy
                        Sneezy has no left subtree
                        process the root and print Sneezy
                        Sneezy has no right subtree so we return

                        done.
```

8.3 Binary Trees

While a tree ADT can have any number of branches theoretically, often we limit the number of branches. A binary tree (one with nodes that can have at most two branches) is a very common ADT.

8.3.1 C Structure for Binary Tree ADT

As with any ADT, the Binary tree needs a structure to hold the pointers to its connected nodes and to hold a pointer to the data. A binary tree node is usually labeled with pointers called 'left' and 'right' to denote the child nodes.

```
struct BinTreeNode
{
        void * data
        struct BinTreeNode * left
        struct BinTreeNode * right
        Tree * myTree;  //not required, but  useful for getting the function pointers
};

struct BinTree
{
        int compare function
```

```
        void destroy function
        Struct BinTreeNode * root;
};

typedef Struct BinTree Tree;
```

Because the Binary Tree ADT stores *void ** data, the user of the tree ADT must supply a function pointer for the destruction of the data and clean up of memory as well as for comparing data elements to determine the position of the data in the tree.

The Tree ADT must guarantee that all nodes that are removed, either individually or when the tree is destroyed, will be properly removed from memory using the supplied function.

Also, because data in a binary tree is kept in sorted order, the user of the ADT must supply a function pointer to a compare operation that can compare the values of two data items. (If you are fuzzy about function pointers, go back to the function pointer review).

Binary tree operations can be divided into two sets. The first set are sometimes called wrapper operations These are the operations that the user of the ADT will call. Wrapper functions typically take the Tree Struct as a parameter, possibly along with other paramters.

The second set of operations are the helper operations. These are the operations that actually do the work of setting up, manipulating, and keeping track of the tree. The helper operations typically take the root node of the tree as a parameter (the **root** member variable of the Tree struct) along with other paramters. Often helper operations for trees are recursive. Below you can find a list of some common wrapper and helper operations on binary trees.

```
//ADT Wrapper Operations

createBinaryTree( compare function pointer, destroy function pointer):BinTree
destroyBinaryTree(BinTree *): void *
addToTree (void * data)
removeFromTree(void * data): Boolean
findInTree(void * data):void *

//Helper Operations

TreeNode * insert(TreeNode * root, TreeDataType data);
TreeNode * delete(TreeNode * root, TreeDataType data);
bool isEmpty(TreeNode * root);
bool hasTwoChildren(TreeNode * root);
int compare (TreeDataType data1, TreeDataType data2);
TreeNode * findMinimum(TreeNode * root);
void printInOrder(TreeNode * root);
```

In addition to wrappers and helpers, most tree ADTs provide operations that will traverse the tree and do something during the traversal. The elegant way to provide node-processing functionality is to provide traversal operations that accept a function pointer. The function is executed once for every node in the tree.

Lets start our exploration of binary trees by looking at the helper functions starting with insertion. We'll ignore the data types and function pointers for now, but will return to them towards the end of this chapter.

8.3.2 Recursive Helper Functions

Insertion

Insertion into a tree has three possible cases:

1. The tree has no nodes, and the value will be inserted as the root node
2. The value to be inserted is larger than the value at the root, and will be inserted in the right subtree of the root node.
3. The value to be inserted is less than the value of the root, and will be inserted into the left subtree of the root node.

Cases 2 and 3 result in a recursive call to the insertion operation.

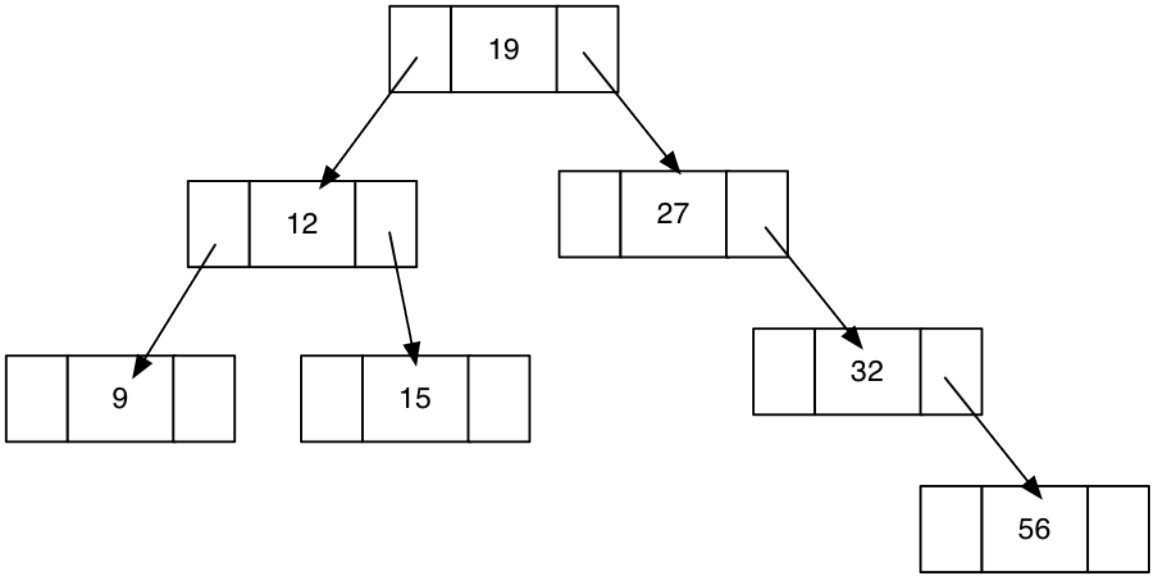
```
insert(TreeNode * root, TreeDataType data, funcPointer to compare operation): TreeNode *

if (the root node is empty)
{
        allocate memory for a TreeNode and assign it to temp
        temp-\textgreater{} data = data
        temp -\textgreater{} left = NULL;
        temp -\textgreater{} right = NULL;
        return temp;
}
if(compareFunction(root-\textgreater{}data, data) shows root data is smaller)
{
        /*recursively call insert on right subtree*/
        root-\textgreater{}right = insert(root-\textgreater{}right,data, comparePointer)
}
else if(compareFunction(root-\textgreater{}data, data) shows root data is larger)
{
        /*recursively call insert on left subtree */
        root-\textgreater{}left =insert(root-\textgreater{}left,data,comparePointer);
}
/* Else there is nothing to do as the data is already in the tree. */
return root;
}
```

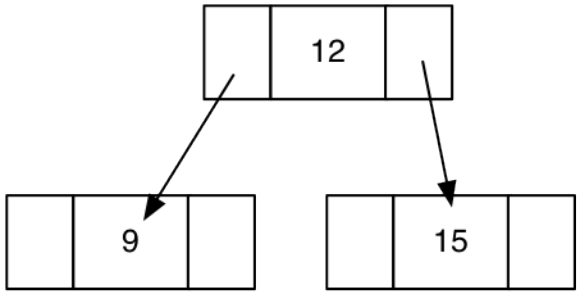
Because a binary tree is stored in sorted order, insertions have to be done by comparing the data to be inserted to the data stored at the current node. Insertions are always done at a leaf node. By supplying a function pointer for the comparison operator, a tree ADT can successfully insert any type of data.

Here is a visual look at the insert operation.

Suppose we wanted to insert 17 into the tree shown.



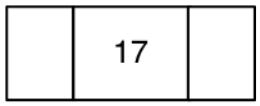
We first compare 17 to the value at the root node (19). Since 17 is less than 19, and in this binary tree we are storing the smallest values in the left subtree, we recursively call the insert procedure on the left subtree of 19.



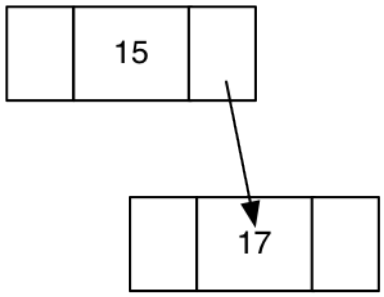
We compare 17 to the root node of the subtree. 17 is greater than 12, so we recursively call the insert procedure the right subtree of 17- which is the node containing 15.

17 is larger than 15, so we recursively call the insert procedure on the right subtree of 15. That subtree is a NULL tree, which activates the base case of the recursive algorithm.

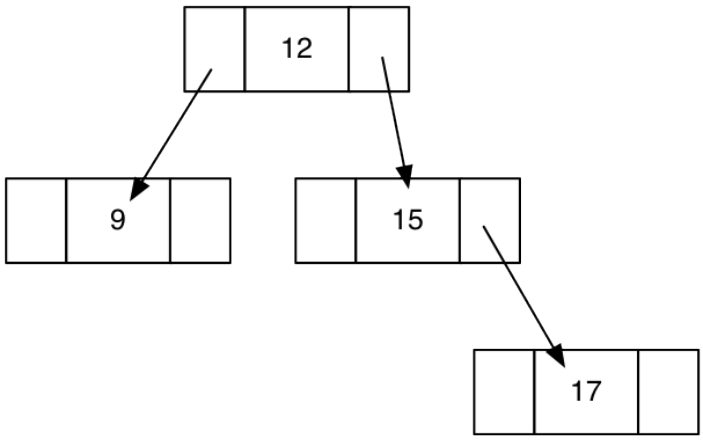
A node is made for 17, the pointer to it is returned as the recursion unwinds.



On the way back up, the tree reassembles each level.



Then



and finally back to the entire tree.

Deletion

Deletion from a binary tree is a bit trickier because you have to be careful to keep the tree in the proper order.

There are three cases to consider when deleting a node from a binary tree:

1) the node has no children

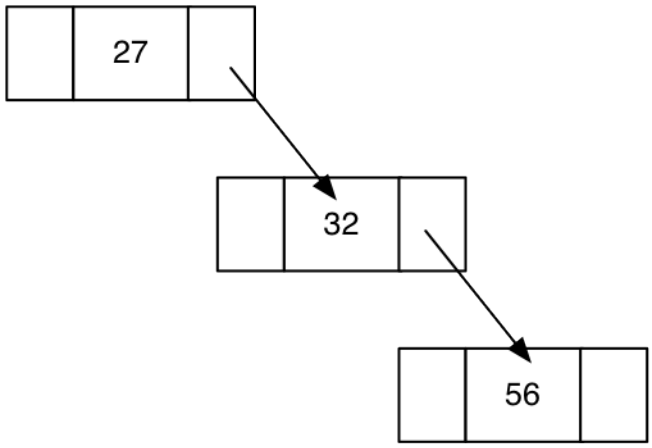
2) the node has only one child (left or right branch)

3) the node has two children

The first case is easy. If the node to be deleted has no children (i.e. it is a leaf node) it can be deleted and the appropriate branch of its parent set to NULL.

In the second case, If the node to be deleted has a right branch child, then the deleted node's parent can be set to point at the child and the tree will remain in the correct order.

Suppose we needed to delete 32 from the subtree below. We could simply remove 32 and set 27 to point at 56 and the properties of the binary tree would not have changed.



If a node has a left child only, the same process can be used to remove the node. Attach the nodes left child to the node's parent and the tree structure will still be valid.

The more complicated case arises when the node to be deleted has two children.

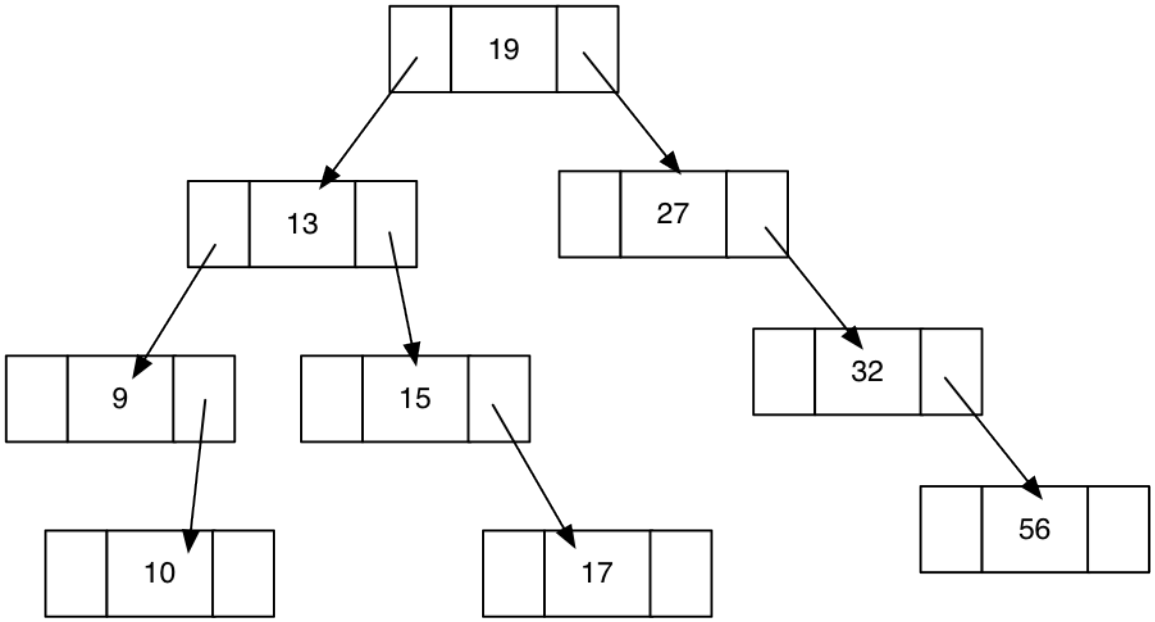
Suppose that we want to delete node 12 from this tree. We can't simply remove 12 and put either one of its children in its place, because we would lose part of a subtree. If 15 replaced 12, there would be no place to reattach 9 and a similar problem exists if 9 were used to replace 12.

The problem is solved by remembering that the right subtree always contains values that are larger than the node, and the left subtree contains values that are greater. So, for any node, the smallest value in the left subtree will be a node that has a value slightly larger than the node being removed, but smaller than all the other nodes in the subtree. That node will also be a leaf node in the tree.

The algorithm is this:

- find the minimum value in the right subtree of the node to be deleted
- copy that value into the node to be deleted (temporarily making a duplicate node)
- recursively call delete on the right subtree – for the value we just copied in. This will activate the base case of the node having no children and will remove the node

In the case of the previous example, we would copy the value 13 into the node that presently holds 12, and then call delete (13) on the subtree rooted at 15. The node at 13 will be removed and the tree will look like this:




```
TreeNode * delete(TreeNode * root, TreeDataType data, compareFP,destroyNodeFP):TreeNode

TreeNode * temp;
TreeDataType removedData;

if the root is empty
        we have a problem and should clean up and exit gracefully
else if (compareFP(root-\textgreater{}data, data) shows that the root data is larger)
{
        root-\textgreater{}left = delete(root-\textgreater{}left, data, compareFP,
            destroyNodeFP);
}
else if (compareFP(root-\textgreater{}data, data) shows that the searched for data is larger
    )
{
root-\textgreater{}right = delete(root-\textgreater{}right,data,compareFP, destroyNodeFP);
}
else /*the comparison is equal and we can delete the node we've found */
{
        if (the root node has two children) /*replace this node with the smallest element in
             right subtree */
        {
                temp = findMinimum(root-\textgreater{}right)
                removedData = root-\textgreater{}data
                destroyFP(root-\textgreater{}data) /*now call destroy function on that data
                    */
                root-\textgreater{}data = temp-\textgreater{}data
                root-\textgreater{}right = delete(root-\textgreater{}right,
                temp-\textgreater{}data,compareFP, destroyFP)
        }

        else  /*this node has zero or one child*/
        {
                temp = root;
                if (root-\textgreater{}left == NULL) /* we have no left child */
                {
                        root = root-\textgreater{}right;
                }

                else if (root-\textgreater{}right == NULL) /*we have no right child */
                {
                        root = root-\textgreater{}left;
```

```
                }
        }
        /*either we've rejoined the subtree, or we're about to delete a leaf node*/

        destroyFP(temp-\textgreater{}data);/*call destroy function on temp-\textgreater{}
            data */
        free(temp);

}
```

8.3.3 Wrapper functions

The wrapper functions for a Tree ADT provide the interface that the user of the ADT interacts with. The wrapper functions are not recursive rather, they call the recursive functions when appropriate.

- createBinaryTree(compareFP, destroyNodeFP)): Tree *
  createTree is used to initialize a tree and to set up the function pointers for the data type that the tree will be storing. When function pointers are used, and data is cast to a void pointer, the same compiled tree library can be used to hold any kind of data.
- destroyBinTree(Tree * toDie)
  destroyBinTree is called when the tree must be released. The destroy tree function will recursively destroy all the tree nodes AND all of the data elements (using the supplied function pointer).
- addToTree(Tree, void *)
  addToTree inserts a value into the tree by calling the recursive insert function.
- removeFromTree(Tree*, void *)
  removeFromTree searches for the data provided in the parameter and, if found, removes that data from the tree. Remove from tree should also free the memory allocated for the node. It is a design choice as to whether removeFromTree also frees the memory for the data.
- findInTree(Tree *, void *):Boolean
  findInTree returns true(1) if the data is found, false(0) if not.

The wrapper functions are usually not complicated and mostly call the recursive helper function for the appropriate operation, passing in the function pointers and parameters to the operation as necessary. See the given code for addToTree for an example.

```
void addToTree (Tree* theTree, void * data)
{
        theTree-\textgreater{}root = insert(theTree-\textgreater{}root, data,theTree-\
            textgreater{}compare);
}
```

Traversals

Another common set of functions for trees are traversal algorithms. A traversal algorithm can also be used to create an **iterator** for a tree. Traversal algorithms were introduced in the introductory section on single-rooted trees. In addition to the in-order and pre-order traversals explored in that section, a tree traversal can be written to be in post-order (the root is processed after the subtrees- left subtree, right subtree, root) or in level-order where the root is processed, then all of the root's immediate children, then all of the root's grandchildren, then all of the root's great grand children, etc. A level-order traversal is also known as a breadth-first traversal.

A recursive algorithm for a level-order traversal of a tree is shown below.

```
for ( the height of the tree)
{
    processLevel(TreeNode * root, height)
}

void processLevel( TreeNode * currentNode, int level )
{
    if ( currentNode  is NULL)
        return;
```

```
    if (level == 1)
        process  currentNode->data //processing could be printing or something else
    else if (level > 1)
    {
        processLevel(currentNode->left, level-1)
        processLevel(currentNode->right, level-1)
    }
```

Often a traversal function will take a function pointer as an argument and will apply that function to the data in each node of the tree. For example, you could write a function that printed a node value, and then combine that function with a traversal function to print the tree in a variety of orders.

The print function is data dependent,but the traversal function is not. For example, a function to print integer data might look like this:

```
void printIntNode(void * data)
{
        printf("\%d␣", data);
}
```

where a function to print values from a tree that was storing strings might look like this

```
void printStringNode(void * data)
{
        printf("\%s␣", data);

}
```

A traversal algorithm could be called, sending it the print function, and any type of data could be successfully printed. For example.

```
printInOrder(myTree-\textgreater root, \&printIntNode);

//or

printInOrder(myTree-\textgreater root, \&printStringNode);
```

The key to remember that the user of the ADT is the one who defines the data stored by the ADT and the operations to compare, destroy, and manipulate that data. The tree ADT simply stores the data and performs the requested operations when given pointers to those operations.

8.4 Types of Binary Trees

A binary tree is **full** if every node in the tree has zero or two children. A full binary tree has the property where the number of leaf nodes (L) is equal to the number of internal nodes (I) + 1. If you are curious about the proof of this propertly see http://www.geeksforgeeks.org/handshaking-lemma-and-interesting-tree-properties/

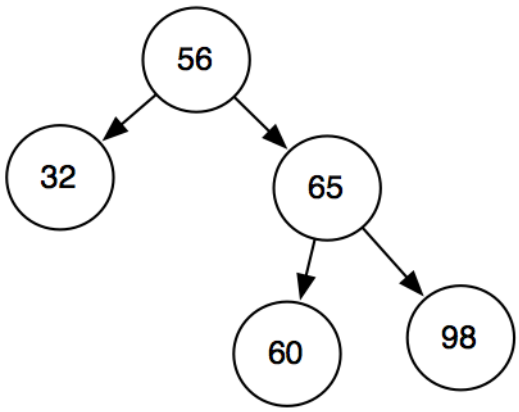


Figure 8.1: A full binary tree

A binary tree is **complete** if all the levels of the tree are full excepting possibly the last level. The last level must have all the nodes as far to the left hand side of the tree as possible. A complete binary tree is used in the implementation of a **heap**.

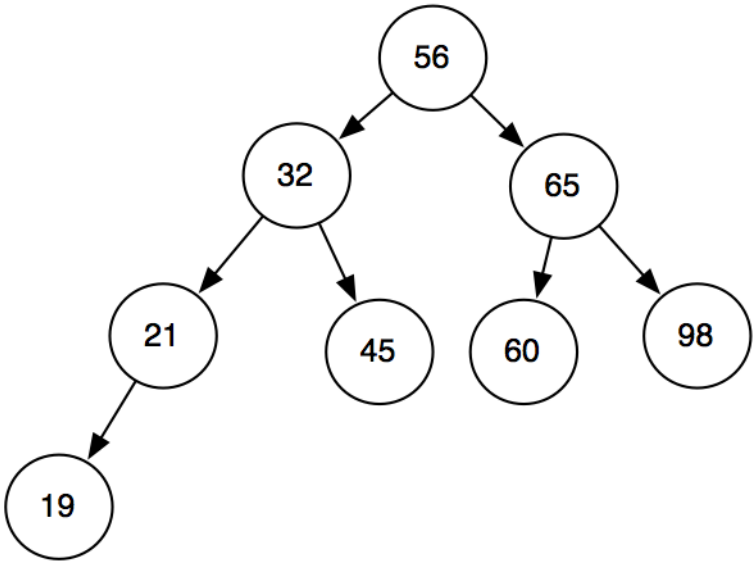


Figure 8.2: A complete binary tree. All of the levels in the tree have two child nodes except for the last level. The child nodes in the last level are as far left as possible.

A **perfect** Binary tree is one in which all internal nodes have two children. The leaf nodes of a perfect binary tree are all at the same level. The number of nodes in a perfect binary tree is $2^h - 1$ where h is the height of the binary tree.

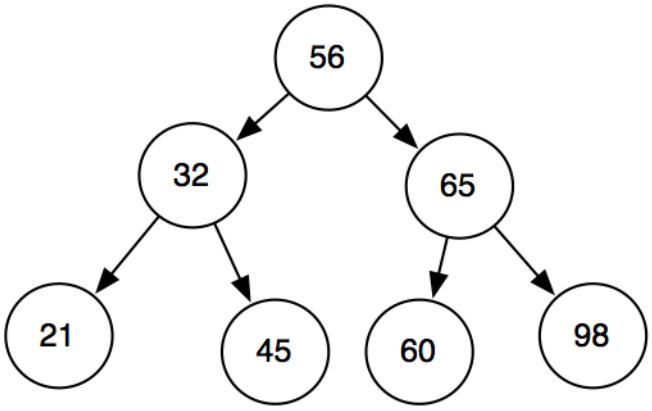
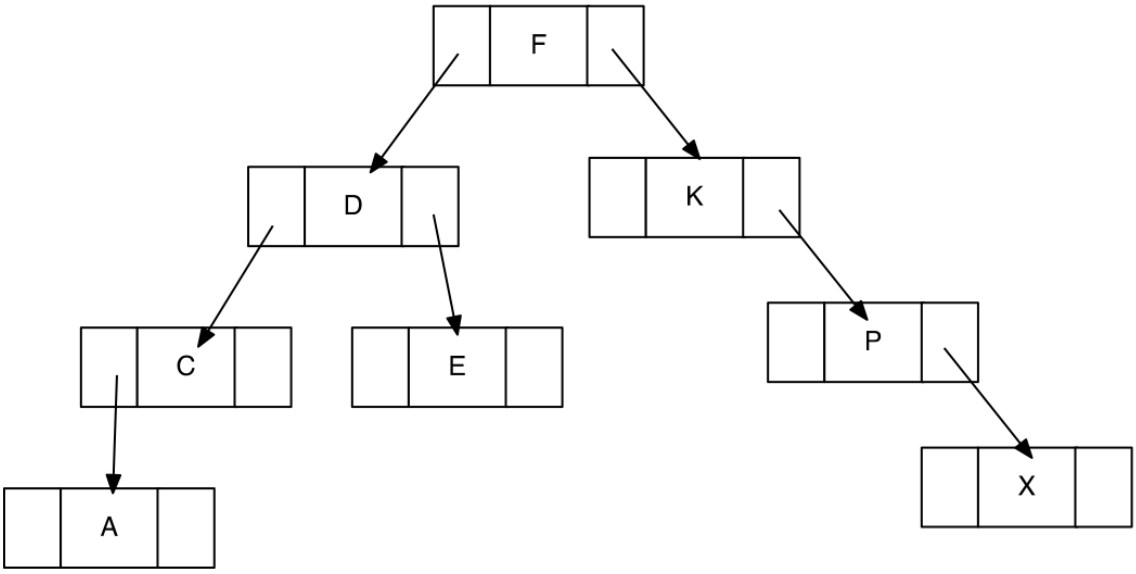


Figure 8.3: A full binary tree

8.5 Extending Activities

- The order that nodes are inserted into a tree can greatly affect the structure of the tree. Draw the binary tree that is constructed by adding the seven names in the following order:
  - Sleepy
  - Bashful
  - Doc

  - Dopey
  - Sneezy
  - Happy
  - Grumpy
- Trace the algorithms for level-order traversal using a hand drawn tree. What order are the nodes of the tree printed in? Can you tweak the algorithm to reverse the order?
- One of the most frequent uses of a binary tree is as a search tree. The object is to search through the data to find out if a particular element is part of the data. Write a recursive find procdure for a binary tree. Start with the root node of the tree. Your procedure can be in c or in pseudocode. Be sure to pass in the compare function pointer.
- 

  Give the in-order traversal of the tree shown above.
  Give the pre-order traversal of the tree.
- Write the pseudocode or c code for a compare operator. A compare operator must return three distinct values, one for when the first parameter is larger, one for when the second parameter is larger, and one for when the two parameters are equal. The convention is to use 1, 0, and -1 for the three values. Zero for equal values, 1 for the first string being larger, -1 for the second.
- The delete operation relies on a recursive function to find the minimum value in a subtree. The minimum value will always be the leftmost leaf of the subtree. Write the findMinimum function (in pseudocode or in C).

Chapter 9

Heaps

9.1 Introduction

A heap is a special kind of complete binary tree. The key property of a heap is that the target element (the minimum or the maximum) is always stored at the root of the heap. This is done by reordering the nodes to ensure that the property is true. A heap can be one of two types, a max heap where the parent node is greater than its children, or a min heap where the parent node is smaller than its children.

To be a valid heap, every element has to follow the heap rule. For max heaps the rule is:

$$value(parent) \geq value(child) \tag{9.1}$$

and for a min heap the rule is:

$$value(parent) \leq value(child) \tag{9.2}$$

A heap is considered to be a partially ordered data structure because the elements within any given level have no defined ordering. Heaps are commonly used as the implementation for a priority queue.

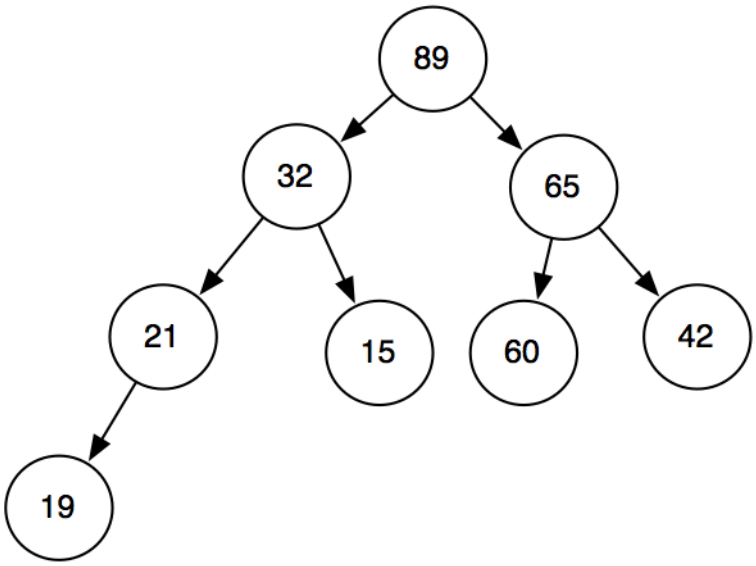


Figure 9.1: A max heap. Notice that the sibling nodes have no ordering within the level

9.2 Operations

A heap should provide the functionality to insert, delete, and retrieve the min/max value. Additionally the heap must provide the standard data structure operations of create, destroy and isEmpty. A robust heap library should allow the library user to choose between min or max heap capabilities when the heap is created. The minimum set of operations is shown below.

- insert
- delete
- findMinorMax
- isEmpty
- create
- destroy

The Heap ADT requires the same abstraction with void * pointers, operations to create nodes, and function pointers to compare, delete and print data as the previous ADTs we have studied. The create, destroy, and isEmpty functions are very similar to the same functions for other ADTs and are not explained in detail here.

The insert and delete functions for a heap are the most complicated operations.

9.2.1 Heap Insert

A heap is a specialized version of a complete tree. This means that all insertions must occur at the left-most side of the bottom level of the tree in order to maintain the property of completeness. The algorithm for adding an element to a heap follows:

```
insert(Heap * heap, void * data)
{
    create a node from the data
    locate the next position in the heap (the left most position with no node)
    add the new node to the located position
    reheapify the heap to maintain the heap property
}
```

The process of reheapification consists of swapping elements until the heap property is satisfied or until the root of the heap is reached. The pseudocode for reheapification is shown below.

```
reheapify(Heap * heap, Node * newNode)
{
     parentNode = get parent node of newNode
     while(newNode->data is greater than parentNode->data  //or less than for a min heap
     {
        swap positions of newNode and Parent Node
        parentNode = get parent node of newNode (has changed because of the swap)
     }
}
```

9.2.2 Heap Delete

The most common way to delete elements from a heap is to remove the value at the root node as part of the the findMinorMax operation. To remove the root node, you find the left-most leaf of the heap, replace the root node with that node, and then reheapify downwards until the heap property is satisfied.

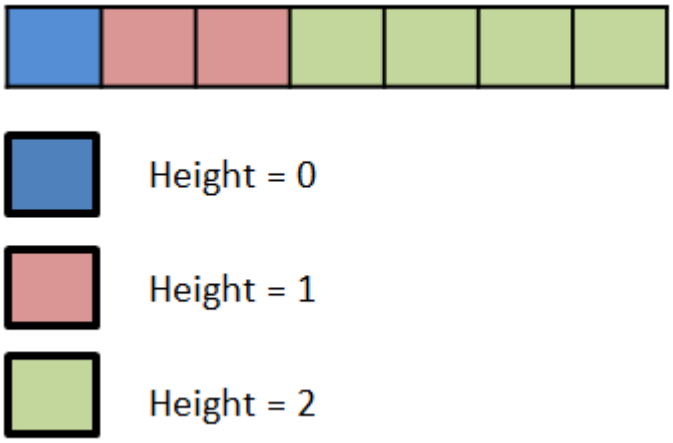
```
 void * findAndDeleteRoot(Heap * heap)
 {
        tempNode = heap->rootNode
        heap->rootNode = the node in the "last" position in the tree //the leftmost, bottom
            node
        outOfPlacePtr = heap->rootNode  // keep track of the node that you're moving into
            place
        remove the node in the last position //after copying it into the root
        while ( outOfPlacePtr->data is less than either left or right child)  //or greater
            than for a min heap
        {
```

```
                swap outOfPlacePtr and child with greatest value
        }
        return tempNode->data
}
```

The delete process for the root node can be used to delete any node in the heap. The node to be deleted is the root node of a subtree, and the delete algorithm can be applied to the subtree instead of the entire heap.

9.3 Array Implementation of a Heap

Heaps can be implemented using a Binary Tree ADT. However, the property that a heap is a complete binary tree allows a heap to be implemented using an array. The array implementation removes the need for pointers as the positions of the child nodes can be easily calculated. The first empty position in the array is the next position to be filled.

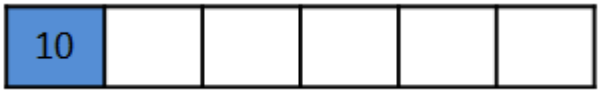


The array is organized so index 0 is height 0, height 1 is indexes 1-2, height 2 is indexes 2-5 and so on. The swaps necessary for inserting or deleting elements can be accomplished simply by calculating the appropriate array index.

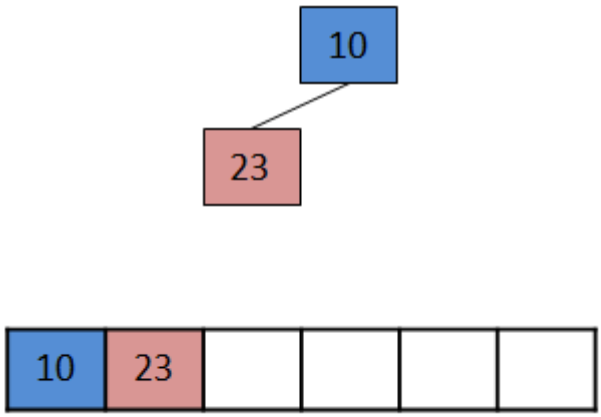
9.3.1 Adding Elements

Adding an element to a heap has a worst case complexity of O(height) or O(log(N)), since there is a maximum of 1 swap per level of the heap. The algorithm for inserting elements does not change when an array is used for implementation. Only the implementation details change. Consider the following example where 10, 23, 3, 32, 17 and 5 are added to a max heap.

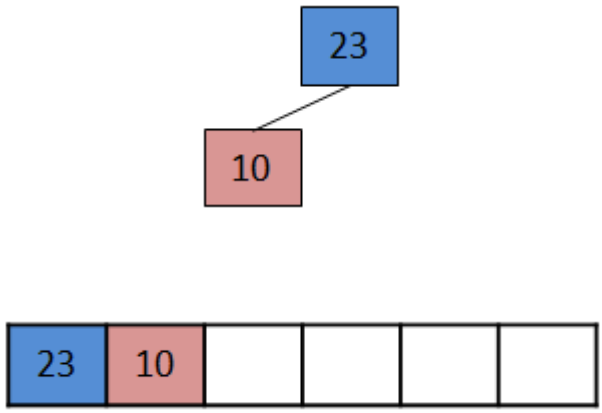
We begin by adding 10 to an empty heap.



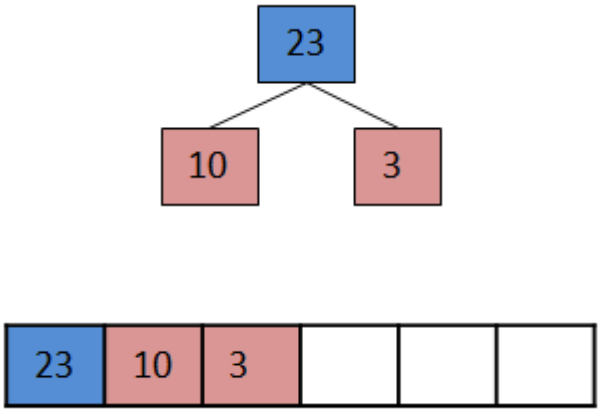
10 is added as the root of the heap at position 0 in the array. It is the first level of the binary tree. We then add 23 to the heap.



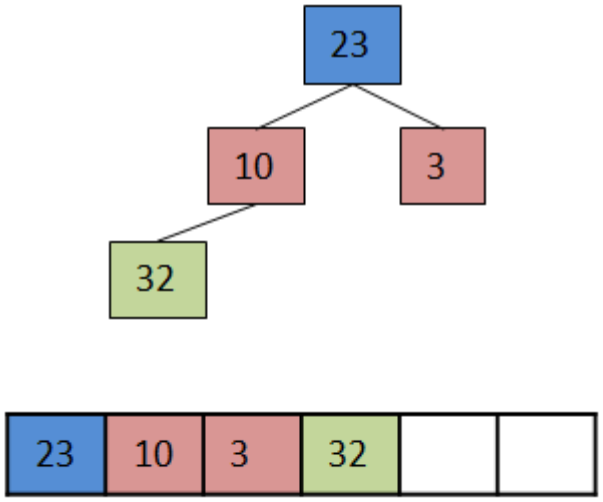
First, 23 is added as 10's child. It is placed in the first empty left-most position in the tree, which is also the first empty position in the array. Because 23 is greater than 10, the heap must be reheapified.



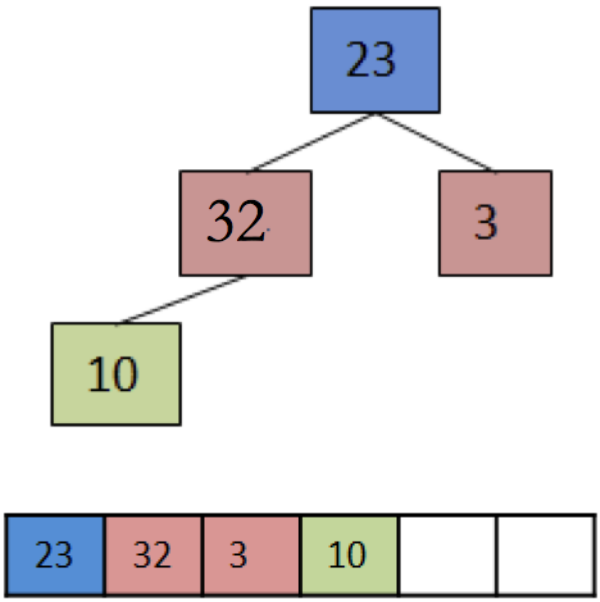
23 is compared to its parent value (10). 23 is larger so the two values are swapped and 23 is the new root of the heap. 10 is the child of 23. We then add 3 to the heap.



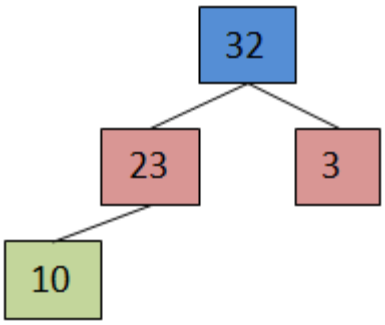
3 is added as the second child of 23. It is added in position 2 of the array. 3 is compared to the value of its parent (23). 3 is smaller than its parent value so no swap is needed. We then add 32 to the heap.

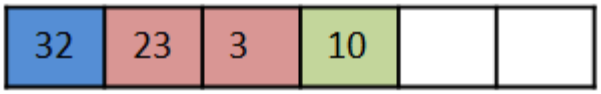


32 is added at the next, left-most position in the heap (the first empty position in the array). 32 is compared with its parent value (10). 32 is larger than 10 so the two values must be swapped.

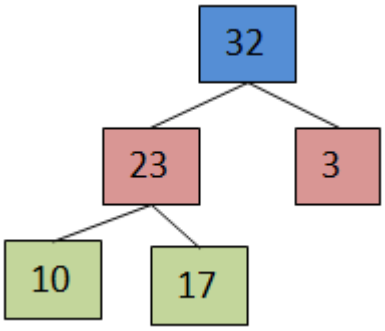


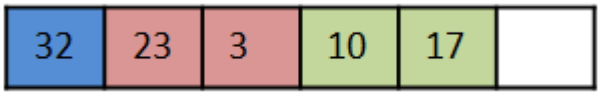
32 iis not the root of the heap, so we again compare 32 with its parent value (23). 32 is larger than 23, so a swap must be made.



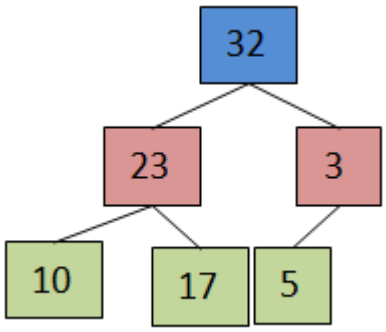


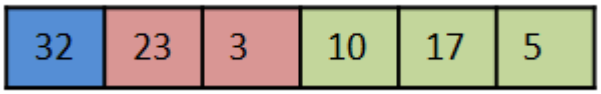
32 is the new root of the heap so no further comparisons are necessary. We then add 17 to the heap.



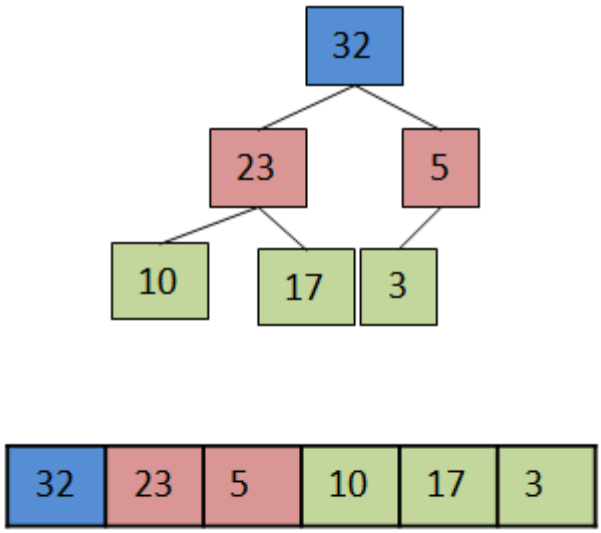


17 is added as a child of 23 to the first, left-most empty position in the tree. 17 is compared to the value of its parent(23). 17 is smaller than its parent value so no swap is required. We then add 5 to the heap.





5 is added to the next position in the heap, as the first child of 3. 5 is compared to its parent value (3). 5 is larger than three so the two values must be swapped.



Finally 5 is compared to its new parent (32). 5 is smaller than 32 so no swap is necessary.

It can be seen that the final array representation is not fully sorted, and not fully segregated by value ranges (5 has a height of 1, higher than 10 and 17). Nonetheless the heap rule holds and the partial ordering of the heap can easily be maintained by swaps.

9.3.2 Removing Elements

Elements of a heap are generally removed from the root, or the largest/smallest element in the structure. The root is then filled with the most recent added element , and the heap is resorted by swapping with the larger child until it satisfies the heap property. If it is necessary to remove an interior item from a heap, the subtree rooted in the item to be removed is handled in the same fashion as the entire heap.

Removing an element will always result in a complete tree because only leaf nodes are actually removed after the value is swapped in to a new spot.

The complexity of a deletion is O(height) or O(log(N)) since there is a maximum of 1 swap necessary per height.

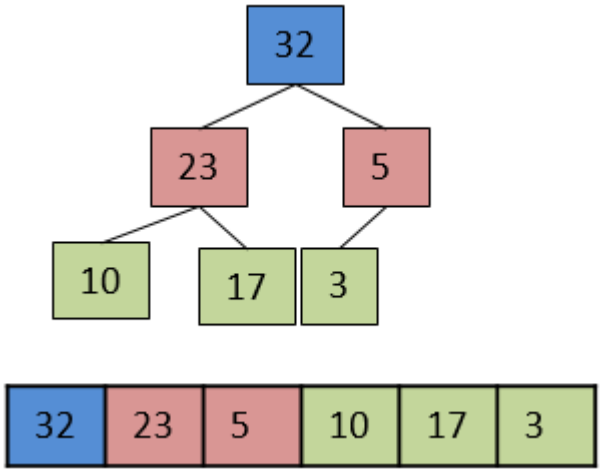
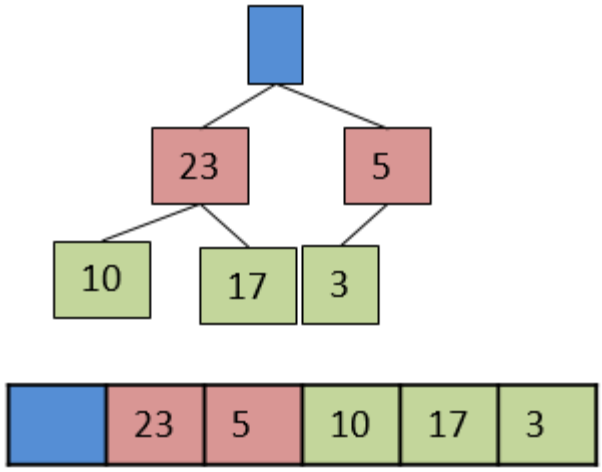
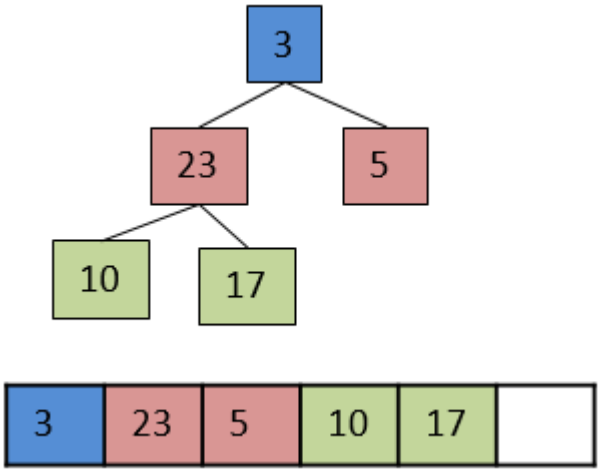


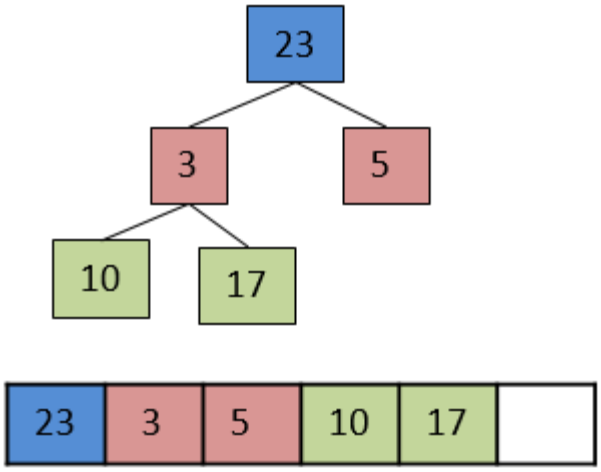
Figure 9.2: The Starting Heap



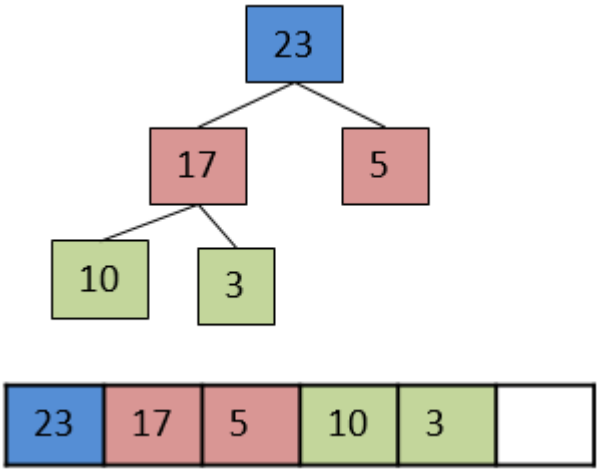
The root is removed, leaving an empty spot in the heap.



The most recently added element is copied to the root position. The most recently added node is deleted. The new root node(3) is compared to both of its child nodes (23, and 5). 3 is less than at least one of its children so it must be swapped. The heap does not follow the heap rule, a sort must be done.



3 is swapped with the larger child, 23. 3 is now compared with its new children (10 and 17). 3 is smaller than at least one child, therefor the heap still does not satisfy the heap property. Another swap must be performed.



3 is swapped with the larger child, 17. The heap property is now satisfied.

9.4 Extending Activities

- Develop a formula to calculate the position of a parent node in an array-implementation of a heap. The only input information to the formula is the position of the current node.
- Create the C struct for a binary-tree based heap. Your struct should have members for the heap, the last added element, the next position, and function pointers to manipulate void* data.
- Heaps do not have to be binary trees. Many variations of heaps are possible. Use internet resources to learn about one of the following different types of heaps: skew heaps, binomial heap, or leftist heap. Write the pseudocode for insert and delete operations for the type of heap you chose.
- The complexity of insert and delete for a binary heap is O(log(N)). Why is that? Create a written explanation, chart, or diagram that explains why log(N) is the complexity for those operations.

Chapter 10

Priority Queue

A priority queue differs from a normal queue because the elements do not follow a FIFO order. Instead elements are removed by the queue in order of priority, with the highest priority elements leaving first. If many items enter a priority queue at the same time the highest priority will be served first followed by the next lowest priority etc. The process gets more interesting when new items are being added sequentially.

10.1 Uses

A priority queue is useful in any situation where scheduling must occur and the items being scheduled are not handled on a first come first served basis. For example, A priority queue would be ideal in an IT department where a lot of people are having a variety of issues. Some issues require immediate attention whereas some others can be put off for a short period of time. If an entire network was to collapse and join the end of IT ticket queue, behind a long list of lost or forgotten user passwords you'd want the network issue to be fixed first since it effects a lot more people. After the network issue is fixed the less urgent issues will be fixed in the same order as before.

Priority queues are useful in scheduling jobs for execution on computing systems. Most operating systems use some kind of priority mechanism to ensure that crucial tasks are scheduled without crippling the less urgent tasks.

10.2 Implementation

A priority queue can be implemented using any ADT that can be constructed so that the largest (or smallest) value is easily retrieved. It is possible to implement a priority queue with a linear ADT, but it would not be very efficient because the entire list would need to be searched to find the highest priority every time the next item was to be removed.

The more effective approach for implementation of a priority queue is to use the Heap ADT. The user experience would be the same as a queue, but the priority queue would run faster and would be able to handle larger sets of data to prioritize.

10.3 Operations

The operations of a priority queue are identical to the operations of a normal queue.

- create()
- destroy()
- isEmpty()
- insert(comparable priority)
- removeMax()

These operations simply wrap the operations for the underlying data structure. For example, if the priority queue were wrapping a linked list, then the insert() function might use the addSorted() function of the linked list.

If the priority queue were implemented to wrap the Heap ADT, the insert() function of the priority queue would wrap the insert() function of the heap.

10.4 Varying Priority and Starvation Prevention

We use the term **starvation** to describe the situation where an element in a priority queue never makes it to the top of the queue and is stuck in the queue forever.

If a large number of high priority items are continuously entering a priority queue that contains lower priority items, the lower priority items will become starved (never getting served/ removed from the queue). This is because the higher priority items will always be served first. In this situation there must be an algorithm in place to prevent the starvation of lower priority items. Many algorithms are possible. The effectiveness of the algorithm depends on the nature of the elements stored in the priority queue and on the context of the application.

One possible solution to starvation is to increase the priority of elements in the queue as their wait time increases. High priority items are still processed quickly and low priority items will wait but only until the low-item's priority increases to a value high to put it at the front of the queue. This makes sure all elements in the queue will eventually be processed, even if the ones that start with a very low priority.

Another possible algorithm is to process some specified number of high priority items followed by processing some specified number of low priority items. If the value of "low priority" is not selected carefully, this approach could result in elements with a middle priority value being starved.

A priority queue ADT should have some mechanism to prevent starvation of the elements in the queue.

10.5 Extending Activities

- Consider the implementation of a priority queue using an array, a linked list, and a heap. For each implementation, provide the pseudocode for the insert and removeMax operation. What is the complexity of those operations in each case?

Chapter 11

Algorithmic Complexity and Complexity Analysis

The efficiency of an algorithm can be measured by estimating how long the algorithm will take to complete for some set of data, and by estimating how much memory the algorithm will require to perform that execution. The context of your computing environment tells you which measure is most important. If you have limited memory you will require an algorithm that uses minimal memory. If you need an answer very quickly, you will require an algorithm with fast execution time. The estimate of the efficiency of an algorithm is known as the algorithm's **complexity**.

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11.0.1 Determining Complexity

We often need to know the complexity of a particular algorithm. We could attempt to experimentally determine the running time of an algorithm, or we could analyse the algorithm theoretically.

Experimental Analysis

In order to experimentally analyse an algorithm, we first must implement the algorithm. This seems like an obvious step, but if we're trying to choose between three competing algorithms, the experimental analysis means that all three algorithms must be implemented. Sometimes that is trivial, sometimes it is now.

After the algorithm is implemented, we must **instrument** the algorithm to enable us to measure the running time. Usually that means using some kind of timer or clock. Below is a sample testing function that uses two fictional functions, Sort() and Time(). Sort() is the function being analysed. Time() is the function used to capture the time in seconds since midnight.

```
float testSort( List * the List)
{
    float seconds = 0.0;
    float time1, time2;
    time1 = Time();
    Sort(theList);
    time2 = Time();
    seconds = time2-time1
    return seconds;
}
```

The testSort function would be run with a variety of lists using different sizes and compositions of data. The results would be analysed statistically to determine what the best, worst, and average running time of the algorithm is, and what data characteristics affect those times.

While this process sounds straightforward, there are several possible issues that can make consistent analysis difficult. For example:

- What language should be used for testing running time of the algorithm? Some languages might offer facilities that significantly slow down, or speed up some operations.
- What compiler should be used? Some languages (like C) have different compilers available. Each compiler is slightly different and will result in different running times.
- What hardware environment should be used? Will the results from running on a new hardware system be the same as running on a system that is several years old?
- What software should be used? Will the results from running on a Windows operating system be the same as the results from running on a linux system?
- How do we select the data to be input? If two different testers make test data, will the results be the same?

Clearly, it is much more difficult to get consistent measurements of running time experimentally than it seems. Most often we content ourselves with a theoretical analysis of the algorithm. This theoretical analysis does not have to be formal (in the sense that it requires a full blown proof).

Theoretical Analysis

Theoretical analysis of algorithms has the advantage that the running time of the algorithm can be evaluated without having to implement the algorithm, while taking into account all possible inputs, independently of hardware and software.

We perform a theoretical analysis by counting the primitive operations in an algorithm. Primitive operations include assignments, comparisons and arithmetic operations. This counting exercise must take into consideration the effect of the size of the input values. The size of the input usually impacts things like the number of iterations of a loop or the length of an array.

Consider the code(shown below) for finding the maximum value in a list of integers. The code assumes that the List ADT has functions for getting the value of an element at a specific place and for finding the length of the list. We'll assume, for the purposes of this analysis, the the running time for all the list operations used in the example is O(1) or constant time.

```
int findMax(List theList)
{
    int currentMax = getFirstElement(theList);
    int i;
    for(i = 1; i<length(theList); i++)
    {
        tempElement = getElement(theList, i);
        if (tempElement > currentMax)
            currentMax = tempElement;
    }
    return currentMax;
}
```

A count of the primitive operations in this code (where n = length of the list) yields the following:

int currentMax = getFirstElement(theList);	2 (assignment and list function call)
int i = 0;	1 (assignment)
for(i = 1; i¡length(theList); i++)	2(n-1)+1 (1 assignment plus n-1 comparions and n-1 increments of i)
tempElement = getElement(theList, i);	2(n-1) (2 operations done n-1 times)
if (tempElement ¿ currentMax)	n-1 (1 comparison done n-1 times)
currentMax = tempElement;	n-1 (1 comparison done n-1 times)
return currentMax;	1

This algorithm only runs through the entire list if the largest value in the list is in the last position of the list. That is called the **worst case**. In the worst case, the algorithm executes 5 + 6(n-1), or 6n -1 primitive statements

(with the assumption that the list functions are the equivalent of a primitive statement).

Lets suppose that there was one specific primitive operation that took longer than the others, and one that was faster. Without doing an analysis to determine exactly the speed of each primitive operation, we can say that the fastest execution speed would occur if all the primitive operations were the fastest operation. We can also say that the slowest execution speed for this algorithm would occur if all the primitive operations were the slowest operation. So the execution speed of the function, is bounded by the fastest/slowest speeds of the primitive operations.

For the purposes of estimating complexity of algorithms, we assume that primitive operations take **constant time** and don't worry too much about exactly how much time that is. Using that knowledge we can say that the algorithm above takes 6(n) constant time units -1 constant time unit. This means that the execution time of the algorithm depends on the length of the list (n). We say that such an algorithm is **Order N** or **O(N)**.

11.0.2 Big O notation

Time complexity is often expressed in Big O notation. The big O is read as on the order of,which means that the most significant factor in the time complexity analysis is shown. The **N** represents the data input into the algorithm.

So if we say that an algorithm is O(N) (read that as order-N),we mean that the upper bound on execution is controlled linearly by the number of input data. We do NOT mean that the algorithm will take exactly N time to complete.

Big O notation signifies an **upper bound** on the complexity/computational time for an algorithm. The term upper bound means that the algorithm might run faster, but won't run slower.

Big O notation usually describes worst case, but sometimes it is given for average and best case too when that can be clearly defined. The complexity measure describes a relationship between the size of the input data (how much data goes in) and the time the algorithm will take to run. Typical Big O values are O(1) or constant time, O(N) or proportional to data size, and $O(n^2)$, which means that execution will take approximately as long as the square of the size of data. We'll learn about each of these types of algorithms as well as others during the course of this class.

11.0.3 Estimating Complexity

Imagine that you are estimating the number of days required for an automobile trip across Canada. Your estimation would likely include the distances between major cities, but you would be unlikely to factor in the short side trips into small centers for gasoline, or trips into museums or tourist sites. The most significant distances are the ones between the cities so those become the basis for your overall estimate of trip time.

When you are estimating time complexity of an algorithm, you also ignore the "side trips", or the parts of the program that don't contribute much to the overall execution time. When calculating computational complexity we ignore things like the programming language used, the characteristics of the compiler, and the speed of the computer that will be used to run the program. The time complexity of the algorithm is based on the characteristics of the algorithm alone.

Big-O lets you compare the complexity of algorithms that do similar operations. It provides a way to estimate the number of operations required to complete an algorithm. We assume that the operation that is executed the most is the one that should be counted, and that we can ignore other operations because the time added wont significantly impact the overall time. We also ignore anything that takes a constant amount of time.

Complexity analysis is used most often in situations where the algorithm is processing data and the set of data can vary in size. For example, suppose you have written a stack library and you want to know how its algorithms compare to those in other libraries. One way to compare is to analyze how much processing effort is required for your stack's algorithms to work with an arbitrary size of data (N). If the stack must go through the entire set of data once, we say it is O(N). If the stack library must go through the set of data in a nested loop, we say it is $O(N^2)$.

For this course we will focus on six categories of computational complexity- O(1), O(logN), O(N), O(NlogN), $O(n^2)$, $O(2^N)$. At the conclusion of the course you will know the characteristics in code that are associated with each category of complexity and you should be able to estimate the complexity for several different types of algorithms from those efficiency classes.

11.0.4 Complexity in the real world

This section presents an easy to understand, real world, example of algorithm complexity using a printed phone book. In this example *N* represents the number of entries in the phone book. The example is copied verbatim from:

http://stackoverflow.com/questions/2307283/what-does-olog-n-mean-exactly

> We will assume our phone book has *businesses*(the *Yellow Pages*) which have unique names and *people*(the *White Pages*) which may not have unique names. A phone number is assigned to at most one person or business. We will also assume that it takes constant time to flip to a specific page.
>
> Here are the running times of some operations we might perform on the phone book, from best to worst:
>
> - **O(1)(worst case):**Given the page that a business's name is on and the business name, find the phone number.
> - **O(1)(average case):**Given the page that a person's name is on and their name, find the phone number.
> - **O(log n):**Given a person's name, find the phone number by picking a random point about halfway through the part of the book you haven't searched yet, then checking to see whether the person's name is at that point. Then repeat the process about halfway through the part of the book where the person's name lies. (This is a binary search for a person's name.)
> - **O(n):**Find all people whose phone numbers contain the digit "5".
> - **O(n):**Given a phone number, find the person or business with that number.
> - **O(nlog n):**There was a mix-up at the printer's office, and our phone book had all its pages inserted in a random order. Fix the ordering so that it's correct by looking at the first name on each page and then putting that page in the appropriate spot in a new, empty phone book.
>
> For the below examples, we're now at the printer's office. Phone books are waiting to be mailed to each resident or business, and there's a sticker on each phone book identifying where it should be mailed to. Every person or business gets one phone book.
>
> - **O(n log n):**We want to personalize the phone book, so we're going to find each person or business's name in their designated copy, then circle their name in the book and write a short thank-you note for their patronage.
> - **O(n^2):**A mistake occurred at the office, and every entry in each of the phone books has an extra "0" at the end of the phone number. Take some white-out and remove each zero.
> - **O(n!):**We're ready to load the phonebooks onto the shipping dock. Unfortunately, the robot that was supposed to load the books has gone haywire: it's putting the books onto the truck in a random order! Even worse, it loads all the books onto the truck, then checks to see if they're in the right order, and if not, it unloads them and starts over. (This is the dreaded bogo sort http://en.wikipedia.org/wiki/Bogosort
> - **O(n^n):**You fix the robot so that it's loading things correctly. The next day, one of your co-workers plays a prank on you and wires the loading dock robot to the automated printing systems. Every time the robot goes to load an original book, the factory printer makes a duplicate run of all the phonebooks! Fortunately, the robot's bug-detection systems are sophisticated enough that the robot doesn't try printing even more copies when it encounters a duplicate book for loading, but it still has to load every original and duplicate book that's been printed.

11.1 Analysis of Algorithms using Big O

When arriving at the Big O complexity for an algorithm, we count the number of primitive operations. We then select the term with the highest order in the resulting sum to use as the complexity. The lower order terms, and any constant terms, will not add to the overall execution time since the higher order term dictates a longer time than required by lower terms.

The following guidelines apply:

- If the sum is a polynomial where the highest degree is d then the algorithm is $O(N^d)$
  - Suppose the sum of the primitive operations is $6n^2 + 5n + 10$
  - We first drop the lowest order terms to arrive at $6n^2$
  - We then drop any constants to arrive at n^2
  - The algorithm would be $O(N^2)$
- Use the smallest possible order to describe the complexity
  - If the sum of the primitive operations is $2n$ say that the algorithm is $O(N)$ rather than saying it is $O(N^2)$
  - Even though $O(N^2)$ is technically correct, because the algorithm will not run slower than that, it is not as informative, since the algorithm will also run an order faster.
- Use the simplest expression of the order
  - If the sum of the primitive operations is $3n + 5$ say that the order of the algorithm is $O(N)$ rather than $O(3N)$

The computational complexity of an algorithm is not an exact measurement of the running time of an algorithm. Rather it is an estimate of the time that allows us to classify the algorithm into one of several **complexity classes**. Complexity classes are divided into three categories, constant, polynomial and exponential. We'll explore several complexity classes in the next few sections.

You begin an analysis of an algorithm by counting. There is no real need to count the executions for every step of an algorithm, since we know that we are only interested in the higher order execution times. In particular, focus the analysis on section of the algorithm that go through the input data item by item.

- Count the number of arithmetic operations performed
- Count the number of comparisons made
- Count the number of variable assignments
- Count the number of array elements accessed
- Count the number of times each loop executes

Example:

Below is an example program that can be used to illustrate how to calculate complexity. The complexity of each line of the program is explained in the table following the source code listing. For this example, assume that N represents the number of data items the algorithm must process.

```
int n=100;
int w = 0;
for(int i=1; i<n; i++)
{
        for(int k=1; k<n+n; k++)
        {
                w = w+1;
        }
}
```

Table 11.1: Big O analysis of simple C program

int n=100; int w=0;	These two assignments happen once each so they are O(1)
for (int i=1; i<n; i++)	This loop will execute n times so the loop is O(N)
for (int k=1; k<n+n; k++)	This loop will execute 2n times. That is still considered to be O(N)
w=w+1;	Assigment happens once for each inner loop iteration. (N outer loops X 2N inner loops) The assignment occurs N*2N times, which is $O(N^2)$

11.1.1 Analyzing recursive functions

A recursive algorithm can be analysed for complexity using the same process of inspection. The only difference is that the repetition in a recursive algorithm happens during the recursion, so counting the number of primitive

operations requires an understanding of how many times the recursion will be invoked.

```
int power (int num, int power) {
    if(power==0)
    {
        return 1;
    }
    else
    {
        return num * power(num, power-1);
    }
}
```

The program shown above uses recursion to calculate the result of applying an exponent (power) to a number (num). An analysis of the algorithm is shown in the table below.

if(power==0)	1 (one comparison)
return 1;	1 (one return)
return num * power(num, power-1);	1 (return) + 1(multiplication) + (power-1)* the running time

The calculations in the algorithm run in constant time (a return statement, a comparison and a multiplication) but those calculations must happen once for every time through the recursion. The recursion happens (power-1) times after the first pass through the algorithm. So the complexity if the algorithm is O(N) where N = power.

11.1.2 O(1) and O(N)

Constant Time- O(1)

If an algorithm runs in constant time the complexity of the algorithm is independent from the size of the input. No matter how big the input is, the algorithm requires the same amount of time to execute.

An algorithm that looks up elements in an array is O(1) because it takes the same amount of time to access any element regardless of where the element is in the array (assuming you know the index of the element). An algorithm that returns a value, such as the date, time, or characteristics of the operating system is O(1). An algorithm that always returns the same value (maybe a zero) regardless of the data involved is also O(1).

- A program to determine characteristics of numbers (odd/even/divisible by 3, power of 2, etc)
- A program that looks something up in a constant sized array or hash table,

O(N)

The performance of an O(N) algorithm is directly proportional to the size of the input data.

An algorithm that runs in O(N) can perform all of its necessary processing by handling each element of data only once. When the data grows in size, the length of time taken by the algorithm increases by a similar proportion. Twice as much data will take twice as long to execute. Reducing the data by 1/3 will reduce the time required by 1/3. Some examples of O(N) algorithms include:

- Looking through an unsorted array or linked list is an O(N) operation.
- Copying a set of data from one memory location to another is O(N). *Realloc* peforms a copy from one memory location to another if necessary when it is called.
- Creating a sum from a list of numbers is an O(N) operation.

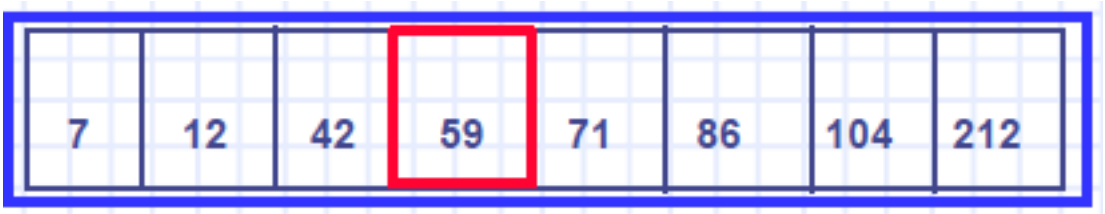
11.1.3 O(logN)

O(logN)suggests that the algorithm will take $\log_2$ N time to complete, where N is the size of the input. This means that the time required increases linearly while the size of the input increases exponentially.

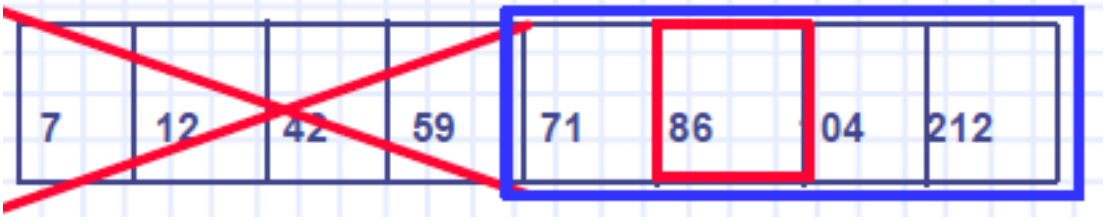
Most often, O(logN) refers to log (base 2) of N. You can think of this as the number of times you can divide the set of things into 2 (the base) before you reach sets of one.An algorithm that os)(logN) often divides the set of data as part of its operation. Operations on binary trees are usually O(logN).

Searching a binary tree (a balanced one) is one example of an algorithm that takes O(logN).If the binary tree has 8 elements in it, thenyou can find any element in the tree in 3 or fewer steps. This makes sense because the maximum depth of a 8 element balanced tree will be 3.

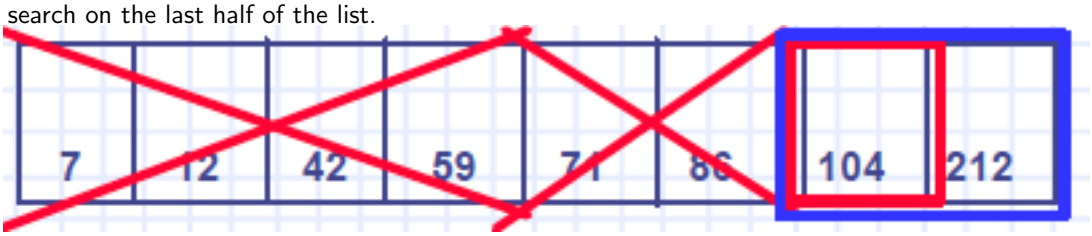
A binary search of an array is also an O(logN) operation. Suppose we were searching the array shown below for the value 89. A binary search begins by examining the middle element in the sorted list.



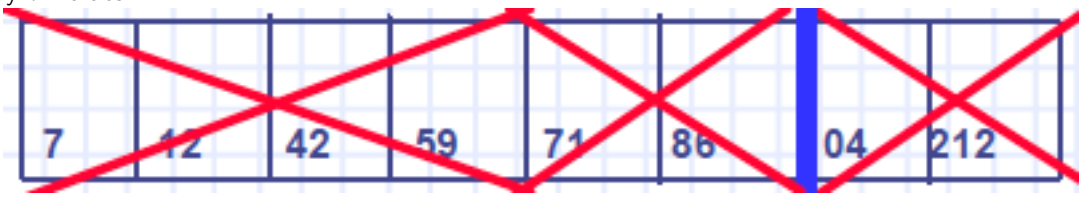
Since our target value (89) is more than the value of the middle element of the list (59) we can immediately disregard the first half of the list for the rest of the search. We then search for the target value in the second half of the list by starting at the mid point of the second half.



The target value is larger than the midpoint value, so we once again ignore the first half of the list and repeat the search on the last half of the list.



This time the midpoint value is larger than the target value, which tells us that our target value is not in the array of values.



The search concluded with 3 comparisons. Log(8) is 3. A binary search is an O(logN) algorithm.

Logarithmic algorithms often use a divide and conquer strategy to manipulate the data. Often there is some kind of selection step in a logarithmic algorithm that permits the algorithm to ignore some part of the data, thus speeding up the algorithm.

More information about O(log N) can be found here: http://stackoverflow.com/questions/2307283/what-does-olog-n-mean-exactly

Some common algorithms that are O(logN) include:

- binary search of a sorted array is O(logN)
- many algorithms using tree data structures are O(logN)
- computing the answer to x^N is O(logN)

11.1.4 O(NlogN)

An O(NlogN) algorithm is one that divides the input data up in a similar fashion to an O(logN) algorithm but then performs an O(N) operation on each of the halves of the data. An O(NlogN) algorithm does not entirely skip the processing for any of the data elements.

Good sorting algorithms are O(NlogN). A good description of O(NlogN) can be found here: http://www.crsr.net/Notes/BigO.html. This article is also a great review of Big O notation in general- it is worth reading

the whole thing.

A mergesort is a great example of an algorithm that os O(NlogN). Recall that a merge sort has two phases:

- Phase 1
  - Divide the list of N elements into two lists of N/2 elements
  - Divide those two lists in half
  - repeat the division until each least is length 1
  - This phase has log N steps
- Phase 2
  - Starting from the lists of length one, create a sorted list from each pair of lists
  - continue merging paired lists, doubling the size of the sorted list with each merge
  - This phase has log N steps or passes.
  - For each of the log N passes, every element in the input data is compared or copied.
  - Each pass is O(N) and there are log N passes, giving O(NlogN) for the sort

11.1.5 $O(n^2)$

An $O(n^2)$ algorithm (or an $O(n^k)$) is one that runs in polynomial time with respect to the size of the input.This means that as the input size increases, the time required by the algorithm increases by some polynomial factor (squared, cubed, etc).

$O(n^2)$ algorithms take more time for large input than logarithmic and linear algorithms, but are still considered to be reasonably good algorithms for some types of problems as long as the value of k is small(typically 2 or 3) Any nested loop that goes through the input data in both loops is an $O(n^2)$ algorithm. An $O(n^2)$ algorithm is one that performs an operation on every data element while looping through the set of data elements.

For example, suppose we implemented a simple algorithm for finding duplicates in an array.

```
void printDuplicates(int * theArray,  int length)
{
    int j, k;
    for (j = 0; j < length; j++)
    {
      for(k=0; k<length; k++)
      {
          if( k!=j && theArray[k] == theArray[j])
              printf("Found␣duplicates␣at␣positions␣%d␣and␣%d", j, k);
      }
}
```

This algorithm selects an element in the array and then compares it to every other element in the array to check for duplicates. It performs this step once for every element in the array. The complexity of this algorithm is $O(N^2)$

- Bubble Sort, Insertion Sort and Selection Sort are $O(n^2)$ algorithms.
- Some text processing algorithms are intuitively $O(n^2)$ such as checking a list for duplicate words.

11.1.6 $O(2^n)$

An algorithm that is $O(2^n)$ will double in complexity each time an additional data element is added. Such an algorithm is said to have exponential complexity and is usually an algorithm that is unlikely to complete in a reasonable time for most sizes of input.

An exponential algorithm experiences rapid growth with the size of the data set and is really only used in situations where the size of data can be controlled or reduced prior to algorithm execution.

These types of algorithms are often the used to approximate solutions to problems that are intractable- or problems that are not presently solvable by computer. (more information on NP-Complete problems can be found here http://www.mathsisfun.com/sets/np-complete.html or http://www.cs.berkeley.edu/%7Edaw/teaching/cs170-s03/Notes/lecture21.pdf

Examples:

- Many classic artificial intelligence problems have a solution that is exponential.
- The travelling salesman problem
- the 8 queens problem
- map colouring
- many graph algorithms (spanning tree, cycle detection).

Understanding Exponential Growth

It is sometimes difficult to understand just how quickly the execution times increase when an algorithm has exponential complexity.

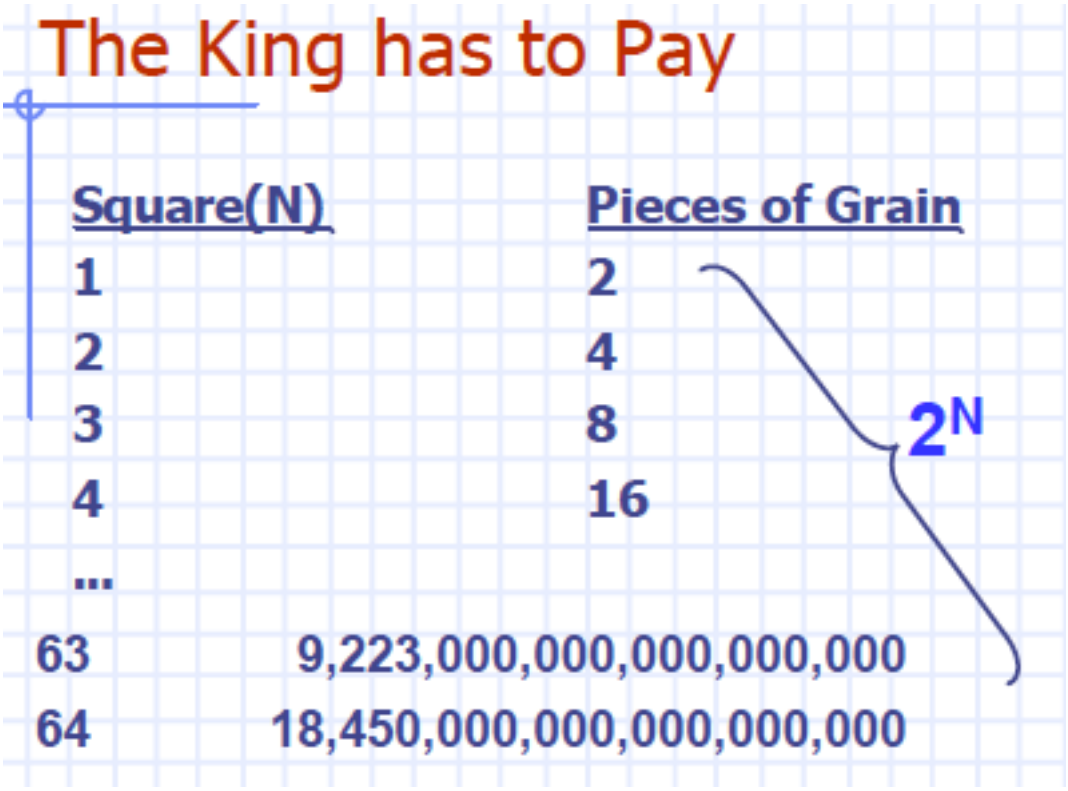
It is perhaps a little easier to consider the exponential increase of personal time. Imagine that you were doing an activity that took you twice as long each time you did it. The first time you did the task it would take 0.5 seconds. The second time it would take 1 second. The chart below shows the time involved for the first few attempts at the task.

attempt number	time to complete in seconds
1	0.5
2	1
3	2
4	4
5	8
6	16
7	32
8	64
9	128
10	256
11	512
12	1024

In 12 repetitions the task goes from something that can be done quickly to something that takes nearly 20 minutes. If you imagine now that the task is something like sorting a list of numbers, and that each attempt represents adding one more element to the list, you can quickly see how dreadful the performance would be with even a short list of numbers.

The following story is often used to illustrate the power of exponential growth.

A peasant once did a great favour for a rich king. The king asked how he could repay the peasant. The peasant asked the king to place two pieces of grain on the first square of a chess board, and to double the amount of grain on each following square. The king, knowing that the chess board only has 64 squares, readily agrees. However, the king does not understand exponents.



So just how much trouble will the king have repaying his debt? Imagine that the king can grow 1 billion pieces of grain ever second. It will take 585 years to grow enough grain for the 64th square on the chess board. It would take over one thousand years to cover the entire chessboard using the peasant's algorithm.

11.1.7 Reasonable vs Unreasonable algorithms

Algorithms that are reasonable have polynomial complexity. Such algorithms might run slowly, but they will eventually finish and return a value. O(N), O(log N) and $O(N^k)$ where k is some constant number are all examples of the complexity of reasonable algorithms.

Even a reasonable algorithm can be unusable if it is $O(N^k)$ and k is a large number.

Unreasonable algorithms have exponential complexity. Examples of exponential complexity are $O(2^n)$, O(N!) and $O(N^N$.

11.2 Additional Resources for Computational Complexity

The internet has excellent resources for learning about computational complexity.

- http://pages.cs.wisc.edu/~vernon/cs367/notes/3.COMPLEXITY.html
- http://stackoverflow.com/questions/107165/big-o-for-eight-year-olds
- http://www.perlmonks.org/?node_id=227909
- http://stackoverflow.com/questions/487258/plain-english-explanation-of-big-o
- http://www.crsr.net/Notes/BigO.html
- http://www.csd.uwo.ca/courses/CS1037a/notes/topic13_AnalysisOfAlgs.pdf

11.3 Extending Activities

- Assume you have a computer that can execute 1 billion instructions per second. How long (in seconds/minutes/days/months/years) would that computer take to execute an algorithm of the complexity give for each of the number of data items shown.

Algorithm order	10 000 data items	1 million data items	1 billion data items
O(log N)			
O(N)			
O(N log N)			
$O(N^2$			
$O(2^N$			

- Create a table showing the count of the primitive operations for the algorithm for adding a node to the end of a singly linked list. The algorithm is given in the Linked List materials for this course. What is the big O complexity for that algorithm?
- Consider these three algorithms for determining whether anyone in a room of people has the same first name as a target person.
  - The target person says their name. All the people with the same name stand up.
  - The target person approaches each person in the room to ask them their name. If the person approached has the same name as the target person the search is successful and stops.
  - The target person asks one person about their name. IIf person one does not have the same name, the target person asks person one to ask the next person (person two). Person two responds to person one, who tells the target person the answer. If that name doesn't match, the target person asks person one to tell person two to ask person three. Person three responds to person two who responds to person one who tells the target person. This algorithm continues until the last person has been asked.

  What is the worst case scenario for these algorithms? For each algorithm, how many questions will be asked in the worst case? What is the Big O complexity class for each algorithm?
- Given four functions with the complexity classes noted, list the functions in order of increasing computational complexity.
  - function 1 = $O(2^N)$
  - function 2 = $O(N^{5/3}$
  - function 3 = O(NlogN)
  - function 4 = $O(N^{logN})$
- Create a table showing the count of primitive operations for a bubble sort. What is the worst case big O complexity class for a bubble sort?